Sphaleron and Axial Charge Production Out of Equilibrium

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Motivation

Novel transport phenomena in the presence of a chirality imbalance have created excitement across the physics community [1]

Chiral Magnetic Effect: $\vec{j}_v \propto j_a^0 \vec{B}$ *j_a^0: axial charge density*   *$\vec{B}$: magnetic field*

High-energy heavy-ion collisions provide an exciting environment

— expect axial charge fluctuations due to sphaleron transitions

— strong magnetic field eB ~ m_π^2 present over the first ~1 fm/c

Since life-time of magnetic field is short [2], expect that most of the effect take place during early-time pre-equilibrium stage

Goal: Develop theoretical description to study anomalous transport in out-of-equilibrium situations based on real-time lattice techniques

Simulation Techniques

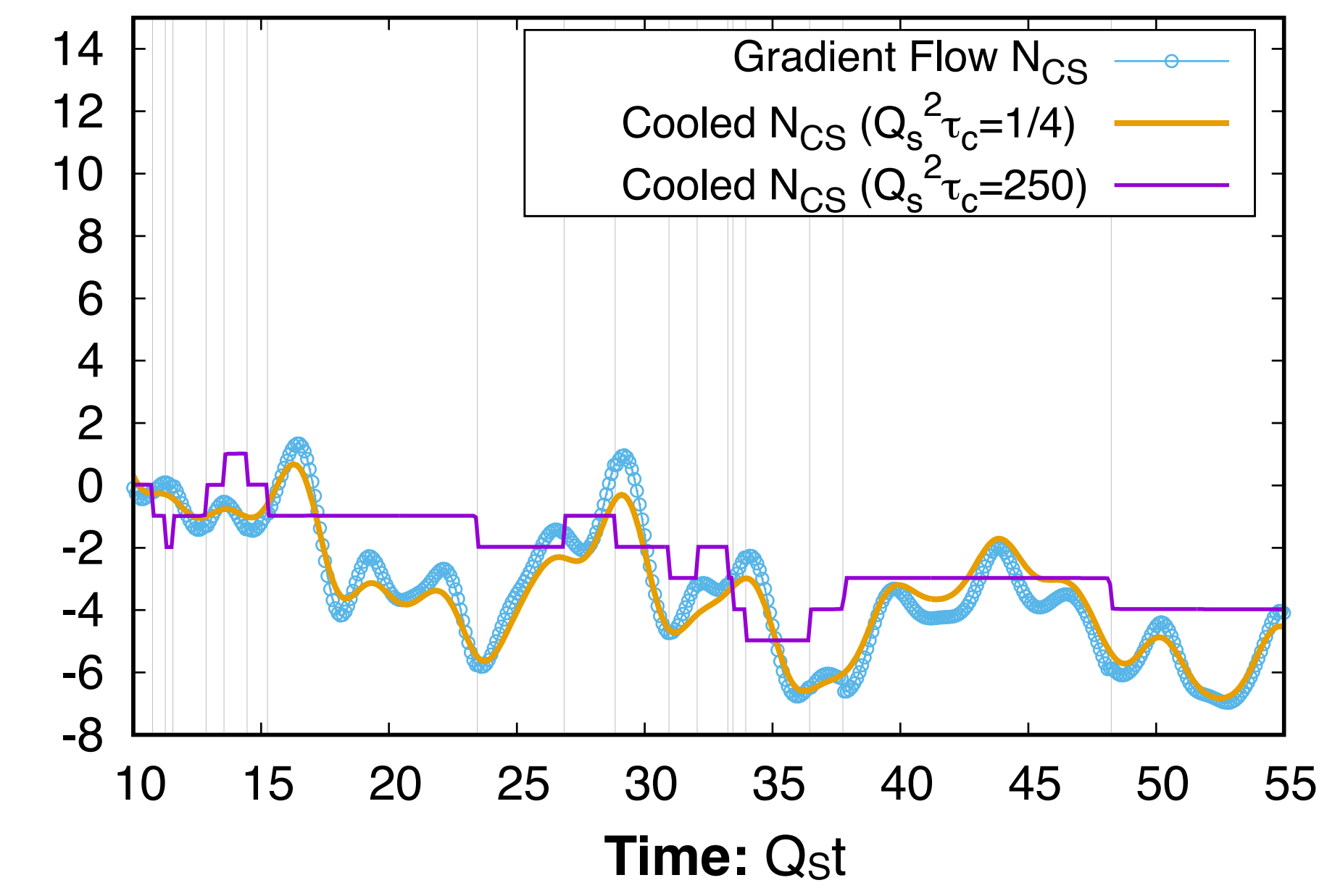
Real-time classical-statistical lattice gauge theory simulations with dynamical fermions [3-6]

- Initially non-perturbatively large phase space density of gluons
- Classical, non-equilibrium description motivated from Color Glass Condensate framework; numerically solve classical Yang-Mills
- Discretize theory on 3D lattice in the Hamiltonian formalism
- Solve Dirac equation on the operator level by mode function expansion $i\gamma^0 \partial_t \hat{\psi} = \left(-i \not{D}_W^s + m\right) \hat{\psi}$
- Extract vector and axial currents to study anomalous transport

Sphalerons Out of Equilibrium

Real-time topological transitions seed anomaly, creating axial charge

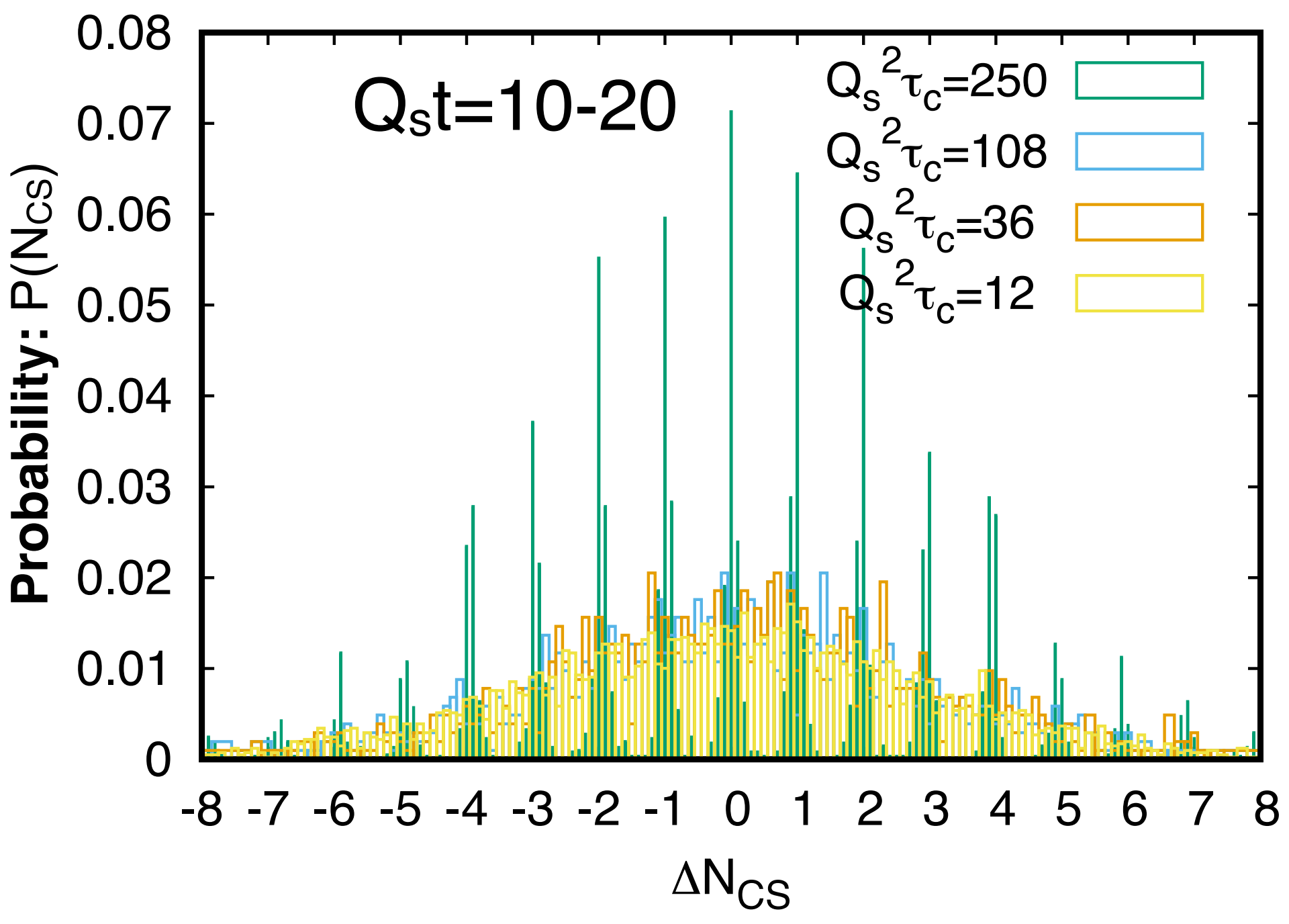
Objective: Understand the rate of these transitions out of equilibrium



Track change in Chern-Simons number in real time

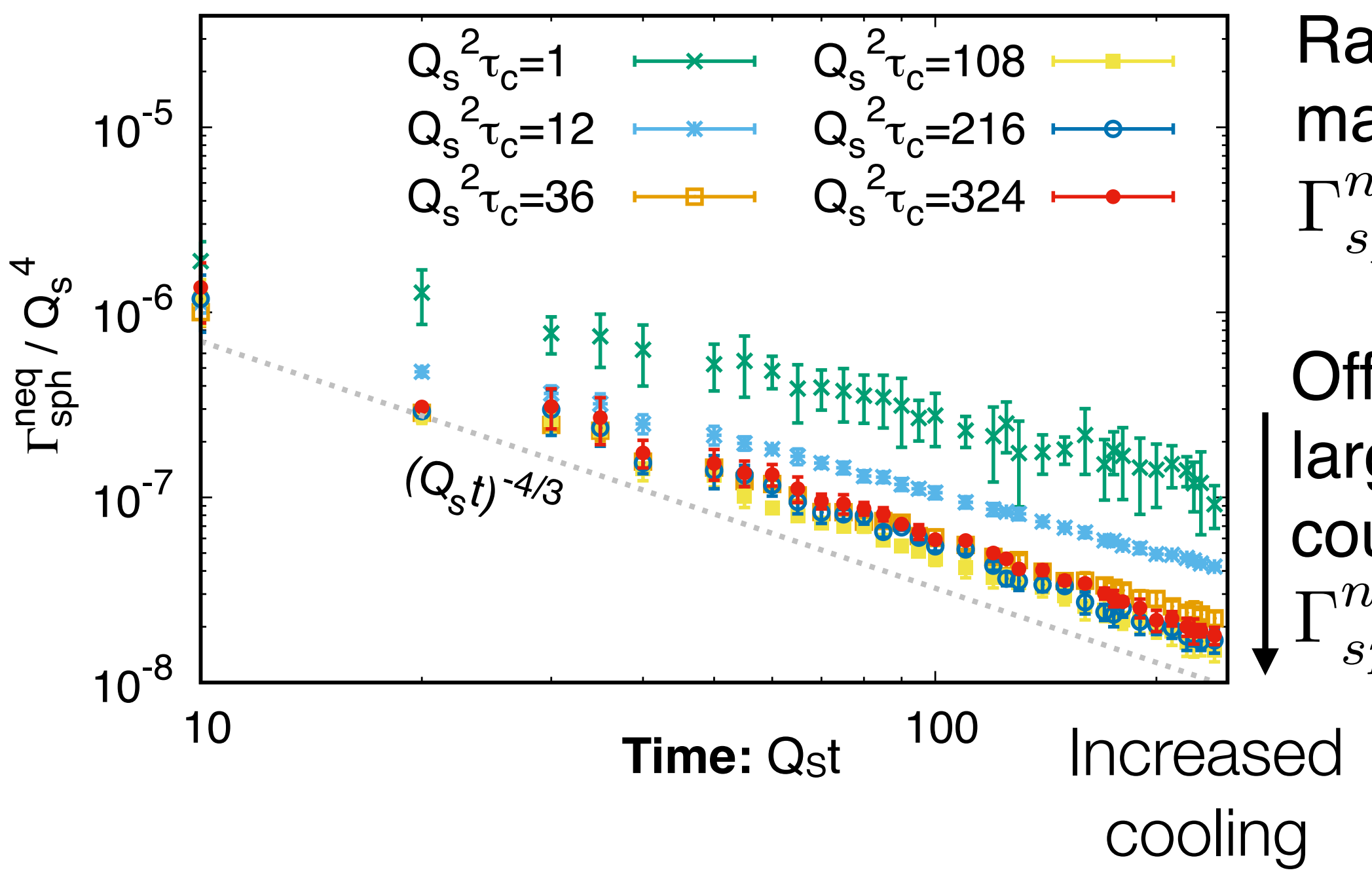
$$\Delta N_{CS} = \frac{1}{8\pi^2}\int d^4x F\tilde{F}$$

Disentangle short range fluctuations from topological transitions with 'cooling'



Significant number of sphaleron transitions on time scales on time scales of a few $1/Q_s$

Strongly time dependent and non-Markovian



Rate of transitions controlled by magnetic screening scale $\Gamma_{sph}^{neq} \sim \sigma^2$

Off-equilibrium rate parametrically larger than equilibrium [4] at weak coupling $\Gamma_{sph}^{neq} \sim \alpha_S^{-5/3} \Gamma_{sph}^{eq}$

Dominant amount of sphaleron transition produced at early times
Expect important pre-equilibrium contribution to CME

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Non-equilibrium Axial Charge Production

Axial charge imbalance in presence of mag. field seeds CME vector current

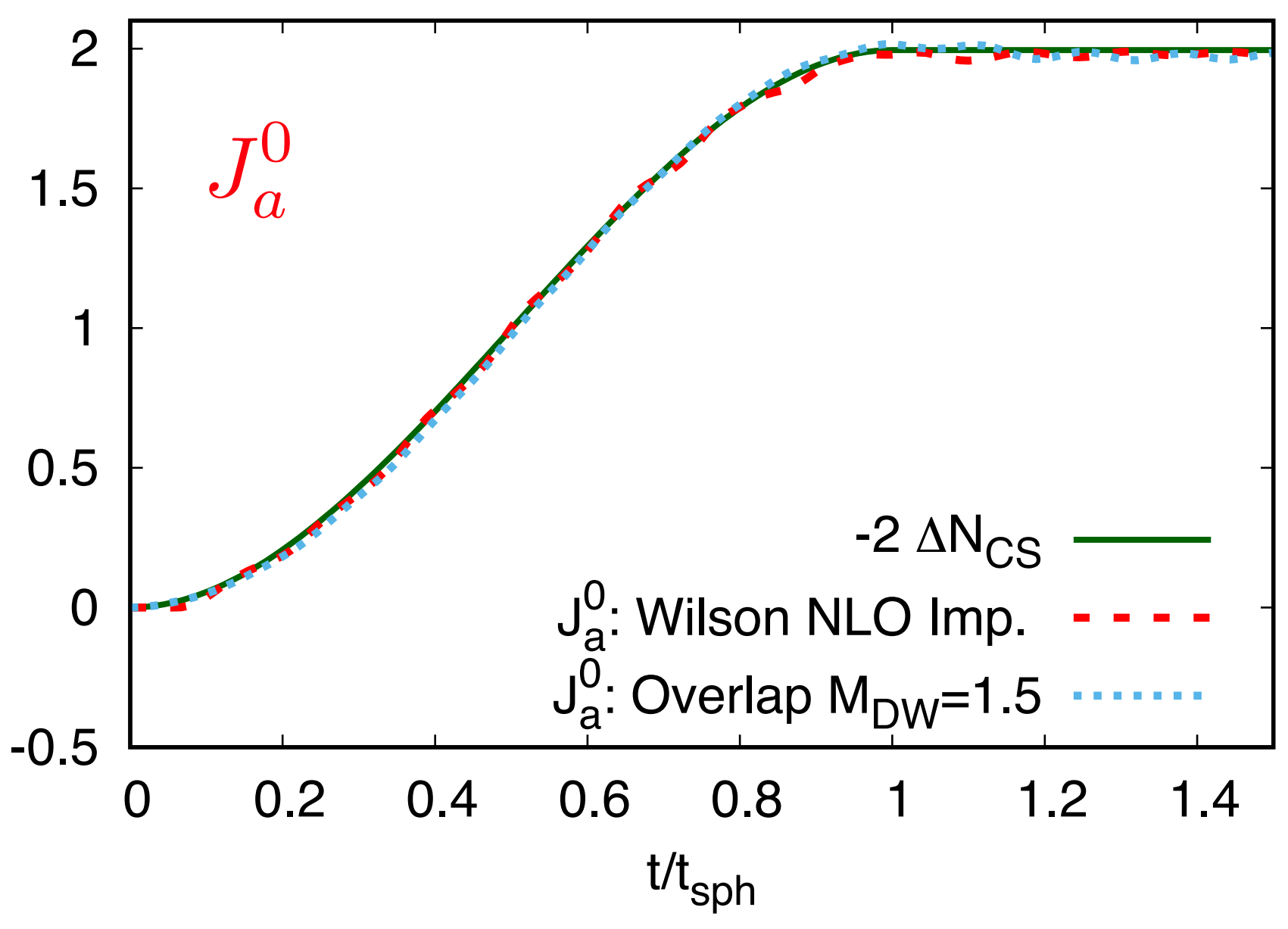
Objective: Understand real-time dynamics of anomaly generated axial charge

Sphaleron transition & axial charge imbalance

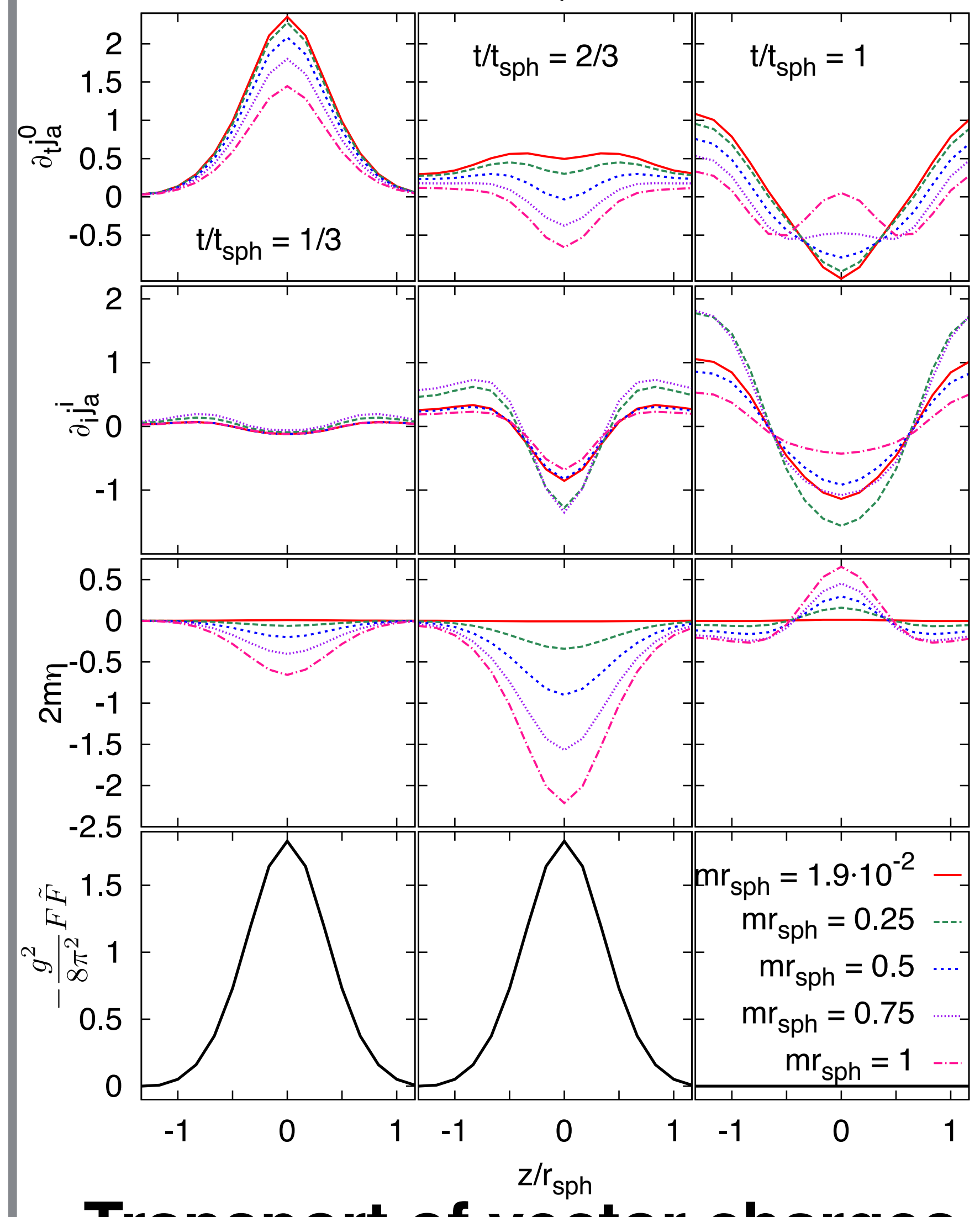
Sphaleron transition induces an imbalance of axial charge due to the anomaly

$$\partial_\mu j_a^\mu(x) = 2im\langle \hat{\bar{\psi}}(x)\gamma_5\hat{\psi}(x)\rangle - \frac{g^2}{8\pi^2}\mathrm{Tr}F_{\mu\nu}(x)\tilde{F}^{\mu\nu}(x)$$

$$\Delta J_a^0 = 2m_f\int d^4x\langle\bar{\psi}i\gamma_5\psi\rangle - 2\Delta N_{CS}$$



Anomaly is consistently reproduced with (tree-level) improved Wilson and Overlap fermions



Light quarks (mr_{sph}<<1)

Initial axial charge follows sphaleron

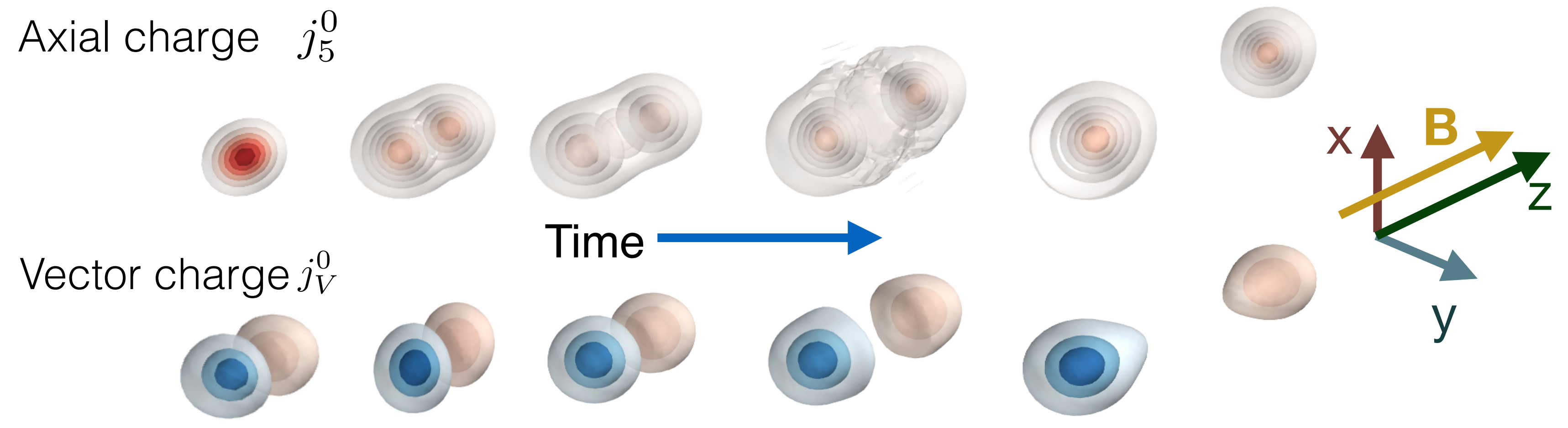
Rate of axial charge density production at the center of the sphaleron is reduced due to axial currents carrying charge away

Heavy quarks (mr_{sph}>>1)

Pseudo scalar density carries away axial charge, signaling chirality changing fermion-fermion interaction

Transport of vector charges - CME

Axial charge imbalance leads to a vector current $j_v^z \propto j_a^0 B^z$ - CME



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Conclusions & Outlook

First ever computation of sphaleron transition rate in an off-equilibrium plasma shows rate to be much greater than in equilibrium QGP

Result strongly suggestive that CME can be generated at early times

First-principles techniques for dynamics of vector and axial charges off-equilibrium developed

Correctly reproduce the chiral anomaly on the lattice

First ab-initio demonstration of CME with chiral fermions

Future plans: Extend simulations to include back-reaction and study quark production in heavy-ion collisions

Initial conditions for anomalous hydrodynamics and kinetic theory

Expect several applications beyond high-energy QCD

Dirac semi-metals, strong field QED, chiral plasma instabilities

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