# Hydrodynamic fluctuations and Hanbury Brown-Twiss interferometry

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## Abstract

The field of high-energy nuclear collisions has witnessed a surge of interest in recent years in the role played by hydrodynamic fluctuations in these collisions. Hydrodynamic fluctuations may have significant effects on systems created at RHIC, FAIR or NICA whose trajectories in the QCD phase diagram pass near a possible critical end point (CEP). To test for the existence of such a CEP, it is vital to understand how the system's proximity to the CEP affects heavy-ion observables, such as the radii obtained from Hanbury Brown-Twiss (HBT) interferometry. Here, we discuss the effects of event-by-event hydrodynamic fluctuations on the HBT radii extracted from the experimentally accessible (normalized) two-particle correlation function. For simplicity, we assume a system characterized by Björken symmetry and study the fluctuations of $R_l^2$ as a function of the rapidity distance $\Delta y$. We show how this quantity is affected by hydrodynamic fluctuations along trajectories in the QCD phase diagram which pass close to the CEP.

## Motivation

HBT interferometry relies on two-particle correlations to infer spatio-temporal characteristics of the freeze-out process in heavy-ion collisions. These characteristics are encapsulated in the HBT radii $R_{ij}^2$. It has recently been shown that the $R_{ij}^2$ may be regarded as event-by-event observables which can be defined, in principle, for individual heavy-ion collisions. This makes them ideal for probing the event-by-event fluctuations of geometry and flow in heavy-ion collisions and the underlying mechanisms which produce these fluctuations.

One such mechanism of much current interest is the stochastically fluctuating hydrodynamics of the quark-gluon plasma. This source of fluctuations is expected to strongly affect the fireball's evolution in the vicinity of a conjectured CEP in the phase diagram of nuclear matter. The event-by-event fluctuations of the HBT radii may therefore provide a unique window into the properties of the QGP and the potential location of a CEP for QCD matter.

## Methodology

We study the effects of hydrodynamic fluctuations on the HBT radii in a system expanding according to the well-known Björken ansatz, which introduces a new set of coordinates $(\tau, \xi)$, defined implicitly by $t = \tau\cosh\xi$, $z = \tau\sinh\xi$, and assumes that all dynamical quantities are independent of the space-time rapidity $\xi$. The hydrodynamic equations of motion are derived from the conservation laws

$$\partial_\mu T^{\mu\nu} = 0, \quad \partial_\mu J_B^\mu = 0. \tag{1}$$

Both the stress-energy tensor $T^{\mu\nu}$ and the baryon current $J_B^\mu$ have fluctuating components which are required to satisfy conditions of the form

$$\begin{aligned} \langle f(x_1)\rangle &= 0 && (2)\\ \langle f(x_1) f(x_2)\rangle &= D\delta^4(x_1 - x_2), && (3)\end{aligned}$$

where $D$ is a coefficient that must be fixed by the fluctuation-dissipation theorem, and the $\langle\cdots\rangle$ represents an average over events.

It is most convenient to treat the system's evolution as consisting of (A), a smooth, background evolution (with Björken symmetry) according to the hydrodynamic equations (1) *without* fluctuations, and (B) local fluctuations of dynamical quantities (e.g., the temperature $T$ and flow velocity $u^\mu$) which propagate on top of the smooth, background evolution, according to the linearized versions of (1) which include fluctuations.

We therefore adopt the following general strategy:

1. Define and compute the longitudinal (squared) HBT radius $R_l^2(y)$ on an event-wise basis at momentum rapidity $y$
2. Relate $R_l^2(y)$ (to linear order) to the fluctuating dynamical quantities of the system. By construction, it follows that $R_l^2(y) \equiv \langle R_l^2(y)\rangle + \delta R_l^2(y)$, since $\langle \delta R_l^2(y)\rangle = 0$, by Eq. (2).
3. Finally, compute the correlator $\langle \delta R_l^2(y_1)\delta R_l^2(y_2)\rangle$, which is non-vanishing, by virtue of Eq. (3).

## Linearized hydrodynamic fluctuations

The equations of motion for a Björken system without fluctuations are well-known, and we do not reproduce them here. The linearized versions of these equations which include fluctuations may be written as

$$\begin{aligned} \tau\frac{\partial}{\partial\tau}\left(\frac{\delta s}{s}\right) + \frac{\partial\omega}{\partial\xi} - \frac{\mu}{T}\frac{\partial f}{\partial\xi} &= 0 && (4)\\ \tau\frac{\partial}{\partial\tau}\left(\frac{\delta n}{s}\right) + \frac{n}{s}\frac{\partial\omega}{\partial\xi} + \frac{\partial f}{\partial\xi} &= 0 && (5)\\ \tau\frac{\partial\omega}{\partial\tau} + (1 - v_\sigma^2)\omega + \frac{v_n^2 Ts}{w}\frac{\partial}{\partial\xi}\left(\frac{\delta s}{s}\right) + \frac{v_s^2\mu s}{w}\frac{\partial}{\partial\xi}\left(\frac{\delta n}{s}\right) &= 0, && (6)\end{aligned}$$

where $\delta s$ and $\delta n$ represent the fluctuating contributions to the entropy density $s$ and the number density $n$, respectively, $w = sT + \mu n$, and $f$ is a stochastically fluctuating source term. $\omega$ represents fluctuations of the local flow velocity, and is defined by $(u^0, u^3) = (\cosh(\xi + \omega), \sinh(\xi + \omega))$.

Without fluctuations, these equations can be solved to obtain the *response functions* $G_X(\xi; \tau, \tau')$, $X \in (\delta s/s, \omega, \delta n/s)$. The full response of the system at freeze-out proper time $\tau_f$ is then obtained by integrating the hydrodynamic response over the system's history:

$$X(\xi, \tau_f) = -\int_{\tau_i}^{\tau_f} \frac{d\tau'}{\tau'} G_X(\xi; \tau, \tau') f(\xi, \tau') \tag{7}$$

The $G_X$ have been computed previously, so we also do not reproduce them here.

## Results

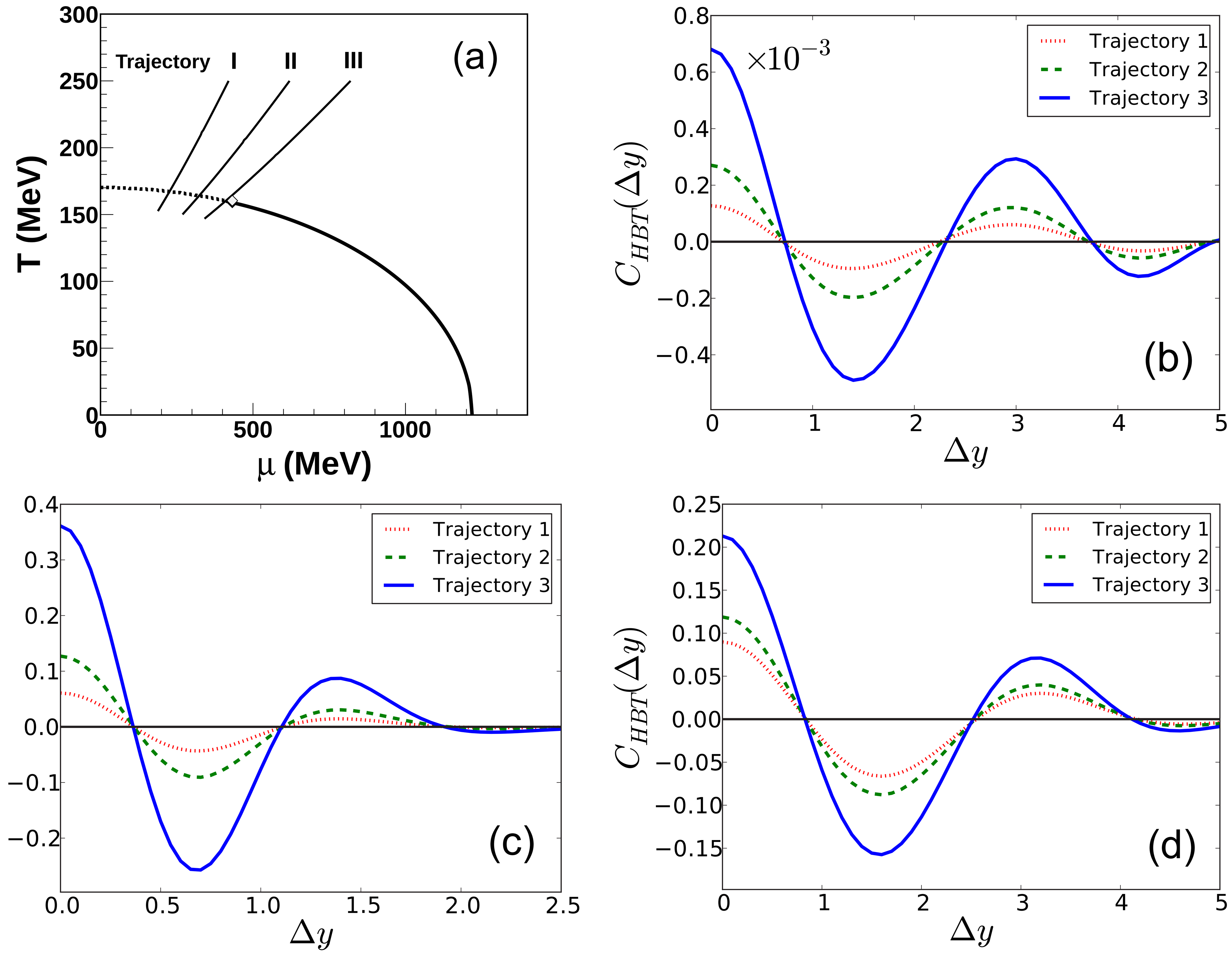


**Figure:** **(a)** - The three trajectories in the conjectured QCD phase diagram for which we compute the two-point function $C_{HBT}(\Delta y)$ for the longitudinal HBT radius $R_l^2(y)$. They are ordered by increasing proximity to the CEP, which we have placed at $(T_c, \mu_c) = (160, 411.74)$ MeV. We model the crossover curve by an ellipse with semi-axes $T_0 = 170$ MeV and $\mu_0 = 1218.48$ MeV. Each trajectory has an initial temperature of $T_i = 250$ MeV and an initial chemical potential of $\mu_i = 420$, 620, or 820 MeV, respectively. **(b)** - The two-point correlations of $R_l^2(y)$ extracted from two-particle correlations of $\pi^+$s, without including fluctuations of $\mu_B$. **(c)** - The two-point correlations of $R_l^2(y)$ extracted from two-particle correlations of $p$ pairs. **(d)** - The two-point correlations of $R_l^2(y)$ extracted from two-particle correlations of $\pi^+$s, with fluctuations of $\mu_B$ included.

Since we include in our equation-of-state only the thermal conductivity $\lambda$, which diverges at the CEP, and ignore shear and bulk viscosities, we choose $D = 2\lambda(\frac{nT}{sw})^2$ in Eq. (3), leading to the strongest correlations on the trajectories which pass closest to the CEP, as seen above.

## Particle emission and the HBT radii

We formulate particle production from the hydrodynamic system in terms of the emission function $S(x, K)$, which is related to the single-particle spectra by

$$\begin{aligned} \frac{dN}{K_T dK_T d\phi dy} &= \int d^4x\, S(x, K) && (8)\\ &= \kappa\, e^{\mu/T}\int d\xi \cosh(y - \xi) m_\perp \\ &\quad\times \exp\left[-\frac{m_\perp}{T}\cosh(y - \xi - \omega)\right], \end{aligned}$$

where $\kappa = d_s A_\perp \tau_f/(2\pi)^3$, $d_s$ is the spin degeneracy, $A_\perp$ the transverse area of the system, and $\tau_f$ the proper time defining the freeze-out surface. In terms of the emission function, the event-by-event longitudinal radius $R_l^2(y)$ is defined to be the $\vec{K}_\perp$-averaged version of

$$R_l^2(K) = \frac{\left[\int d^4x\, (z - v_L t)^2 S(x, K)\right]}{\int d^4x\, S(x, K)} - \frac{\left[\int d^4x\, (z - v_L t) S(x, K)\right]^2}{\left[\int d^4x\, S(x, K)\right]^2}, \tag{9}$$

with $v_L = \tanh y$. Using Eqs. (7), (8), and (9), we can determine the event-by-event response $\delta R_l^2(y) \equiv R_l^2(y) - \langle R_l^2(y)\rangle$. We then construct the observable quantity

$$\begin{aligned} C_{\mathrm{HBT}}(\Delta y) &= \left\langle\frac{dN}{dy}\right\rangle \frac{\langle \delta R_l^2(y_2)\delta R_l^2(y_1)\rangle}{\langle R_l^2(y_2)\rangle\langle R_l^2(y_1)\rangle} && (10)\\ &\equiv \left\langle\frac{dN}{dy}\right\rangle \left(\frac{\sigma_l}{\langle R_l^2\rangle}\right)^2. && (11)\end{aligned}$$

as an unambiguous way of quantifying the effects of hydrodynamic fluctuations on $R_l^2(y)$, where the factor of $\left\langle\frac{dN}{dy}\right\rangle$ is included to cancel dependence on the transverse system size $A_\perp$.

## Conclusions

We have analyzed the effects of hydrodynamic fluctuations on the HBT radii in the highly idealized scenario of Björken flow. Although such a model cannot yield quantitatively precise predictions for heavy-ion observables, it can offer us some intuition for the overall size of the fluctuations' effects on event-by-event interferometric analyses.

Within the limitations of our simplified model, we estimate (for $\pi^+$ pairs) that the relative widths at $\Delta y = 0$ of the event-wise $R_l^2$ distribution ($\sigma_l/\langle R_l^2\rangle$) is, for $p + p$ collisions, of order 85%, while the same effect is of order 12% in $A + A$ collisions.

We conclude that a proper treatment of hydrodynamic fluctuations is essential to correctly understanding the event-wise distributions of HBT radii, particularly in small collision systems. We also find that systematic studies of HBT distributions could help to locate a CEP in the QCD phase diagram.

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## References and Acknowledgements

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