Confinement in SU(2) Yang-Mills Theory from Correlated Instanton-Dyon Ensemble

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Abstract

In the framework of a correlated ensemble of instanton-dyons, namely the constituents of the caloron field with non-trivial holonomy, we present a substantial numerical study of confinement properties in SU(2) Yang-Mills theory at finite temperature, obtaining important observables such as the effective holonomy potential or free energy and static-quark potentials from Polyakov loop correlators and spatial Wilson loops, among others.

Confinement at Finite Temperature

The order parameter of the deconfinement phase transition is given by the expectation value of the traced ***Polyakov Loop*** at spatial infinity (a.k.a. ***Holonomy***)

$$L_\infty \equiv \lim_{|\vec{x}|\to\infty} \frac{1}{2}\mathrm{Tr}L(\vec{x}),$$

where $L(\vec{x}) = \mathcal{P}\exp\left(i\int_0^\beta \mathrm{d}x_4 A_4(\vec{x},x_4)\right)$.

It is related to the free energy of a single static quark as

$$\langle \mathrm{Tr}L(\vec{x})\rangle \propto e^{-\beta F_q} \begin{cases} = 0 \longrightarrow \text{CONFINED PHASE} \\ \neq 0 \longrightarrow \text{DECONFINED PHASE} \end{cases}$$

and to the static quark-antiquark potential

$$\left\langle \mathrm{Tr}L(\vec{x})\,\mathrm{Tr}L^\dagger(\vec{y})\right\rangle \propto e^{-\beta F_{q\bar{q}}}$$

The Instanton-Dyon Ensemble

- We take as gauge configuration the caloron field with non-trivial holonomy

$$A_4^{\mathrm{KvBLL}}\big|_{|\vec{x}|\to\infty} = \frac{v}{2}\tau^3 \quad \text{with} \quad \begin{matrix} v \equiv 2\pi T\nu \\ \nu\in[0,1] \end{matrix}$$

- Semiclassical expansion of the YM action around the caloron field, parametrized by the "constituent" instanton-dyon (L and M) positions:

$$\mathcal{Z} = \int \mathcal{D}A_\mu\, e^{-S[A_\mu]} \longrightarrow A_\mu(x) = A_\mu^{\mathrm{KvBLL}}(x) + a(x)$$

$$\longrightarrow \mathcal{Z} = e^{-VP(\nu)}\int (\mathrm{d}^3r_L\, f_L)\,(\mathrm{d}^3r_M\, f_M)\, T^6 \det(\hat{G})$$

where dyon fugacities are defined as

$$f_j = \Gamma S^2 e^{-\nu_j S}\nu_j^{\frac{8}{3}\nu_j - 1}$$

- Generalize to arbitrary number of dyons and antidyons $L, M, \bar{L}, \bar{M}$:

$$\underbrace{\det(G)\det(\bar{G})}_{\textit{Uncorrelated}}\underbrace{e^{-V_{D\bar{D}}}}_{\textit{Correlated}}$$

- Dyon-antidyons interactions $V_{D\bar{D}}$ given by [2,3]
  - ➢ Of the same kind:

$$V_{L\bar{L}} = -2\bar{\nu}S\left(\frac{1}{\zeta_L} - 1.632e^{-0.704\zeta_L}\right) \qquad \text{where}$$

$$V_{M\bar{M}} = -2\nu S\left(\frac{1}{\zeta_M} - 1.632e^{-0.704\zeta_M}\right) \qquad \begin{matrix}\zeta_j = 2\pi\nu_j T r_{j\bar{j}} \\ r_{j\bar{j}} = |\vec{r}_j - \vec{r}_{\bar{j}}|\end{matrix}$$

  - ➢ Repulsive core: $V_{j\bar{j}}^C = \frac{\nu_j V_c}{1 + e^{(\zeta_j - \zeta_j^0)}}$
  - ➢ Classical long range interaction (Abelian Electric/Magnetic)

$$V_{ij} = \frac{S}{2\pi T r_{ij}}(e_i e_j + m_i m_j - 2h_i h_j)$$

	M	$\bar{M}$	L	$\bar{L}$
e	1	1	-1	-1
m	1	-1	-1	1
h	1	1	-1	-1

- The partition function of the ensemble is:

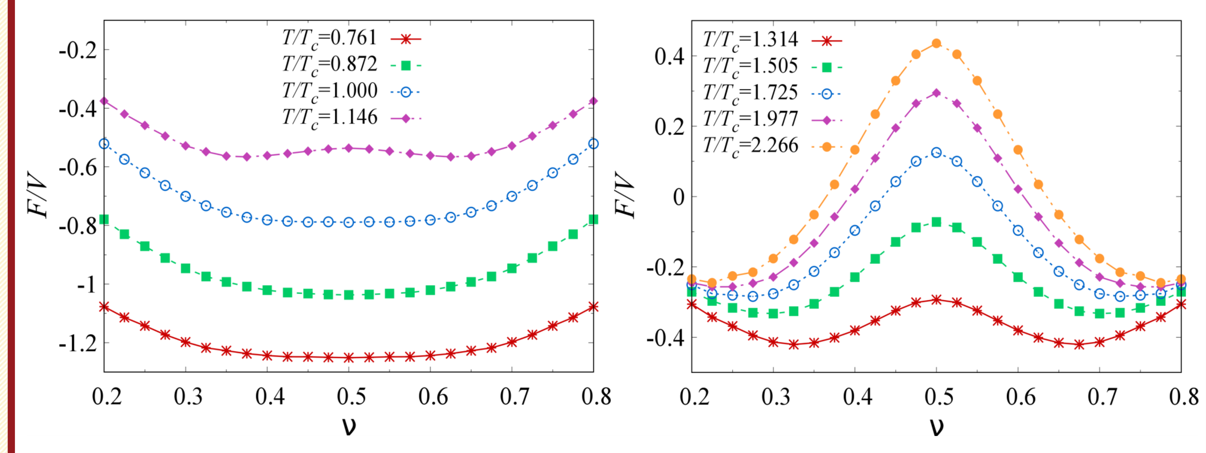
$$\mathcal{Z} = e^{-VP(\nu)}\sum_{\substack{N_M,N_L,\\ N_{\bar{L}},N_{\bar{M}}}} \frac{1}{N_L!N_M!N_{\bar{L}}!N_{\bar{M}}!}\int \prod_{l=1}^{N_L} f_L T^3\,\mathrm{d}^3r_{L_l}\prod_{m=1}^{N_M} f_M T^3\,\mathrm{d}^3r_{M_m}$$
$$\times\prod_{\bar{l}=1}^{N_{\bar{L}}} f_{\bar{L}} T^3\,\mathrm{d}^3r_{\bar{L}_{\bar{l}}}\prod_{\bar{m}=1}^{N_{\bar{M}}} f_{\bar{M}} T^3\,\mathrm{d}^3r_{\bar{M}_{\bar{m}}}\det(G_D)\det(G_{\bar{D}})\,e^{-V_{D\bar{D}}}$$

Free energy $F = -T\log\mathcal{Z}$

Given that the holonomy L_∞ of the caloron field A_μ^{KvBLL} takes the simple form

$$L_\infty = \cos(\pi\nu),$$

we plot the free energy density or ***holonomy potential*** as a function of the holonomy parameter ν, to show the 2nd order phase transition from the confined $(\nu_{\min} = \frac{1}{2})$ to the deconfined phase.



Temperature dependence is parametrized through the instanton action and the 1-loop running of the coupling constant:

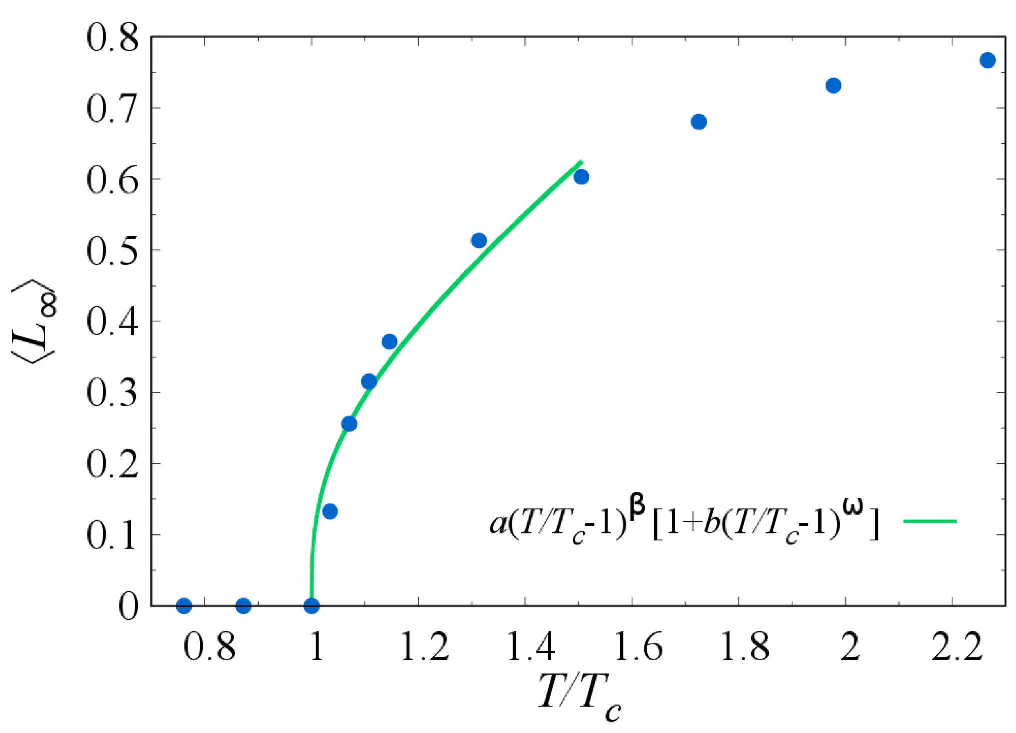
$$S(T) = \frac{8\pi^2}{g^2(T)} = \frac{22}{3}\log\left(\frac{T}{\Lambda}\right) \quad \Rightarrow \frac{\Lambda}{T_c} = \exp\left[-\frac{3}{22}S(T_c)\right]$$

The phase transition was found at S=7, thus fixing the scale parameter to $\Lambda = 0.385T_c$.

The order parameter $\langle L_\infty\rangle$ and universality

The Svetitsky-Yaffe conjecture relates *SU*(2) pure gauge theory in (3 + 1) dimensions to the 3*D* Ising model by categorizing both in the same universality class. Therefore, the order parameter must follow the power law

$$\langle L_\infty\rangle \sim (T/T_c - 1)^\beta[1 + (T/T_c - 1)^\omega]$$



Data points fitted to the Ising model's critical exponents $\beta \approx 0.3265$ and $\omega \approx 0.84$.

Static quark-antiquark potential

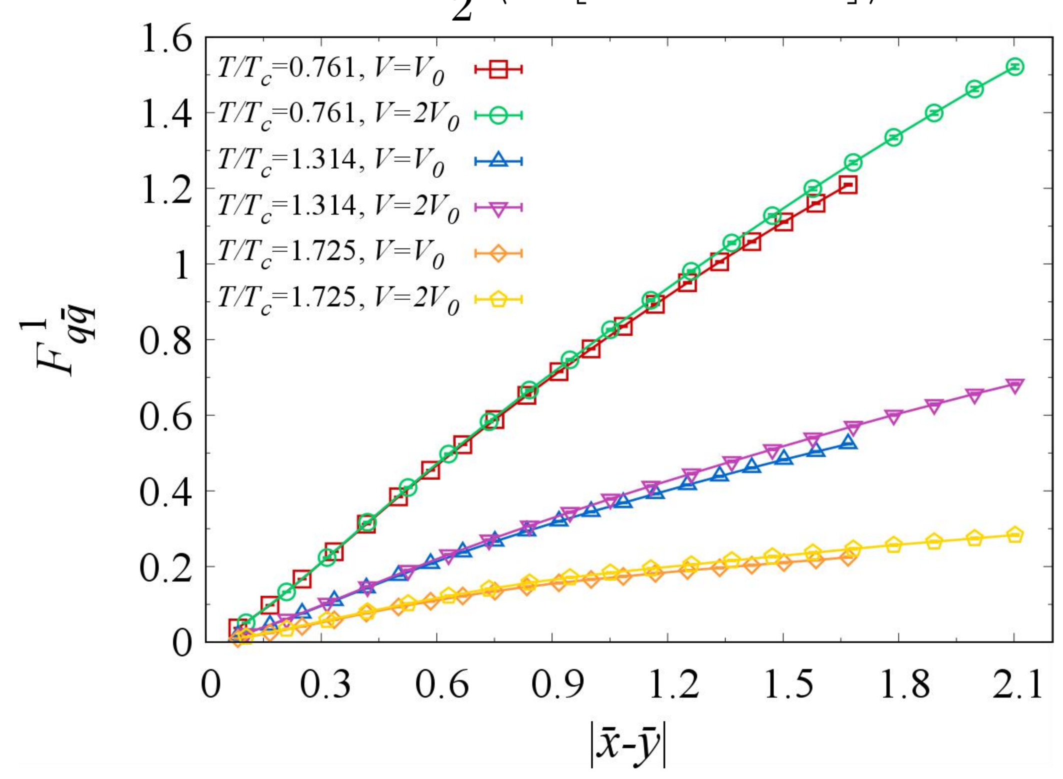
For color sources (quarks) in the fundamental representation, quark and antiquarks interact via *singlet* and *triplet* channels $\mathbf{2}\otimes\bar{\mathbf{2}} = \mathbf{1}\oplus\mathbf{3}$.

The color average static quark-antiquark potential is thus given by

$$e^{-F_{q\bar{q}}^{\mathrm{avg}}} \equiv \frac{1}{4}\left\langle \mathrm{Tr}\,L^{\dagger f}(\vec{x})\,\mathrm{Tr}\,L^f(\vec{y})\right\rangle \Longrightarrow e^{F_{q\bar{q}}^{\mathrm{avg}}} = \frac{1}{4}e^{-F_{q\bar{q}}^1} + \frac{3}{4}e^{-F_{q\bar{q}}^3},$$

where the singlet contribution is defined as

$$e^{-F_{q\bar{q}}^1} \equiv \frac{1}{2}\left\langle \mathrm{Tr}\left[L^{\dagger f}(\vec{x})L^f(\vec{y})\right]\right\rangle$$



The confinement criterion must satisfy

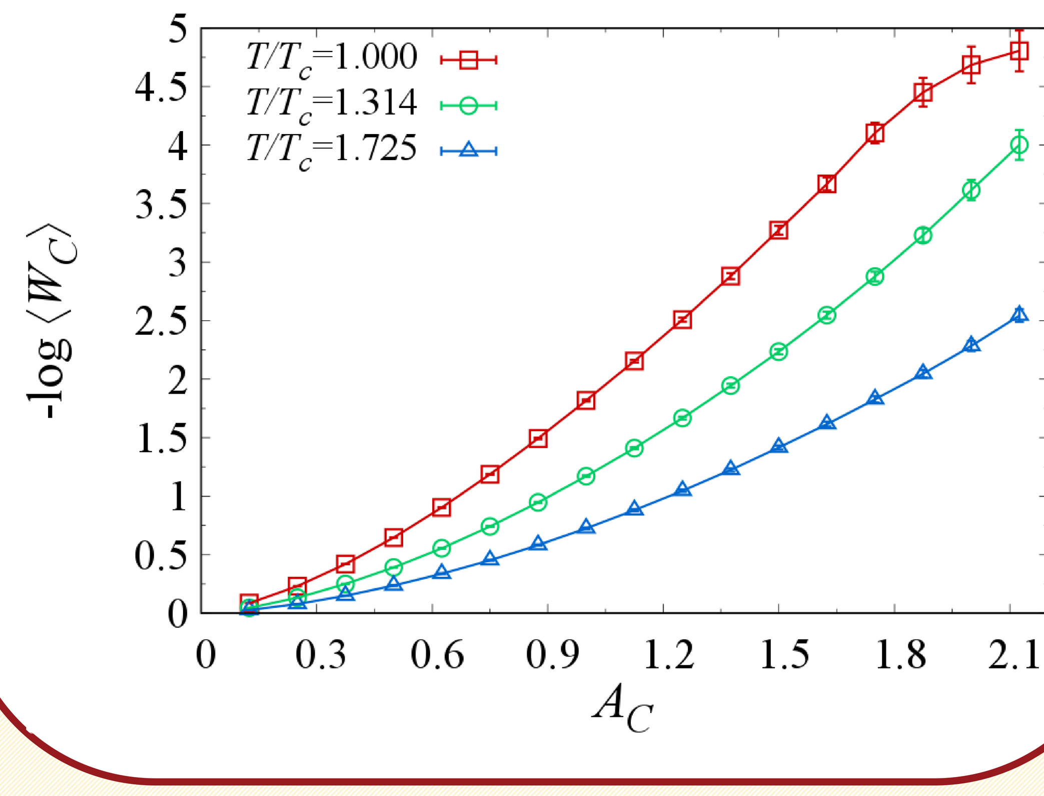
$$F_{q\bar{q}}\big|_{|\vec{x}-\vec{y}|\to\infty} \approx \sigma_E|\vec{x}-\vec{y}|$$

Spatial Wilson Loop

At finite temperature, the spatial Wilson loop defined as

$$W_C \equiv \frac{1}{2}\mathrm{Tr}\,\mathcal{P}\exp\left[i\oint_C \mathrm{d}x_i A_i(x)\right],$$

does not provide a good measure for confinement, since even above T_c, there is still area law $\langle W_C\rangle \sim e^{-\sigma_M A_C}$. However, in the zero temperature limit, the electric σ_E and magnetic σ_M string tensions should coincide.



Summary. The mechanism of confinement remains a significant challenge to our understanding and is generally believed to pertain to certain nontrivial topological configurations of the gluonic sector. The correlated ensemble of instanton-dyons correctly produces the various essential features of the confinement dynamics from above to below the transition temperature. Given the success, it appears reasonable to believe that such ensemble of topological objects, may indeed hold the key of the confinement mechanism.

References

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[3] R. Larsen and E. Shuryak, Phys. Rev. D **92**, no. 9, 094022 (2015) [arXiv:1504.03341 [hep-ph]].