Upsilon Ground State Formation and Dissociation inside Quark-Gluon Plasma

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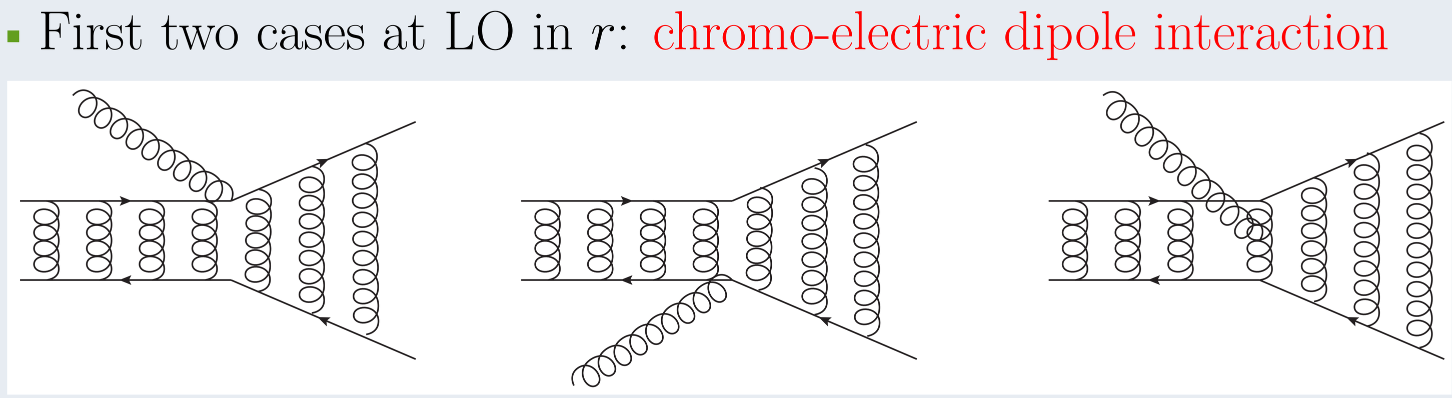
Introduction

- **Use heavy quarkonium to probe QGP**
- Debye screening is complicated by dynamical screening, initial-state effect and recombination
- Want **a dynamical approach beyond rate equation**: keep track of quarkonium formation and dissociation throughout in-medium evolution
- Design a linearized Boltzmann transport framework to record changes in the number of heavy quarks and quarkonia, with calculations of **formation and dissociation rates** as inputs
- **Effective field theory treatment: potential NRQCD**
- **Advantage: organize terms by physical importance**
- Dissociation rate has been calculated by studying the imaginary part of the singlet potential [4, 5] but not formation!
- Here I present calculations of both dissociation and formation by computing the scattering amplitude

Power Counting

- Upsilon 1S state: $M \approx 4.5$ GeV (b-quark), $Mv \sim 1/r \approx 1.5$ GeV, $Mv^2 \approx 0.5$ GeV, $T \sim 175 - 400$ MeV
- Separation of scales: $M \gg Mv \gg Mv^2 \sim T \sim m_D$
- Expansion in relative velocity v or $1/M$: NRQCD
- Expansion in relative distance r: pNRQCD ; expand in $m_D r$ for inelastic scattering and Tr for gluon absorption or radiation
- Probability of finding dynamical gluon inside quarkonium (third case below) is suppressed by v^2
- First two cases at LO in r: chromo-electric dipole interaction



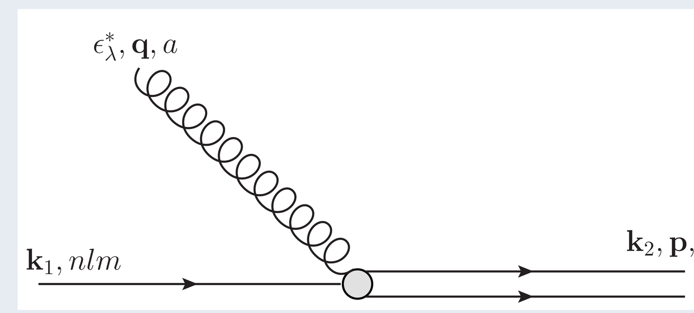
Lagrangian

$$\mathcal{L}_{pNRQCD} = \int \mathrm{d}^3 r \mathrm{Tr}\Big[\mathrm{S}^\dagger\big(i\partial_0 - H_s\big)\mathrm{S} + \mathrm{O}^\dagger\big(iD_0 - H_o\big)\mathrm{O}$$
$$+ V_A\Big(\mathrm{O}^\dagger \boldsymbol{r}\cdot g\boldsymbol{E}\mathrm{S} + \mathrm{S}^\dagger \boldsymbol{r}\cdot g\boldsymbol{E}\mathrm{O}\Big) + \frac{V_B}{2}\mathrm{O}^\dagger\Big\{\boldsymbol{r}\cdot g\boldsymbol{E}, \mathrm{O}\Big\} + \cdots\Big]$$

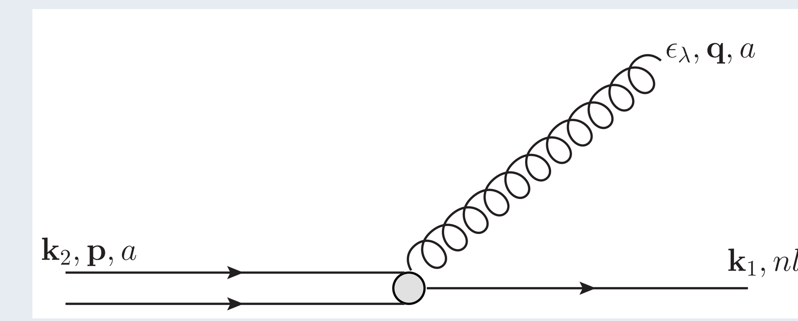
- **Degrees of freedom**: color singlet $\mathrm{S}(\boldsymbol{R}, \boldsymbol{r}, t)$ octet $\mathrm{O}(\boldsymbol{R}, \boldsymbol{r}, t)$
- Normalization in the color index space $\mathrm{S} = \frac{1}{\sqrt{N_c}}S$, $\mathrm{O} = \frac{1}{\sqrt{T_F}}O^a T^a$
- **Equation of motion**: Schrödinger equation with
$$H_{s,o} = \frac{\boldsymbol{P}^2}{4M} + \frac{\boldsymbol{p}^2}{M} + V^{(0)}_{s,o} + \frac{V^{(1)}_{s,o}}{M} + \frac{V^{(2)}_{s,o}}{M^2} + \cdots$$
for a weakly coupled theory in Coulomb gauge, $V_s^{(0)} = -C_F\frac{\alpha_s}{r}$, $V_o^{(0)} = \frac{1}{2N_c}\frac{\alpha_s}{r}$; **at finite temperature, Debye screening**
- **Interaction vertex**: dipole interaction at LO of r
$$gV_A\sqrt{\frac{T_F}{N_c}}\int \mathrm{d}^3 r O^{\dagger a}\boldsymbol{r}\cdot\boldsymbol{E}^a S + \mathrm{h.c.}$$

Dissociation and Formation Rates

- Gluo-dissociation

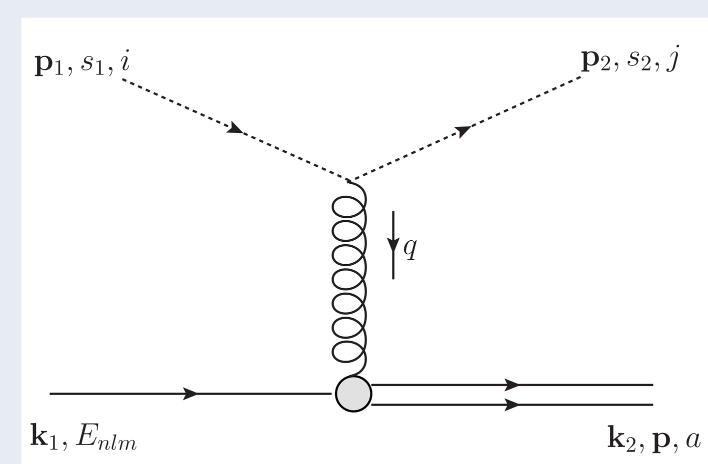


- Gluo-formation

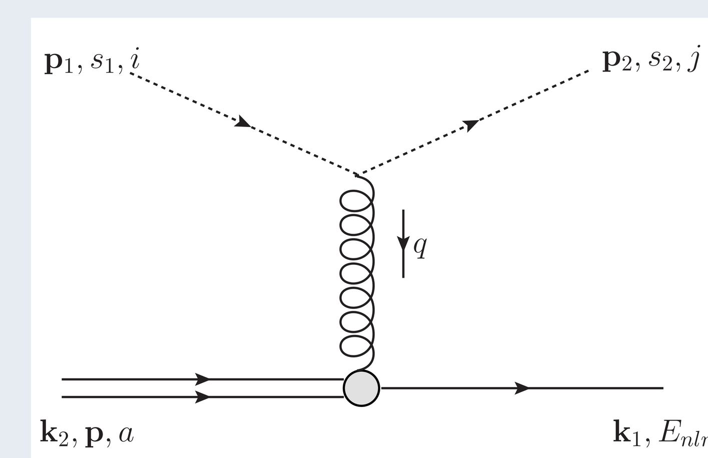


$$\Gamma^d_{gluo} = \int\frac{\mathrm{d}^3 q}{(2\pi)^3}n_B(q)\frac{g^2V_A^2C_F}{6\pi}M^{3/2}q\sqrt{q+E_{nlm}}\big|\langle\psi_{nlm}|r_i|\Psi_{\boldsymbol{p}}\rangle\big|^2_{p=\sqrt{M(q+E_{nlm})}}$$
$$\Gamma^f_{gluo} = \frac{1}{N_C^2-1}\frac{M}{p}\frac{1}{3\pi}g^2V_A^2C_F\big(1+n_B(q)\big)q^3\big|\langle\psi_{nlm}|r_i|\Psi_{\boldsymbol{p}}\rangle\big|^2_{q=\boldsymbol{p}^2/M-E_{nlm}}$$

- Light-quark-inelastic-scattering dissociation

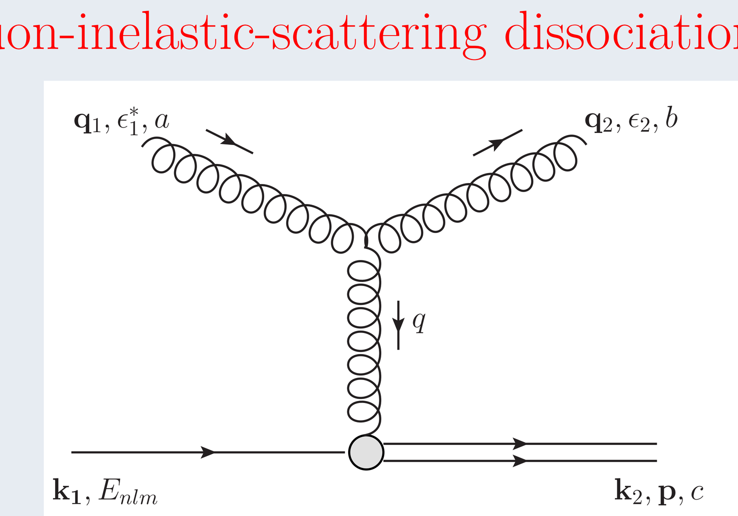


- Light-quark-inelastic-scattering formation

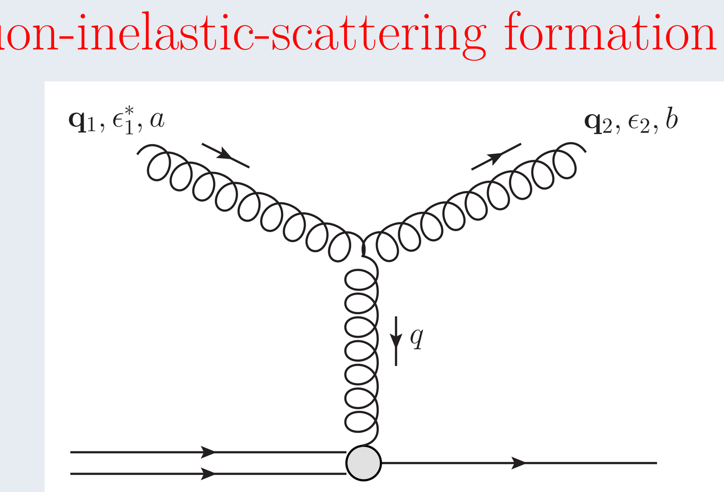


$$\Gamma^d_{\mathrm{inel},q} = \int\frac{\mathrm{d}^3p_1}{(2\pi)^3 2E_{p_1}}\int\frac{\mathrm{d}^3p_2}{(2\pi)^3 2E_{p_2}}n_F(E_{p_1})\big(1-n_F(E_{p_2})\big)\frac{2}{3}g^4V_A^2C_F\big|\langle\psi_{nlm}|\boldsymbol{r}|\Psi_{\boldsymbol{p}}\rangle\big|^2$$
$$\Big[\frac{E_{p_1}E_{p_2}+\boldsymbol{p}_1\cdot\boldsymbol{p}_2}{\boldsymbol{q}^2} + \frac{2q_0^2}{(q^2+i\epsilon)^2}\Big(E_{p_1}E_{p_2} - \boldsymbol{p}_1\cdot\hat{q}\boldsymbol{p}_2\cdot\hat{q}\Big)\Big]\frac{M|\boldsymbol{p}|}{2\pi}\Big|_{|\boldsymbol{p}|=\sqrt{M(E_{p_1}+E_{nlm}-E_{p_2})}}$$
$$\Gamma^f_{\mathrm{inel},q} = \frac{1}{N_C^2-1}\int\frac{\mathrm{d}^3p_1}{(2\pi)^3 2E_{p_1}}\int\frac{\mathrm{d}^3p_2}{(2\pi)^3 2E_{p_2}}n_F(E_{p_1})\big(1-n_F(E_{p_2})\big)\frac{2}{3}g^4V_A^2C_F\big|\langle\psi_{nlm}|\boldsymbol{r}|\Psi_{\boldsymbol{p}}\rangle\big|^2$$
$$\Big[\frac{E_{p_1}E_{p_2}+\boldsymbol{p}_1\cdot\boldsymbol{p}_2}{\boldsymbol{q}^2} + \frac{2q_0^2}{(q^2+i\epsilon)^2}\Big(E_{p_1}E_{p_2} - \boldsymbol{p}_1\cdot\hat{q}\boldsymbol{p}_2\cdot\hat{q}\Big)\Big](2\pi)\delta(E_{p_1}+\frac{\boldsymbol{p}^2}{M}-E_{nlm}-E_{p_2})$$

- Gluon-inelastic-scattering dissociation



- Gluon-inelastic-scattering formation



In Coulomb gauge, transverse gluons can scatter with a heavy quark in all three channels

Here s- and u-channels are neglected because they are suppressed by v^2

$$\Gamma^d_{\mathrm{inel},g} = \int\frac{\mathrm{d}^3q_1}{(2\pi)^3 2E_{q_1}}\int\frac{\mathrm{d}^3q_2}{(2\pi)^3 2E_{q_2}}n_F(|\boldsymbol{q}_1|)\big(1-n_F(|\boldsymbol{q}_2|)\big)\frac{M|\boldsymbol{p}|}{2\pi}\overline{|M|^2}\Big|_{|\boldsymbol{p}|=\sqrt{M(E_{q_1}+E_{nlm}-E_{q_2})}}$$
$$\Gamma^f_{\mathrm{form},g} = \frac{2\pi}{N_C^2-1}\int\frac{\mathrm{d}^3q_1}{(2\pi)^3 2E_{q_1}}\int\frac{\mathrm{d}^3q_2}{(2\pi)^3 2E_{q_2}}n_F(|\boldsymbol{q}_1|)\big(1-n_F(|\boldsymbol{q}_2|)\big)\overline{|M|^2}\delta(E_{q_1}+\frac{\boldsymbol{p}^2}{M}-E_{q_2}-E_{nlm})$$
$$\overline{|M|^2} = \frac{1}{3}g^4V_A^2N_CC_F\big|\langle\psi_{nlm}|r_i|\Psi_{\boldsymbol{p}}\rangle\big|^2\Big[\Big(\frac{(E_{q_1}+E_{q_2})^2}{\boldsymbol{q}^2} + \frac{q_0^2(\boldsymbol{q}_1+\boldsymbol{q}_2)^2}{(q^2+i\epsilon)^2} + 2\frac{(\boldsymbol{q}_1^2-\boldsymbol{q}_2^2)^2}{\boldsymbol{q}^2(q^2+i\epsilon)}\Big)(1+(\hat{q}_1\cdot\hat{q}_2)^2)$$
$$+8\frac{q_0^2(\boldsymbol{q}_1^2+\boldsymbol{q}_2^2)}{(q^2+i\epsilon)^2}(1-(\hat{q}_1\cdot\hat{q}_2)^2)\Big]$$

$Q\bar{Q}$ Transport in a QCD Medium

- Initial production: pQCD and nuclear PDF
- Medium properties are taken from hydro-dynamics
- Temperature-dependent potential fitted from lattice results of free energy
- At each time step, determine if a quarkonium decays, if a $Q\bar{Q}$ pair recombines according to the rates calculated above and if a heavy quark scatters
- **For formation, quark pair needs to be treated as two wave packets rather than plane waves so the rate depends on their distance**
- Sample the initial and final light degrees of freedom, update the record and go to next step

This work is supported by DOE research grant DE-FG02-05ER41367.

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