Improvements to the $T = 0$ QCD EOS and rotating NS phenomenology

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BASED ON PAPERS

[1] T. Gorda, "Global properties of rotating neutron stars with QCD equations of state," *Astrophys. J.* **832** no. 1, (2016) 28, arXiv:1605.08067 [astro-ph.HE].

[2] I. Ghişoiu, T. Gorda, A. Kurkela, P. Romatschke, M. Säppi, and A. Vuorinen, "On high-order perturbative calculations at finite density," *Nucl. Phys.* **B915** (2017) 102–118, arXiv:1609.04339 [hep-ph].

ABSTRACT

I discuss recent progress in the determination of the **QUARK MATTER EQUATION OF STATE** (EOS) and its applications to the **PHENOMENOLOGY OF NEUTRON STARS** (NSs). The current state-of-the-art matched QCD EOS comes from the work of Kurkela et al. [1], in which the authors **MATCHED** chiral effective theory (ChEFT) at low densities to perturbative QCD (pQCD) at high densities. Since then, the $T = 0$, massless-quark, pQCD EOS has been **THEORETICALLY IMPROVED BEYOND $O(g^4)$**. I will discuss these improvements, as well as improvements to the **PHENOMENOLOGY OF NSS**. In particular, I will detail NS phenomenology **TAKING ROTATION INTO ACCOUNT**, something that was not done in the original work of Ref. [1]).

NSS: MATCHING

OUTER LAYERS of NSs: ChEFT [2] or quantum Monte Carlo [3] applicable and can yield insights into static properties (e.g. EOS) and some transport properties of NSs. Currently, low-density calculations **VALID UP TO ∼1.1 TIMES THE NUCLEAR SATURATION DENSITY** $n_s \approx 0.16/\text{fm}^3$, corresponding to a baryon chemical potential of about $\mu_B \approx 0.97$ GeV [2].

Deep in the core, controlled, direct theoretical calculations are **NOT POSSIBLE**: densities at the center of star are not large enough to use pQCD. In the state-of-the-art pQCD calculations at zero temperature [4], the errors associated with varying the mass scale reach 30% at around $\mu_B = 2.6$ GeV. The value of μ_B in the cores of NSs lie within a subset of this 0.97 – 2.6 GeV range.

One can hope to reach the intermediate values of μ_B by **MATCHING** the low-density EOS from the low-energy EFTs to the pQCD results in a **THERMODYNAMICALLY CONSISTENT WAY** to investigate the (static) makeup of NSs. This has been carried out in the work of Kurkela et al. [1] and Fraga et al. [5], using 2-3 interpolating polytropes. These works also incorporated the $2M_\odot$ constraint from [6, 7].

In these works, the authors used their matched EOSs to analyze **NON-ROTATING NSS ONLY**.

Here, we used the EOS of [1] in the form $P(\epsilon)$, along with the publicly-available RNS code [8] to construct NSs with **ANY ω BELOW THE MASS-SHEDDING (KEPLER) LIMIT**.

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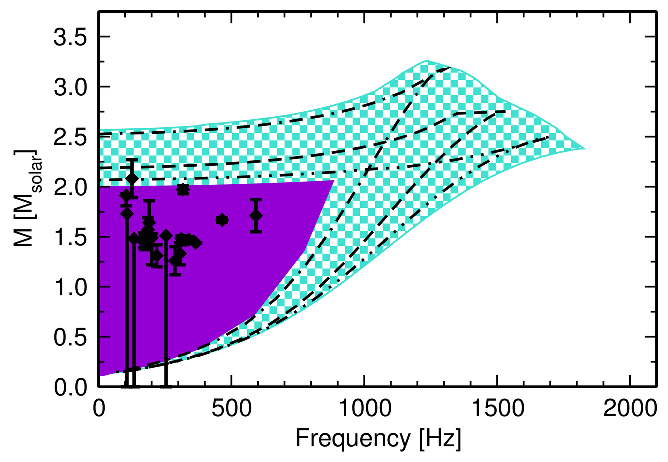
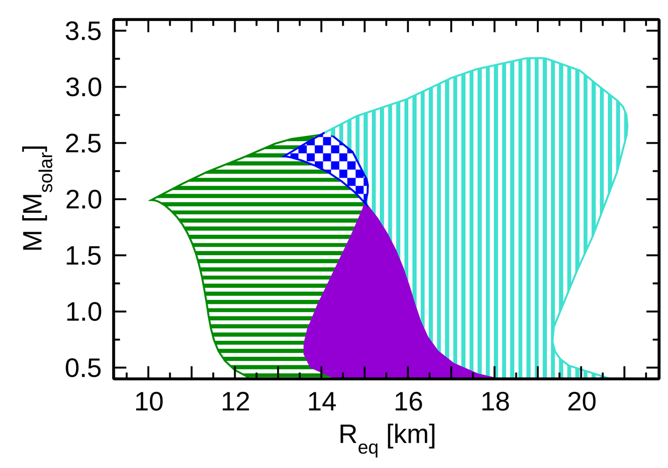
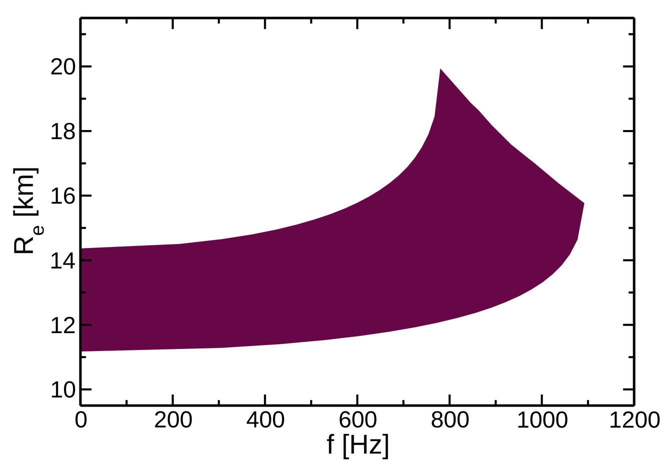
ROTATING NSS WITH MATCHED EOSS: PLOTS



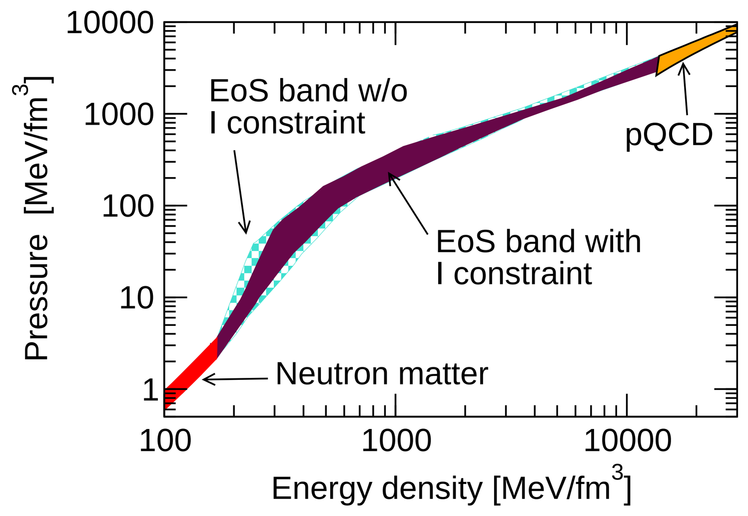
MASS VS. FREQUENCY region for all of the possible EOSs. The inner, solid region is allowed for every equation of state, and the outer, checkered band shows where the possible boundaries are for each EOS. The dashed lines are the outer boundaries of the mass–frequency regions for three sample EOSs. Data points for NSs with $f > 100$ Hz, taken from a table in Ref. [9], are also plotted.



MASS VS. EQUATORIAL RADIUS regions for non-rotating stars (horizontal stripes) and mass-shedding stars (vertical stripes). The upper, checkered region is an overlap between the non-rotating and mass-shedding regions. The lower, solid region is only accessible to non-mass-shedding, rotating NSs.



CIRCUMFERENTIAL, EQUATORIAL RADIUS VS. FREQUENCY curves for a $1.4M_\odot$ star.



A plot illustrating how much the EOS band from Ref. [1] would be restricted by a **HYPOTHETICAL MEASUREMENT** of $I = 1.5 \times 10^{45}$ g cm^2 with a **PRECISION OF 10%** for PSR J0737-3039A.

ROTATING NS RESULTS: EXCLUSION REGIONS

- Maximum non-rotating mass is $2.5M_\odot$; rotating is $3.25M_\odot$
- Maximum allowed radius is 21 km.
- f constraint: upper-right corner of purple region in M vs. f plot: $f > 883$ Hz will start eliminating EOSs.
- lower-f NSs could also rule out EOSs if their masses were sufficiently low (e.g., $f = 716$ Hz starts constraining for $M < 1M_\odot$)
- For $1.4M_\odot$ star, $f > 780$ Hz will start reducing EOS band.
- Most stringent constraints on EOS band would come from **EVEN A RELATIVELY IMPRECISE MEASUREMENT** of I of the double pulsar PSR J0737-3039A.

PQCD IMPROVEMENTS

Extending state-of-the-art zero-temperature result [4] to higher orders presents a considerable technical challenge: Part of the problem lies in understanding how to handle the contributions of the **SOFT MOMENTUM SCALES** to the quantity.

Here, we present a **NEW TECHNICAL TOOL** for perturbative calculations at $T = 0$ and $\mu > 0$ that enables a high-order determination of many important thermodynamic quantities.

USEFUL AS THERE IS A VAST AMOUNT OF LITERATURE ON VACUUM AMPLITUDES THAT CAN BE DIRECTLY TAKEN OVER.

CUTTING RULES

Sometimes referred to as the **NAIVE REAL-TIME FORMALISM** [10], and connected to the much earlier work of Ref. [11], where a connection between certain statistical-physics quantities and scattering amplitudes was proposed. It has since then been developed, e.g., in Refs. [12, 13].

STATEMENT: At $T = 0$, the general structure inside a Euclidean n-point function integral expression is (for a general function f with no poles):

$$\int_{\widetilde{Q}} \frac{f(q_0, q)}{q^2 + E^2} = \int_{Q} \frac{f(q_0, q)}{q^2 + E^2} - \int_{\vec{q}} \frac{\theta(\mu - E)}{2E} f(\mathrm{i}\, E_\alpha, q)$$

GENERAL REMARKS

- In the $\widetilde{Q}$ integral, there is a shift in the q^0 component by an amount $\mathrm{i}\,\mu$. (Standard notation.)
- Note that the structure is: **VACUUM + MATTER.**
- This substitution is to be carried out on **EVERY FERMIONIC PROPAGATOR**

This procedure results in a **SUM OVER "CUTS"** of the original diagram F up to the number of loops N:

$$F(\{P_k\}, \mu) = F_{0\text{-cut}}(\{P_k\}) + F_{1\text{-cut}}(\{P_k\}, \mu) + \cdots + F_{N\text{-cut}}(\{P_k\}, \mu).$$

An important additional rule is that those cuts that **DIVIDE THE ORIGINAL GRAPH INTO DISCONNECTED PIECES** are to be **THROWN OUT**.

The general "naive real-time formalism" has been **PROPOSED** only for **VACUUM DIAGRAMS**, and even there no proof to all orders exists. Instead, the validity of the replacement has only been checked on a case-by-case basis up to partial three-loop order. In contrast, **OUR PROOF** of the zero-temperature cutting rules covers **ALL EUCLIDEAN n-POINT FUNCTIONS** up to an arbitrary order in perturbation theory.

NOTATION OF PROOF

- Set of all **PROPAGATORS**: $P \equiv \{1, 2, \ldots, M\}$,
- Set of all possible **CHOICES OF LOOP MOMENTA**: S. Each element $S_r \in S$ corresponds to some subset of N indices from P. The sets S_r are limited only by momentum conservation.

Simplifying **ASSUMPTIONS** (which **CAN BE RELAXED** at the expense of more cumbersome notation only):

- No structure in numerator of Feynman integral F
- No individual propagator is raised to a power higher than one
- There is only one chemical potential appearing in the graph

STRUCTURE OF THE PROOF

1. **VACUUM CASE**, the general integral is
$$\int_{-\infty}^{\infty} \prod_{i=1}^{N} \frac{dq_0^i}{2\pi} \prod_{\alpha=1}^{M} \frac{1}{(r_0^\alpha)^2 + E_\alpha^2},$$
which we prove evaluates to
$$\sum_{S_r \in S} \prod_{i \in S_r} \frac{1}{2E_i} \prod_{\alpha \in P \setminus S_r} \frac{1}{(r_0^\alpha(S_r))^2 + E_\alpha^2(S_r)} \bigg|_{\{q_0^i = iE_i\}},$$
where $P \setminus S_r$ denotes the propagators that do not belong to the set S_r and the explicit forms of the R_α in terms of the momenta are dictated by S_r. **NOTE**: Each set $S_r \in S$ is counted only once. Relabellings within S_r don't matter.
2. **GENERALIZATION TO NONZERO DENSITY**
$$\frac{1}{2E_i} \to \frac{\theta(E_i - \mu)}{2E_i}$$
FOR INTERNAL FERMION LINES only, via the residue theorem. **NOTE:** Different $S_r \in S$, imply different numbers of fermionic momenta and thus different numbers of θ-function factors
3. **CONNECTION TO ORIGINAL RULES**:
$$\theta(E_i - \mu) = 1 - \theta(\mu - E_i).$$
Then multiply out and rearrange.

CRUX OF THE PROOF lies in step (1), which relies on the residue theorem, the crucial realization that we cannot take residues of a set of propagators whose momenta are linearly dependent, and the independence of the original integral F on the choice of integration momenta $S_r \in S$.

REFERENCES

[1] A. Kurkela, E. S. Fraga, J. Schaffner-Bielich, and A. Vuorinen *Astrophys. J.* **789** (2014) 127, arXiv:1402.6618 [astro-ph.HE].

[2] I. Tews, T. Krüger, K. Hebeler, and A. Schwenk *Phys. Rev. Lett.* **110** no. 3, (2013) 032504, arXiv:1206.0025 [nucl-th].

[3] S. Abbar, J. Carlson, H. Duan, and S. Reddy *Phys. Rev.* **C92** no. 4, (2015) 045809, arXiv:1503.01696 [astro-ph.HE].

[4] A. Kurkela, P. Romatschke, and A. Vuorinen *Phys.Rev.* **D81** (2010) 105021, arXiv:0912.1856 [hep-ph].

[5] E. S. Fraga, A. Kurkela, and A. Vuorinen *Eur. Phys. J.* **A52** no. 3, (2016) 49, arXiv:1508.05019 [nucl-th].

[6] P. Demorest, T. Pennucci, S. Ransom, M. Roberts, and J. Hessels *Nature* **467** (2010) 1081–1083, arXiv:1010.5788 [astro-ph.HE].

[7] J. Antoniadis *et al. Science* **340** (2013) 6131, arXiv:1304.6875 [astro-ph.HE].

[8] N. Stergioulas and J. Friedman *Astrophys. J.* **444** (1995) 306, arXiv:astro-ph/9411032 [astro-ph]. http://www.gravity.phys.uwm.edu/rns/.

[9] P. Haensel, M. Bejger, M. Fortin, and L. Zdunik *Eur. Phys. J.* **A52** no. 3, (2016) 59, arXiv:1601.05368 [astro-ph.HE].

[10] J. O. Andersen, E. Braaten, and M. Strickland *Phys. Rev.* **D62** (2000) 045004, arXiv:hep-ph/0002048 [hep-ph].

[11] R. Dashen, S.-K. Ma, and H. J. Bernstein, "S Matrix formulation of statistical mechanics," *Phys. Rev.* **187** (1969) 345–370.

[12] A. I. Bugrii and V. N. Shadura, "Three loop contributions to the free energy of lambda phi**4 QFT," arXiv:hep-th/9510232 [hep-th].

[13] J. Frenkel, A. V. Saa, and J. C. Taylor, "The Pressure in thermal scalar field theory to three loop order," *Phys. Rev.* **D46** (1992) 3670–3673.