

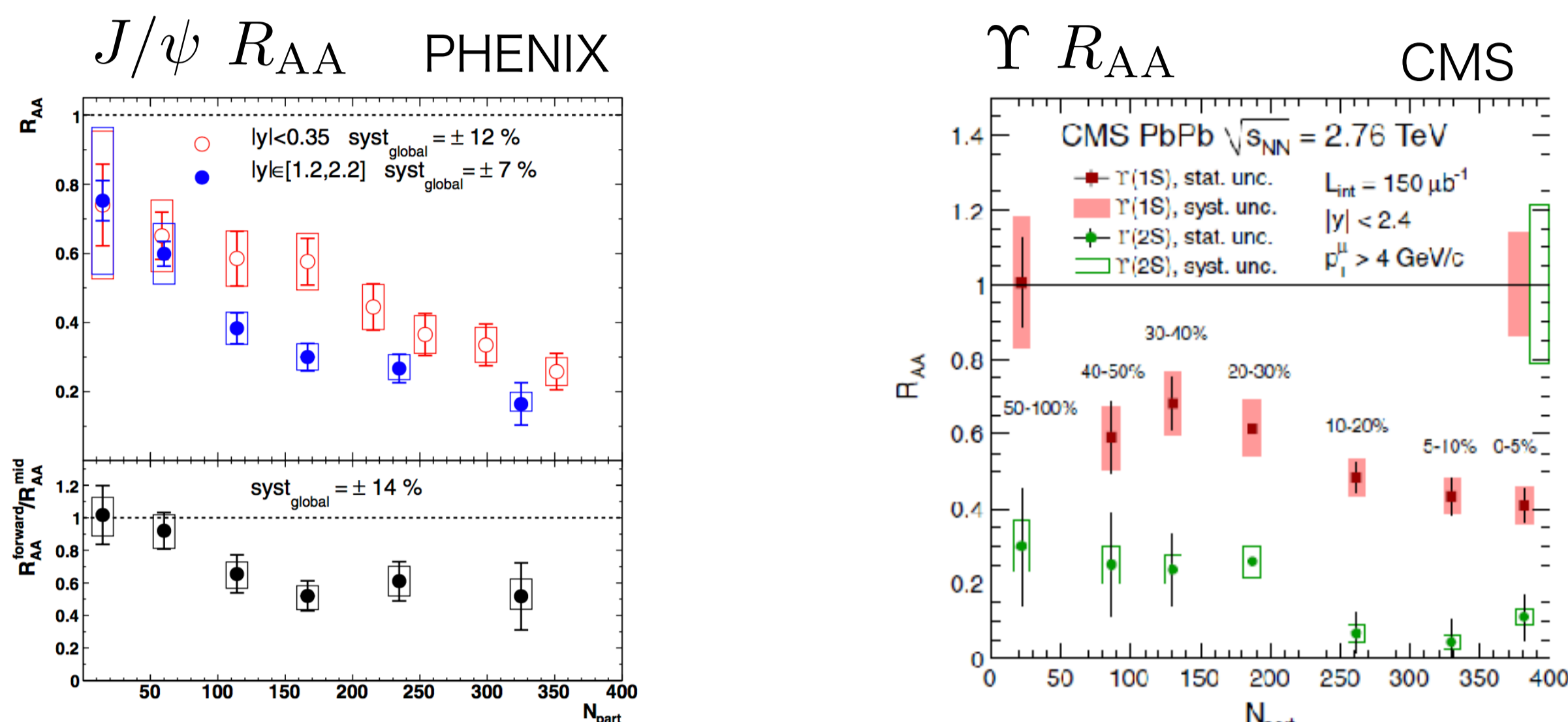
# Time Evolution of Heavy Quarkonium in Quark-Gluon Plasma

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In heavy-ion collision experiment, quarkonium suppression is the one of important probe for QGP formation. Recently lattice QCD and perturbative calculation reveal that quarkonium potential has an imaginary part. Stochastic potential model naturally explains the imaginary part as the result of the interaction between quarkonium and QGP, or environment.

## 1. Introduction

### Experimental Results of $J/\psi$ and $\Upsilon$ suppression



In central collisions, suppression is stronger.  
Excited state is much more suppressed.

### Static Potential Model

Matsui and Satz, Phys. Lett. B178 (1986) 416

Confining potential (T=0)

$$V = -\frac{\alpha}{r} + \sigma r$$

Debye screened potential (T>Tc)

$$V = -\frac{\alpha}{r} \exp\left(-\frac{r}{r_D}\right)$$

### Real-time Potential Model

Laine et al., JHEP 03 (2007) 054

Wilson Loop perturbative calculation

$$V = -\frac{g^2 C_F}{4\pi} \left[ m_D + \frac{\exp(-m_D r)}{r} \right] - \frac{ig^2 T C_F}{4\pi} \phi(m_D r)$$

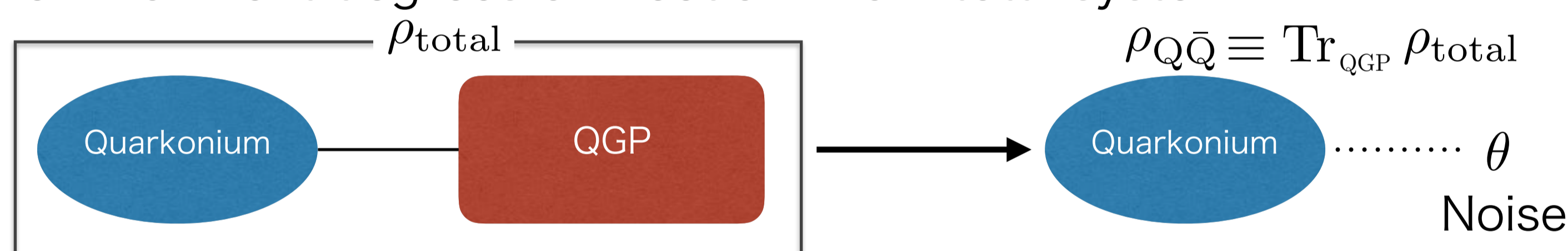
$$\phi(x) = 2 \int_0^\infty \frac{dz z}{(z^2 + 1)^2} \left[ 1 - \frac{\sin(zx)}{zx} \right]$$

→ How should we interpret the imaginary part?

## 2. Stochastic potential model

Akamatsu and Rothkopf, phys. rev. D 085 (2012), 105011

When we consider a quarkonium in QGP as an open quantum system, we can extract quarkonium information by tracing out environment degrees of freedom from total system.



$$H(\mathbf{X}) \rightarrow H(\mathbf{X}) + \Theta(\mathbf{X}; t)$$

$$\mathbf{X} = \{\mathbf{x}, \mathbf{y}\}$$

$$H(\mathbf{X}) = 2M - \frac{\nabla_{\mathbf{X}}^2}{2M} + V(\mathbf{X})$$

$$\Theta(\mathbf{X}; t) = \theta(\mathbf{x}, t) - \theta(\mathbf{y}, t)$$

$$\langle \theta(\mathbf{x}, t) \rangle = 0$$

$$\langle \theta(\mathbf{x}, t) \theta(\mathbf{x}', t') \rangle = D(|\mathbf{x} - \mathbf{x}'|) \frac{\delta_{tt'}}{\Delta t}$$

### Unitary time evolution

$$\Psi_{Q\bar{Q}}(t + \Delta t) = e^{-iH\Delta t} \Psi(t)$$

$$e^{-i(H+\Theta)\Delta t} = 1 - i(H(\mathbf{X}) + \Theta(\mathbf{X}; t))\Delta t - \frac{1}{2!} \{\Theta(\mathbf{X}, t)\}^2 (\Delta t)^2 + \mathcal{O}((\Delta t)^3/2)$$

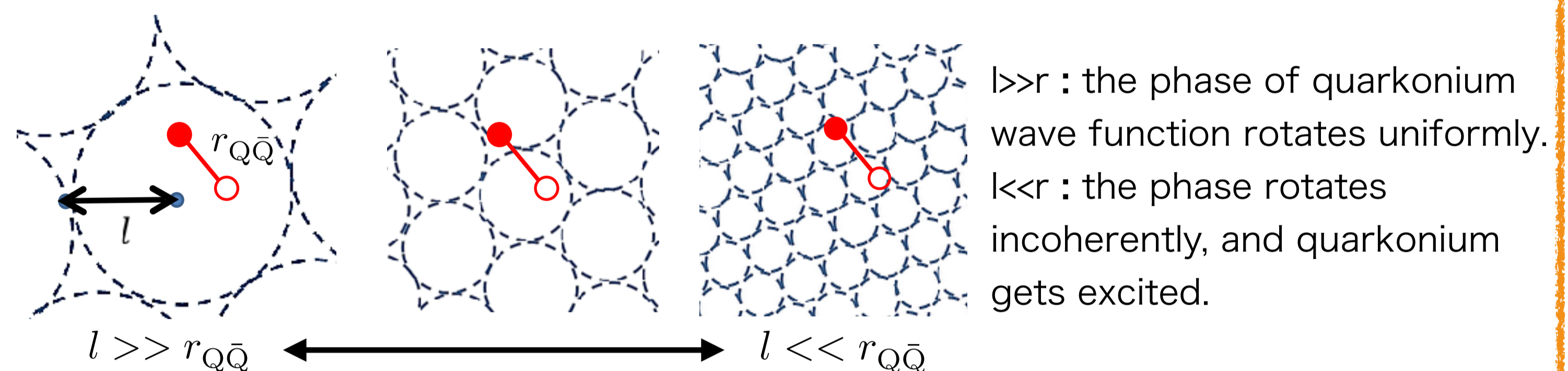
$$= 1 - i [ H(\mathbf{X}) - I\{D(0) - D(|\mathbf{x} - \mathbf{y}|\}) + \Xi(\mathbf{X}, t) ] \Delta t + \mathcal{O}((\Delta t)^3/2)$$

Stochastic Schrödinger equation

$$i \frac{\partial}{\partial t} \Psi_{Q\bar{Q}}(\mathbf{X}, t) = [H(\mathbf{X}) - i\{D(0) - D(|\mathbf{x} - \mathbf{y}|\}) + \Theta(\mathbf{X}, t)] \Psi_{Q\bar{Q}}(\mathbf{X}, t)$$

### Decoherence

We approximate  $D(r) = \gamma \exp\left(-\frac{r^2}{l^2}\right)$  with correlation length  $l$ .  
The noise in potential brings about the phase rotation.



### Master Equation

$\rho_{Q\bar{Q}}$  is defined as  $\rho_{Q\bar{Q}}(X, X', t) \equiv \langle \Psi_{Q\bar{Q}}(X, t) \Psi_{Q\bar{Q}}^*(X', t) \rangle_{\Theta}$

Master equation after tracing out the c.m.s. motion

$$\frac{\partial \rho_{Q\bar{Q}}(\mathbf{r}, \mathbf{r}', t)}{\partial t} = \frac{h(\mathbf{r}) - h(\mathbf{r}')}{i\hbar} \rho_{Q\bar{Q}}(\mathbf{r}, \mathbf{r}', t) + \frac{f(\mathbf{r}, \mathbf{r}')}{\hbar} \rho_{Q\bar{Q}}(\mathbf{r}, \mathbf{r}', t)$$

By expanding the density matrix with a complete set of in-medium Hamiltonian, we get the time evolution of admixture,

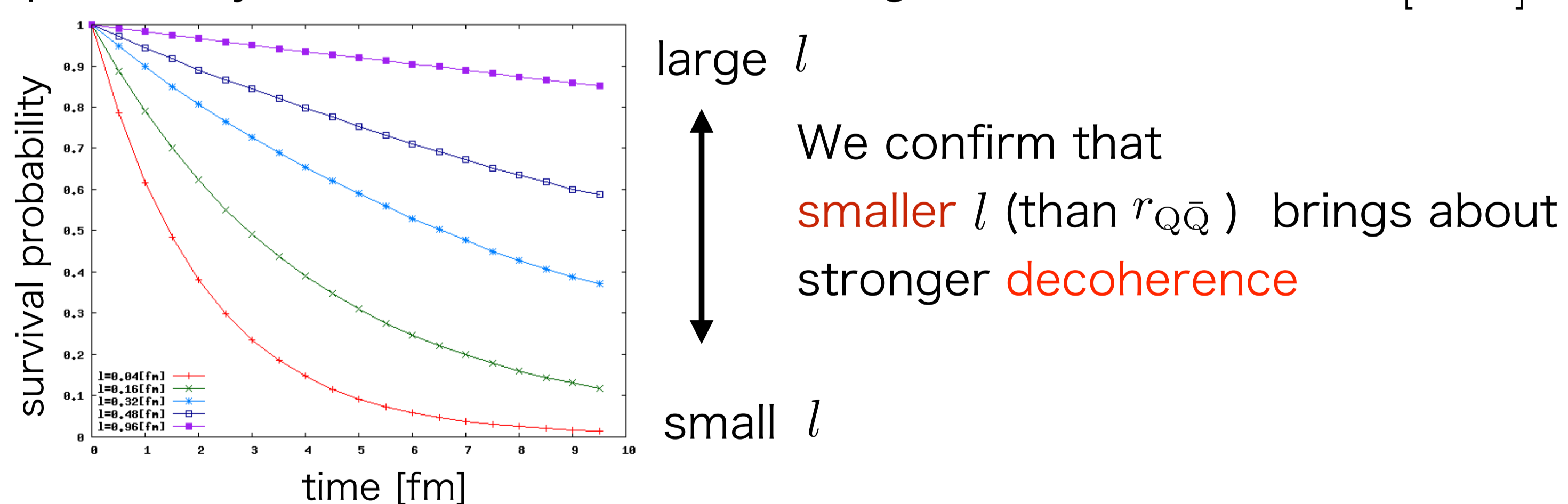
$$\rho_{Q\bar{Q}}(\mathbf{r}, \mathbf{r}', t) = \sum_{nm} c_{nm}(t) \psi_n(\mathbf{r}) \psi_m^*(\mathbf{r}')$$

$$\rightarrow \dot{c}_{nm}(t) = \frac{\epsilon_n - \epsilon_m}{i} c_{nm}(t) + \sum_{kl} \gamma_{nk, lm} c_{nm}(t) - \frac{1}{2} \sum_k \{ \gamma_{nk} c_{km}(t) + c_{nk}(t) \gamma_{km} \}$$

## 3. Results

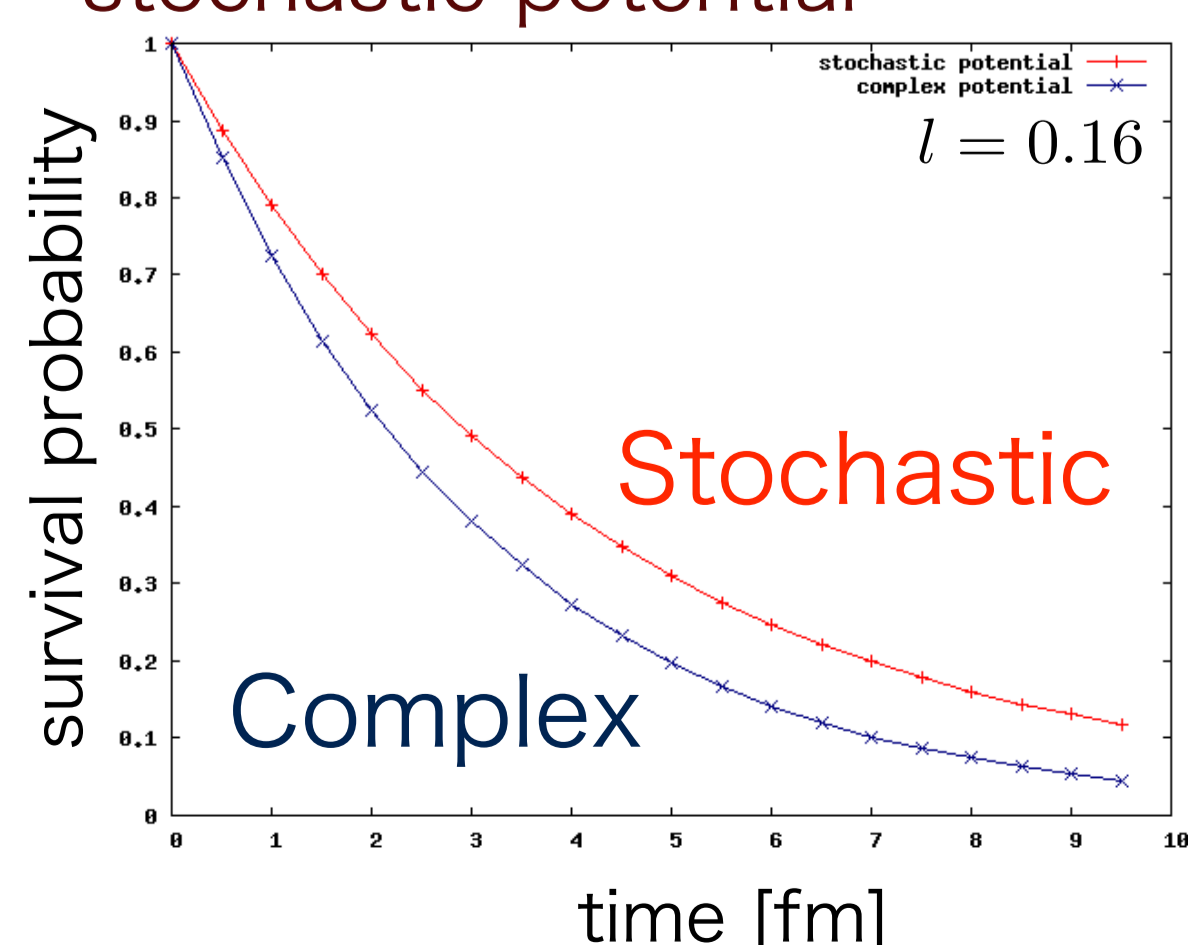
### 3-1. One-dimensional numerical calculation in a static QGP

By solving Stochastic Schrödinger equation, we calculate the survival probability of bottomonium in-medium ground state. ( $T = 0.4$  [GeV])



large  $l$   
We confirm that  
smaller  $l$  (than  $r_{Q\bar{Q}}$ ) brings about  
stronger decoherence  
small  $l$

Difference between the results using complex potential and stochastic potential



If we start from  $c_{00} = 1$  and  $c_{ii} = 0$ , the slope at  $t=0$  is

$$\dot{c}_{00} \sim \gamma_{00,00} - \gamma_{00} \quad (\text{stochastic})$$

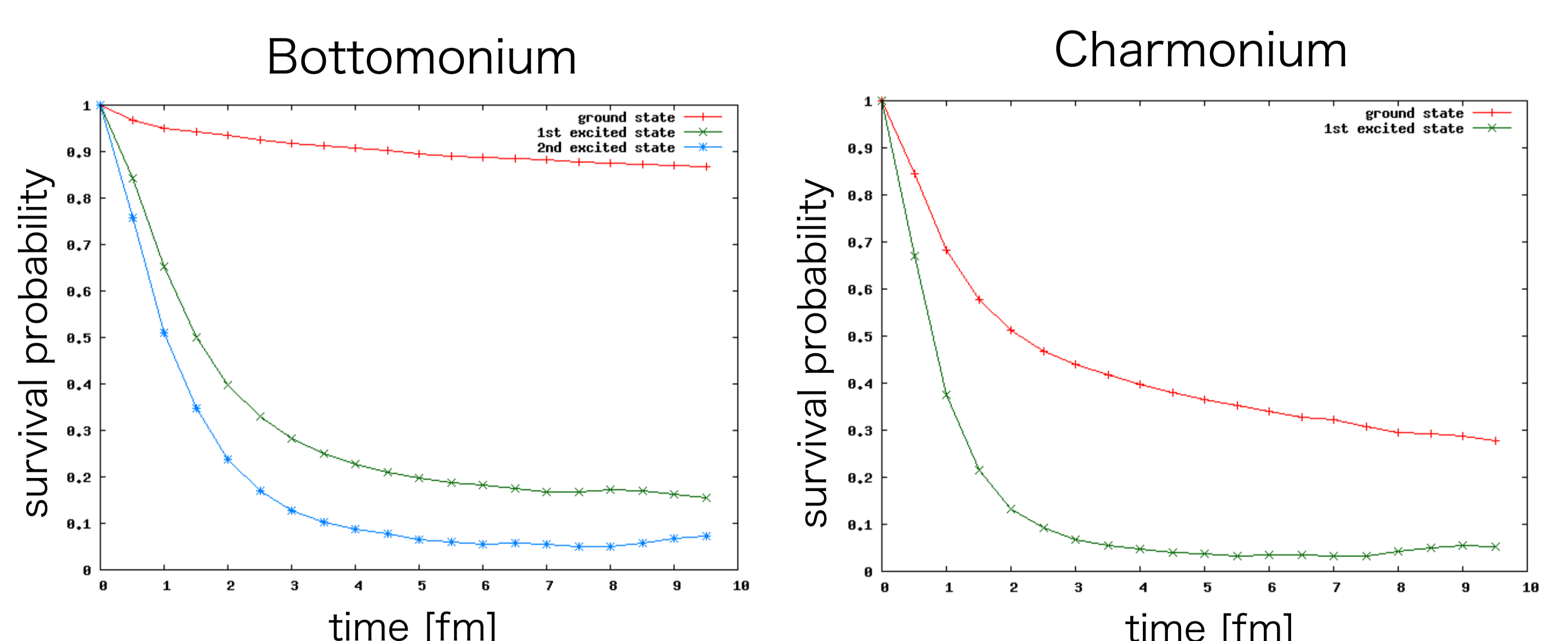
$$\dot{c}_{00} \sim -\gamma_{00} \quad (\text{complex})$$

Complex potential  
overestimates the decay rate  
by including the scattering  
from ground state to itself.

### 3-2. One-dimensional numerical calculation

In Bjorken-Expanding QGP

By solving Stochastic Schrödinger equation, we calculate the survival probability of states. Initial states are picked up from eigenstates of zero-temperature confining potential. Initial temperature set to  $T_0 = 0.4$  [GeV].



Excited states are affected earlier by decoherence than ground state.

Charmonium is more affected by decoherence than bottomonium because of their radius.

These results are consistent qualitatively with experimental results  $R_{AA}$