Effective Model of QCD Monopoles Around T_c

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(With E. Shuryak, in preparation)

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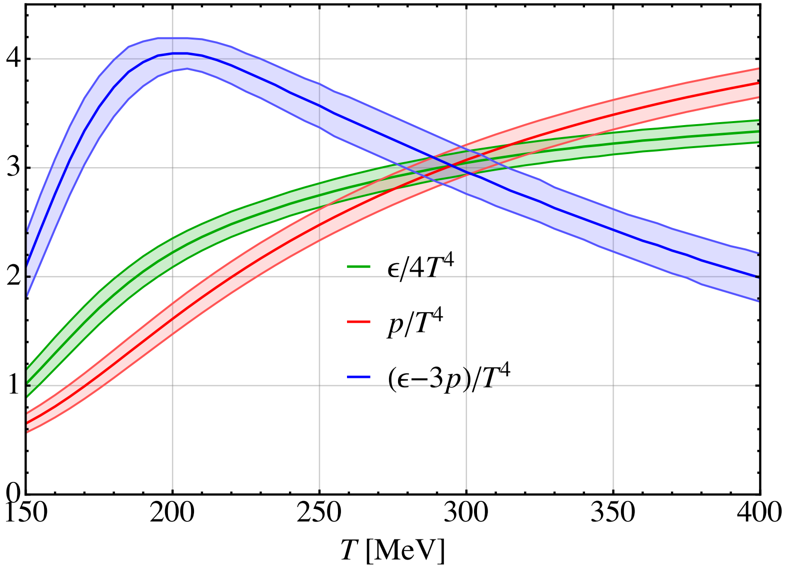
Outline

Lattice QCD Equation of State



Results adapted from HotQCD Collaboration (2014)

Outline

Magnetic Scenario of QCD

Dual Superconductivity Model of the QCD Vacuum

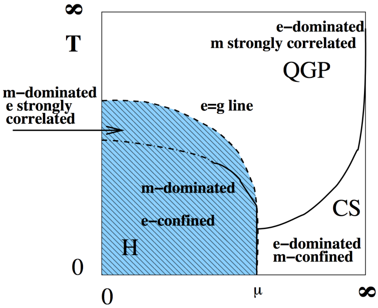
Nambu (1974), Mandelstam (1976), 't Hooft (1981)

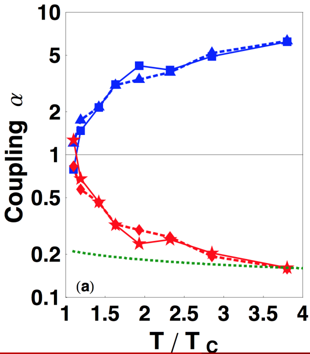
- Electric quasiparticles (quarks and gluons) and magnetic quasiparticles (monopoles, etc.)
- Confinement is due to the Bose-Einstein condensation (BEC) of magnetic (quasi)particles
- Lattice studies have identified electric flux-tubes and monopole currents

Can we make an effective model for the magnetic component of QCD above T_c?

Magnetic Scenario of QCD

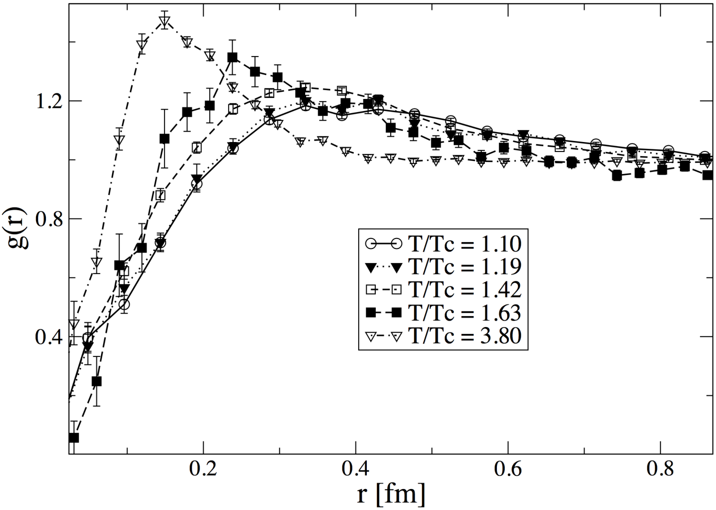
- Condition for making Dirac strings invisible: $\frac{eg}{4\pi} = \text{integer}$
  Dirac (1931)
  - $\alpha_s = e^2/4\pi$ in QCD runs with T and μ
  - $\alpha_m = g^2/4\pi$ runs oppositely
- Classical studies of electric-magnetic plasma have seen this behavior of the coupling Liao and Shuryak (2007)





Monopoles on the Lattice

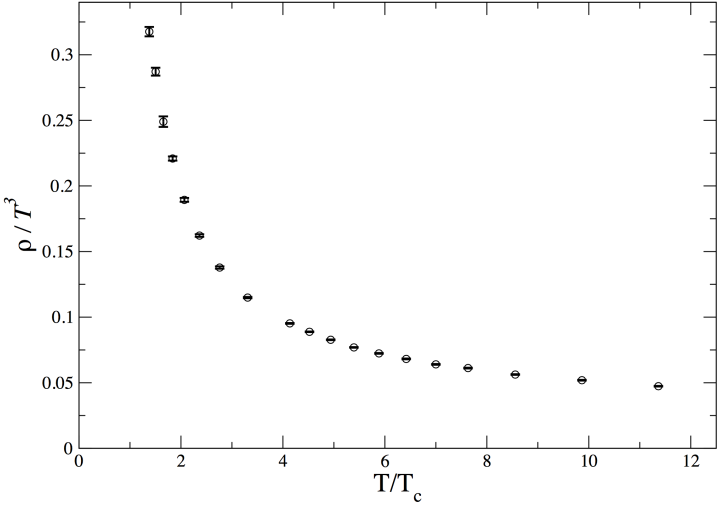
Antimonopole-monopole correlations stronger at higher temperatures



D'Alessandro and D'Elia (2007)

Monopoles on the Lattice

Density: $\rho_m / T^3 \sim \log(T)^{-2} \rightarrow$ Monopoles important near T_c



D'Alessandro and D'Elia (2007)

Outline

Goals

- Classical monopoles $\rightarrow$ Quantum monopoles (BEC)
  - Reproduce behavior of lattice monopoles, without other degrees of freedom
  - Identify the contribution of magnetic monopoles to the thermodynamics of QCD around T_c
- Quantify BEC critical temperature behavior of one- and two-component quantum Coulomb systems

Outline

Path-Integral Monte Carlo

- Density matrix at finite temperature from path integrals:
  Feynman (1953), Matsubara (1951)

$$\rho(x_i, x_j, \beta) = \int_0^\beta \mathcal{D}x(\tau) \exp\left\{-S_E[x(\tau)]\right\}$$

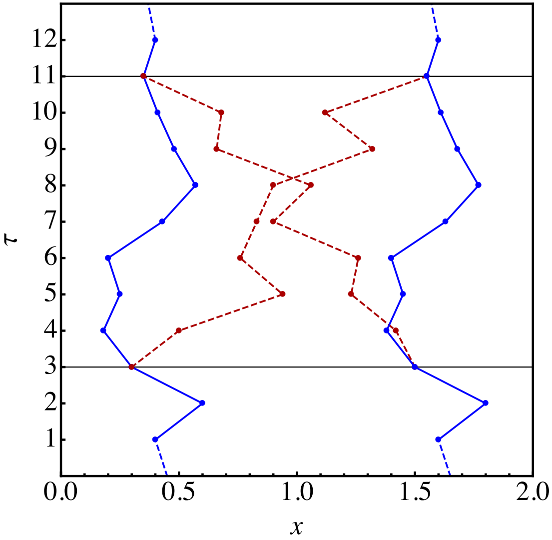
  where $\beta = 1/T$
- Partition function: $Z = \mathrm{Tr}[\rho] \rightarrow$ periodic paths
- Discretize paths and sample configurations using Markov chain Monte Carlo (MCMC)
  Ceperley (1995) (Review w/ focus on application to liquid helium)
  - The configuration weight is given by the Euclidean action,

$$\pi(\{\vec{x}\}) = e^{-S_{\mathsf{E}}(\{\vec{x}\})}$$

  - Sample the partition function $\rightarrow$ thermodynamics

Path-Integral Monte Carlo

- The imaginary-time paths of bosons can be permuted



Outline

Finding T_c, Method 1: Permutation Cycles

- Measure $n_k(T)$, the probability of finding a particle in a k-permutation cycle as a function of temperature
- Densities of k-cycles go as

$$\rho_k(T) \equiv n_k(T)/k = \exp(-\hat{\mu}(T)k) \times f(k)\,,$$

where $f(k)$ is some decreasing function $\sim 1/k^{\alpha}$

- Critical temperature is where permutation cycles are no longer exponentially suppressed, i.e. $\hat{\mu}(T) = 0$

D'Alessandro, D'Elia, and Shuryak (2010)

Finding T_c, Method 2: Superfluid Fraction

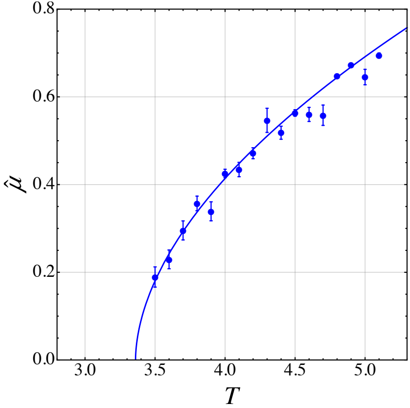
- The superfluid fraction of a condensate is related to the spatial winding number distribution

$$\frac{\rho_s}{\rho} = \frac{\langle W^2 \rangle}{2\lambda\beta N}$$

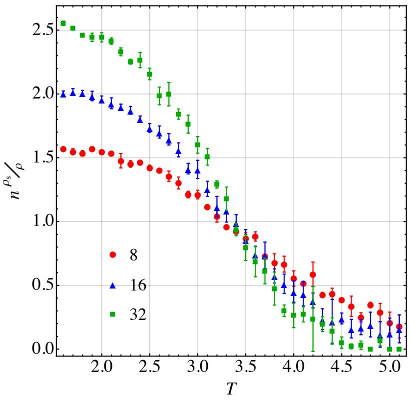
- Thermodynamic limit $\rightarrow$ expect the superfluid fraction to go from 0 (above T_c) to 1 (below T_c)
- Study finite-size scaling of this quantity $\rightarrow$ determine critical point by the intersection of the superfluid fraction lines

Pollock and Ceperley (1987)

Finding T_c: Examples



Permutation-cycle calculation



Finite-size scaling of the superfluid fraction

Outline

Study of Quantum Coulomb Systems

- Find the dependence of T_c on α, the coupling, with interaction potential:

$$V_{\text{int}}(r_{ij}) = \alpha \frac{q_i q_j}{r_{ij}}$$

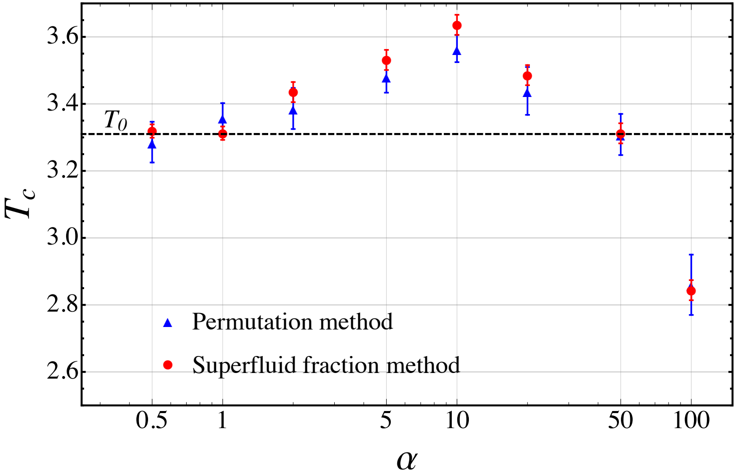
- Critical temperature of an ideal Bose gas:

$$T_0 = \left(\frac{2\pi\hbar^2}{mk_{\text{B}}}\right)\left(\frac{n}{\zeta\left(\frac{3}{2}\right)}\right)^{\frac{2}{3}}$$

- $|q| = m = n = 1 \rightarrow$ scale is fixed by the temperature T
- Ideal gas BEC temperature in these units:

$$T_0 = 2\pi\left(\frac{1}{\zeta\left(\frac{3}{2}\right)}\right)^{\frac{2}{3}} = 3.3125$$

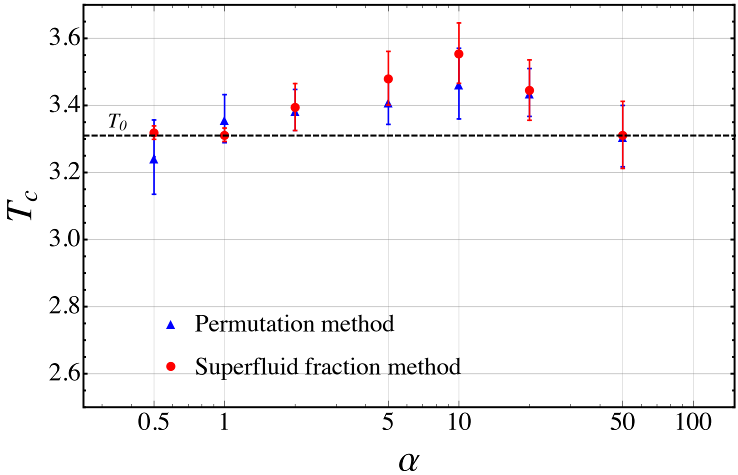
One-Component Coulomb System



Same behavior seen by varying density of hard spheres Grüter, et al. (1997)

Analytically explained by M. Holzmann, G. Baym, J. P. Blaizot, and F. Laloe (2001)

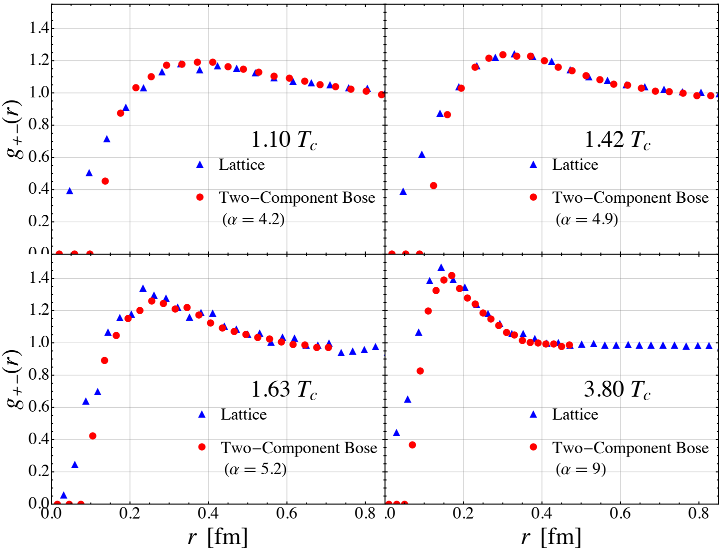
Two-Component Coulomb: Critical Temperature



Same qualitative behavior as one-component case, but the increase in T_c at moderate coupling is slightly smaller

Outline

Matching Radial Distribution Functions



Lattice data from D'Alessandro and D'Elia (2007)

Fixing Parameters

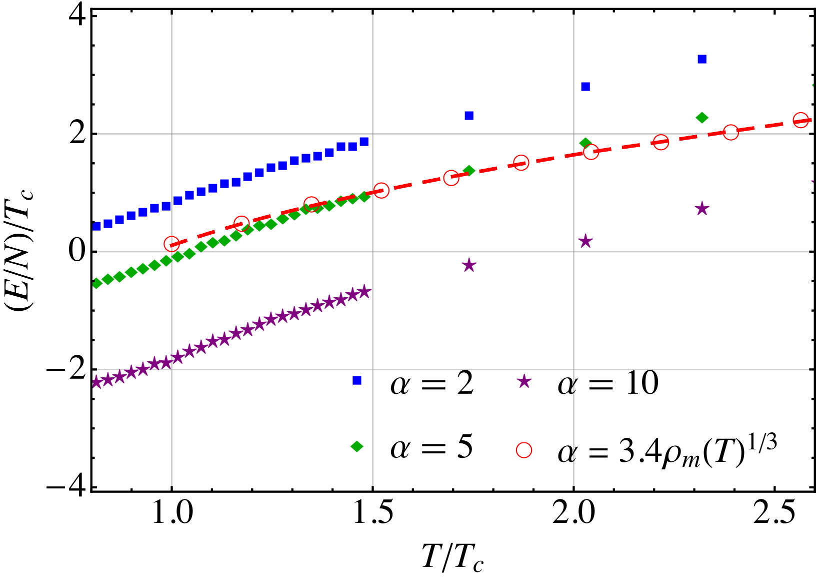
- Scaling relation for α, the simulation coupling, to match lattice results:

$$\boxed{\alpha(T/T_c) \approx 3.4\, \rho_m^{1/3}(T/T_c)}$$

- $\rho_m(T)$ is the monopole density (in fm^{-3})
  D'Alessandro and D'Elia, 2007
- One unit of length in our simulations is $\rho_m^{-1/3}(T)$ (in fm)
- $T_c =$ 155 MeV $\approx$ 3.45 in our units of temperature

Outline

Energy Per Monopole on the Physical Line

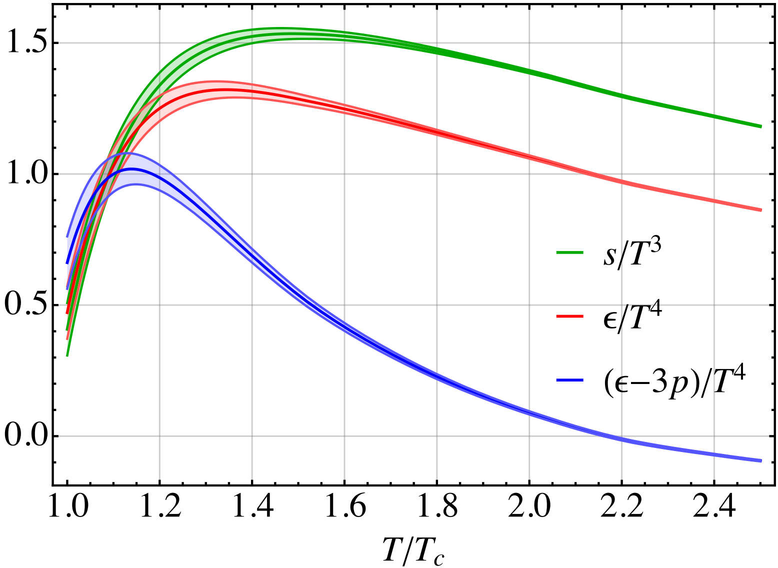


Equation of State

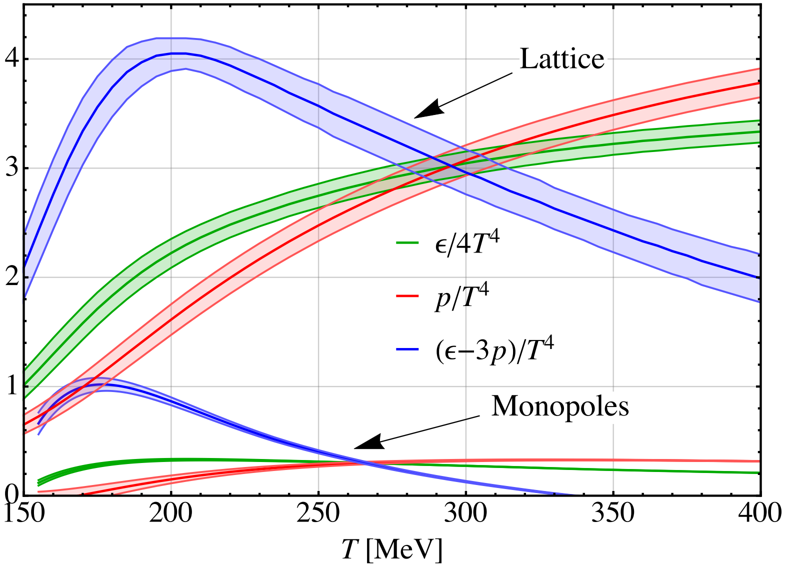
- Studies on the lattice found a large entropy density and trace anomaly at 170-250 MeV
- May not all be from quarks and gluons $\rightarrow$ other contributions to the thermodynamics of QCD-like theories at T_c (hadrons, instantons, etc.)
- What portion comes from monopoles?

Wuppertal-Budapest Collab. (2013)
HotQCD Collab. (2014)
etc.

Monopole Contribution to Equation of State



Comparison to Lattice QCD



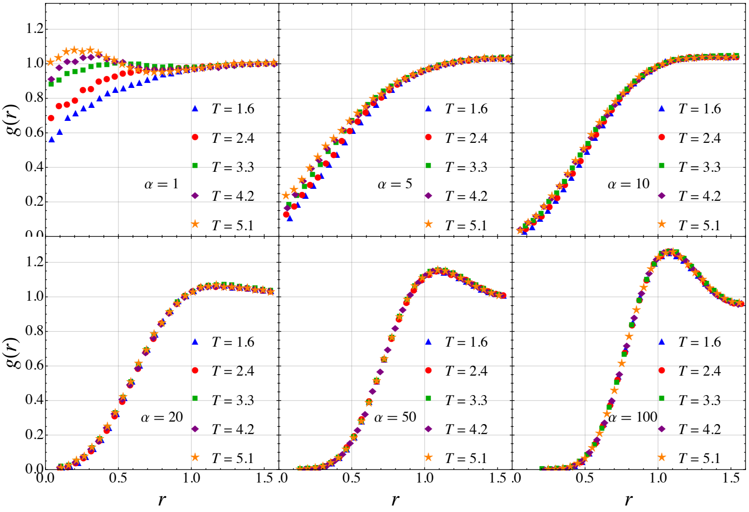
Lattice results from HotQCD Collaboration (2014)

Conclusions and Outlook

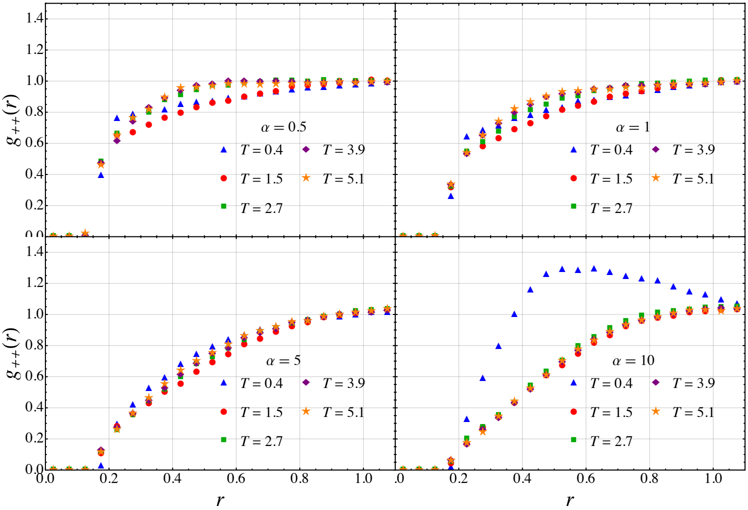
- Found parameters of a two-component Coulomb quantum Bose gas that give an effective model of QCD monopoles
- Identified the contribution of monopoles to QCD equation of state
- Found the effect of coupling on the critical temperature of Coulomb quantum Bose gases
- Can we use the parameters of this model to simulate *real-time* properties of QGP, such as viscosity, jet quenching, and heavy quark propagation?

Backup Slides

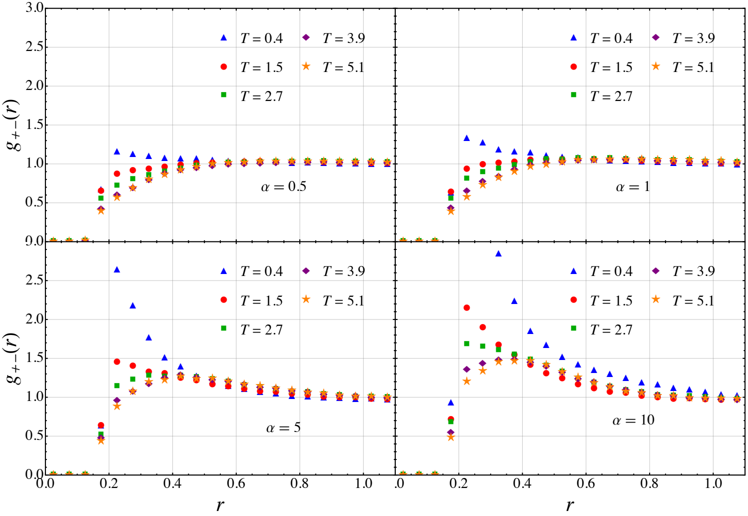
One-Component Radial Distribution Functions



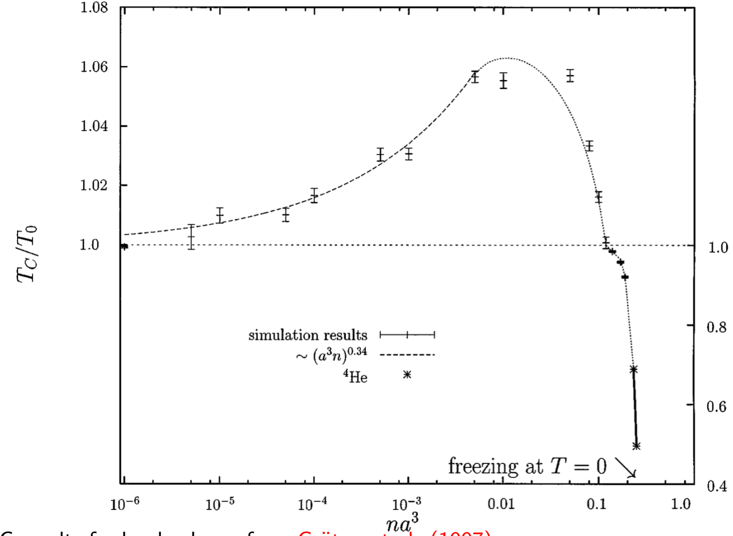
Two-Component Radial Distribution Functions



Two-Component Radial Distribution Functions

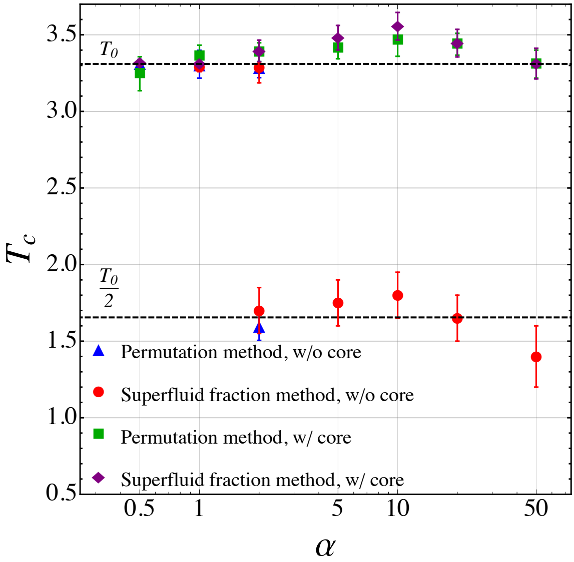


Critical Temperature of Hard Spheres

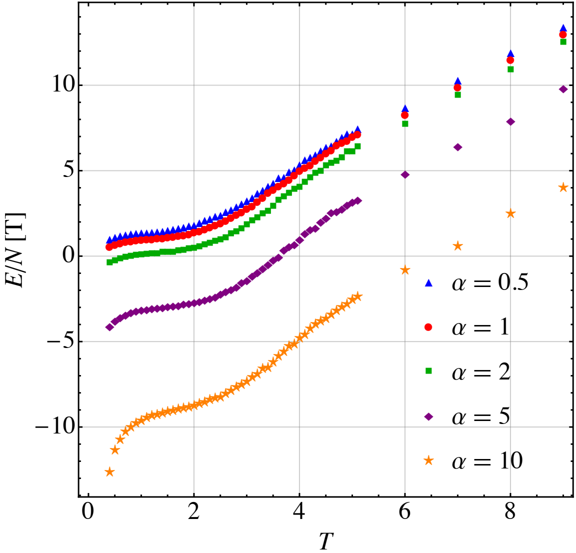


PIMC results for hard spheres from Grüter, et al. (1997)

Two-Component Critical Temperature (incl. core)



Two-Component Internal Energy



Primitive Approximation

- In the primitive approximation, the density matrix between time slice i and $i+1$ is given by

$$\begin{aligned}\rho(R_i, R_{i+1}, \tau) &= \langle R_i | e^{-\tau \hat{T}} e^{-\tau \hat{V}} | R_{i+1} \rangle \\ &= \langle R_i | e^{-\tau \hat{T}} | R_{i+1} \rangle \, e^{-\tau V}\end{aligned}$$

- The kinetic matrix element for N particles can be computed using the eigenfunction expansion of the kinetic operator,

$$\langle R_i | e^{-\tau \hat{T}} | R_{i+1} \rangle = \frac{1}{(4\pi\lambda\tau)^{3N/2}} \exp\left\{ -\frac{(R_i - R_{i+1})^2}{4\lambda\tau} \right\}$$

where $\lambda = (2m)^{-1}$

Partition Function

- For periodic paths, such that $x_i = x_j$,

$$Z = \sum_n e^{-\beta E_n} = \mathsf{Tr}[e^{\beta \hat{H}}] = \mathsf{Tr}[\hat{\rho}]$$

- The expectation value of the operator $\mathcal{O}$ is

$$\langle \mathcal{O} \rangle = \frac{\mathsf{Tr}[\mathcal{O}\hat{\rho}]}{\mathsf{Tr}[\hat{\rho}]} = \frac{\mathsf{Tr}[\mathcal{O}\hat{\rho}]}{Z}$$

- For N bosons in the primitive approximation,

$$Z = \frac{1}{N!(4\pi\lambda\tau)^{3N/2}} \prod_{i=1}^{M-1} \prod_{n=1}^{N} \sum_P \int dx_{i,n} \times \exp\left\{ -\frac{(x_{i,n} - x_{P,i+1,n})^2}{4\lambda\tau} - \tau V(x_{i,n}) \right\}$$

Thermodynamic Energy

- The general partition function is

$$\mathcal{Z} = \frac{1}{N!}(4\pi\lambda\tau)^{-3NM/2}\int \mathcal{D}R\exp\left\{-\tau\left[\frac{(R_i - R_{i+1})^2}{4\lambda\tau^2} + V\right]\right\}$$

- The thermodynamic estimate for the energy is

$$\begin{aligned}\langle E\rangle &= \frac{1}{\mathcal{Z}}\frac{\partial}{\partial\beta}\mathcal{Z} \\ &= \left\langle \frac{3N}{2\tau} - \frac{1}{M}\sum_{i=0}^{M-1}\frac{(R_i - R_{i+1})^2}{4\lambda\tau^2} + V + \tau\frac{\partial}{\partial\tau}V\right\rangle\end{aligned}$$

  - The last potential term vanishes in the primitive approximation.

Finding T_c: Permutation Cycles

As done by D'Alessandro, et al. (2010) on the lattice, we can use permutation cycles to study the transition temperature in PIMC

- The partition function of a *non-interacting* ideal gas of bosons can be broken up into a product of contributions of k-cycles – for example, the permutation $(1, 2, 3) \rightarrow (2, 3, 1)$ is considered a 3-cycle

$$Z = \frac{1}{N!} \sum_{P} \prod_{k} z_k^{n_k} ,$$

  where n_k is the number of k-cycles present in the system

- Expanding these contributions,

$$z_k(T) = \int \mathrm{d}y_1 .. \mathrm{d}y_k \, \langle y_2, y_3, ..., y_k, y_1 | e^{-\beta \hat{H}} | y_1, y_2, ..., y_k \rangle$$

$$= \int \mathrm{d}y_1 \, \langle y_1 | e^{-k\beta \hat{H}} | y_1 \rangle \equiv z_1(T/k)$$

- Using the partition function for a non-relativistic free particle in a box,

$$z_k(T) = \frac{V}{\lambda_B^3 k^{3/2}} \,,$$

where λ_B is the thermal de Broglie wavelength and V is the volume of the box

- The total partition function is

$$Z = \frac{1}{N!} \sum_P \sum_k \left(\frac{V}{\lambda_B^3 k^{3/2}} \right)^{n_k}$$

- This quantity is not easily computed for fixed particle number, but this problem is avoided if we go to the Grand Canonical ensemble,

$$\mathcal{Z} = \prod_k \left(\frac{V e^{\mu k/T}}{\lambda_B^3 k^{5/2}} \right)$$

- From this partition function, we can extract the density of k-cycles

$$\rho_k \equiv \frac{\langle n_k \rangle}{V} = \frac{e^{\mu k/T}}{\lambda_B^3 k^{5/2}}$$

- The total particle density is

$$\frac{N}{V} = \sum_k k\rho_k = \sum_k \frac{e^{\mu k/T}}{\lambda_B^3 k^{5/2}}\,,$$

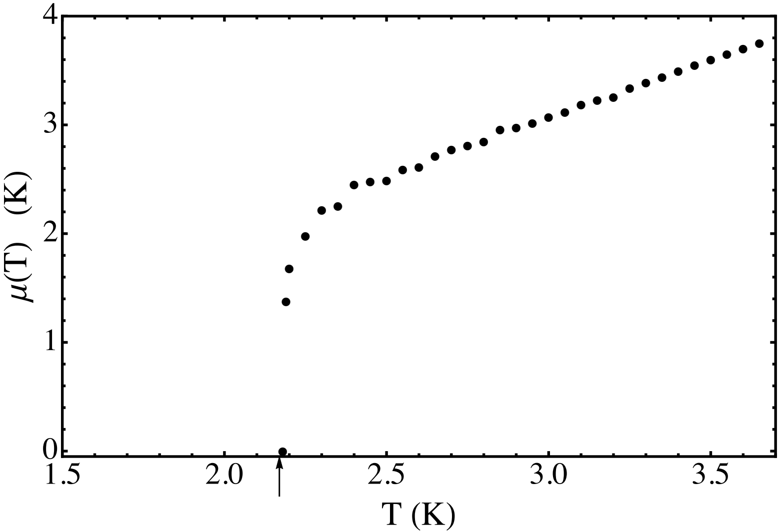
which has an upper limit of $\mu = 0$. This approach is fully valid for any non-interacting gas above T_c

- Even though we assumed non-interaction, we expect the densities of cycles to go roughly as

$$\rho_k = e^{-\hat{\mu}k} f(k)\,,$$

where $\hat{\mu} = -\mu/T$ and $f(k)$ is some decreasing function of k of the form $f(k) \sim 1/k^{\alpha}$

Finding T_c, Test of Method 1: Liquid ^{4}He



1% accuracy to experimentally-determined T_c!