Bose kinetics out of thermal equilibrium

Kai Zhou[1,2], Zhe Xu[2], Pengfei Zhuang[2] and Carsten Greiner[1]

[1]Institut für Theoretische Physik, Goethe-Universität Frankfurt, Max-von-Laue-Straße 1, 60438 Frankfurt am Main, Germany
[2]Department of Physics, Tsinghua University, Beijing 100084, China

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Motivation

(1) In HIC, fast thermalization remains challenging.
(2) CGC $\rightarrow$ over-populated Glasma: contains more gluons than can be accommodated by a Bose distribution.
(3) BEC formation? Lots of studies lately, like classical-field theory or Fokker-Planck approach. Kinetic simulation?
(4) Turbulent scaling solution? Self-similarity?

Bose statistics in kinetic simulation

Consider a homogeneous massless boson system,

$$\left(\frac{\partial}{\partial t} + \frac{\mathbf{p}_1}{E_1}\frac{\partial}{\partial \mathbf{r}}\right) f_1 = \frac{1}{2E_1}\int d\Gamma_2 \frac{1}{2}\int d\Gamma_3 d\Gamma_4 |\mathcal{M}_{34\to12}|^2 \times [f_3f_4(1+f_1)(1+f_2) - f_1f_2(1+f_3)(1+f_4)] \times (2\pi)^4\delta^{(4)}(p_3+p_4-p_1-p_2)\,, \quad (1)$$

where $f_i = f_i(\mathbf{r},\mathbf{p}_i,t)$ and $d\Gamma_i = d^3p_i/(2E_i)/(2\pi)^3$.
Define an effective cross section involving the Bose factors

$$\sigma_{22}^{eff} = \frac{1}{4s}\int d\Gamma_1 d\Gamma_2 |\mathcal{M}_{34\to12}|^2(1+f_1)(1+f_2) \times (2\pi)^4\delta^{(4)}(p_1+p_2-p_3-p_4)\,, \quad (2)$$

we can obtain the collision probability [1],

$$P_{22} = \frac{\Delta N_{coll}}{\Delta N_3 \Delta N_4} = v_{rel}\frac{\sigma_{22}^{eff}}{N_{\text{test}}}\frac{\Delta t}{\Delta^3 x}\,, \quad (3)$$

where $v_{rel} = s/2E_3E_4$ denotes the relative velocity.
To avoid numerically integrating σ_{22}^{eff} we introduce a new scheme where we define the differential collision probability,

$$\frac{dP_{22}}{d\Omega^*} = \frac{v_{rel}}{N_{\text{test}}}\frac{d\sigma_{22}}{d\Omega^*}(1+f_1)(1+f_2)\frac{\Delta t}{\Delta V}\,. \quad (4)$$

Differential stochastic sampling [2] :

(1) Choose a normalized reference function $dF/d\Omega^*$
(2) Sample $\tilde{\Omega}^*$ for each pair and calculate $dP_{22}/d\tilde{\Omega}^*$
(3) Sample a random number ξ within $[0, dF/d\tilde{\Omega}^*]$
$\rightarrow$ A collision occurs, if $\xi < dP_{22}/d\tilde{\Omega}^*$.
In practice, we choose $dF/d\Omega^* = (d\sigma_{22}/d\Omega^*)/\sigma_{22}$.

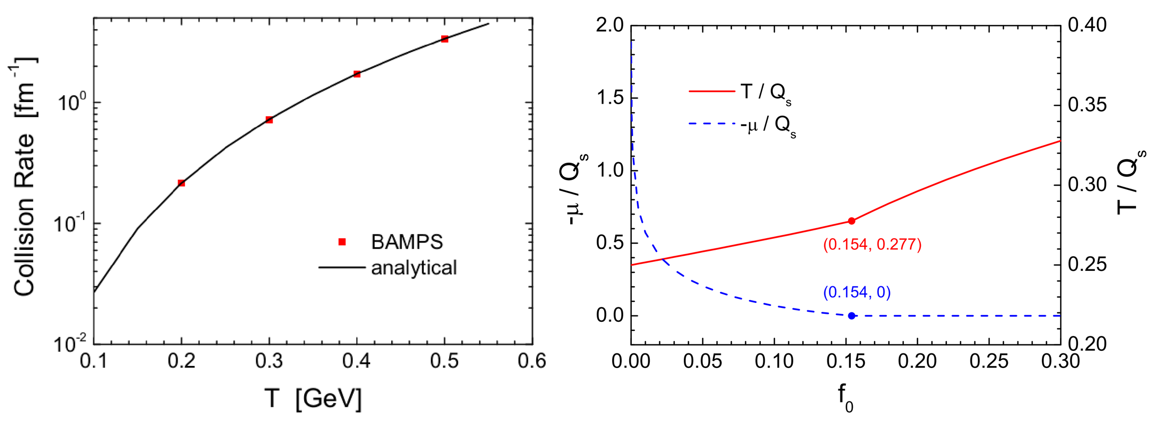

Figure 1: (left) Collision rate test in static box (size $3\,\mathrm{fm}\times3\,\mathrm{fm}\times3\,\mathrm{fm}$) with an equlibrium initial condition obeying Bose-Einstein distribution (with $\mu = 0$), istropic $\sigma = 10$ mb, $N_{\text{test}} = 1600$ and degeneracy factor $g = 1$. (right) Dependence of the equilibrium μ and T on f_0.

CGC-like initial condition : $f_{\text{init}}(\mathbf{p}) = f_0\theta(Q_s - |\mathbf{p}|)$.
Assuming binary collisions only (number conservation),
$f_0 < f_0^c = 0.154$: under-populated, no condensation
$f_0 > f_0^c = 0.154$: over-populated, with condensation

Under-populated case

No BEC will appear. The final T^{eq} and μ^{eq} can be evaluated through number and energy conservation, see Fig.1 (right). During the simulation, f is calculated at a set of equidistant momenta p_i with $\Delta p = 2.5$ MeV beginning from 5 MeV by counting test particle numbers within $[p_i - \Delta p/2, p_i + \Delta p/2]$. Interpolation (for $p > 7.5$ MeV and $p \neq p_i$) and extrapolation (for $p < 7.5$ MeV) using power-law fit (at first 10 p_i from 7.5 MeV) are used to get value of f.

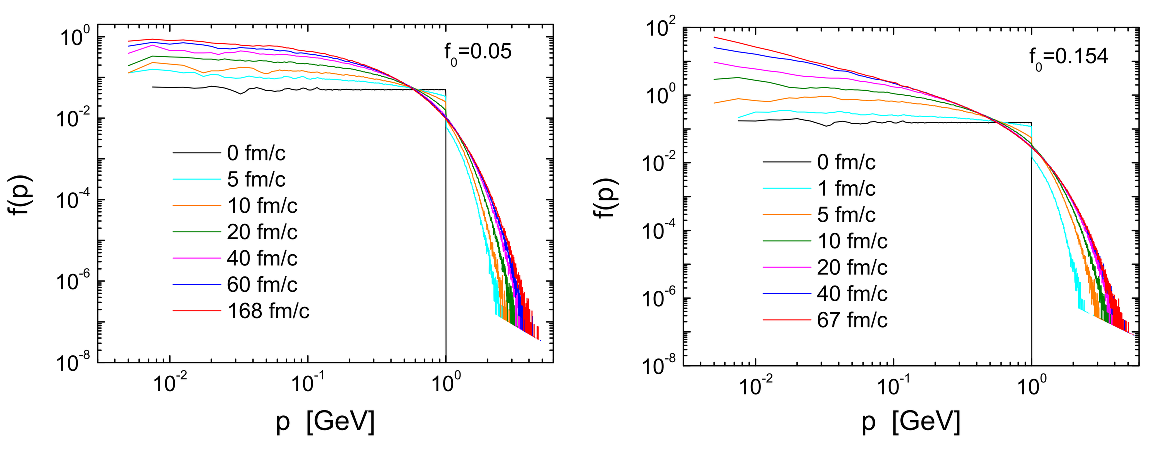

Figure 2: Time evolution of f for underpopulated cases. For $f_0 = 0.05$ (left), $N_{\text{test}} = 691000$. For $f_0 = 0.154$ (right), $N_{\text{test}} = 230000$.

Perfect agreements: simulation and analytical results

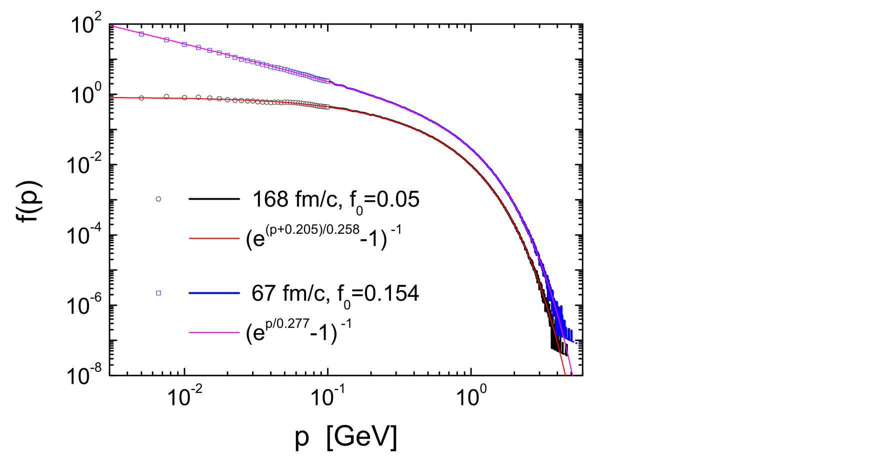

Figure 3: Comparisions between simulation with analytical f^{eq}.

$\rightarrow$ correct implementation of Bose statistics in simulation.
Note: shorter equlibration time for larger f_0; and for $f_0 = f_0^c$ the distribution f^{eq} diverges at $p \rightarrow 0$ ($\sim 1/p$).

Over-populated case

HTL modified [3] pQCD LO matrix element is used,

$$|\mathcal{M}_{gg\to gg}|^2 \approx 144\pi^2\alpha_s^2\frac{s^2}{t(t-m_D^2)} \quad (5)$$

screening mass $m_D^2 = 16\pi N_c\alpha_s\int\frac{d^3p}{(2\pi)^3}\frac{1}{p}f$, coupling $\alpha_s = 0.3$.

Before onset :

Differential stochastic sampling (described above) is used. Transportation of Energy and Particles from around $p \sim Q_s$ to infrared and ultraviolet region. (flow in p space)

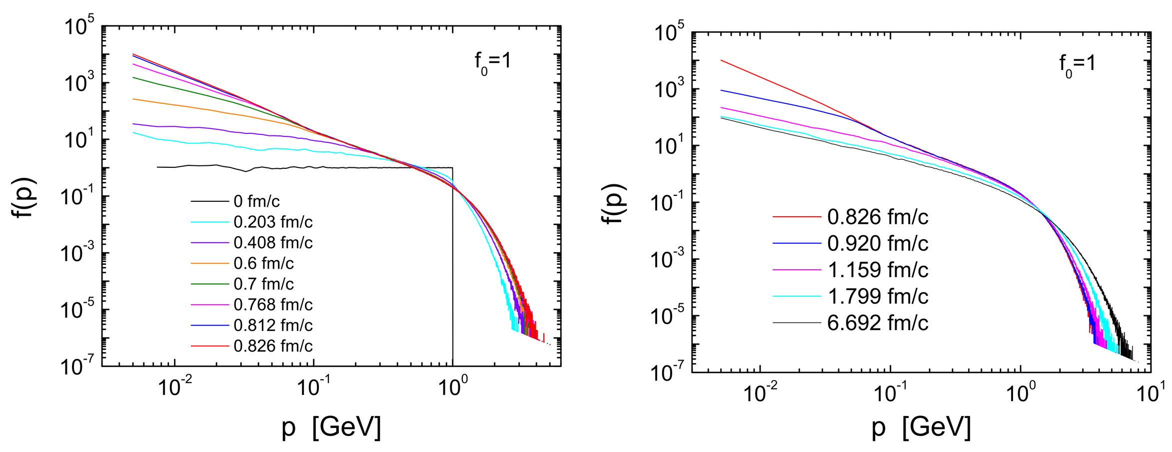

Figure 4: Time evolution of f before (left) and after (right) the onset of BEC, with $N_{\text{test}} = 2400$ and $g = 16$.

After onset :

($f \rightarrow 1/p^2$ in very infrared) @ $t_c = 0.826fm/c$
Decompose : $f = f^g + f_c$ (with $f_c = (2\pi)^3 n_c\delta^3(\mathbf{p})$) $\rightarrow$ equations for $g+g \rightarrow g+g$ and $g+c \rightarrow g+g$, and also

$$\frac{\partial f_1^c}{\partial t} = \frac{1}{2E_1}\int d\Gamma_2\frac{1}{2}\int d\Gamma_3 d\Gamma_4 |\mathcal{M}_{34\to12}|^2(2\pi)^4\delta^{(4)}(\ldots) \times [f_3^g f_4^g f_1^c(1+f_2^g) - f_1^c f_2^g(1+f_3^g)(1+f_4^g)]\,.$$

Integrate over the condensate momenta $\mathbf{p}_1$ we obtain

$$\frac{\partial n_c}{\partial t} = \frac{n_c}{64\pi^3}\int dE_3 dE_4\,[f_3f_4 - f_2(1+f_3+f_4)] \times E\left[\frac{|\mathcal{M}_{34\to12}|^2}{s}\right]_{s=2mE} \quad (6)$$

where $E = E_3 + E_4$, $p = |\mathbf{p}_3 + \mathbf{p}_4|$, and $s = E^2 - p^2$. For massless bonsons, a finite condensation rate needs finite ratio $|\mathcal{M}_{34\to12}|^2/s$ at $s = 0$, which is fulfilled for our choice

$$\left[\frac{|\mathcal{M}_{34\to12}|^2}{s}\right]_{s=0} = 144\pi^2\alpha_s^2\left[\frac{s}{t(t-m_D^2)}\right]_{s=0} = 144\pi^2\alpha_s^2\frac{1}{m_D^2}$$

(1) $g+g \rightarrow g+c$:
Approximation : $\delta^{(3)}(\mathbf{p}_1) \approx \frac{\theta(\varepsilon-p_1)}{4\pi p_1^2\varepsilon}$ and $n_c(t_c) = n_{E<\varepsilon}(t_c)$
(we set $\varepsilon = 2.5$ MeV, the collision rate is unchanged)$\rightarrow$

$$\sigma_c^{eff} = \pi n_c\left[\frac{|\mathcal{M}_{34\to12}|^2}{s}\right]_{s=0}\frac{1}{2p}[1+f^g(p)]\frac{1}{4\varepsilon} \times\left(\frac{2}{E-p} - \frac{1}{\min\{\varepsilon,(E+p)/2\}}\right)\theta\left(\varepsilon - \frac{E-p}{2}\right) \quad (7)$$

$\rightarrow P_{g+g\to g+c}$ with Eq.(3) $\rightarrow$ standard stochastic method [1].
(2) $c+g \rightarrow g+g$, same as $g+g \rightarrow g+g$:
Differential stochastic sampling. Contribution proportional to $f_4^g f_3^g f_2^g$ cancelled with condensation processes in (1).

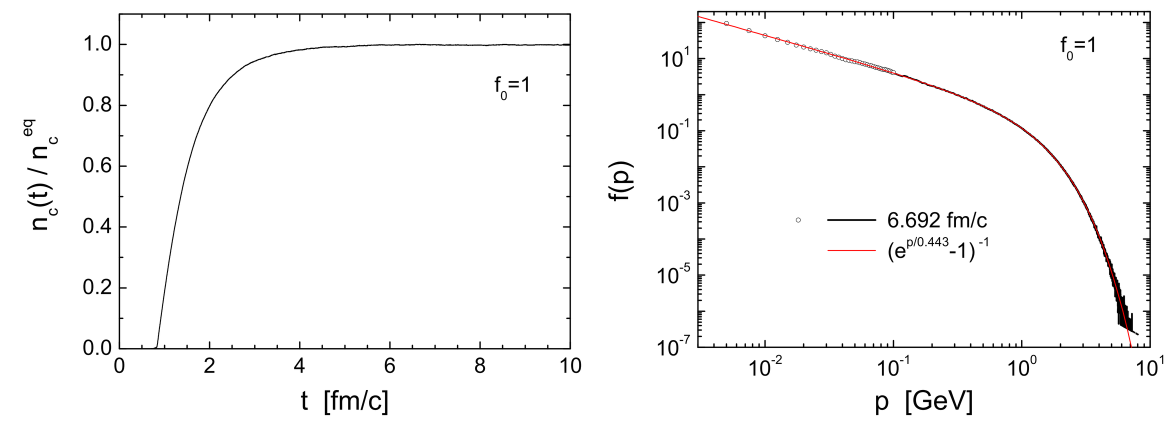

Figure 5: (left) Condensation growth and (right) Time evolution of f.

(Note: before t_c, about 1% of n_c generated via $g+g \rightarrow g+g$).

Scaling analysis

There exists a stationary turbulence region [4], where f is in a scaling invariant power-law form: $f(p) \sim p^{-r}$.
Follow derivations in Ref. [4] we obtain for $|\mathcal{M}|^2$ in Eq.(5), $r = 2$ for particle cascade and $r = 7/3$ for energy cascade.

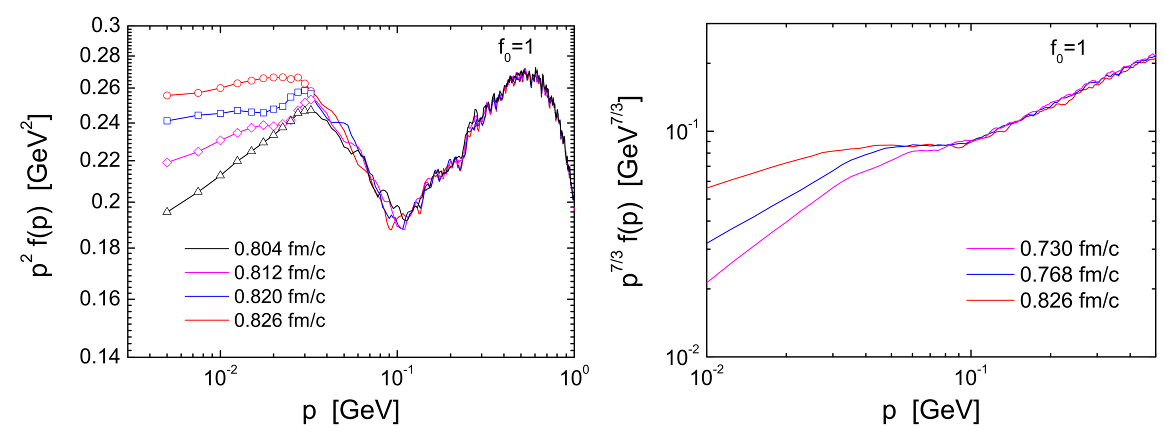

Figure 6: Depiction of $r = 2$ (left) and $r = 7/3$ (right) power-law form of the distribution. Plateau of $p^r f(p)$ implies the power-law form of f.

Besides, a power-law scaling with $r = 7/4$ is also observed following the energy cascade region:

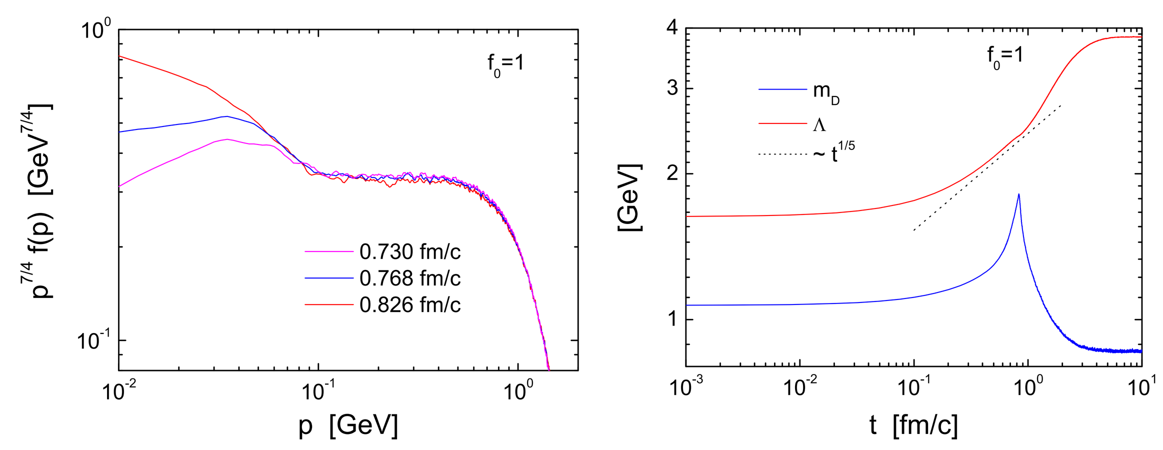

Figure 7: (left) Depiction of r=7/4 power-law form of distribution and (right) Time evolution of hard scale Λ and Debye screening mass m_D.

The hard momentum scale which is defined as

$$\Lambda^2(t) = \frac{\int d\Gamma 4p^2 E f^g(\mathbf{p},t)}{\int d\Gamma E f^g(\mathbf{p},t)},$$

would show scaling behavior $\Lambda(t) \sim t^\beta$ for self-similar solution [4] with $f^g(\mathbf{p},t) = t^\alpha f_S(t^\beta\mathbf{p})$.
For $|M|^2$ in Eq.(5), we derived: $\alpha = -4/5$ and $\beta = -1/5$, and is confirmed by numerical simulation in Fig.(5).

Conclusions

(1) A differential stochastic method is presented to solve Boltzmann equations for bosons. Rate tested.
(2) Kinetic simulation for dynamics of non-equilibrium boson system in under- and over-populated case.
(3) Dynamics beyond onset of BEC in over-populated case is studied kinetically.
(4) Turbulent scaling power-law form of distribution and self-similarity is analyzed from simulation.

References

[1] Z. Xu and C. Greiner, Phys. Rev. C **71** (2005), arXiv:hep-ph/0406278.
[2] Z. Xu, K. Zhou, P. Zhuang, and C. Greiner, Phys. Rev. Lett. **114**, 182301 (2015), arXiv:1410.5616.
[3] P. Aurenche, F. Gelis, and H. Zaraket, JHEP **05**, 043 (2002), hep-ph/0204146.
[4] R. Micha and I. I. Tkachev, Phys. Rev. **D70**, 043538 (2004), hep-ph/0403101.

zhou@th.physik.uni-frankfurt.de