Electromagnetic Fields From Quantum Sources in Heavy-Ion Collisions

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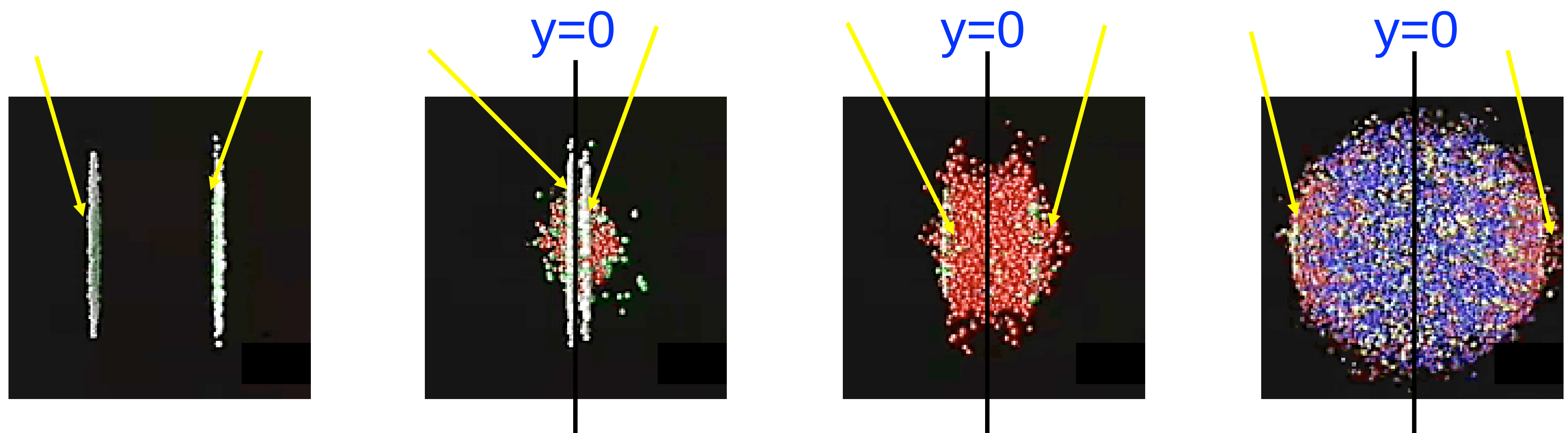
http://arxiv.org/pdf/1604.04572v2.pdf

Valence electric charges



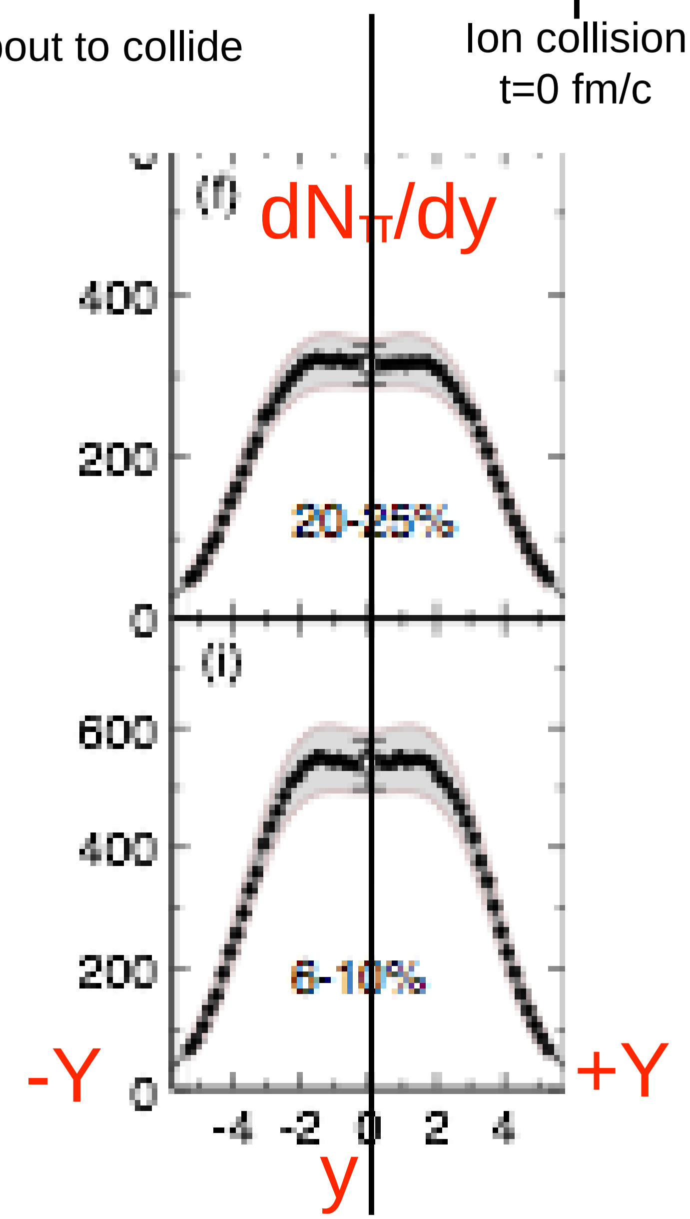
Ions about to collide

Ion collision
t=0 fm/c

Quarks, gluons freed
t=0.2 fm/c

Plasma created
t=0.5-1 fm/c

Plasma decays
t~10 fm/c



Most of the plasma is formed around rapidity y=0

Number of valence quarks in plasma decreases with energy: “baryon stopping”.

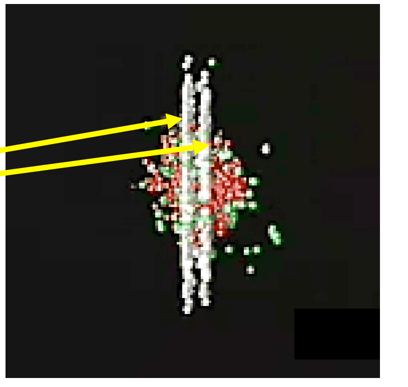
Therefore, we can consider fields within the plasma created by external sources.

EM fields in Quark Gluon Plasma

$$\nabla \times E = -\frac{\partial B}{\partial t} \qquad \nabla \cdot B = 0$$

$$\nabla \times B = \frac{\partial E}{\partial t} + J \qquad \nabla \cdot E = \rho = 0$$

$$\boldsymbol{J} = \sigma(\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B}) + \boldsymbol{J_{ext}}$$



$$\frac{\partial \boldsymbol{B}}{\partial t} = \nabla \times (\boldsymbol{v} \times \boldsymbol{B}) + \frac{1}{\sigma}(\nabla^2 \boldsymbol{B} - \frac{\partial^2 \boldsymbol{B}}{\partial t^2} + \nabla \times \boldsymbol{J_{ext}})$$

Lattice calculations for $T \sim T_c$: $\sigma = 5.8$ MeV, very small compared to the typical QCD scale of 200 MeV. We neglect the terms on the left hand side. This means we neglect the effect of the plasma.

We are aware that these terms cannot be neglected in general; however, this is not the focus of our talk.

Similarly we ignore initial conditions.

We want to focus on the difference between the classical and quantum sources.

Main Idea

- Unlike all present calculations, we treat the sources as wave packets, not as point particles.
- The main difference between the point charges and the wave packets is that the wave packets diffuse as required by the uncertainty principle.
- The difference between the magnetic field created by the point charges (“the classical sources”) and the wave packets (“the quantum sources”) is the main subject of this work.
- We consider the field created by a single charge

Non-relativistic Approximation

Want to treat diffusion of the wave packet non-relativistically so that we can solve for the currents analytically

$$\psi(\boldsymbol{r}_0, 0) = \frac{1}{\pi^{3/4} a^{3/2}} e^{-\frac{r_0^2}{2a^2}}$$

Model wave function as Gaussian of width a=1fm

$$\psi(\boldsymbol{r}_0, t_0) = \int \frac{d^3k}{(2\pi\hbar)^{3/2}} e^{\frac{i\boldsymbol{k}\cdot\boldsymbol{r}_0}{\hbar}} e^{-\frac{ik^2 t_0}{2m\hbar}} \psi_{\boldsymbol{k}}(0) = \frac{1}{\pi^{3/4}(a + i\hbar t_0/ma)^{3/2}} e^{-\frac{r_0^2}{2(a^2 + i\hbar t_0/m)}}$$

Calculate the charge and current densities and apply Galilean transformation

$$\rho(\boldsymbol{r}, t) = \frac{e}{\pi^{3/2}(a^2 + (\lambda t/a)^2)^{3/2}} e^{-\frac{b^2 + (z - vt)^2}{a^2 + (\lambda t/a)^2}}$$

To return to classical point particles take $\lambda = \hbar/m \to 0$ Then take $a \to 0$

Neither of these limits are good approximations in our case.
Because λ and a are of the same magnitude, the diffusion rate is too fast to be treated non-relativistically, so we must treat the diffusion relativistically.

Method

Model wave function as a Gaussian satisfying Klein-Gordon equation

$$\Psi(t_0, \boldsymbol{r_0}) = \int \frac{d^3k}{(2\pi\hbar)^{3/2}} e^{i\boldsymbol{k}.\boldsymbol{r_0}} e^{\frac{i\varepsilon_k t_0}{\hbar}} \sqrt{\tfrac{m}{\varepsilon_k}} \psi_{\boldsymbol{k}}(0)$$

Calculate current and charge density in the charge rest frame.

$$\boldsymbol{j}(t_0, \boldsymbol{r_0}) = -\frac{ei\hbar}{2m}[\Psi^* \nabla \Psi - (\nabla \Psi^*)\Psi]$$

$$\rho(t_0, \boldsymbol{r_0}) = \frac{ei\hbar}{2m}[\Psi^* \partial_t \Psi - (\partial_t \Psi^*)\Psi]$$

Use ρ and j to find retarded vector and scalar potentials

$$\boldsymbol{A}(t_0, \boldsymbol{r_0}) = \frac{1}{4\pi} \int \frac{\boldsymbol{j}(t_R, \boldsymbol{r}')d^3r'}{|\boldsymbol{r_0} - \boldsymbol{r}'|}$$

$$\varphi(t_0, \boldsymbol{r_0}) = \frac{1}{4\pi} \int \frac{\rho(t_R, \boldsymbol{r}')d^3r'}{|\boldsymbol{r_0} - \boldsymbol{r}'|}$$

Find fields and boost

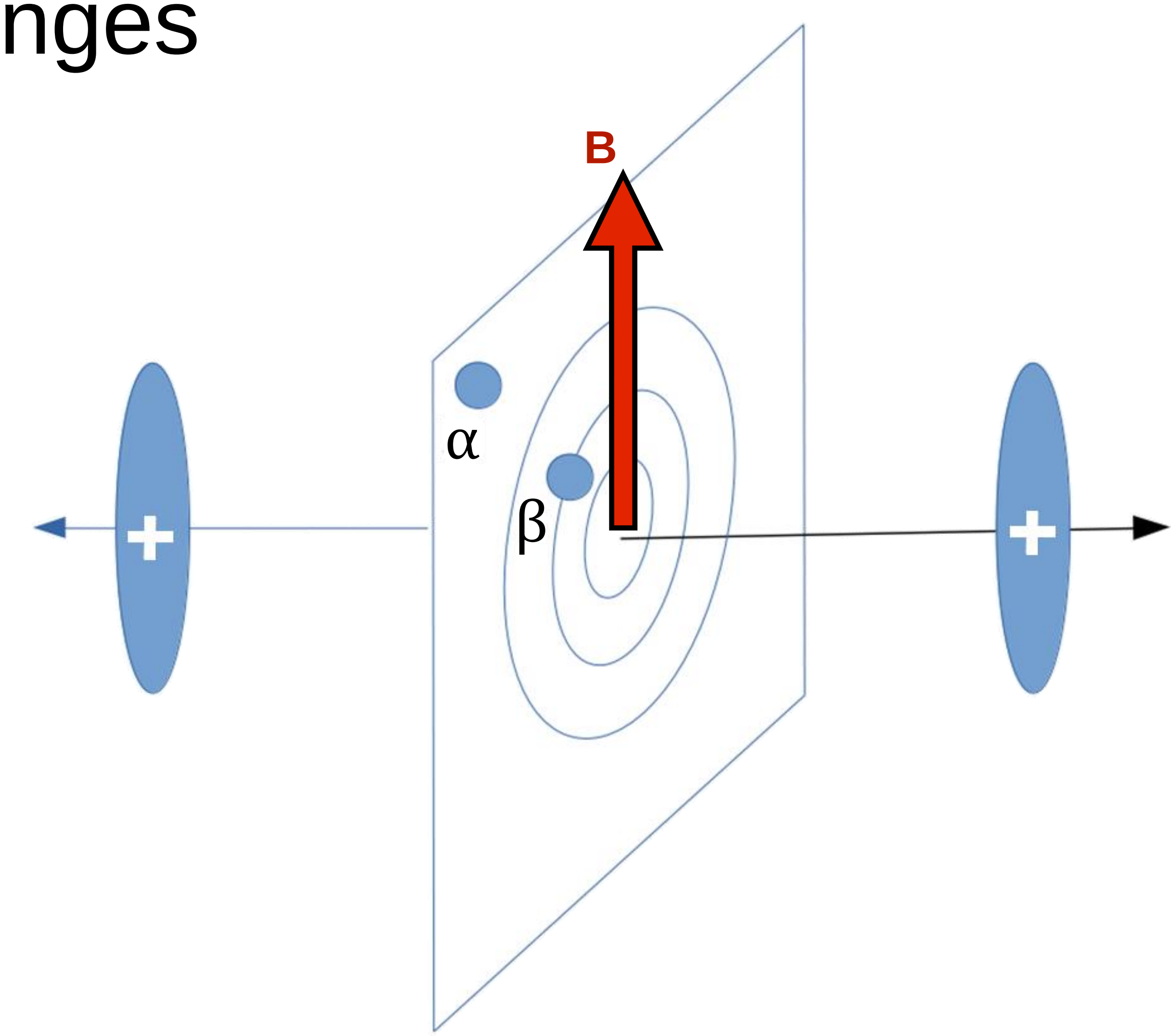
$$\boldsymbol{E}_{||} = \frac{\gamma(z - vt)}{\sqrt{\xi^2 + \gamma^2(z - vt)^2}} E_r(\gamma(t - vz), \sqrt{\xi^2 + \gamma^2(z - vt)^2})\hat{\boldsymbol{z}}$$

$$\boldsymbol{B} = \frac{\gamma v \xi}{\sqrt{\xi^2 + \gamma^2(z - vt)^2}} E_r(\gamma(t - vz), \sqrt{\xi^2 + \gamma^2(z - vt)^2})\hat{\boldsymbol{\theta}}$$

Fields

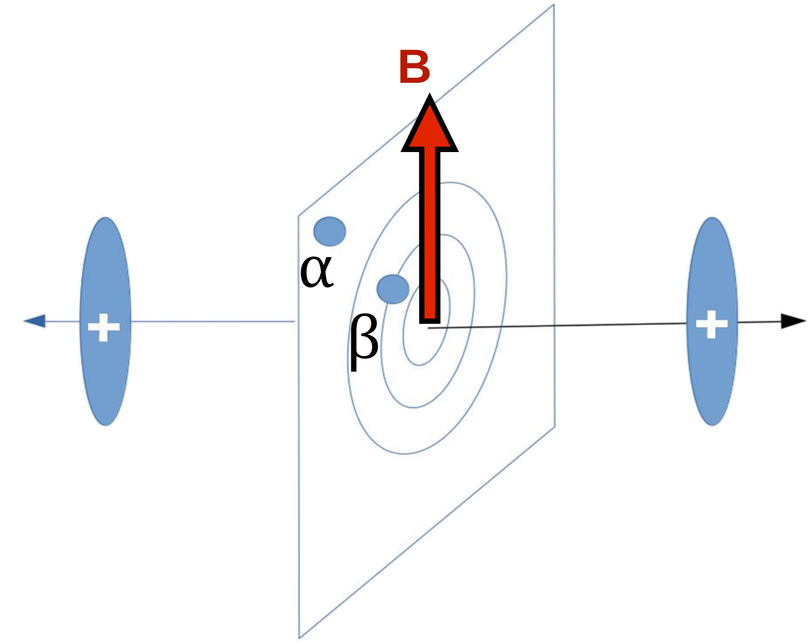
Magnitude of fields from quantum sources significantly different from classical

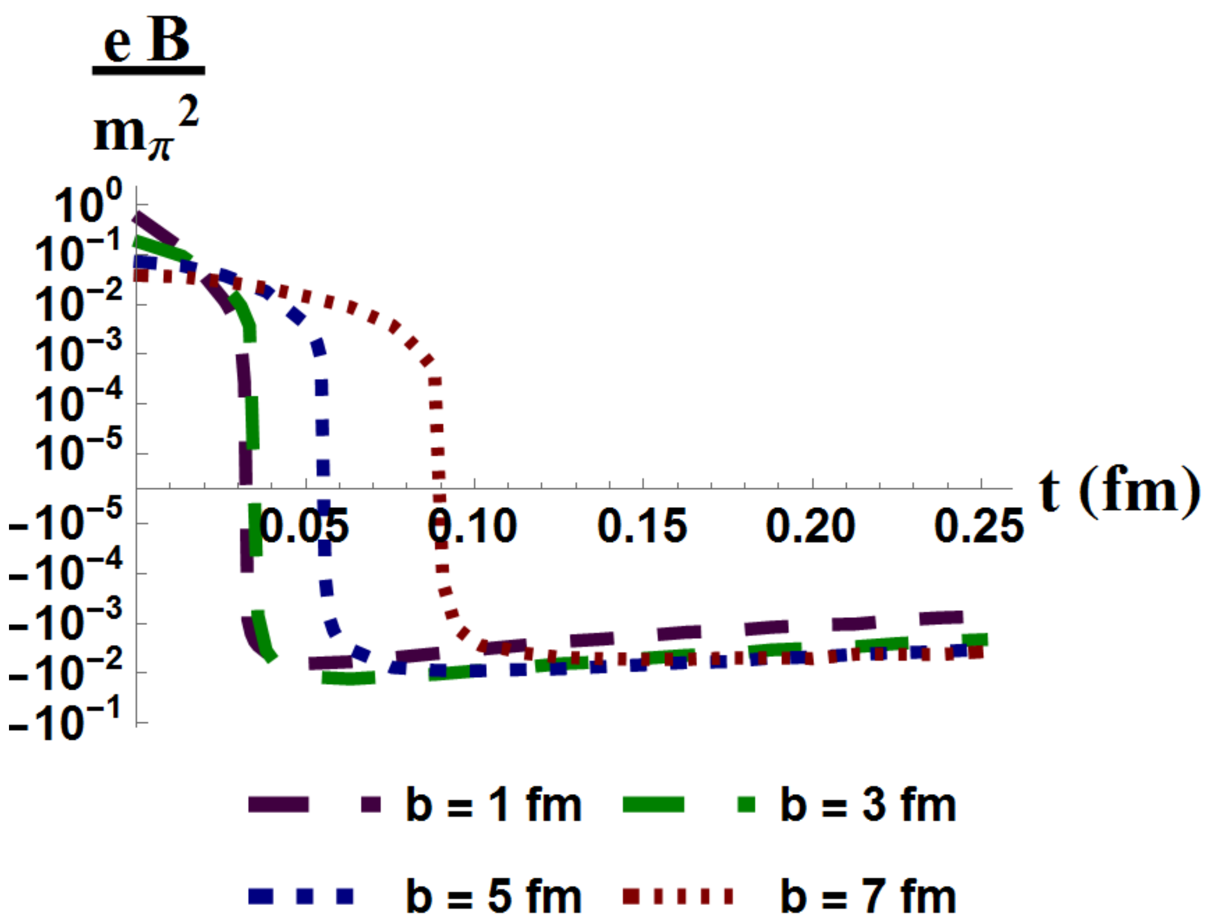
Under certain conditions sign of fields from quantum sources changes

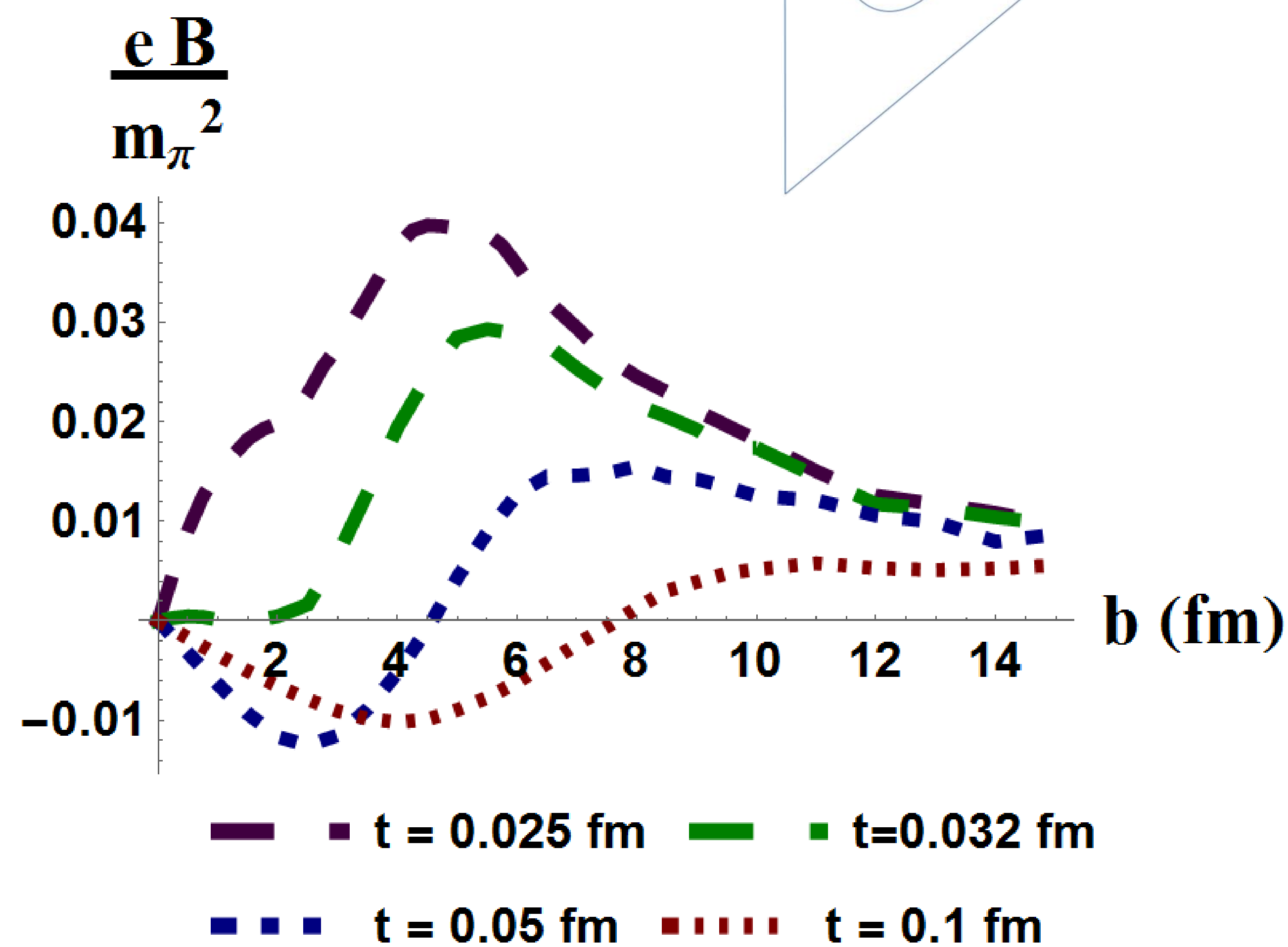


Results

$$\boldsymbol{E}(\boldsymbol{r},t)=\int\left\{\frac{\rho(\boldsymbol{r}',t')\boldsymbol{R}}{R^3}+\frac{\boldsymbol{R}}{R^2}\frac{\partial\rho(\boldsymbol{r}',t')}{\partial t'}-\frac{1}{R}\frac{\partial\boldsymbol{j}(\boldsymbol{r}',t')}{\partial t'}\right\}$$





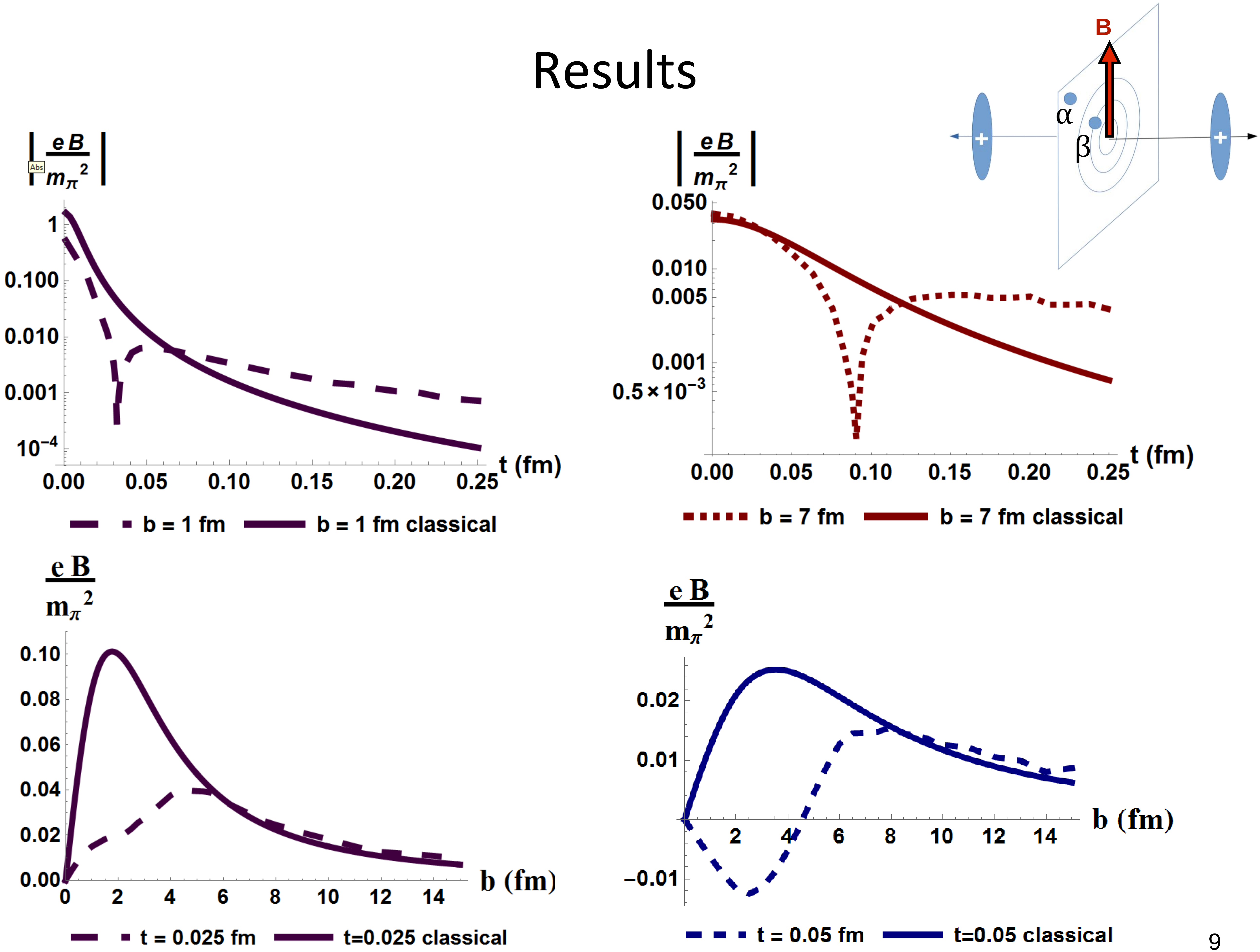


Here we plot the fields in the lab frame.

In the rest frame, if the charge were point-like, the magnetic field would vanish and the electric field would be represented by a Coulomb field.

However, due to the quantum diffusion B is not zero even in the rest frame.

Results



B
α
β
$\left|\frac{eB}{m_\pi^2}\right|$
Abs
1
0.100
0.010
0.001
10^{-4}
0.00
0.05
0.10
0.15
0.20
0.25
t (fm)
b = 1 fm
b = 1 fm classical
$\left|\frac{eB}{m_\pi^2}\right|$
0.050
0.010
0.005
0.001
0.5×10^{-3}
0.00
0.05
0.10
0.15
0.20
0.25
t (fm)
b = 7 fm
b = 7 fm classical
$\frac{eB}{m_\pi^2}$
0.10
0.08
0.06
0.04
0.02
0.00
0
2
4
6
8
10
12
14
b (fm)
t = 0.025 fm
t=0.025 classical
$\frac{eB}{m_\pi^2}$
0.02
0.01
−0.01
2
4
6
8
10
12
14
b (fm)
t = 0.05 fm
t=0.05 classical

Taking Spin Into Account

Solution to Dirac equation

$$\Psi(\boldsymbol{r},t) = \frac{1}{\sqrt{2}}\sum_{\lambda}\int \frac{d^3k}{(2\pi)^{3/2}} e^{i\boldsymbol{k}\cdot\boldsymbol{r}} e^{-i\varepsilon_k t}\psi_{\boldsymbol{k}}(0)u_{\boldsymbol{k}\lambda}$$

$$u_{\boldsymbol{k}+} = \sqrt{\frac{\varepsilon_k + m}{2\varepsilon_k}}\begin{pmatrix}\chi_+ \\ \frac{\boldsymbol{\sigma}\cdot\boldsymbol{k}}{\varepsilon_k+m}\chi_+\end{pmatrix}, \quad u_{\boldsymbol{k}-} = \sqrt{\frac{\varepsilon_k + m}{2\varepsilon_k}}\begin{pmatrix}\chi_- \\ \frac{\boldsymbol{\sigma}\cdot\boldsymbol{k}}{\varepsilon_k+m}\chi_-\end{pmatrix}$$

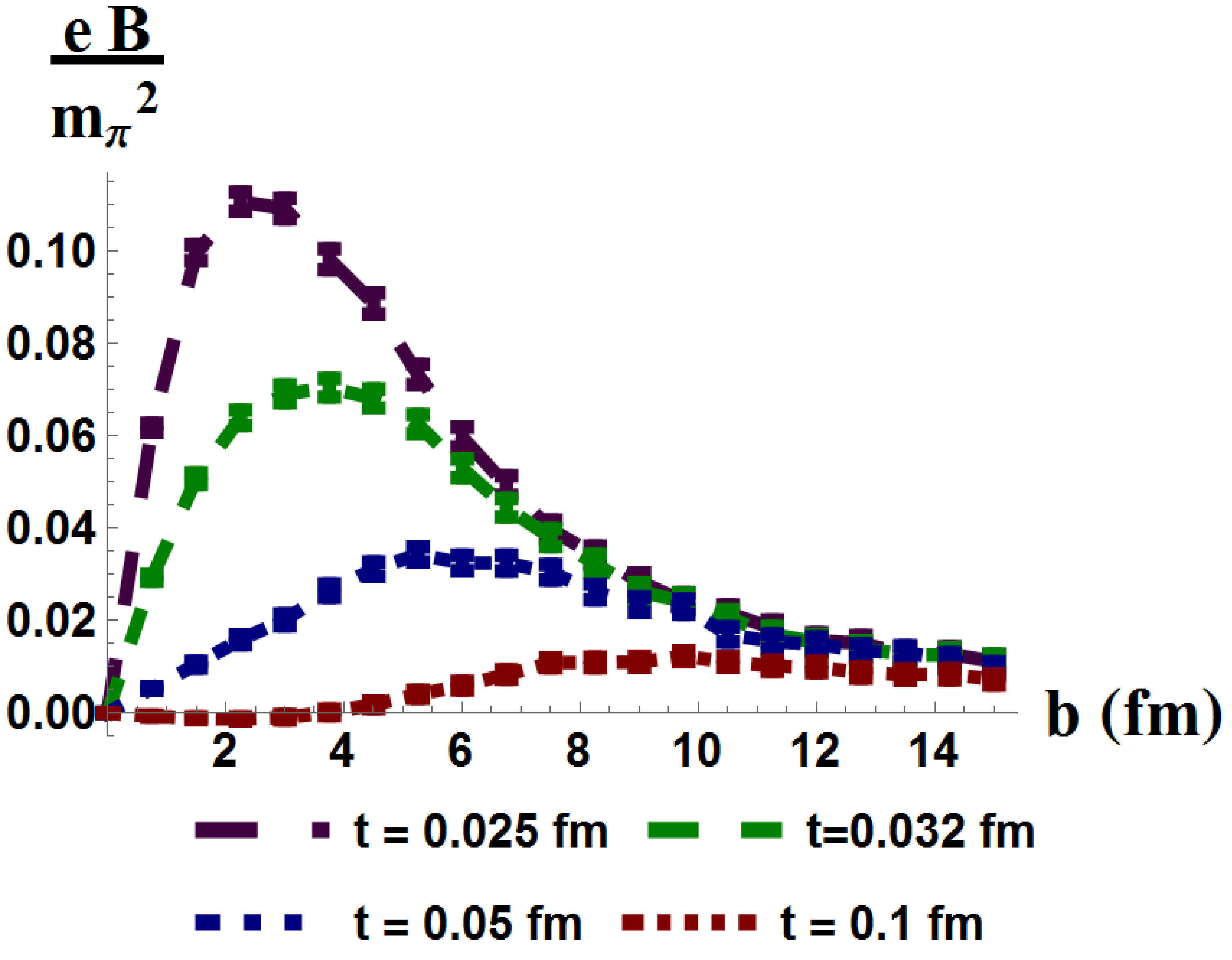
Note that χ are helicity eigenstates

$$\boldsymbol{j} = e\Psi^\dagger\boldsymbol{\alpha}\Psi$$

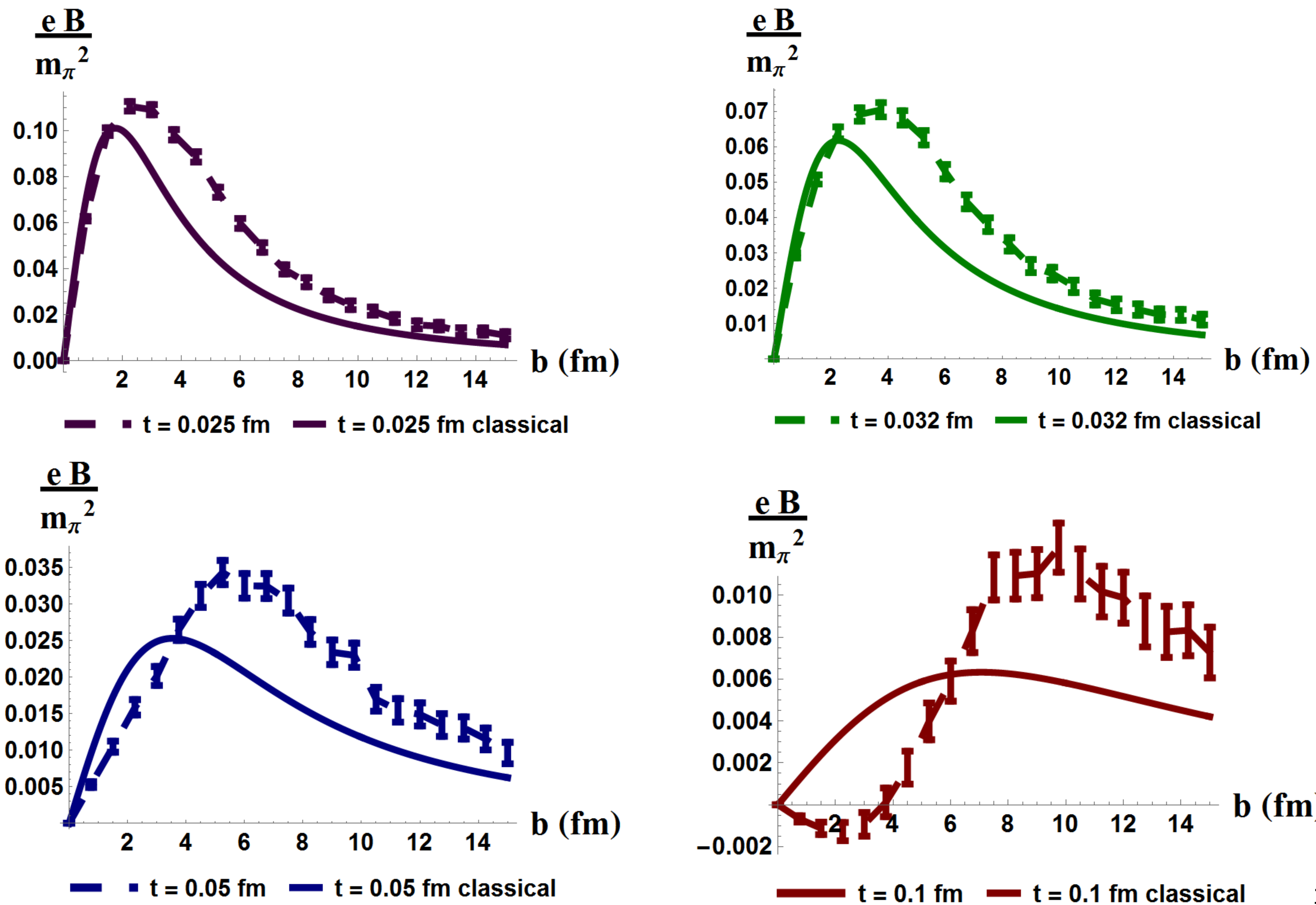
$$\rho = e\Psi^\dagger\Psi$$

$$\boldsymbol{E}(\boldsymbol{r},t) = \int\left\{\frac{\rho(\boldsymbol{r}',t')\boldsymbol{R}}{R^3} + \frac{\boldsymbol{R}}{R^2}\frac{\partial\rho(\boldsymbol{r}',t')}{\partial t'} - \frac{1}{R}\frac{\partial\boldsymbol{j}(\boldsymbol{r}',t')}{\partial t'}\right\}$$

Preliminary Spin results



Preliminary Spin results



Convective and Spin Currents

Using the Gordon Identity

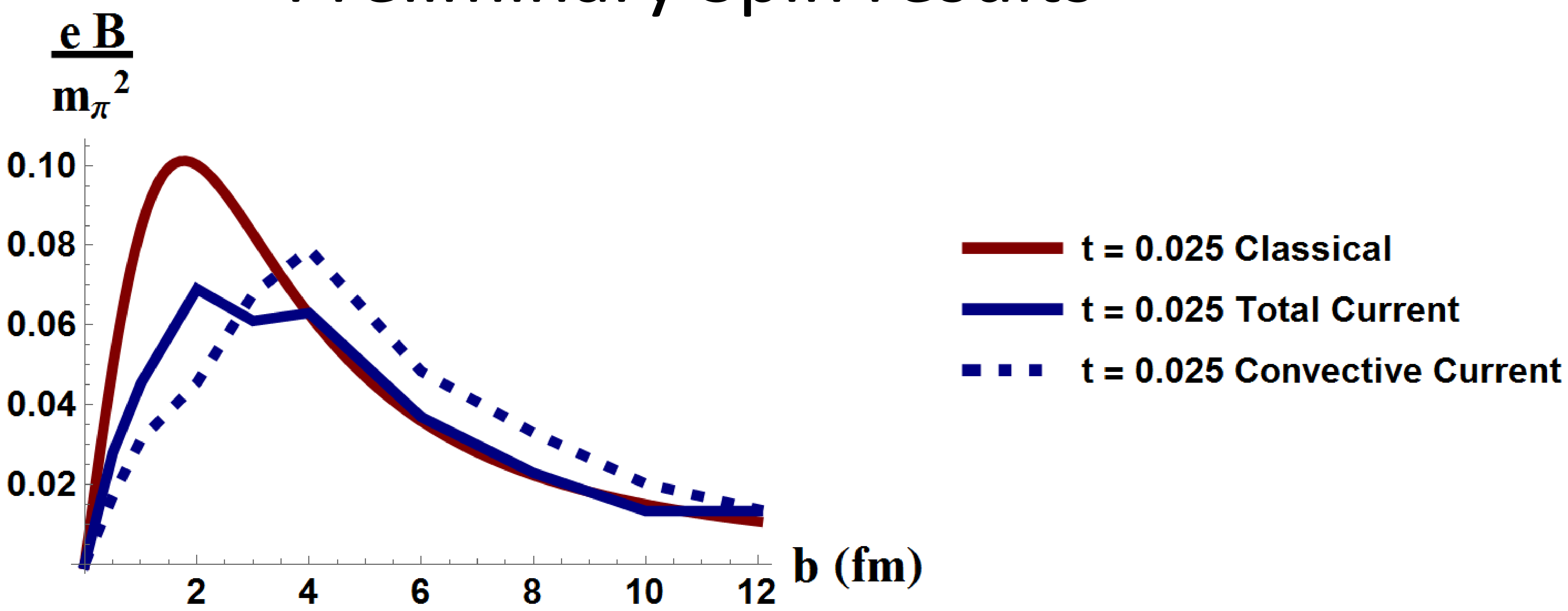
$$\bar{\Psi}_2\gamma^\mu\Psi_1 = \frac{1}{2m}\left[\bar{\Psi}_2 i\partial^\mu\Psi_1 - (i\partial^\mu\bar{\Psi}_2)\Psi_1\right] - \frac{1}{2m}i\partial_\nu(\bar{\Psi}_2\sigma^{\mu\nu}\Psi_1)$$

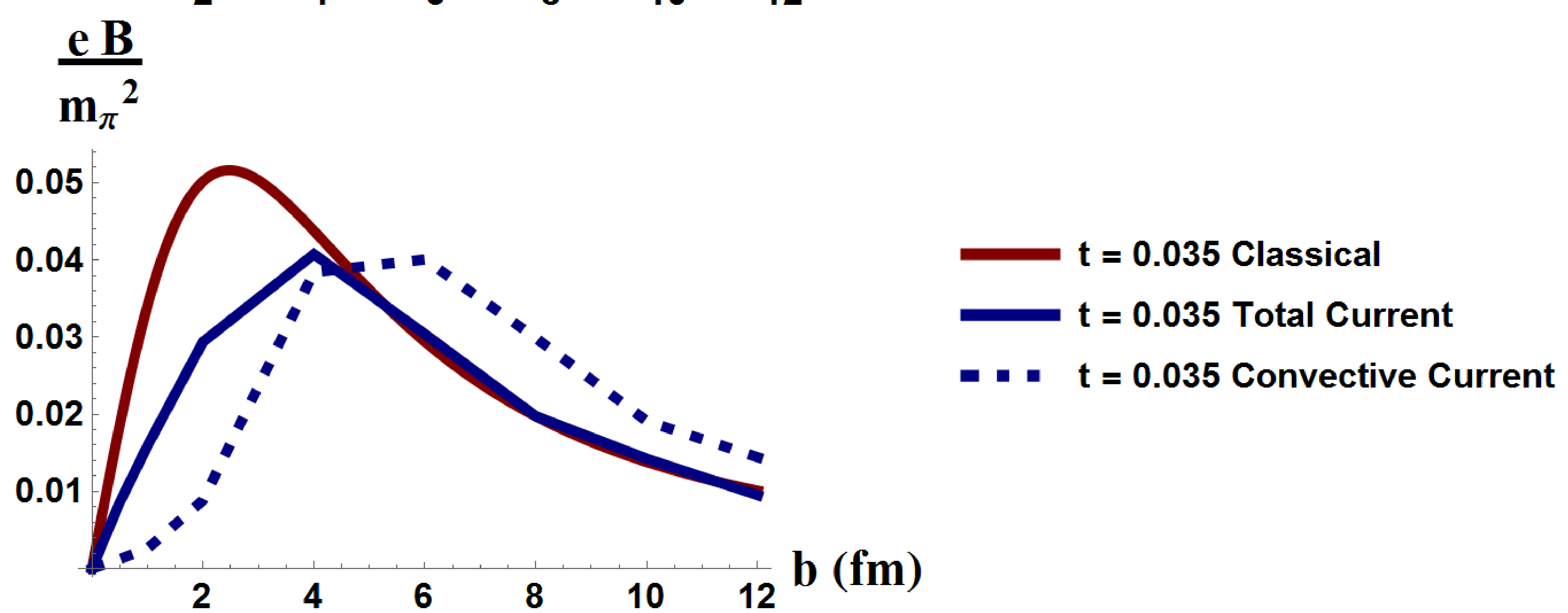
We can split the charge and current densities into convective and spin parts

$$\rho_c = \frac{ie}{2m}(\bar{\Psi}\dot{\Psi} - \dot{\bar{\Psi}}\Psi)\,, \qquad \boldsymbol{j}_c = \frac{ie}{2m}\left[(\boldsymbol{\nabla}\bar{\Psi})\Psi - \bar{\Psi}(\boldsymbol{\nabla}\Psi)\right]\,,$$

$$\rho_s = -\frac{e}{2m}\boldsymbol{\nabla}\cdot(\Psi^\dagger\boldsymbol{\Sigma}\Psi)\,, \qquad \boldsymbol{j}_s = \frac{e}{2m}\partial_t(\Psi^\dagger\boldsymbol{\Sigma}\Psi) - \frac{ie}{2m}\boldsymbol{\nabla}\times(\Psi^\dagger\gamma_0\boldsymbol{\Sigma}\Psi)$$

Preliminary Spin results





Conclusion

We argue that using the point-like approximation is inadequate for calculation of EM field created by valance quarks in quark-gluon plasma because the effect of quantum diffusion changes the qualitative behavior of the field.

Future Work

$$\frac{\partial \boldsymbol{B}}{\partial t} = \nabla \times (\boldsymbol{v} \times \boldsymbol{B}) + \frac{1}{\sigma}(\nabla^2 \boldsymbol{B} - \frac{\partial^2 \boldsymbol{B}}{\partial t^2} + \nabla \times \boldsymbol{J_{ext}})$$

Remove the slashes!

Consider all parts of the equation as well as include initial conditions put all the pieces together into a complete model