Universality & All N gauge/string duality

Daniel Krefl

based on

arXiv: 1304.7873 (with A. Schwarz)
arXiv: 1506.03907 (with R. Mkrtchyan)
arXiv: 1508.04219

But first a little personal intro …

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- Simons fellow at UC Berkeley
  (Perturbative quantum geometry for top. strings in the NS limit)

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- Fellow at CERN
  (... ? ...)

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- Climbing & Hiking

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- Skiing & Boarding

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Beyond Academia:

- Climbing & Hiking
- Skiing & Boarding
- Martial arts

CV

Beyond Academia:

- And some Tec related projects in my remaining spare-time …

```java
 *
 * @param target : target value
 */

public void FORCE_training(double target) { // Currently only for one output !

    E = Ao_o.getQuick(0) - target;

    DenseDoubleMatrix1D Ar  = mt.concat(Ar_o, Ai_o);
    DenseDoubleMatrix2D mAr = mt.Matrix1Dto2D(Ar);

    // Compute new Wo
    P_o.zMult(mAr, Wo, -E, 1, false, false);

    // Compute new P matrix
    double d = 1 + Ar.zDotProduct(P_o.zMult(Ar, null, 1, 0, false));

    DenseDoubleMatrix2D tmp = (DenseDoubleMatrix2D) P_o.zMult(mAr.zMult(mAr, null, 1, 0, false, true).zMult(P_o, null, 1,
    mt.multiplyConst(tmp, -1/d);

    P_o = (DenseDoubleMatrix2D) mt.add(P_o, tmp);

    //System.out.println(P_o);
}


/**
```

```c
magma_init();

// Vars
double aux_work[1];
double *w1, *h_R, *h_work;
magma_int_t info, lwork, liwork, *iwork, aux_iwork[1];

double *G = NULL;          // GPU data pointer
double *E = NULL;

magma_dmalloc( &G, d*d );
magma_dmalloc_cpu( &E, d*d );
magma_dmalloc_cpu( &h_R, d*d );
magma_dmalloc_cpu( &w1, d);

// Copy M to GPU
magma_dsetmatrix( d, d, M, d, G, d );

// Query for workspace sizes
magma_dsyevd_gpu( MagmaVec, MagmaLower,
                  d, G, d, w1,
                  h_R, d,
                  aux_work,  -1,
                  aux_iwork, -1,
                  &info
               );

lwork  = (magma_int_t) aux_work[0];
liwork = aux_iwork[0];

magma_dmalloc_cpu( &h_work, lwork );
iwork = ( magma_int_t *) malloc ( liwork * sizeof ( magma_int_t ));
```

Back to physics ...

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Less known that there exists a notion of "universal Lie algebra".
(Technically, it is a certain tensor category) [Vogel '99]

Universality

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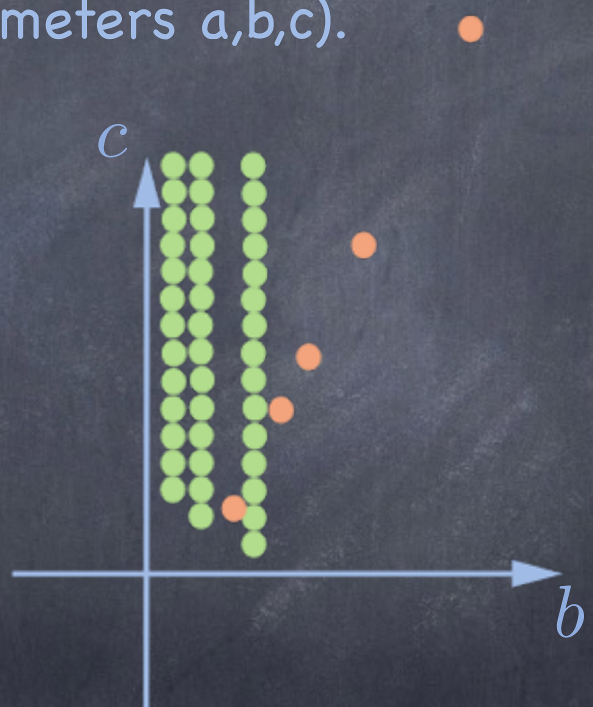
The usual Lie algebras correspond to special points in the moduli space (parameterized by the three Vogel parameters a,b,c).



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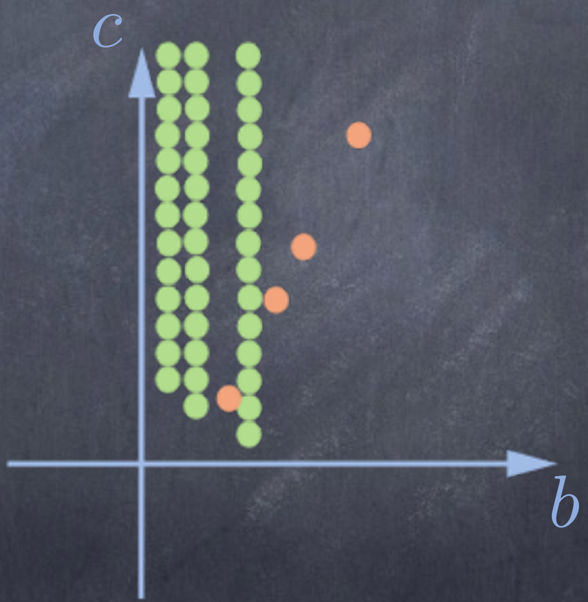
Universal expressions of (some?) quantities can be found, like for example:

$$\dim \mathfrak{g} = \frac{(a-2t)(b-2t)(c-2t)}{abc}, \quad t := a+b+c$$



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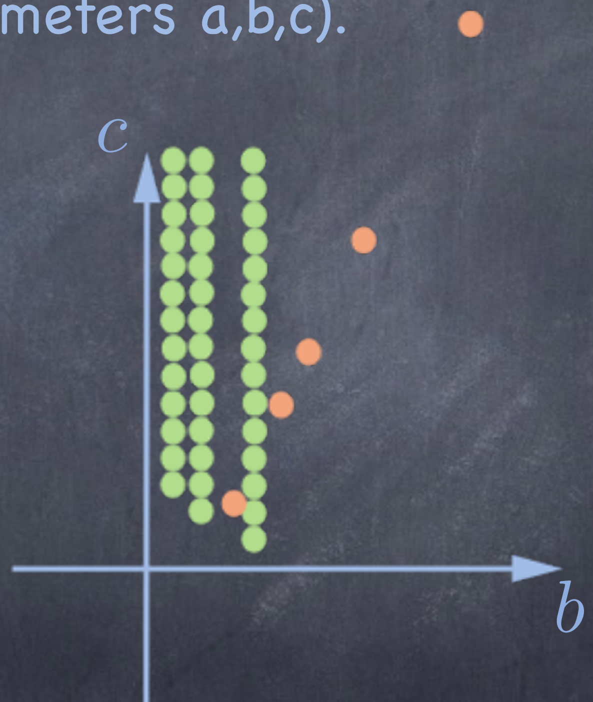

⇝ Idea: "Universal" gauge theory [Mkrtchyan+Veselov '12]

(Categorification)

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However: One can show that actually [D.K.+Mkrtchyan '15]

$$Z \sim \int_{\mathbb{R}_+} \frac{dx}{x} \frac{\sinh(x(t-\delta))}{\sinh(xt)\sinh(x\delta)} \frac{\sinh\left(\frac{x(a-2t)}{2}\right)\sinh\left(\frac{x(b-2t)}{2}\right)\sinh\left(\frac{x(c-2t)}{2}\right)}{\sinh\left(\frac{xa}{2}\right)\sinh\left(\frac{xb}{2}\right)\sinh\left(\frac{xc}{2}\right)}$$

(see also previous related expressions of [Mkrtchyan+Veselov '12])

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$\rightsquigarrow$ This expression is actually striking !
(i.e. provides an analytic continuation in ALL parameters)

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⇝ Sheds some light on the refined Chern-Simons theory [D.K.+Schwarz '13]

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$\rightsquigarrow$ A powerful application: Finite N duality
[D.K.+Mkrtchyan '15; D.K. '15]

All N gauge/string duality

Toy model: [Gopakumar+Vafa '98]

Well known that in the t'Hooft large N limit (for SU, Sp/SO):

$$Z(N,\delta) \to Z_{Coni}(t, g_s)$$

↑

Topological string partition function for the conifold
(respectively, orientifold thereof)

⇝ Gauge/string duality at large N

All N gauge/string duality

Toy model: [Mkrtchyan '12+'14; D.K.+Mkrtchyan '15]

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Allows us to easily go to finite (but continuous) N !
(Essentially boils down to sum over residue, but a bit tricky to arrive there ...)

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[Mkrtchyan '12+'14; D.K.+Mkrtchyan '15]

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"Perturbative poles"

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"Non-Perturbative poles"

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$\rightsquigarrow$ Non-perturbative expansion (valid at all continuous N)

All N gauge/string duality

Toy model: [D.K. '15]

Using some Mellin-Barnes integral tricks, one can calculate as well Z_{Coni} non-perturbatively.

⇝ for U(N):

$$\log Z_{CS}(N,\delta) = \log Z_{Coni}(1/\delta, e^{-2\pi i N}) - \frac{i\pi N^2}{4} + \frac{i\pi N(N^2-1)}{6\delta} - \frac{i\pi N}{12}\left(\frac{1}{\delta}+\delta\right)$$

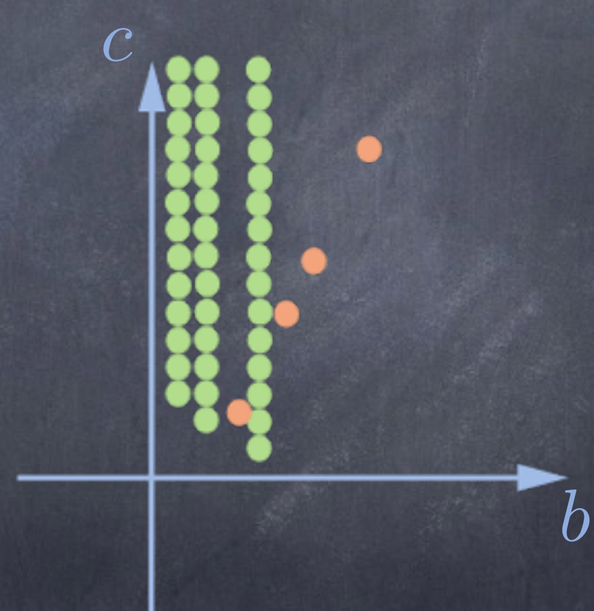
This holds for all N !

(The derivation is completely analytical, hence constitutes a proof of non-perturbative equivalence between CS and Top. strings on the conifold)

Future

Current topics (difficult):

Universality from strings ?



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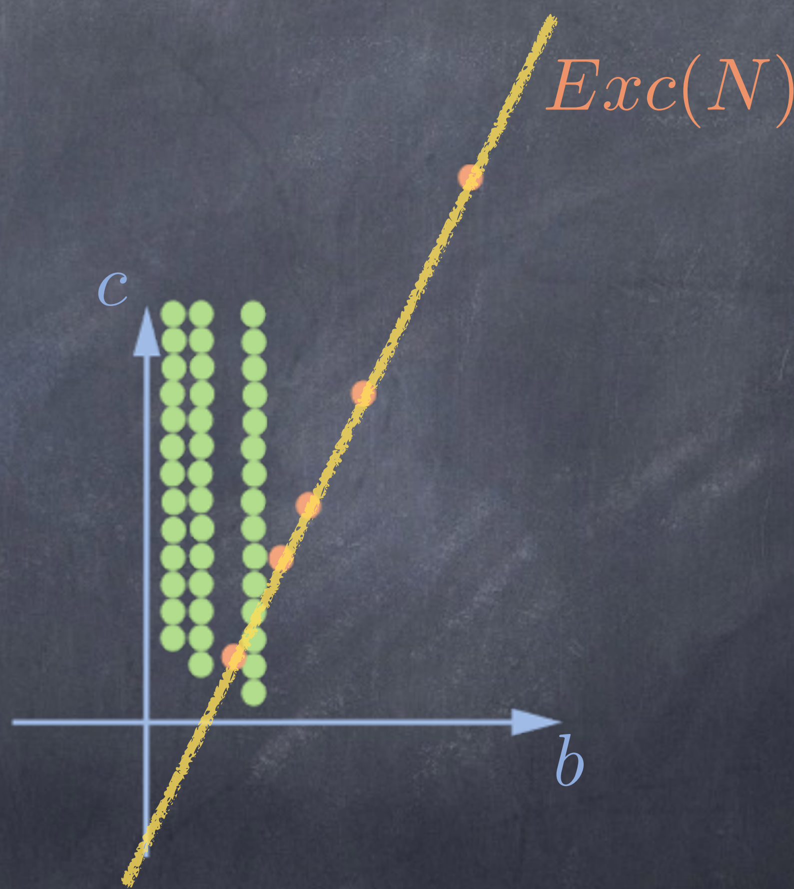
Exceptional top. string ?



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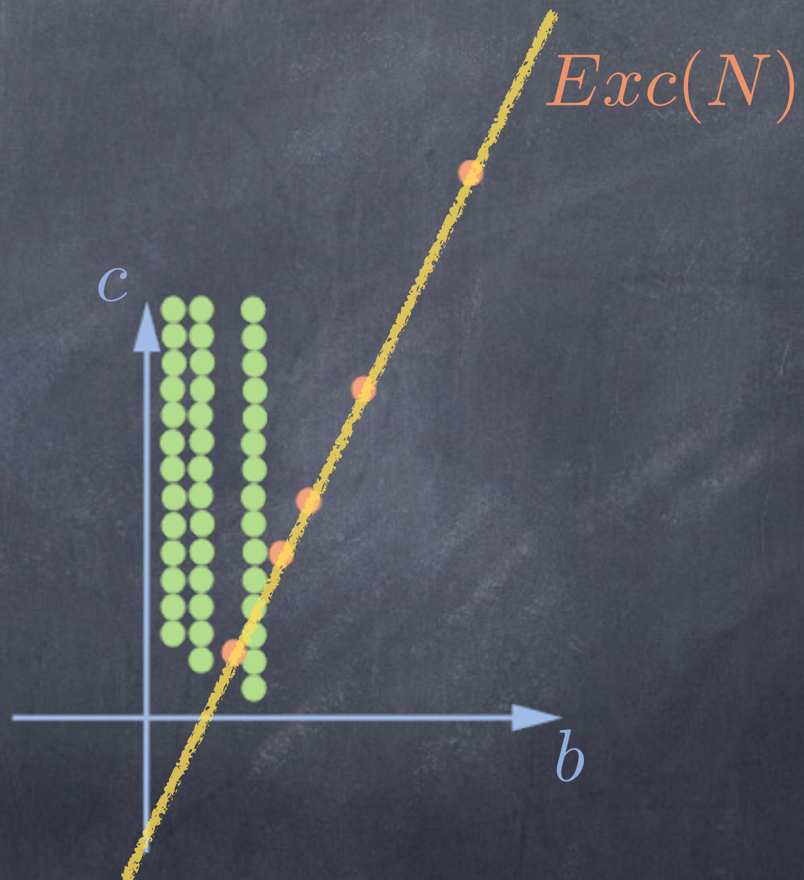
In general:

This provides a hint that there may exist a powerful re-formulation of gauge theory !



... Thank you ...