CERN Theory Retreat

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Les Houches
Profile

• Born in Athens

• Doctorat from Ecole Normale Supérieure (University of Paris)

  string thermodynamics & cosmology

• Previous position: MPI for Physics, Munich

• In my 2nd year as Fellow at the TH

Interested in both mathematical aspects of string theory & applications

• string perturbation theory at loop level (BPS saturated amplitudes)

• spontaneous breaking of supersymmetry & perturbative corrections

• topological amplitudes & connection to SUSY gauge theories
Overview of Recent Research

String theory - Supergravity - SUSY gauge theory (& techniques in Number theory)

• Radiative corrections to gauge couplings in non-supersymmetric string vacua
  
  arXiv: 1407.8023, 1509.00027
  with: C. Angelantonj and M. Tsulaia

• New methods for one-loop amplitudes based on Poincaré series
  
  arXiv: 1110.5318, 1203.0566, 1304.4271, 1502.00007
  with: C. Angelantonj and B. Pioline

• Proposal for refinement of topological string & connection to Nekrasov partition function
  
  arXiv: 1302.6993, 1309.6688, 1508.01477
  with: I. Antoniadis, S. Hohenegger, K.S. Narain and A. Zein Assi

• Gauged supergravity (non geometric fluxes) & string uplift
  
  arXiv: 1307.0999, 1202.6366
  with: C. Condeescu and D. Lüst
A Gauge theory - String theory connection

Localisation techniques may be used to compute the partition function of supersymmetric gauge theory on the $\Omega$ background

- 5d (rigid) supersymmetric gauge theory with $\text{N}=2$ on $\mathbb{R}^4 \times S^1$
- $\text{SU}(N)$ gauge group

$$Z_{\text{Nek}}(\epsilon_+, \epsilon_-) = \text{Tr} \left( -1 \right)^F e^{-2\epsilon_- J_3^-} e^{-2\epsilon_+ (J_3^3 + J_R^3)} e^{-\beta H}$$

Hilbert space on $\mathbb{R}^4$

representations of $\text{SO}(4) \sim \text{SU}(2)_+ \times \text{SU}(2)_-$

translations on $S^1$

Nekrasov 2002
Localisation techniques may be used to compute the partition function of supersymmetric gauge theory on the $\Omega$ background.

- 5d (rigid) supersymmetric gauge theory with $N=2$ on $R^4 \times S^1$
- SU($N$) gauge group

\[ Z_{\text{Nek}}(\epsilon_+, \epsilon_-) = \text{Tr} \left( (-1)^F e^{-2\epsilon_- J^3_-} e^{-2\epsilon_+ (J^3_+ + J^3_R)} e^{-\beta H} \right) \]

~ vacuum amplitude on a Melvin like background

fibration of $R^4$ over $S^1$  

“$\Omega$ background”
A Gauge theory - String theory connection

Non-trivial relation with N=2 String theory

\[ \int d^4 x \ F_g(\phi) \ R^2_{(-)} \ F_{(-),\text{grav}}^{2g-2} \]

- class of higher derivative gravitational couplings
- 2 self-dual Riemann tensors and (2g-2) self-dual graviphoton field strengths
- \( F_g \) computed by the free energy of the topological string (twisted version)

Field theory limit for \( \varepsilon_+ = 0 \) reproduces the perturbative gauge theory partition function

\[ \sum_{g \geq 0} g_s^{2g-2} \ F_g \bigg|_{\text{F.T.}} = \log Z_{\text{Nek}}^{\text{pert}} (\varepsilon_+ = 0, \varepsilon_- = g_s) \]

Antoniadis, Gava, Narain, Taylor 1993
A Gauge theory - String theory connection

Is it possible to find a generalisation of $F_g$ that produces the gauge theory partition function in the full $\Omega$ background?

$$\frac{1}{\epsilon_-^2 - \epsilon_+^2} \sum_{g,n} \epsilon_-^{2g} \epsilon_+^{2n} F_{g,n} \bigg|_{F.T.} = \log Z_{\text{Nek}}^{\text{pert}} (\epsilon_+, \epsilon_-)$$

- this is indeed the case for generalised couplings of the form

$$\int d^4 x \ F_{g,n}(\phi) \ R_{(-)}^{2g-2} F_{(-),\text{grav}}^{2n} F_{(+),T}$$

Antoniadis, I.F., Hohenegger, Narain, Zein Assi 2013

vector multiplet of Kähler modulus $T$ on heterotic $K3 \times T^2$
A Gauge theory - String theory connection

This relation is true for the full non-perturbative gauge theory partition function

\[
\frac{1}{\epsilon_-^2 - \epsilon_+^2} \sum_{g,n} \epsilon_-^{2g} \epsilon_+^{2n} F_{g,n} \bigg|_{\text{F.T.}} = \log Z_{Nek}(\epsilon_+, \epsilon_-)
\]

- verified by instanton calculations in dual type I theory on K3 x T^2
- D9 branes with D5-instantons wrapping the internal space
- string disk diagrams

Antoniadis, I.F., Hohenegger, Narain, Zein Assi 2014

Bulk insertions of self-dual field strengths

ADHM moduli at the boundary
A Gauge theory - String theory connection

\[
\frac{1}{\epsilon_+^2 - \epsilon_-^2} \sum_{g,n} \epsilon_-^{2g} \epsilon_+^{2n} F_{g,n} \Bigg|_{\text{F.T.}} = \log Z_{\text{Nek}}(\epsilon_+, \epsilon_-)
\]

reproduces the full $\Omega$-deformed ADHM action

The field theory limit of these couplings indeed reproduces the full non-perturbative gauge theory partition function

Antoniadis, I.F., Hohenegger, Narain, Zein Assi 2014
Questions:

- what is so special about the T-vector of K3 x T^2?
- contamination by hypermultiplet moduli - loss of BPS property
- can these couplings be turned topological in some appropriate (local) limit?

\[
\int d^4 x \, F_{g,n}(\phi) \, R^{2}_{(-)} \, F^{2g-2}_{(-),\text{grav}} \, F^{2n}_{(+),T}
\]

Answers:

- exactly calculable in string length, but one could also use any (matter) vector multiplet
- holomorphicity property is replaced by set of differential equations for genus-g correlators
- specific conditions for the couplings to acquire holomorphy (local limit)
- recover generalised holomorphic anomaly equation proposed in the literature (refinement)
Further Questions / Directions

- Better understand the conditions in terms of CY geometry
- Mass deformations, N=2*

Thank you