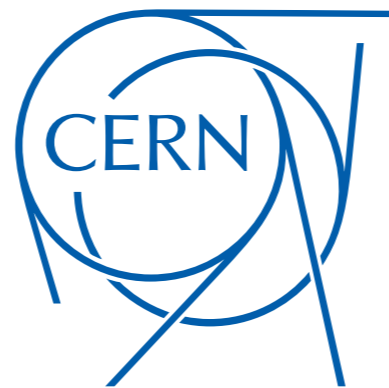


# Muon $g-2$ : leading hadronic contribution from the lattice

Marina Krstić Marinković



## Before coming to CERN ...

- 2003-2009 BSc/MSc at University of Belgrade, Serbia
- 2009-2013 PhD at Humboldt University, Berlin, Germany
- 2012-2014 Postdoc at the University of Southampton, UK

## My current interests

- Hadronic vacuum polarisation from the lattice
- Isospin breaking corrections, QED+QCD
- Heavy quark physics
- Quantum simulation of gauge theories

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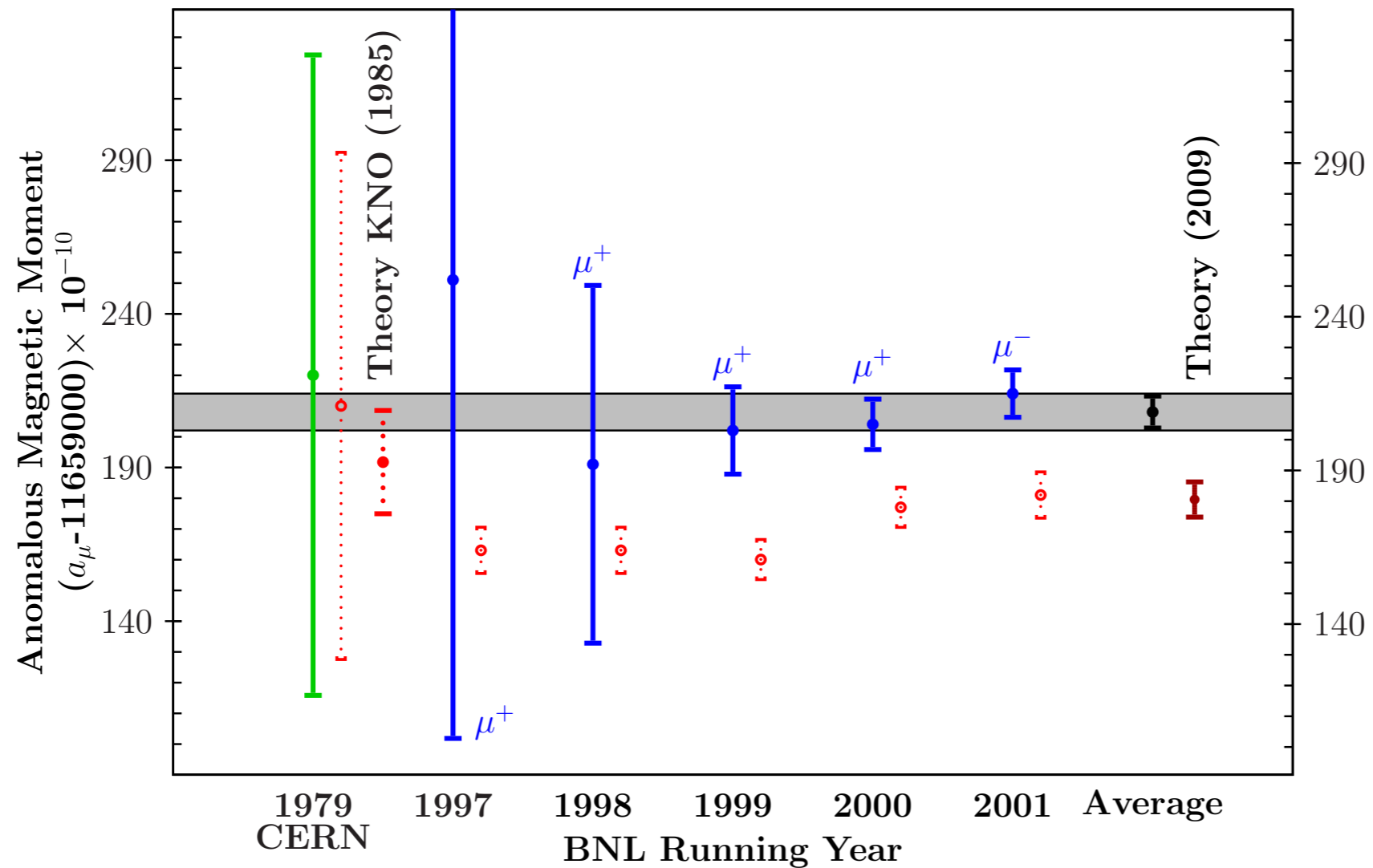
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## My current interests

- Hadronic vacuum polarisation from the lattice
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- 
- Heavy quark physics (Southampton+KEK+Tsukuba..., theoretical explorations)
  - Quantum simulation of gauge theories (initial discussions ... )

# $a_\mu$ as a stringent test of the SM

- Evolution of the (th - exp) tension [[Jegerlehner, Nyffeler 0902.3360](#)]



# $a_\mu$ as a stringent test of the SM

- $a_\mu^{exp} = 11659208.0(6.3) \times 10^{-10} (0.54\text{ppm})$  [BNL, 2006-2008]
- Current theoretical and experimental estimates:
  - ➔ discrepancy: 2.9- 3.6 stand. dev. discrepancy ( $e^+e^-$ ,  $\tau$  - data)
  - ➔  $a_\mu^{exp} - a_\mu^{th,SM} = 287(63)(51) \times 10^{-11}$  [Jegerlehner, Nyffeler 0902.3360]
- New experiments (J-PARC, Fermilab E989) expected to perform 4x more precise measurement
- Improved precision of the theoretical estimates with dominating uncertainty required

# $a_\mu$ as a stringent test of the SM

SM Contribution	Value $\pm$ Error ( $\times 10^{11}$ )	Ref
QED (5 loops)	$116584718.951 \pm 0.080$	[Aoyama et al., 2012]
HVP LO	$6923 \pm 42$	[Davier et al., 2011]
	$6949 \pm 43$	[Hagiwara et al., 2011]
HVP NLO	$-98.4 \pm 0.7$	[Hagiwara et al., 2011]
		[Kurz et al., 2014]
HVP NNLO	$12.4 \pm 0.1$	[Kurz et al., 2014]
HLbL	$105 \pm 26$	[Prades et al., 2009]
Weak (2 loops)	$153.6 \pm 1.0$	[Gnendiger et al., 2013]
SM Tot (0.42 ppm)	$116591802 \pm 49$	[Davier et al., 2011]
	(0.43 ppm)	$116591828 \pm 50$ [Hagiwara et al., 2011]
	(0.51 ppm)	$116591840 \pm 59$ [Aoyama et al., 2012]
Exp (0.54 ppm)	$116592089 \pm 63$	[Bennett et al., 2006]
Diff (Exp – SM)	$287 \pm 80$	[Davier et al., 2011]
	$261 \pm 78$	[Hagiwara et al., 2011]
	$249 \pm 87$	[Aoyama et al., 2012]

- Lattice provides *ab initio* setup for the computation of hadronic contribution(s)
- Completely independent from the current phenomenological determinations
- HVP leading order: largest uncertainty!
- HLbL next, very important to compute, requires QED

# Hadronic contributions to $a_\mu$ from the lattice

- Computing hadronic vacuum polarisation(HVP) contribution as a part of RBC&UKQCD
- Computing leading isospin breaking correction (LIBE) to HVP
- Prospects: disconnected contribution, improving LIBE to HVP  $\rightarrow$  QED+QCD

## The RBC & UKQCD collaborations

### BNL and RBRC

Tomomi Ishikawa  
Taku Izubuchi  
Chulwoo Jung  
Christoph Lehner  
Meifeng Lin  
Taichi Kawanai  
Christopher Kelly  
Shigemi Ohta (KEK)  
Amarjit Soni  
Sergey Syritsyn

### CERN

Marina Marinkovic

### Columbia University

Ziyuan Bai  
Norman Christ  
Xu Feng

Luchang Jin  
Bob Mawhinney  
Greg McGlynn  
David Murphy  
Daiqian Zhang

### University of Connecticut

Tom Blum

### Edinburgh University

Peter Boyle  
Luigi Del Debbio  
Julien Frison  
Richard Kenway  
Ava Khamseh  
Brian Pendleton  
Oliver Witzel  
Azusa Yamaguchi

### Plymouth University

Nicolas Garron

### University of Southampton

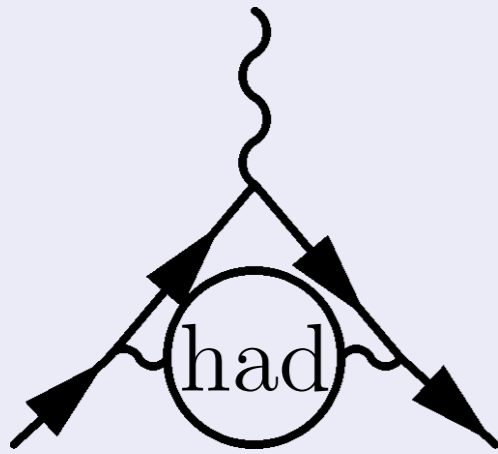
Jonathan Flynn  
Tadeusz Janowski  
Andreas Juettner  
Andrew Lawson  
Edwin Lizarazo  
Antonin Portelli  
Chris Sachrajda  
Francesco Sanfilippo  
Matthew Spraggs  
Tobias Tsang

### York University (Toronto)

Renwick Hudspith

# Hadronic vacuum polarisation

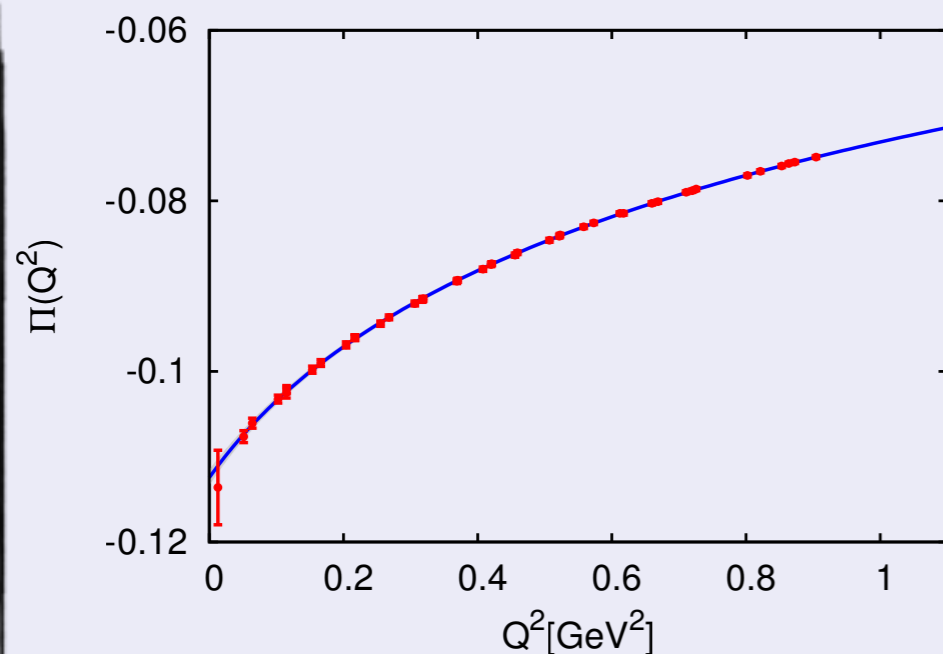
- Can be computed in Euclidean space-time [Blum, 2003; Lautrup et al., 1971]



- $a_{\mu}^{HLO} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^{\infty} dQ^2 f(Q^2) \times \hat{\Pi}(Q^2)$

- $f(Q^2) = m_{\mu}^2 Q^2 Z^3(Q^2) \frac{1 - Q^2 Z(Q^2)}{1 + m_{\mu}^2 Q^2 Z^2(Q^2)}$

- $Z(Q^2) = (\sqrt{(Q^2)^2 + 4m_{\mu}^2 Q^2} - Q^2) / (2m_{\mu}^2 Q^2)$



- $\hat{\Pi}(Q^2) = \Pi(Q^2) - \Pi(0)$

- $\Pi_{\mu\nu}(Q) = a^4 \sum_x e^{iQx} \langle J_{\mu}^{em}(x) J_{\nu}^{em}(0) \rangle$

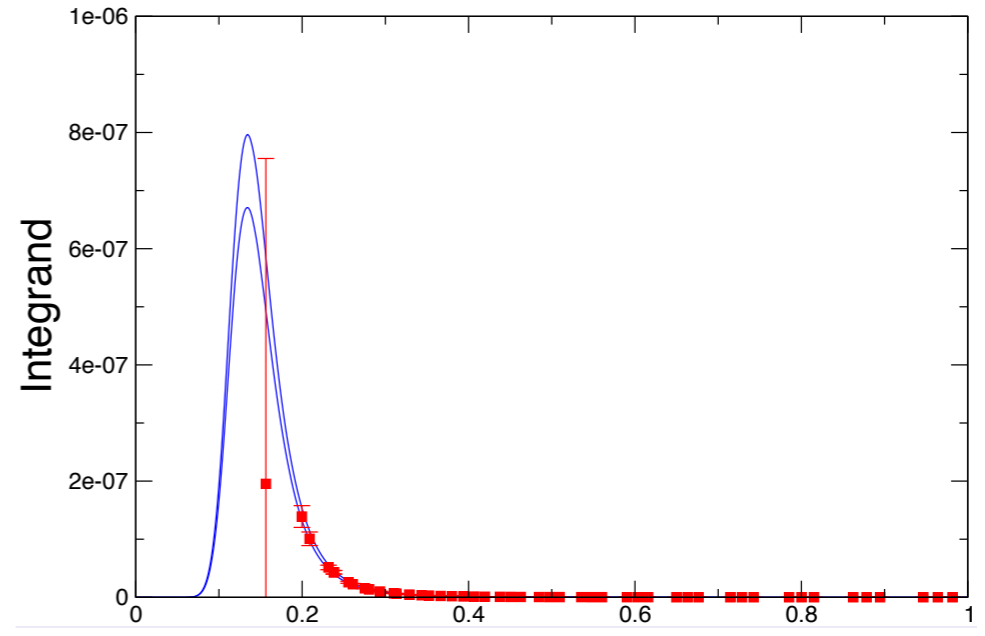
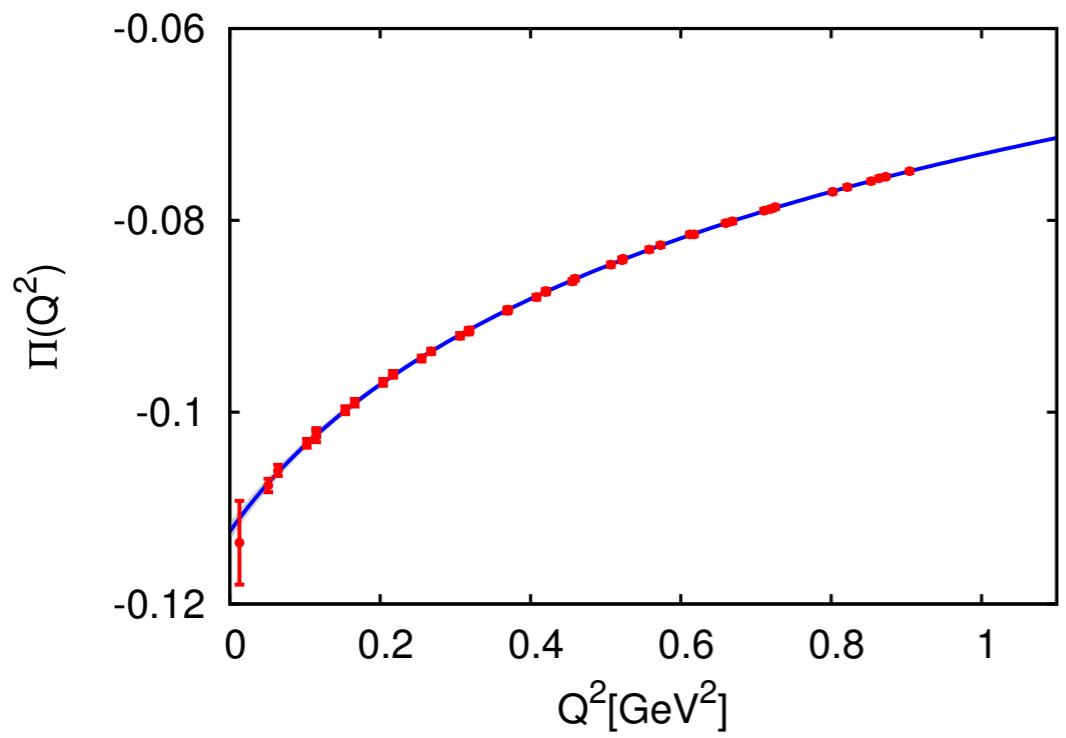
- $\Pi_{\mu\nu}(Q) = (Q^2 \delta_{\mu\nu} - Q_{\mu} Q_{\nu}) \Pi(Q^2)$



# Hadronic vacuum polarisation

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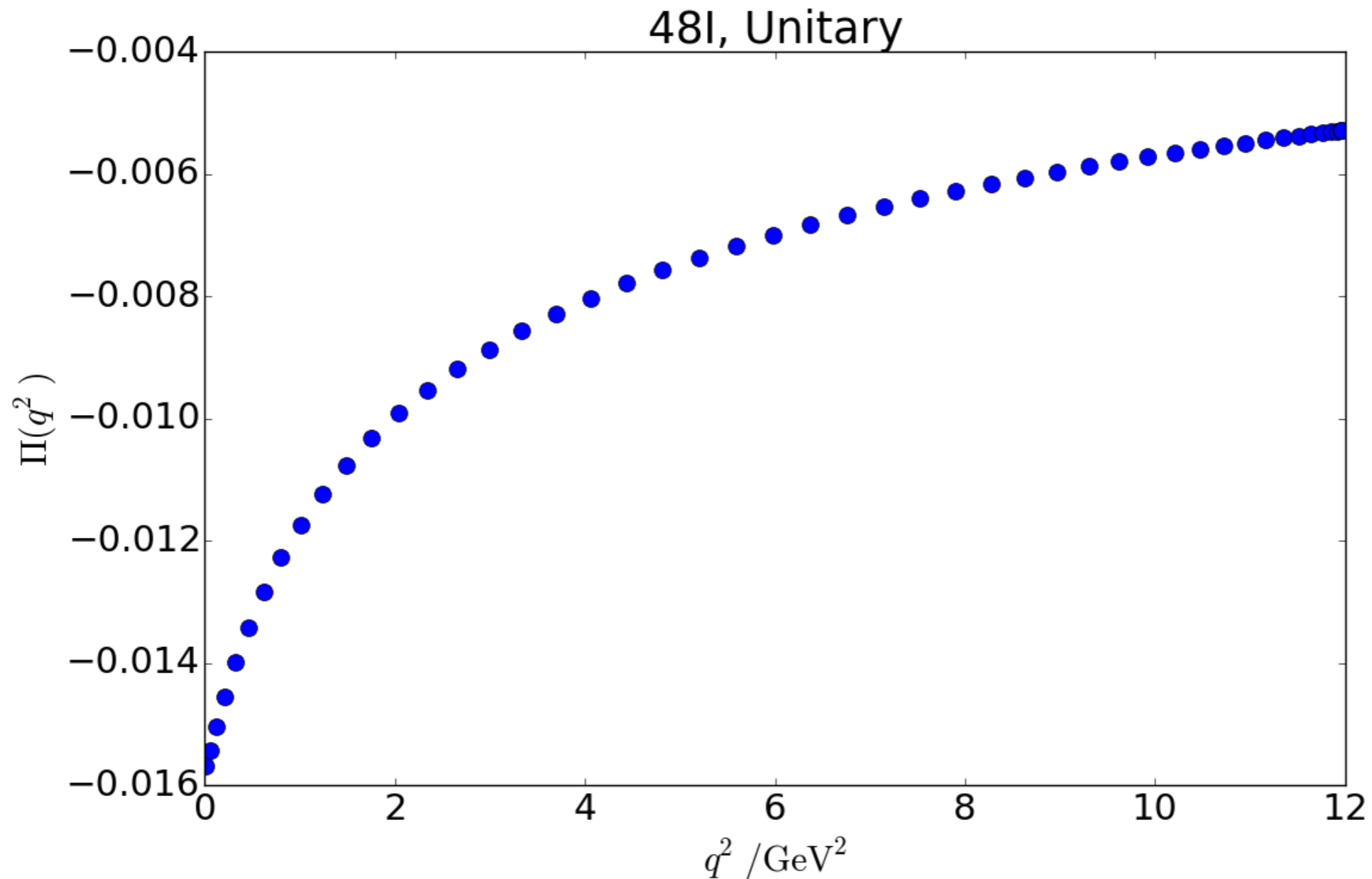
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[RBC-UKQCD, Boyle et al. '12]

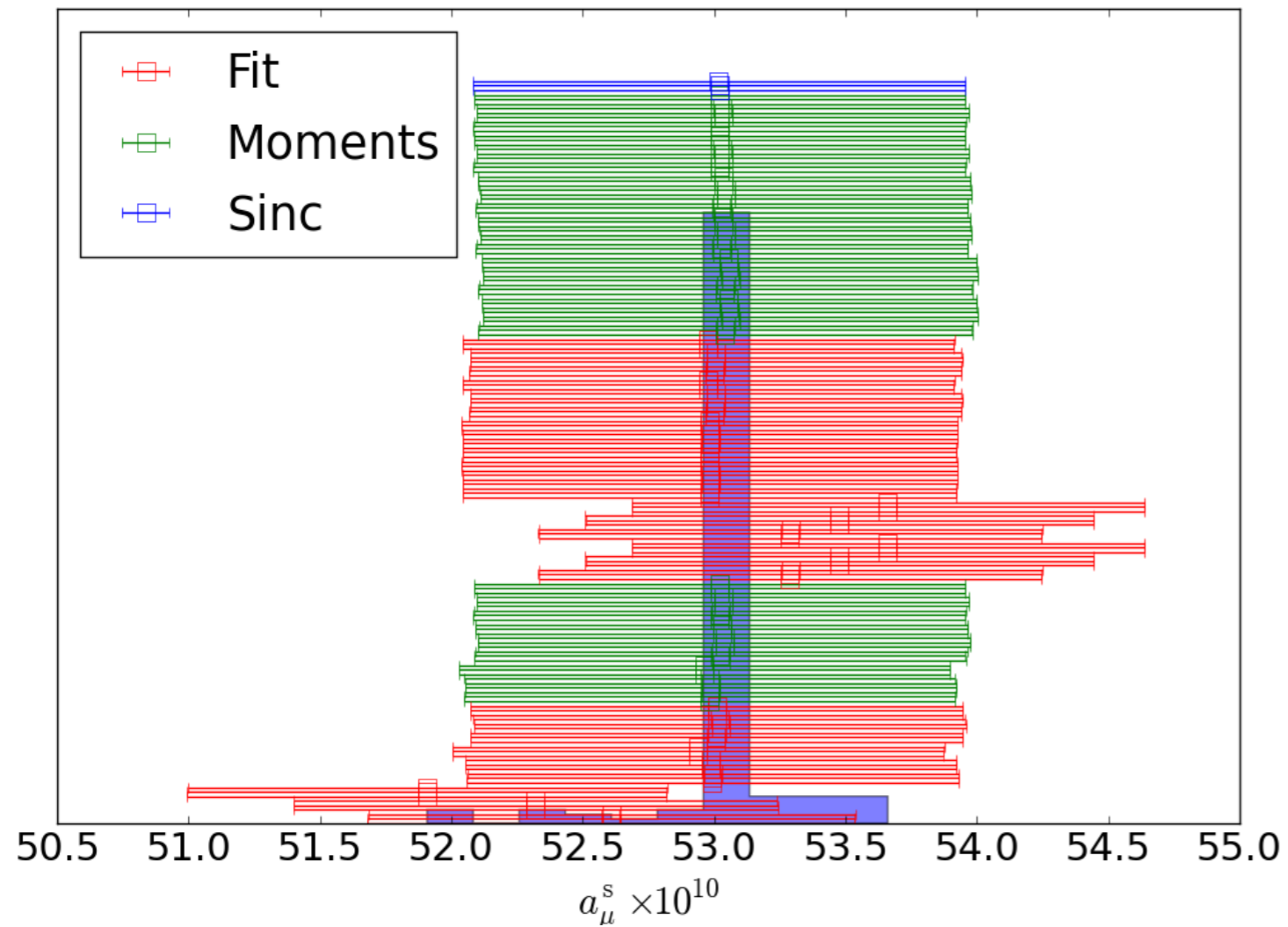
## Strange HVP: Matt Spraggs's talk at Lattice 2015 [RBC/UKQCD 2015]

- Strange contribution to HVP, 2+1 flavor Möbius DWF, physical quark mass ensemble



## Strange HVP: Matt Spraggs's talk at Lattice 2015 [RBC/UKQCD 2015]

Histogram of results from various strategies. Results insensitive



- remarkable agreement with HPQCD 2+1+1 staggered fermion result: 53.41(59)

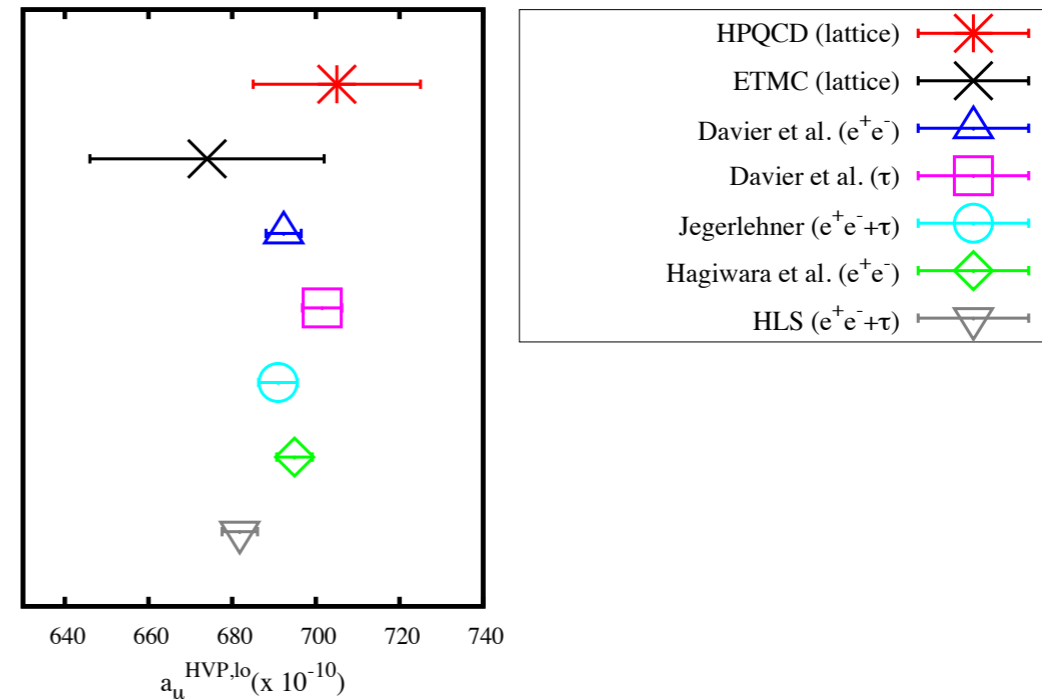
**Light HVP (u,d): needs quite a bit more work ...**

# Computing IB correction to the HVP

- Once the aimed precision for the connected HVP from the lattice is achieved (in the isosymmetric theory)  $\rightarrow$  the effects we neglected so far might become important:
  - disconnected contribution,
  - isospin breaking corrections,
  - charm in the sea, ...

- In the phenomenological determination of  $a_\mu^{had}$  model calculation of [Jegerlehner,Szafron '11]

$\rightarrow$  correctly applied IB correction reduced the discrepancy between  $e^+e^-$  and  $\tau$ -estim.



- It would be good to have a model independent

[Plot: B. Chakraborty, LATTICE 2015]

(non-perturbative) estimate of IB effects: lattice QCD+QED

# Computing IB correction to the HVP

$$\Delta O = \left\{ e^2 \frac{\partial}{\partial e^2} + [g_s^2 - (g_s^0)^2] \frac{\partial}{\partial g_s^2} + [m_f - m_f^0] \frac{\partial}{\partial m_f} + [m_f^{cr} - m_0^{cr}] \frac{\partial}{\partial m_f^{cr}} \right\} O$$

[Gasser, Rusetsky, Scimemi '03, RM123 '13]

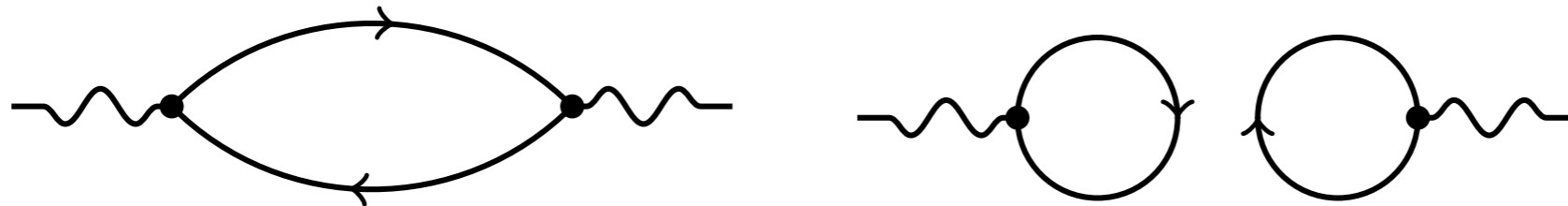
- RM123 method [[arXiv:1303.4896](https://arxiv.org/abs/1303.4896)] for computing leading isospin breaking corrections (LIBE)
  - ➔ Expanding an observable (in the isospin broken theory) with respect to the isosymmetric QCD result
- For a start: applying it to the connected part of the HVP
- Main advantage w. respect to simulating QED+QCD:
  - ➔ Diagrams obtained individually (before multiplying with  $O(\alpha_{em})$ ,  $O(m_u - m_d)$  coeff.)
  - ➔ No extrapolation in  $\alpha_{em}$
- Main disadvantage: one needs to compute many diagrams, mainly 3pt and 4pt functions
  - ➔ Can be overcome with careful organisations of the computation [[DD-SCOR code](#), [N.Tantalo](#)]
  - ➔ Code base for lattice QCD with isospin breaking corrections

# Prospects I: conceptual improvements

- Reducing finite volume effects - they are expected to be main source of systematics
- Currently: global zero mode subtracted:  $A_\mu(k=0) \equiv 0$ 
  - ➔ Violates reflection positivity and does not have a well defined  $T \rightarrow \infty$  limit [1406.4088]
- Removing the zero mode of the field on each time slice separately [Hayakawa, Uno '08]
  - ➔ this explicitly violates the hypercubic symmetry of the lattice -> no trace of the violation in the inf. vol limit [1406.4088]
- Charged particles in QED/QED+QCD with  $C^*$  BCs  $\rightarrow$  FV effects for masses even smaller
  - ➔ [Alberto, Agostino, ... arXiv:1509.01636]
- Eager to try this out for LIBE of HVP as well
- Getting the disconnected contributions (beyond el-quenched)

# Prospects II: Disconnected contribution

- Connected and disconnected contribution to the HVP



- Disconnected:
  - Computationally very demanding
  - ChPT estimate  $\propto 10\%$  [Della Morte, Juettner '10]
  - Direct estimates from the lattice in progress [Guelpers et al. '14]
  - **Several other progress reports @ Lattice2015, Kobe**
  - **Together with postdoc@IST,Lisbon [N. Cardoso]: setting up evaluation of disconnected on GPUs**

# Conclusions & outlook

- Lattice gives an independent theory prediction of hadronic contributions  $a_\mu$
- HVP front: strange contribution under control, light still needs work
- Phenomenologically - IB plays an important role in the th.-exp. discrepancy
- First attempt to extract the IB correction to the HVP from first principles
- Difficult task, but RM123 method should give good signal over noise ratio
  
- Other topics I am interested in:
  - **Heavy quark physics** from non-perturbative perspective
  - **Quantum simulations of gauge theories** motivated by the sign problem at finite  $\mu$

- Feel free to come and discuss any of these or sth. else further...
- And just one more thing ...



# Excited QCD 2016

Costa da Caparica, Lisbon, Portugal  
6-12 March 2016

## QCD at high temperatures & finite densities

- heavy-ion collisions, jets, diffraction, hadronisation
- quark-gluon plasma
- holography, colour-glass condensate
- compact stars, applications to astrophysics

## QCD at low energies

- excited hadrons
- new resonances
- glueballs, multiquarks



## More information

<https://indico.cern.ch/event/453434/>

✉ [excitedqcd@th.physik.uni-frankfurt.de](mailto:excitedqcd@th.physik.uni-frankfurt.de)

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