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Formal / mathematical aspects of string theory
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  - String perturbation theory, instanton calculus, dualities...

Exact results in SUSY gauge theories in various dimensions

Math applications: automorphic forms, algebraic geometry...
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\[ \text{Diagram: } \quad + \quad + \quad + \quad + \ldots \]
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The down side is that the integral over the moduli space of curves of (super) Riemann surfaces of genus $h$ is almost impossible to compute!
For $h \leq 3$, the moduli space is a fundamental domain $\mathcal{F}_h$ in the Siegel upper-half plane $\mathcal{H}_h$. For $h = 1$, still relatively tame:
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The trick is to represent the integrand as sum over images under the modular group, and unfold the integration domain against the sum.
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Higher loop modular integrals

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- One significant complication is that the integral typically diverges when the Riemann surfaces becomes degenerate, i.e. acquires a node.
- These divergences reflect the infrared divergences from massless supergravity states.
In fact, all supergravity Feynman diagrams emerge from degenerations of Riemann surfaces:

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The unfolding method at higher genus

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This includes e.g. the two-loop correction to $D^4 R^4$ coupling in type II string theory compactified on a torus, and establishes relations with Eisenstein series which Obers and I had conjectured in 1999.
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Higher derivative couplings are more singular and require more sophisticated treatment. In fact, the study of the $D^6 R^4$ coupling has led to a new representation of the Kawazumi-Zhang invariant for genus two curves, which seems to baffle mathematicians...