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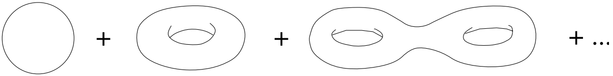
  - Black hole microstate counting, wall-crossing...
  - Exact results in SUSY gauge theories in various dimensions
  - Math applications: automorphic forms, algebraic geometry...

String perturbation theory

- One of the virtues of string theory is that only one diagram is needed at each loop order.



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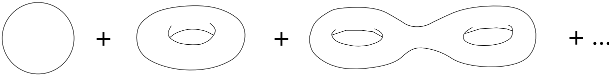
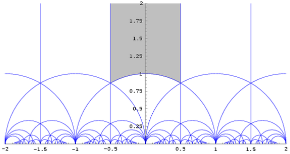


- The down side is that the integral over the moduli space of curves of (super) Riemann surfaces of genus h is almost impossible to compute !

One-loop modular integrals

- For $h \leq 3$, the moduli space is a fundamendal domain $\mathcal{F}_h$ in the Siegel upper-half plane $\mathcal{H}_h$. For $h = 1$, still relatively tame:



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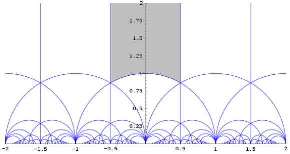

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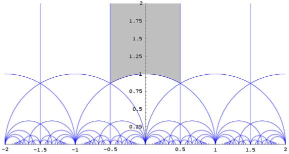

- In collaboration with C. Angelantonj, I. Florakis (2011-15), we have developed methods to reduce a large class of one-loop modular integrals to ordinary Schwinger-type integrals.
- The trick is to represent the integrand as sum over images under the modular group, and unfold the integration domain against the sum.

Higher loop modular integrals

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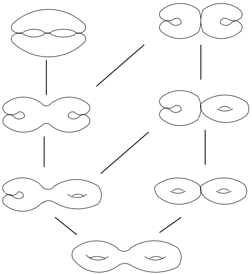
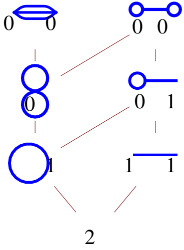
- One significant complication is that the integral typically diverges when the Riemann surfaces becomes degenerate, i.e. acquires a node.
- These divergences reflect the infrared divergences from massless supergravity states.

Nodal curves and Feynman diagrams

- In fact, all supergravity Feynman diagrams emerge from degenerations of Riemann surfaces:





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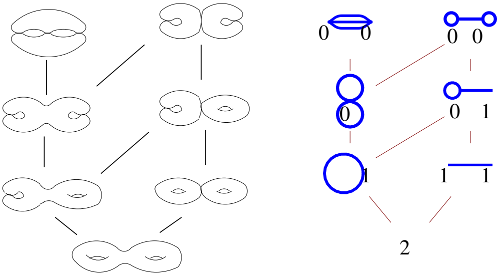

- In mathematics, these decorated Feynman diagrams are known as tropical Riemann surfaces.

The unfolding method at higher genus

- By introducing a suitable cut-off, we managed to extend the unfolding method to a class of two-loop modular integrals where the integrand is regular near the separating degeneration and grows at most polynomially near the non-separating degeneration.

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- This includes e.g. the two-loop correction to $D^4 R^4$ coupling in type II string theory compactified on a torus, and establishes relations with Eisenstein series which Obers and I had conjectured in 1999.
- Higher derivative couplings are more singular and require more sophisticated treatment. In fact, the study of the $D^6 R^4$ coupling has led to a new representation of the Kawazumi-Zhang invariant for genus two curves, which seems to baffle mathematicians...