

Dark Matter in the sky and in the lab

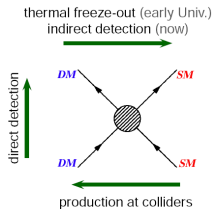
Mathias Garny (CERN)



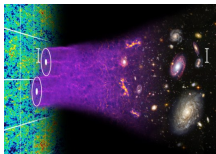
TH division retreat, Les Houches, 04.11.15

Topics

- ▶ Interplay of dark matter searches (mainly indirect detection; γ -ray spectral features; electroweak effects)



- ▶ Large-scale structure (perturbation theory)



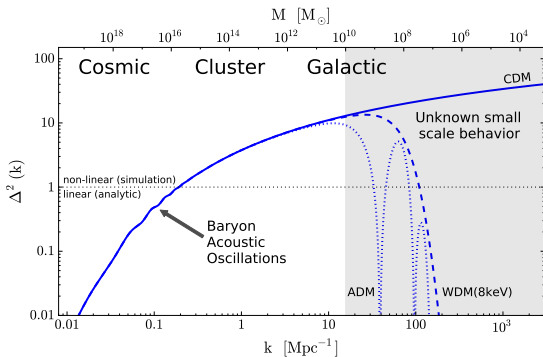
- ▶ Physics of the Early Universe (leptogenesis, nonequilibrium QFT)

Can we understand the weakly non-linear regime?

Power spectrum of density contrast $\delta(\mathbf{x}, z) = \rho(\mathbf{x}, z)/\bar{\rho}(z) - 1$

$$\langle \delta(\mathbf{k}, z)\delta(\mathbf{k}', z) \rangle = \delta^{(3)}(\mathbf{k} + \mathbf{k}')P(k, z)$$

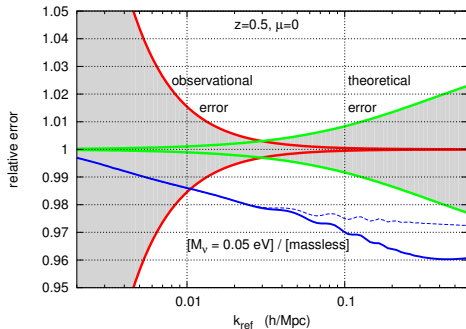
$$\Delta^2(k, z) = 4\pi k^3 P(k, z)$$



What can we learn? (for example)

Euclid forecast vs theoretical errors

Audren, Lesgourgues, Bird et. al. 1210.2194



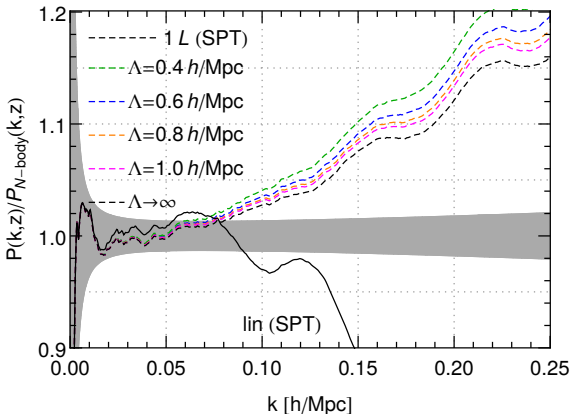
$$\sigma(M_\nu) \simeq \begin{cases} 25\text{meV} & \text{fiducial (2\%th. err. at } k = 0.4h/\text{Mpc, } z = 0.5) \\ 14\text{meV} & \text{th. err. } / = 10, k_{\text{max}} = 0.6h/\text{Mpc} \end{cases}$$

$$M_\nu = \sum m_\nu \geq \begin{cases} 0.056\text{eV (nh)} \\ 0.095\text{eV (ih)} \end{cases}$$

theoretical uncertainties from bias, RSD, non-linear evolution, ...

Perturbation theory for large scale structure

$P_{\delta\delta}(k, z=0)/P_{N\text{-body}}$, SPT with cutoff

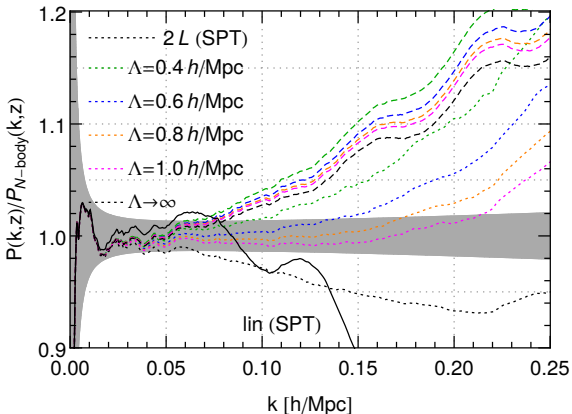


Idea: expand in density contrast $\delta = \rho/\bar{\rho} - 1$

- ▶ Solve coupled continuity, Euler, Poisson equation, ideal fluid (SPT)
- ▶ Initial $\delta(\mathbf{x}, t_0)$ Gaussian random field with two-point corr. $P_0(k)$

Perturbation theory for large scale structure

$P_{\delta\delta}(k, z=0)/P_{N\text{-body}}$, SPT with cutoff



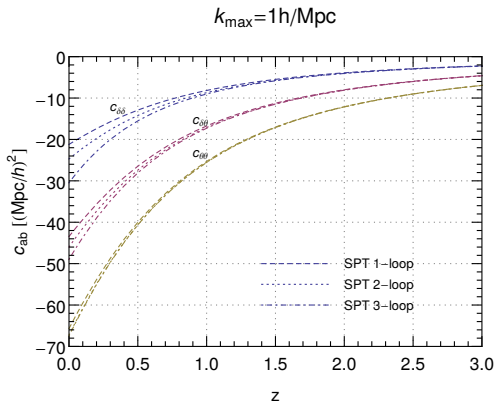
Idea: expand in density contrast $\delta = \rho/\bar{\rho} - 1$

- ▶ Solve coupled continuity, Euler, Poisson equation, ideal fluid (SPT)
- ▶ Initial $\delta(\mathbf{x}, t_0)$ Gaussian random field with two-point corr. $P_0(k)$

Power spectrum on large scales: SPT

Power spectrum for $q \rightarrow 0$, for density contrast δ and velocity div. θ

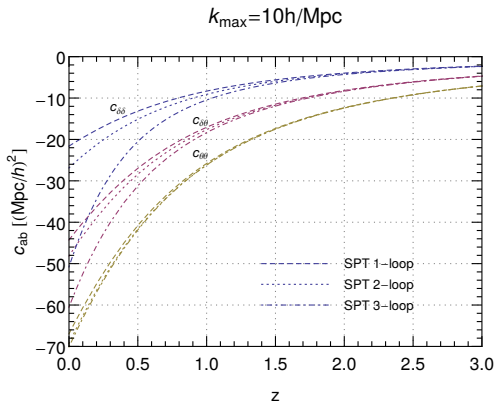
$$P_{ab}(q, z) \rightarrow \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} P_{lin}(q, z) + c_{ab}(z) q^2 P_{lin}(q, z) + \dots + d_{ab} q^4 + \dots$$



Power spectrum on large scales: SPT

Power spectrum for $q \rightarrow 0$, for density contrast δ and velocity div. θ

$$P_{ab}(q, z) \rightarrow \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} P_{lin}(q, z) + c_{ab}(z) q^2 P_{lin}(q, z) + \dots + d_{ab} q^4 + \dots$$



Power spectrum on large scales: SPT

Power spectrum for $q \rightarrow 0$

$$P_{ab}(q, z) \rightarrow \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} P_{lin}(q, z) + c_{ab}(z) q^2 P_{lin}(q, z) + \dots + d_{ab} q^4 + \dots$$

$$c_{ab}^{1-loop} = - \begin{pmatrix} \frac{61}{105} & \frac{25}{21} \\ \frac{25}{21} & \frac{9}{5} \end{pmatrix} \frac{4\pi}{3} \int_{k < \Lambda} dk P_{lin}(k, z)$$

Power spectrum on large scales: SPT

Power spectrum for $q \rightarrow 0$

$$P_{ab}(q, z) \rightarrow \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} P_{lin}(q, z) + c_{ab}(z) q^2 P_{lin}(q, z) + \dots + d_{ab} q^4 + \dots$$

$$c_{ab}^{1-loop} = - \begin{pmatrix} \frac{61}{105} & \frac{25}{21} \\ \frac{25}{21} & \frac{9}{5} \end{pmatrix} \frac{4\pi}{3} \int_{k < \Lambda} dk P_{lin}(k, z)$$

$$\begin{aligned} c_{ab}^{L-loop} &\sim - \int_{k < \Lambda} dk P_{lin}(k, z) \left[\int_{p < k} dp p^2 P_{lin}(p, z) \right]^{2L-2} \\ &\sim - \int_{k < \Lambda} dk P_{lin}(k, z) [D_+(z)^2 \ln^3(k/k_{eq})]^{2L-2} \quad \text{Eisenstein-Hu} \\ &\sim - k_{eq}^2 \frac{(3L)!}{2^{3L}} D_+(z)^{2L} \quad \text{where } D_+ \sim 1/(1+z) \end{aligned}$$

Asymptotic series

Power spectrum on large scales: SPT

Possible strategies

- ▶ Truncate series at smallest term $\Rightarrow \mathcal{O}(25\%)$ uncertainty at $z = 0$

Power spectrum on large scales: SPT

Possible strategies

- ▶ Truncate series at smallest term $\Rightarrow \mathcal{O}(25\%)$ uncertainty at $z = 0$
- ▶ Pade or Borel-Pade ansatz

Power spectrum on large scales: SPT

Possible strategies

- ▶ Truncate series at smallest term $\Rightarrow \mathcal{O}(25\%)$ uncertainty at $z = 0$
- ▶ Pade or Borel-Pade ansatz
- ▶ Extract fctns $c_{\delta\delta}(z)$, $c_{\theta\theta}(z)$ from large-volume N-body ('EFTofLSS')

Power spectrum on large scales: SPT

Possible strategies

- ▶ Truncate series at smallest term $\Rightarrow \mathcal{O}(25\%)$ uncertainty at $z = 0$
- ▶ Pade or Borel-Pade ansatz
- ▶ Extract fctns $c_{\delta\delta}(z)$, $c_{\theta\theta}(z)$ from large-volume N-body ('EFTofLSS')

What do we expect?

- ▶ Decoupling arguments of virialized structures (Peebles '80): **small UV sensitivity**
- ▶ Response fctn from N-body support this *Nishimichi, Bernardeau, Taruya 1411.2970*

Wilsonian approach I

Power spectrum for $q \rightarrow 0$

$$P_{ab}(q, z) \rightarrow \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} P_{lin}(q, z) + c_{ab}(z)q^2 P_{lin}(q, z) + \dots + d_{ab}q^4 + \dots$$

Wilsonian picture

$$c_{ab}(z) = \int d\Lambda \frac{\delta c_{ab}}{\delta \Lambda}$$

... if we would expand perturbatively

$$\frac{\delta c_{ab}^{L-loop}}{\delta \Lambda} \sim -P_{lin}(\Lambda, z) [\ln^3(\Lambda/k_{eq})]^{2L-2}$$

\Rightarrow Goal: non-perturbative equation

Non-perturbative equation for the long-wave power spectrum

$$(\partial_\eta + 2)c = \{\Omega, c\} - \begin{pmatrix} 0 & 1 \\ 1 & 2 \end{pmatrix} C_{\theta\theta}(z) - \begin{pmatrix} 2 & 1 \\ 1 & 0 \end{pmatrix} C_{\delta\theta}(z),$$

where

$$\begin{aligned} C_{\theta\theta}(z) &= 4\pi \int dk f_{\theta\theta}^{\mathcal{O}_0}(k, z) \\ &= \frac{4\pi}{3} \left(-4 - \frac{\partial}{\partial\eta} + 5 \frac{\partial}{\partial\kappa} \right) \int dk P_{\theta\theta}^\kappa(k, z) \Big|_{\kappa=0} \end{aligned}$$

$P_{\theta\theta}^\kappa(k, \eta)$ is the **full non-perturbative velocity power spectrum** in the presence of (hypothetical) spatial curvature $K = \mathcal{H}^2 \kappa$.

Non-perturbative equation for the long-wave power spectrum

$$(\partial_\eta + 2)c = \{\Omega, c\} - \begin{pmatrix} 0 & 1 \\ 1 & 2 \end{pmatrix} C_{\theta\theta}(z) - \begin{pmatrix} 2 & 1 \\ 1 & 0 \end{pmatrix} C_{\delta\theta}(z),$$

where

$$\begin{aligned} C_{\theta\theta}(z) &= 4\pi \int dk f_{\theta\theta}^{\mathcal{O}_0}(k, z) \\ &= \frac{4\pi}{3} \left(-4 - \frac{\partial}{\partial\eta} + 5 \frac{\partial}{\partial\kappa} \right) \int dk P_{\theta\theta}^\kappa(k, z) \Big|_{\kappa=0} \end{aligned}$$

$P_{\theta\theta}^\kappa(k, \eta)$ is the **full non-perturbative velocity power spectrum** in the presence of (hypothetical) spatial curvature $K = \mathcal{H}^2 \kappa$.

MG, Konstandin, Porto, Sagunski 1508.06306

$$\Rightarrow \frac{\delta C_{ab}}{\delta\Lambda} \sim [-4 - \partial_\eta + \partial_\kappa] P_{\theta\theta}^\kappa(\Lambda, z) + [\dots] P_{\delta\theta}^\kappa(\Lambda, z) \Big|_{\kappa=0}$$

Using $P_{\theta\theta}^\kappa$ from small-scale N-body \Rightarrow small UV sensitivity

Derivation

- ▶ OPE ($\psi = (\delta, \theta)$ at time $\eta = \log(D)$, $\Phi = \text{grav. potential}$)

$$\psi_a(\mathbf{x}, \eta)\psi_b(\mathbf{y}, \eta) \xrightarrow{\mathbf{x} \rightarrow \mathbf{y}} \sum_{\mathcal{O}} f_{ab}^{\mathcal{O}}(|\mathbf{x} - \mathbf{y}|, \eta) \mathcal{O}[\Phi, \partial\Phi, \dots]\left(\frac{1}{2}(\mathbf{x} + \mathbf{y}), \eta\right),$$

Derivation

- ▶ OPE ($\psi = (\delta, \theta)$) at time $\eta = \log(D)$, $\Phi = \text{grav. potential}$)

$$\psi_a(\mathbf{x}, \eta)\psi_b(\mathbf{y}, \eta) \xrightarrow{\mathbf{x} \rightarrow \mathbf{y}} \sum_{\mathcal{O}} f_{ab}^{\mathcal{O}}(|\mathbf{x} - \mathbf{y}|, \eta) \mathcal{O}[\Phi, \partial\Phi, \dots]\left(\frac{1}{2}(\mathbf{x} + \mathbf{y}), \eta\right),$$

- ▶ Response to long mode $\delta_L \Rightarrow$ response to bkg curvature $\kappa = \frac{5}{3}\delta_L$

e.g. Baldauf et al 1106.5507, Sherwin, Zaldarriaga 1202.3998

$$f_{\theta\theta}^{\mathcal{O}_0}(k, \eta) = \left(-\frac{5}{3} - \frac{1}{3}k \frac{\partial}{\partial k} - \frac{1}{3} \frac{\partial}{\partial \eta} + \frac{5}{3} \frac{\partial}{\partial \kappa} \right) P_{\theta\theta}^{\kappa}(k, \eta)|_{\kappa=0}$$

\rightarrow closely related to 'equal-time consistency relations'

Valageas 1311.4286, Kehagias, Perrier, Riotto 1311.5524, Konstandin, Porto, Sagunski 1411.3225

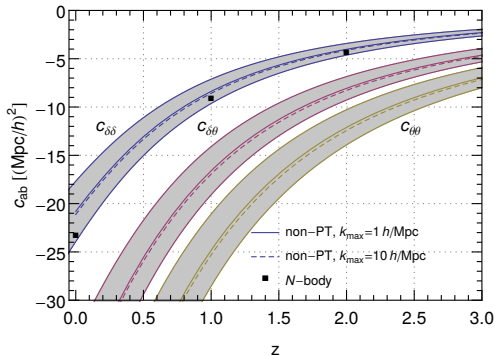
- ▶ Directional long mode can be absorbed in Bondi-type bkg

Numerical check

Solution of non-pt eq for the long-wave power spectrum (lines)

$$P_{ab}(q, z) \rightarrow \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} P_{lin}(q, z) + c_{ab}(z) q^2 P_{lin}(q, z) + \dots$$

compared to direct measurements of $c_{\delta\delta}$ from N-body (black squares)



Bands = error estimate due to VKPR approx. for curvature response

Wilsonian approach II

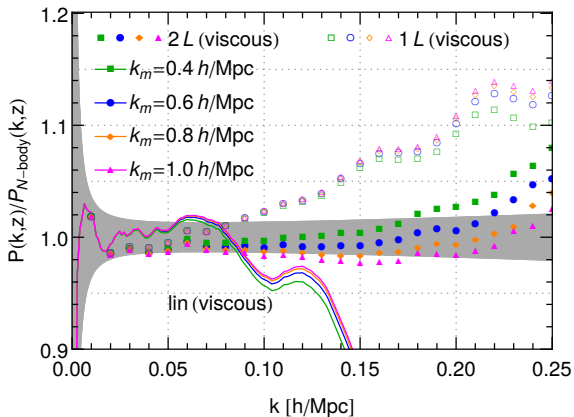
- ▶ Coarse-grained (Navier-Stokes) theory where modes below $k < k_m$ are included
- ▶ Effective viscosity and pressure which depend on k_m in such a way that

$$\begin{aligned}c_{ab}(z) &= \int d\Lambda \frac{\delta c_{ab}}{\delta \Lambda} \\ &\stackrel{!}{=} \int_0^{k_m} d\Lambda \frac{\delta c_{ab}}{\delta \Lambda} + c_{ab}^{\text{visc/pressure}}(z)\end{aligned}$$

- ▶ The scale k_m is an **artificial matching scale**, and the physical result should be independent
- ▶ Any residual dependence on k_m quantifies the **theoretical error due to perturbative approximations**

Wilsonian approach II

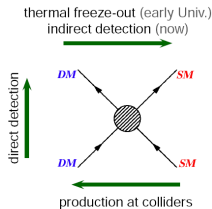
$P_{\delta\delta}(k, z=0)/P_{N\text{-body}}$, viscous theory



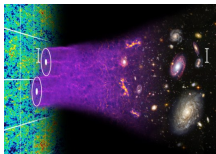
Effective viscosity/pressure from integrating out modes $k > k_m$
Residual dependence on k_m is an estimate for the theoretical error

Topics

- ▶ Interplay of dark matter searches (mainly indirect detection; γ -ray spectral features; electroweak effects)



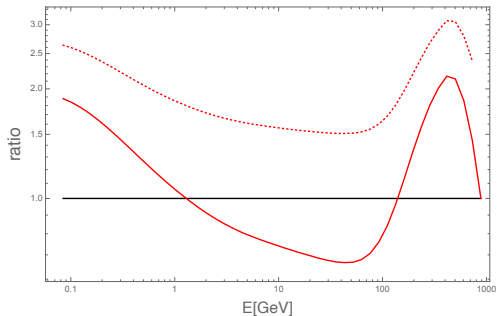
- ▶ Large-scale structure (perturbation theory)



- ▶ Physics of the Early Universe (leptogenesis, nonequilibrium QFT)

$$\chi\chi \rightarrow f\bar{F} + \gamma, Z, W, h, H, A, H^\pm$$

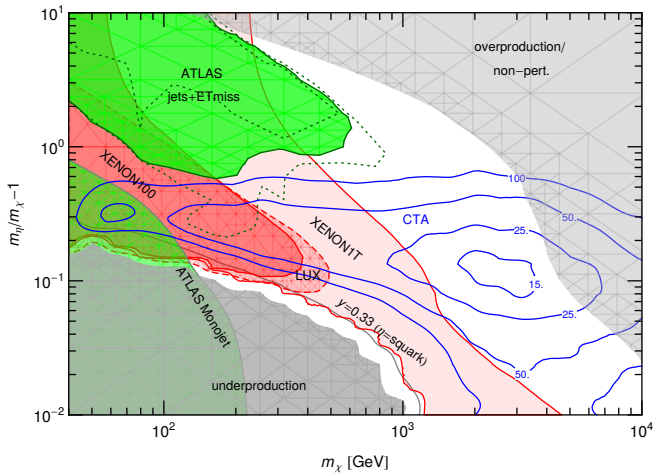
Modification of ν_μ spectrum (MSSM)



Preliminary – Bringmann, Calore, Galea, MG

Prospects (coloured t -channel mediator)

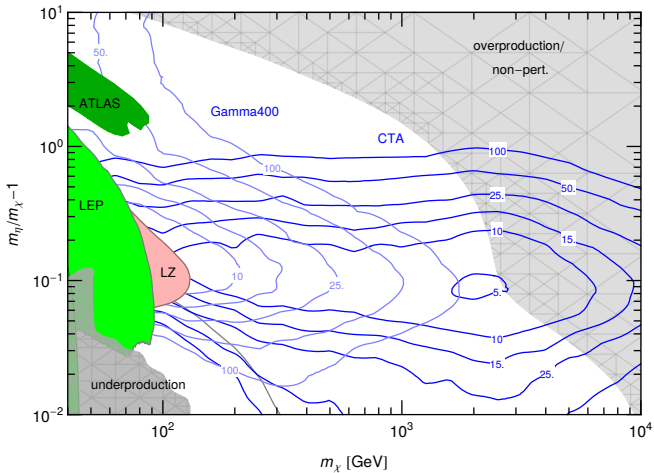
DM coupling to u -quark (prospects)





Prospects (uncolored t -channel mediator)

DM coupling to leptons (prospects)



UV dependence

- ▶ Variance $\sigma_{ab}^2 = 4\pi/3 \int_0^{k_{max}} dk P_{ab}$
- ▶ For velocity and cross spectrum, **saturation around $\text{few} \times k_{nl}$**
- ▶ \Rightarrow **small dependence on UV modes** with $k > \text{few} \times k_{nl}$

