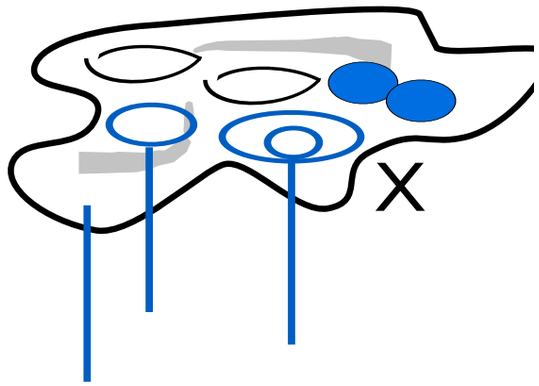
Geometry of Topological Strings & Branes

WL/ TH Retreat 2015

Motivation: string compactifications to 4d

Typical brane + flux configuration on a Calabi-Yau space X:



closed string (bulk) moduli t

open string (brane location + bundle) moduli u

3+1 dim world volume with effective N=1 SUSY theory

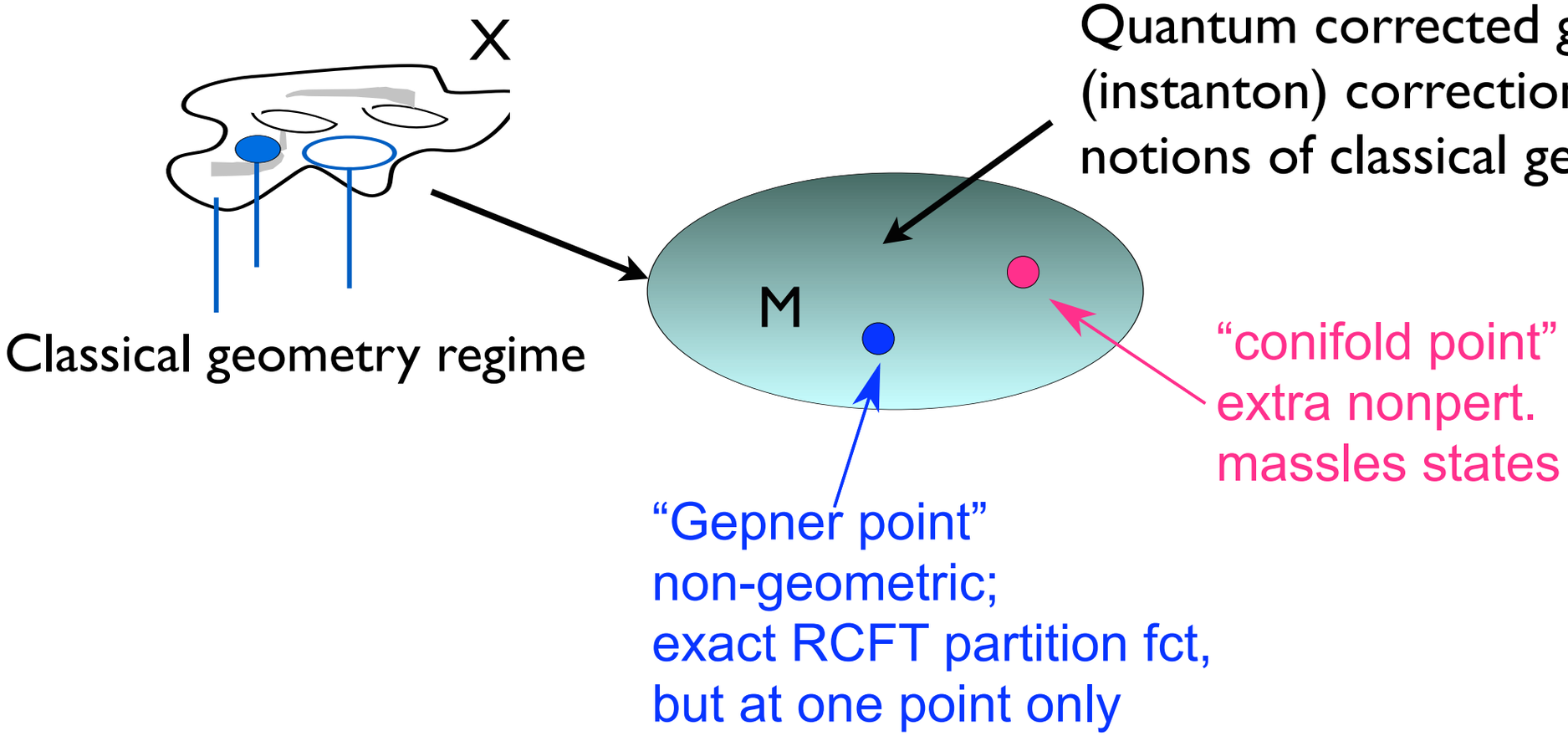
What are the exact effective superpotential, the vacuum states, gauge couplings (general F-terms), etc, as functions of moduli ?

$$\mathcal{W}_{\text{eff}}(\Phi, t, u) \; = \; ?$$

....well developed geometrical techniques mostly for non-generic brane configurations (non-compact, -intersecting) branes only !
(mirror symmetry, localization, integrable matrix models...)

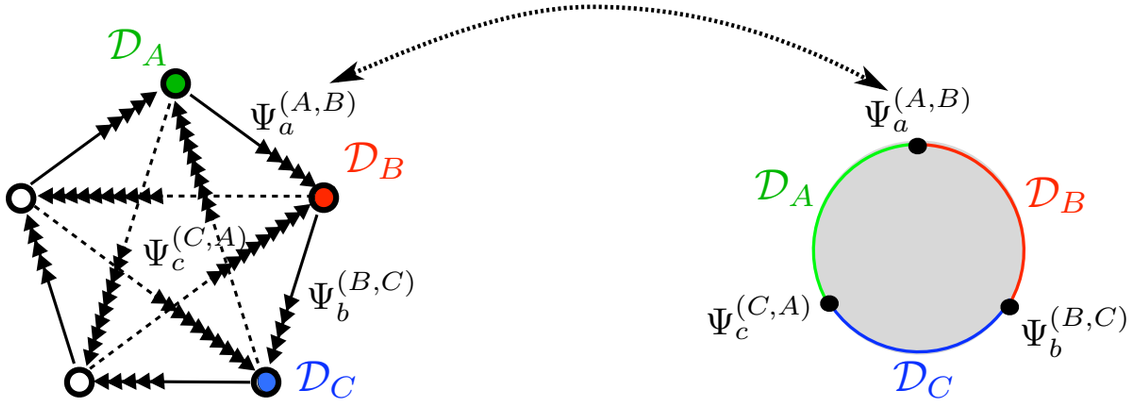
Classical versus Stringy Quantum Geometry

Classical geometrical notions ("branes wrapping p-cycles", gauge bundle configurations on top of them) makes sense only at weak coupling/large radius!



Most of string phenomenology deals with (semi-)classical regime!

Deformation families of 2d TCFTs over whole M

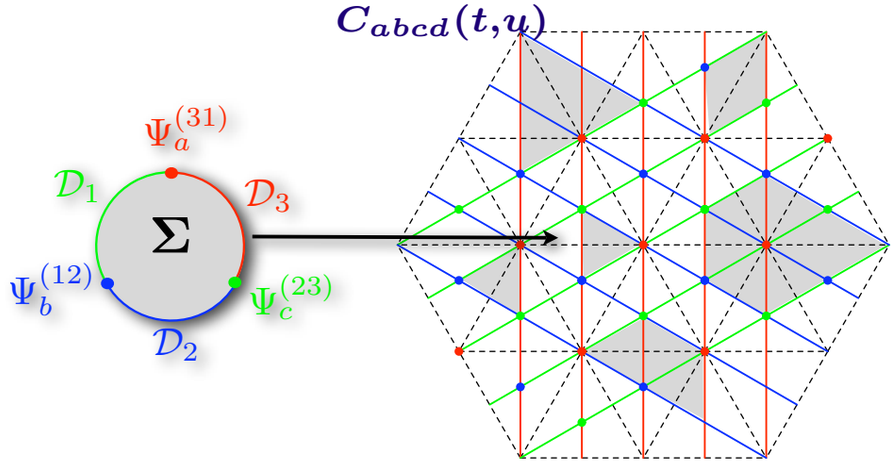


Superpotential ~ closed paths on quiver

$$\mathcal{W}_{eff}(T,u,t) = T_a T_b T_c \underbrace{\langle \Psi_a^{(A,B)} \Psi_b^{(B,C)} \Psi_c^{(C,A)} \rangle}_{C_{abc}(t,u)} + T_a T_b T_c T_d \underbrace{\langle \Psi_a^{(A,B)} \Psi_b^{(B,C)} \Psi_c^{(C,D)} \Psi_d^{(D,A)} \rangle}_{C_{abcd}(t,u)} + \ldots$$

count polygonal instantons

$$C_{abc} \sim e^{-S_{\text{inst}}} \sim q^{\Delta_{abc}} + \ldots..$$



- math. framework: Homological Mirror Symmetry (Kontsevich): map complicated problem (A-model, Fukaya category) to simpler one (B-model, category of coh. sheaves)
- phys. framework: boundary LG based on matrix factorizations

$$Q(x) \cdot Q(x) \;=\; W_{LG}(x)\, 1$$