RECENT RESULTS FROM LATTICE QCD

WWND2016

Claudia Ratti
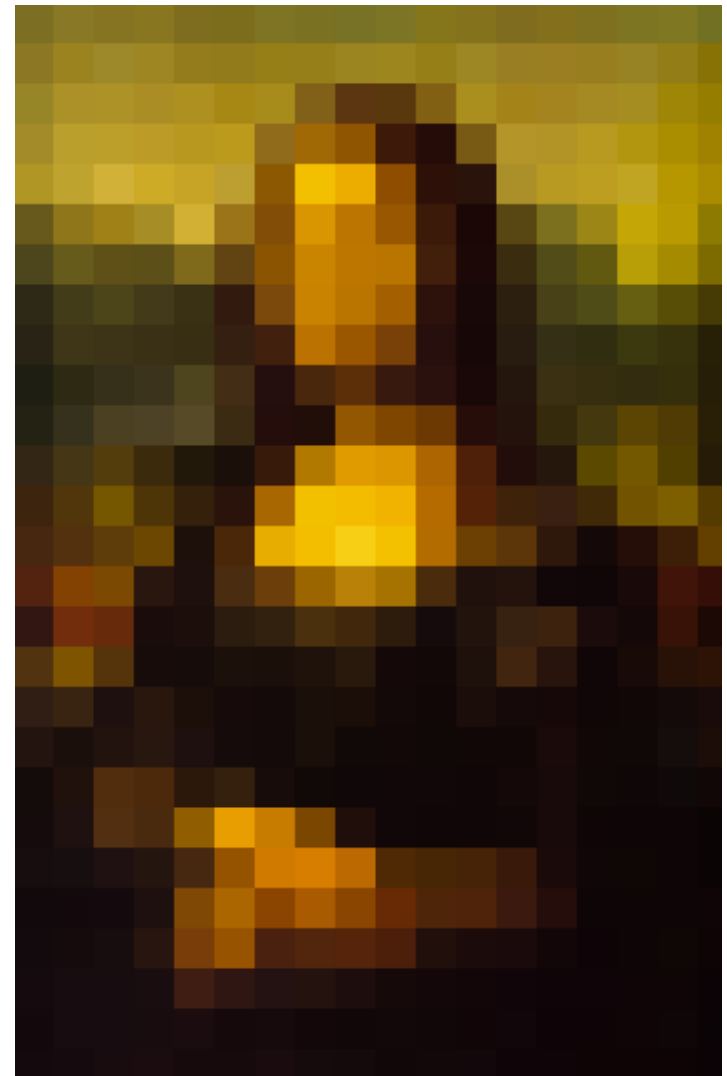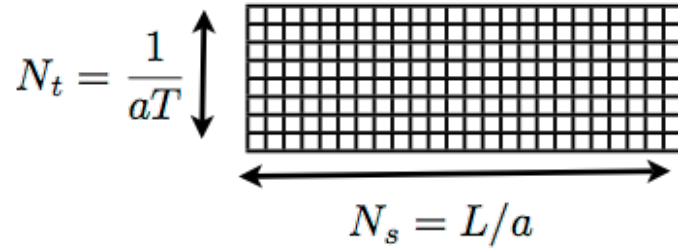University of Houston (USA)

Lattice QCD

- Best first principle-tool to extract predictions for the theory of strong interactions in the non-perturbative regime

- Uncertainties:
  - Statistical: finite sample, error $\sim 1/\sqrt{\text{sample size}}$
  - Systematic: finite box size, unphysical quark masses

- Given enough computer power, uncertainties can be kept under control

- Results from different groups, adopting different discretizations, converge to consistent results

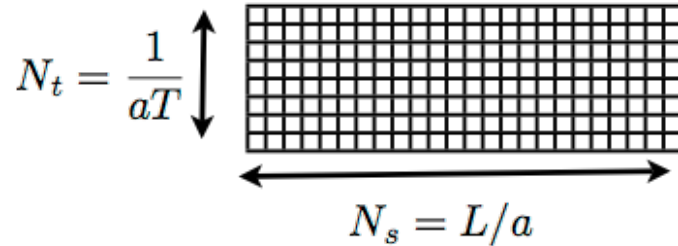- Unprecedented level of accuracy in lattice data

Importance of continuum limit

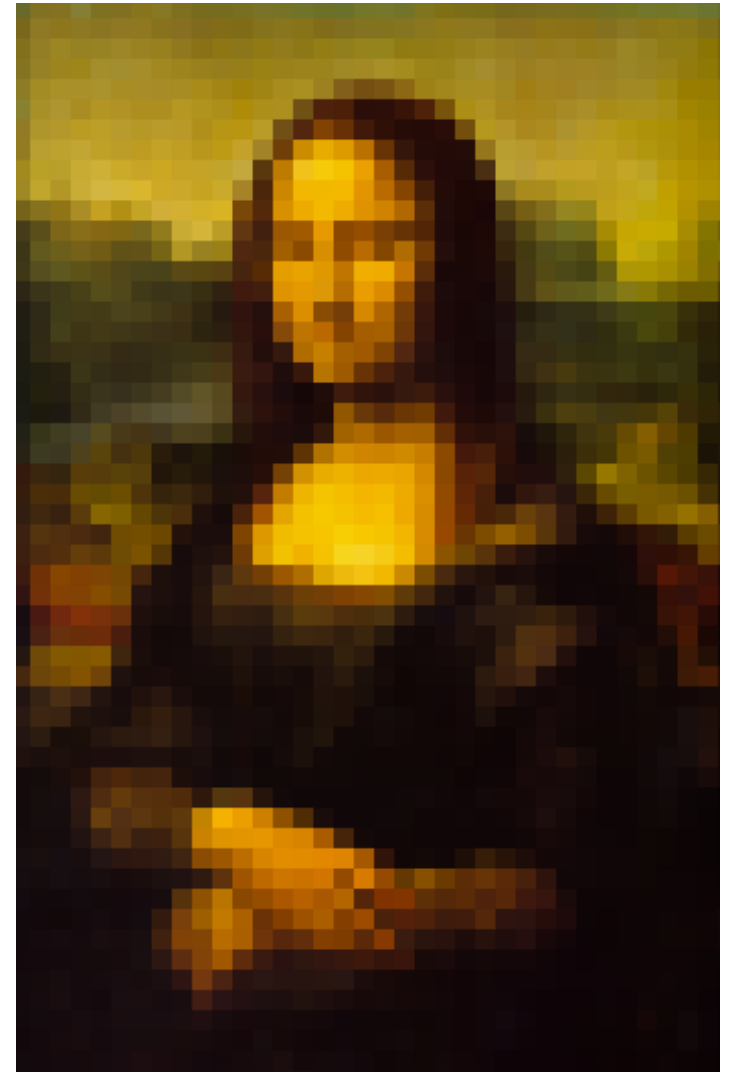□ Lattice action: parametrization used to discretize the Lagrangian of QCD on a space-time grid

$$N_t = \frac{1}{aT}$$

$$N_s = L/a$$

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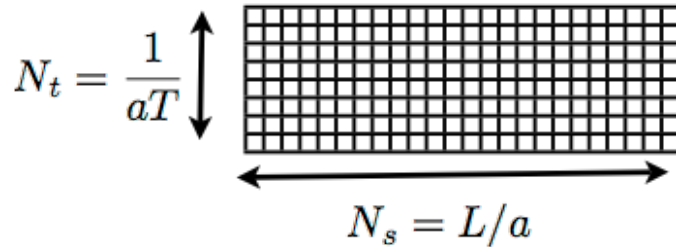
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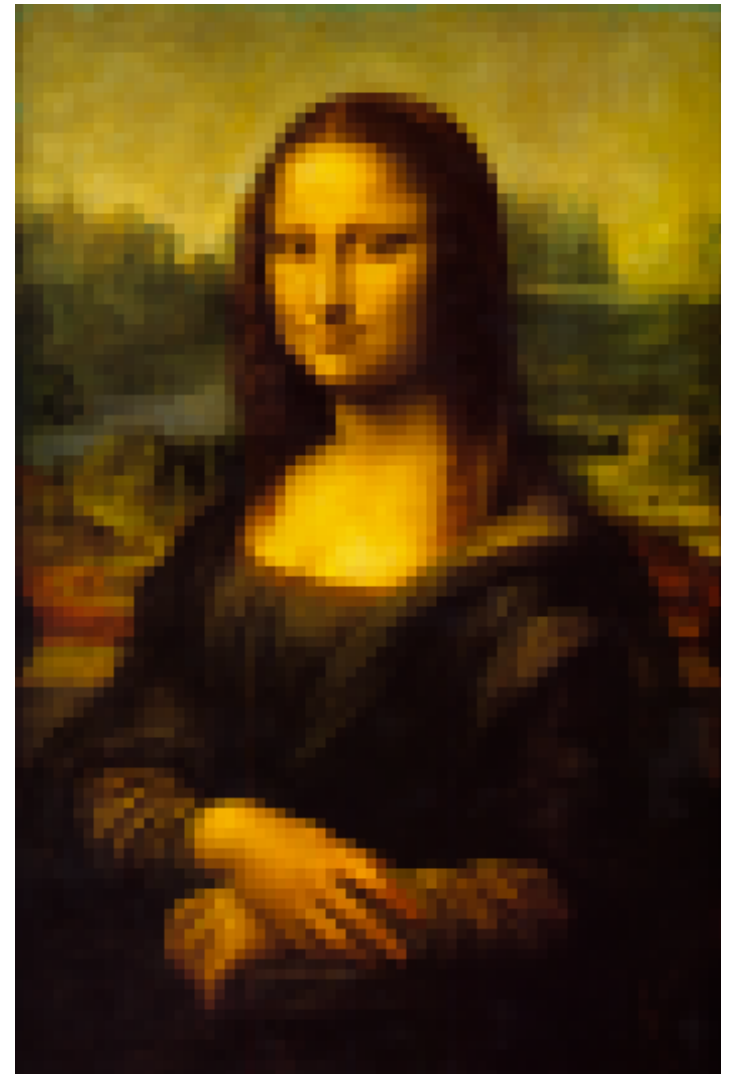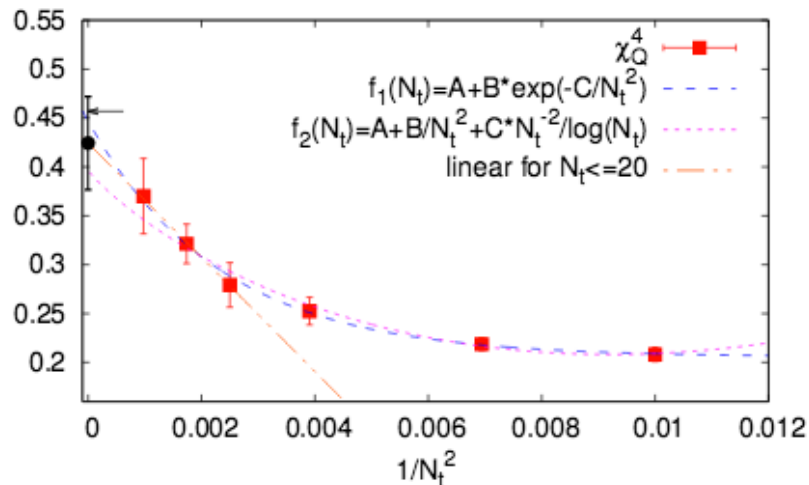
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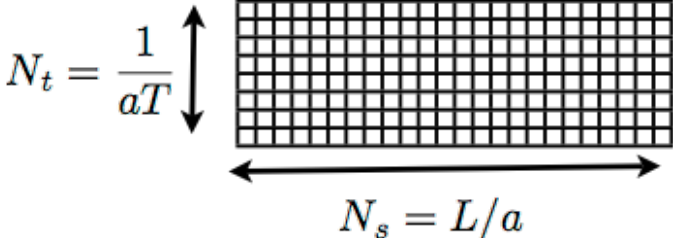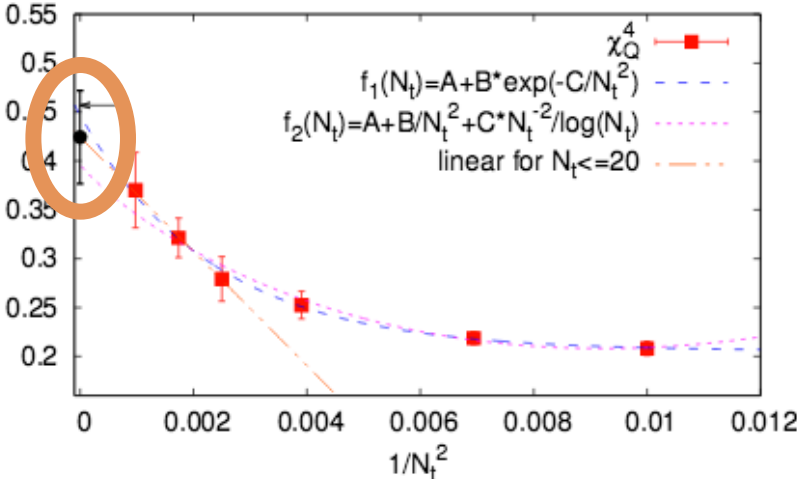- Lattice action: parametrization used to discretize the Lagrangian of QCD on a space-time grid

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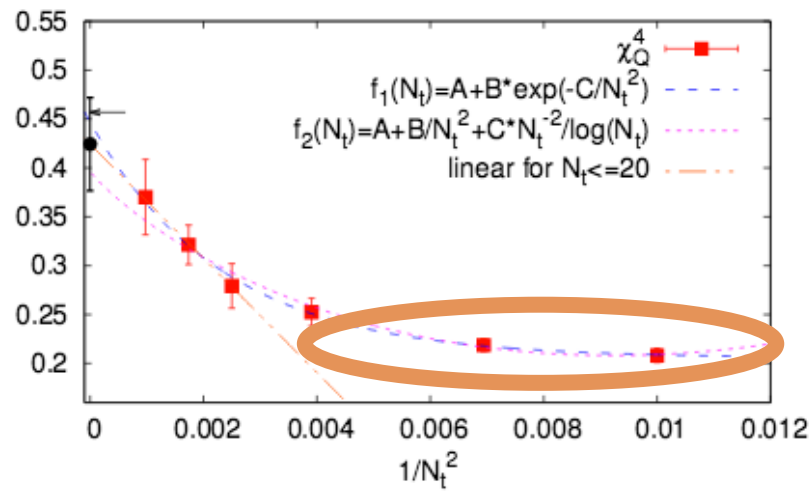
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Importance of continuum limit

□ Observables are affected by discretization effects differently

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- In quantitative predictions, finite-N_t results can lead to misleading information

Importance of continuum limit

□ Observables are affected by discretization effects differently

□ In quantitative predictions, finite-N_t results can lead to misleading information

□ Message: continuum extrapolated data always preferable

Low temperature phase: HRG model

Dashen, Ma, Bernstein; Prakash, Venugopalan, Karsch, Tawfik, Redlich

- Interacting hadronic matter in the ground state can be well approximated by a non-interacting resonance gas

- The pressure can be written as:

$$p^{HRG}/T^4 = \frac{1}{VT^3} \sum_{i \in mesons} \ln \mathcal{Z}^M_{m_i}(T,V,\mu_{X^a}) + \frac{1}{VT^3} \sum_{i \in baryons} \ln \mathcal{Z}^B_{m_i}(T,V,\mu_{X^a})$$

where

$$\ln \mathcal{Z}^{M/B}_{m_i} = \mp \frac{V d_i}{2\pi^2} \int_0^\infty dk k^2 \ln(1 \mp z_i e^{-\varepsilon_i/T}) \ ,$$

with energies $\varepsilon_i = \sqrt{k^2 + m_i^2}$, degeneracy factors d_i and fugacities

$$z_i = \exp\left(\left(\sum_a X_i^a \mu_{X^a} \right)/T \right) \ .$$

X^a: all possible conserved charges, including the baryon number B, electric charge Q, strangeness S.

- Up to which temperature do we expect agreement with the lattice data?

High temperature limit

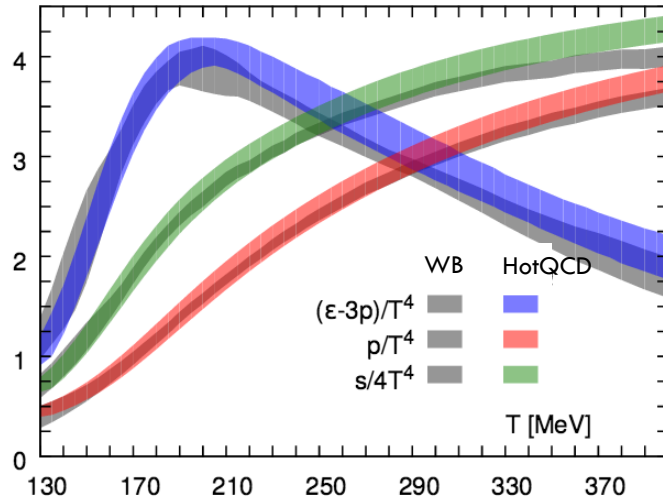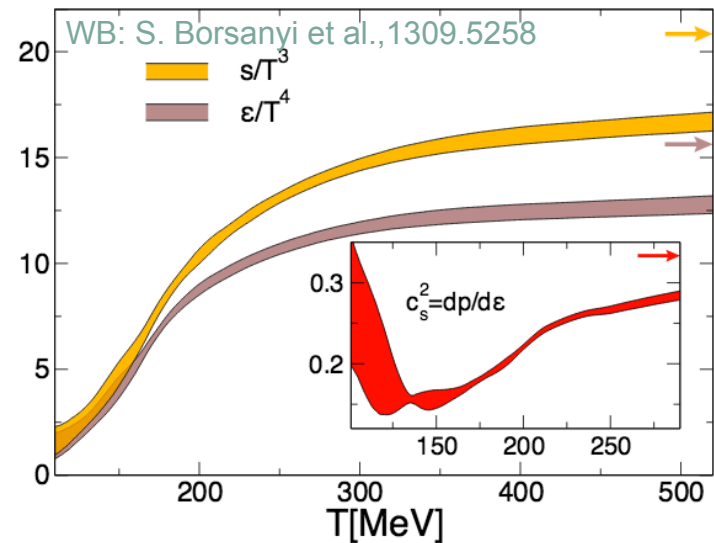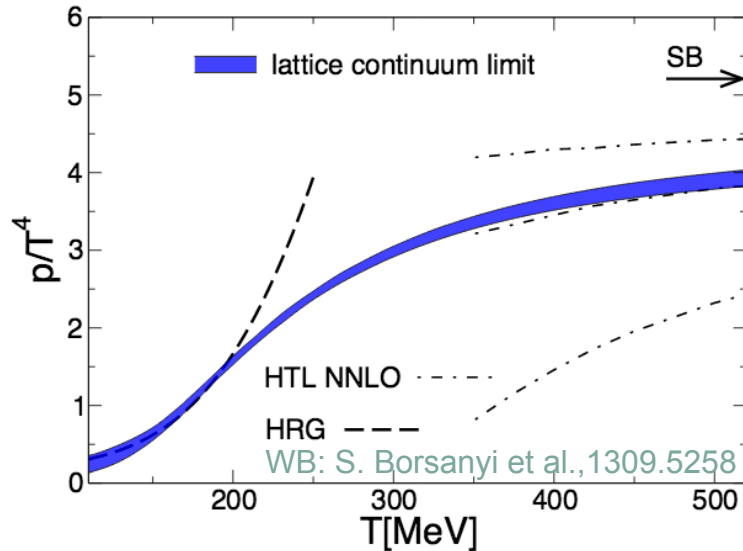☐ QCD thermodynamics approaches that of a non-interacting, massless quark-gluon gas:

$$\left(\frac{P}{T^4}\right)_{\text{ideal}} = \frac{8\pi^2}{45} + \sum_{f=u,d,s} \left[\frac{7\pi^2}{60} + \frac{1}{2}\left(\frac{\mu_f}{T}\right)^2 + \frac{1}{4\pi^2}\left(\frac{\mu_f}{T}\right)^4\right]$$

☐ We can switch on the interaction and systematically expand the observables in series of the coupling *g*

☐ Resummation of diagrams (HTL) or dimensional reduction are needed, to improve convergence

Braaten, Pisarski (1990); Haque et al. (2014); Hietanen et al (2009)

☐ At what temperature does perturbation theory break down?

QCD Equation of state at $\mu_B=0$

□ EoS available in the continuum limit, with realistic quark masses

□ Agreement between stout and HISQ action for all quantities

WB: S. Borsanyi et al., 1309.5258, PLB (2014)
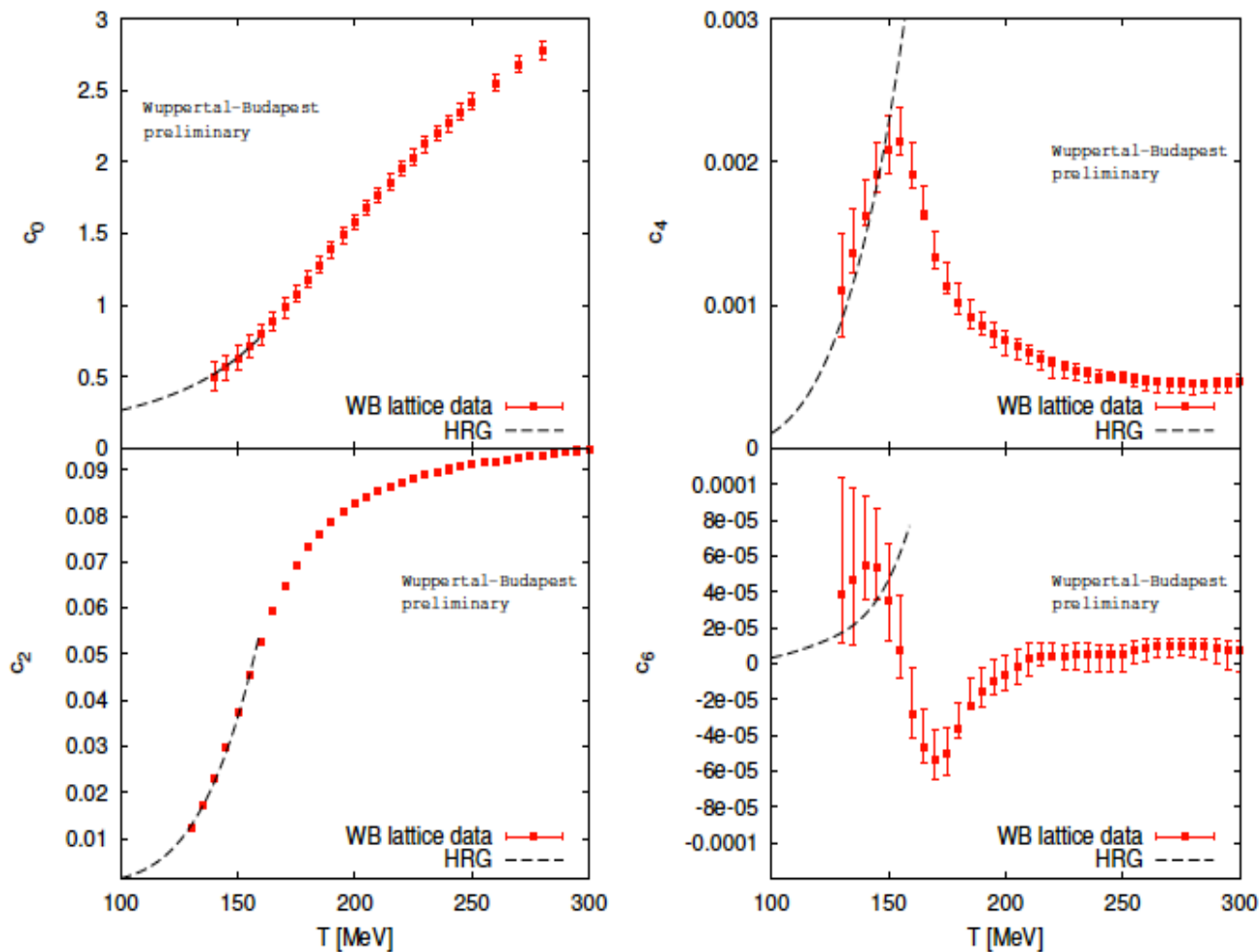HotQCD: A. Bazavov et al., 1407.6387, PRD (2014)

Sign problem

□ The QCD path integral is computed by Monte Carlo algorithms which samples field configurations with a weight proportional to the exponential of the action

$$Z(\mu_B, T) = \text{Tr} \left(e^{-\frac{H_{\text{QCD}} - \mu_B N_B}{T}} \right) = \int \mathcal{D}U e^{-S_G[U]} \det M[U, \mu_B]$$

□ detM[μ_B] complex → Monte Carlo simulations are not feasible

□ We can rely on a few approximate methods, viable for small μ_B/T:

◘ Taylor expansion of physical quantities around μ=0 (Bielefeld-Swansea collaboration 2002; R. Gavai, S. Gupta 2003)

◘ Reweighting (complex phase moved from the measure to observables) (Barbour et al. 1998; Z. Fodor and S, Katz, 2002)

◘ Simulations at imaginary chemical potentials (plus analytic continuation) (Alford, Kapustin, Wilczek, 1999; de Forcrand, Philipsen, 2002; D'Elia, Lombardo 2003)
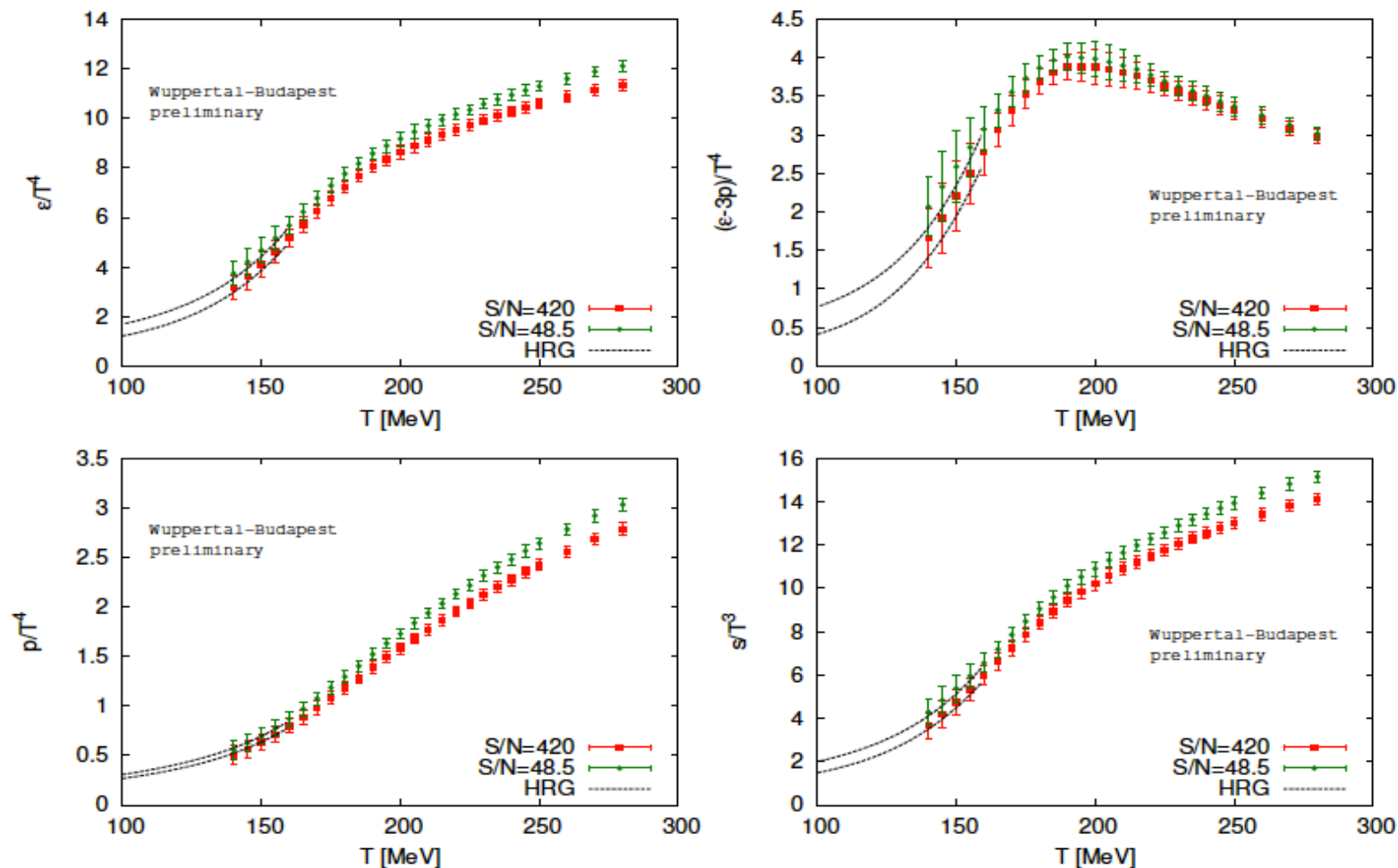
Equation of state at $\mu_B > 0$

- Expand the pressure in powers of μ_B

$$\frac{p(\mu_B)}{T^4} = c_0 + c_2 \left(\frac{\mu_B}{T}\right)^2 + $$
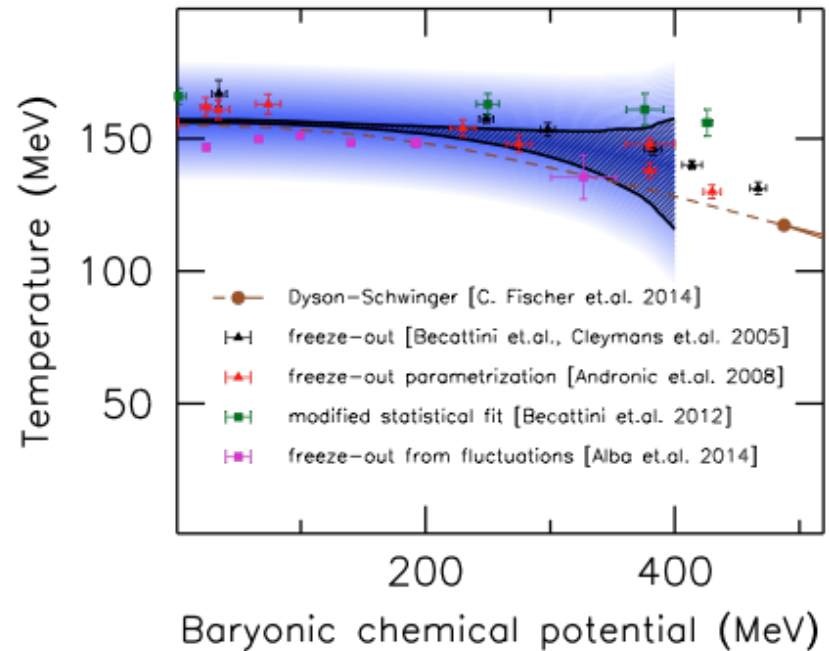$$+ c_4 \left(\frac{\mu_B}{T}\right)^4 + c_6 \left(\frac{\mu_B}{T}\right)^6 + \mathcal{O}(\mu_B^8)$$
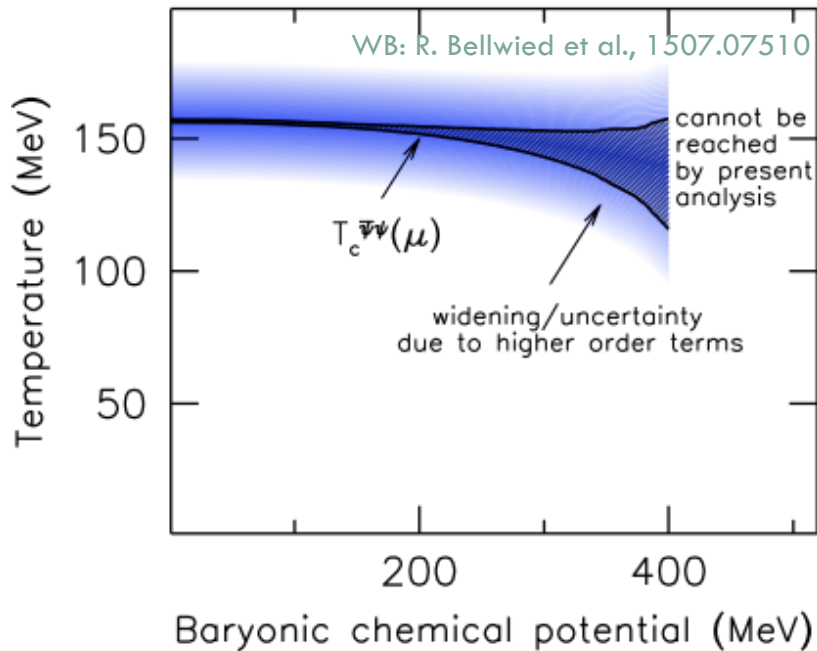
- Continuum extrapolated results for c_2, c_4, c_6 at the physical mass

WB: S. Borsanyi et al., preliminary

Equation of state at $\mu_B > 0$

- Calculate the EoS along the constant S/N trajectories

QCD phase diagram

WB: R. Bellwied et al., 1507.07510

$T_c^{\bar{\Psi}\Psi}(\mu)$

widening/uncertainty
due to higher order terms

cannot be
reached
by present
analysis

- Dyson-Schwinger [C. Fischer et.al. 2014]
- freeze-out [Becattini et.al., Cleymans et.al. 2005]
- freeze-out parametrization [Andronic et.al. 2008]
- modified statistical fit [Becattini et.al. 2012]
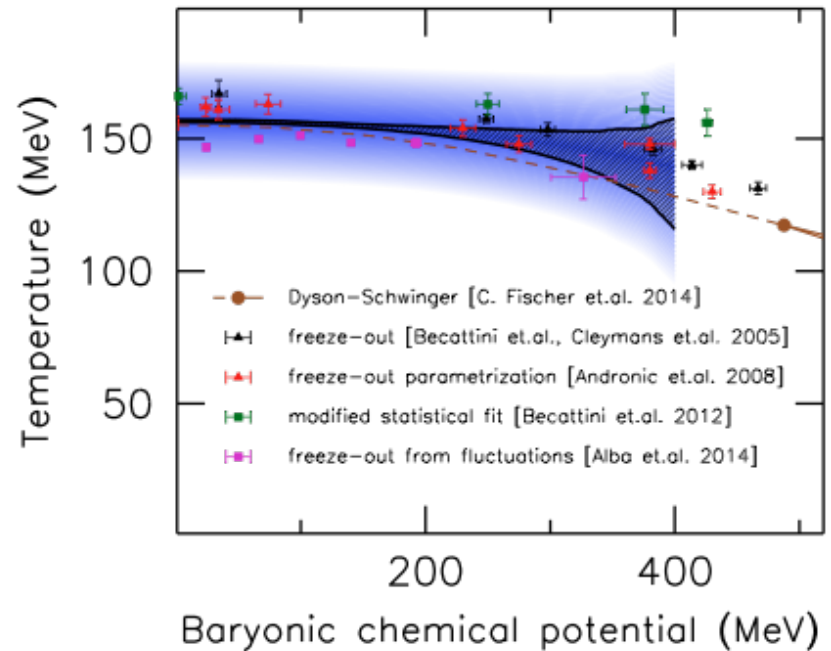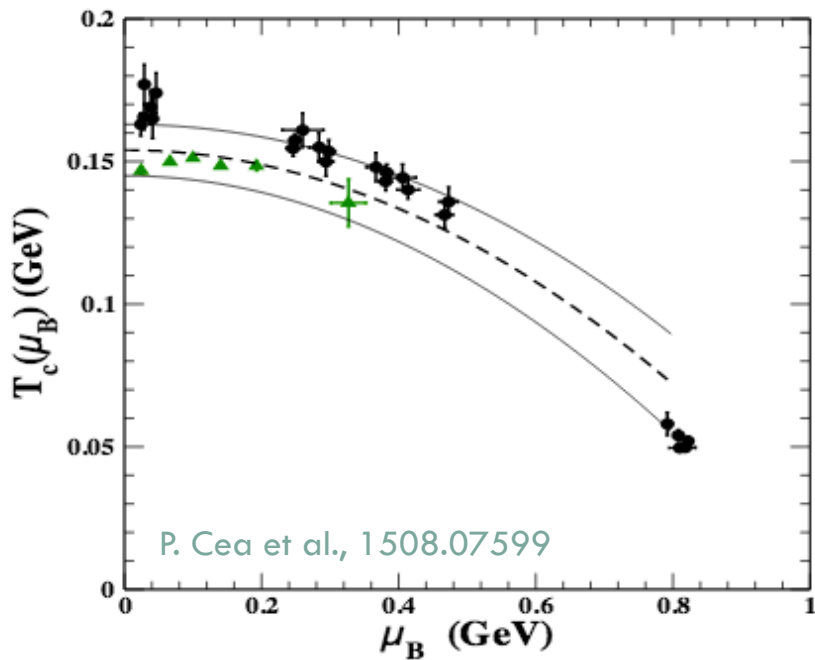- freeze-out from fluctuations [Alba et.al. 2014]

Curvature κ defined as:

$$\frac{T_c(\mu_B)}{T_c(\mu=0)} = 1 - \kappa \left(\frac{\mu_B}{T_c(\mu_B)} \right)^2 + \lambda \left(\frac{\mu_B}{T_c(\mu_B)} \right)^4 \cdots$$

Recent results:

$$\kappa = 0.0149 \pm 0.0021$$

WB: R. Bellwied et al., 1507.07510

QCD phase diagram
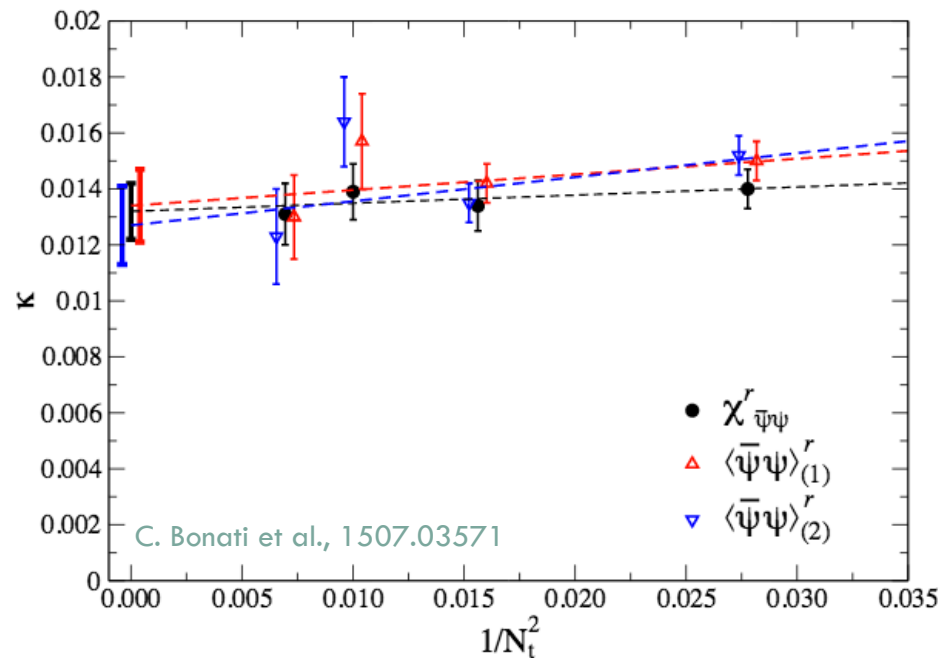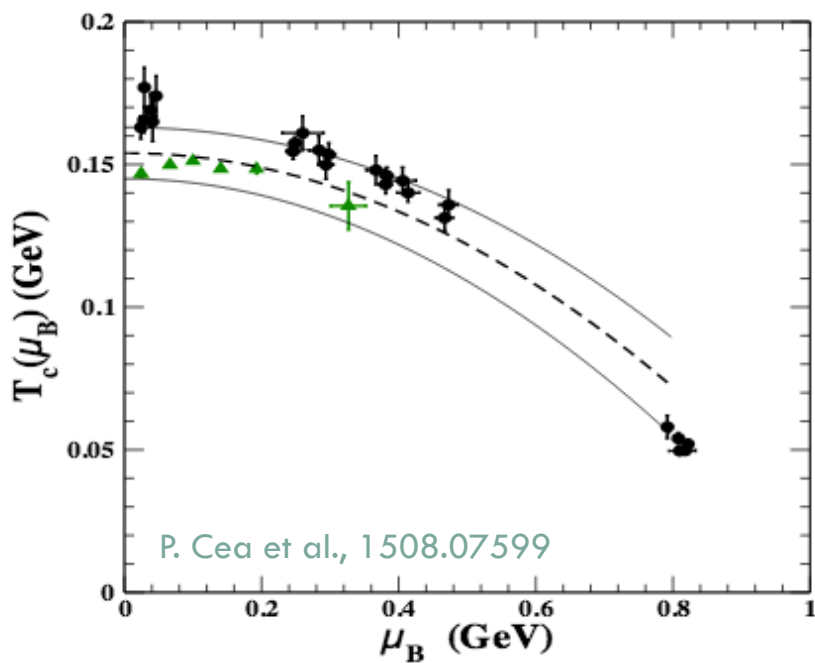

P. Cea et al., 1508.07599

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$$\kappa = 0.020(4)$$

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QCD phase diagram

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C. Bonati et al., 1507.03571

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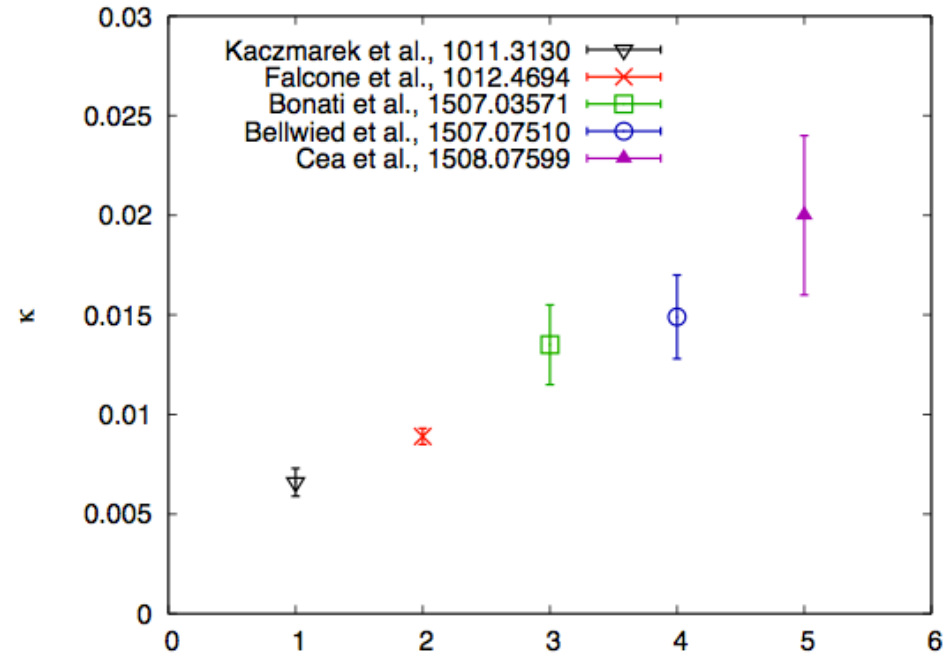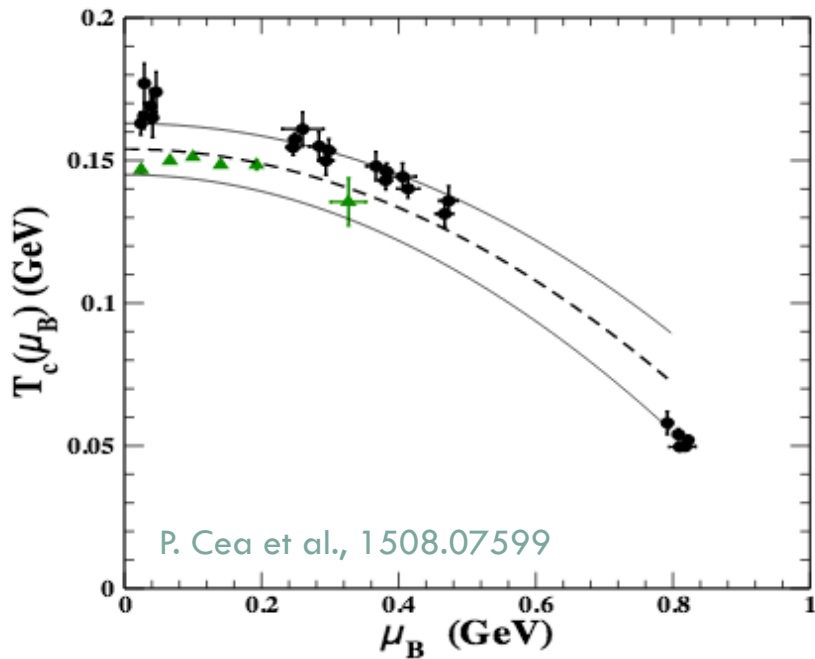
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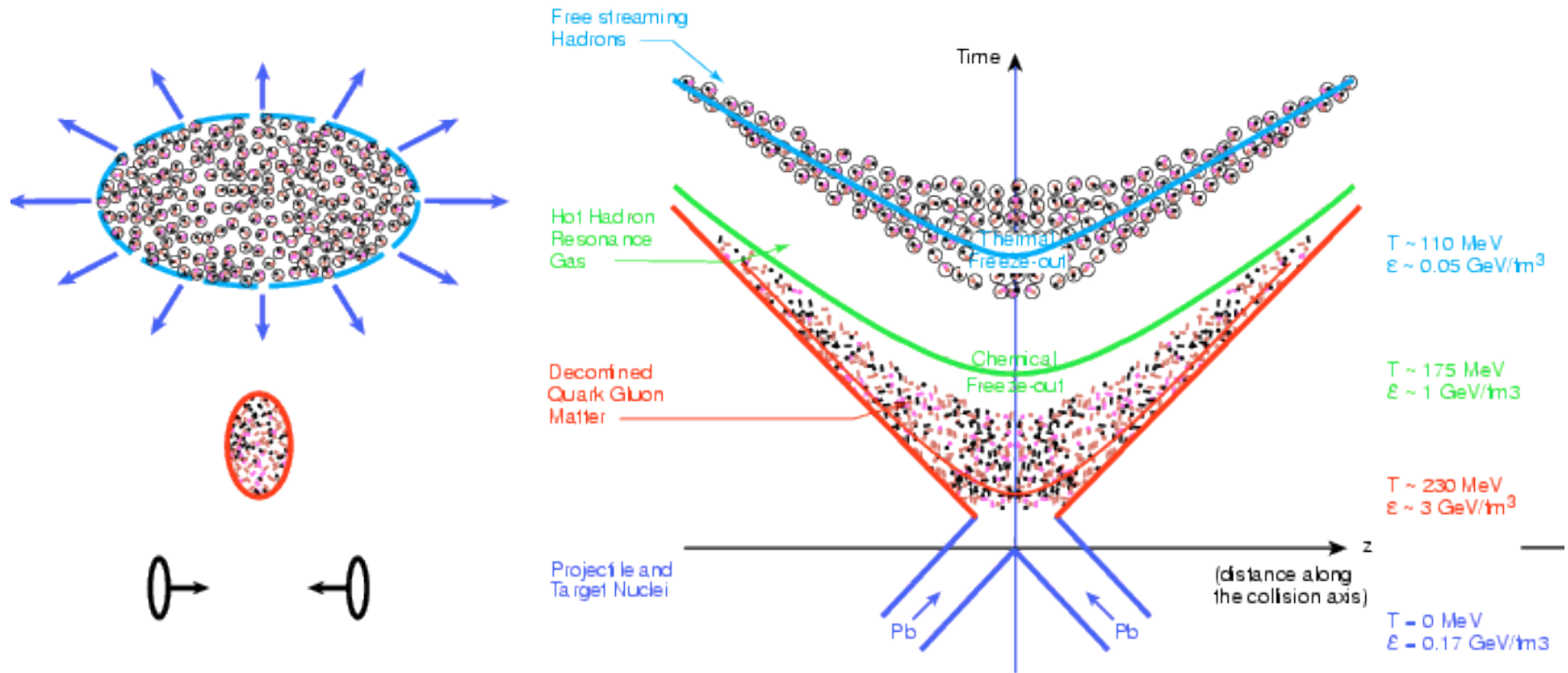
$$\kappa = 0.0135(20)$$

C. Bonati et al., 1507.03571

QCD phase diagram

P. Cea et al., 1508.07599

Kaczmarek et al., 1011.3130
Falcone et al., 1012.4694
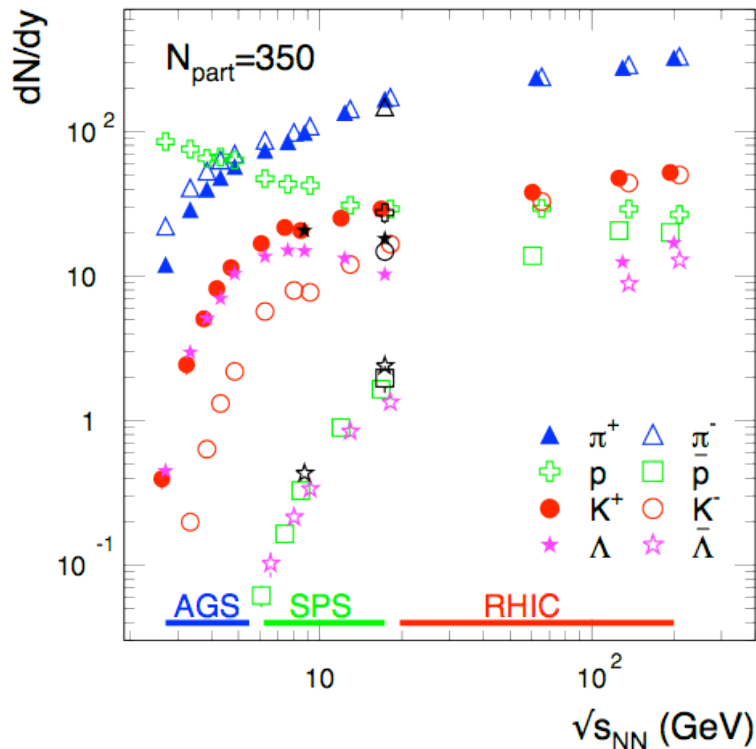Bonati et al., 1507.03571
Bellwied et al., 1507.07510
Cea et al., 1508.07599

1. Kaczmarek et al., Nf=2+1, p4 staggered action, Taylor expansion, μ_s=0, N_t=8

2. Falcone et al., Nf=2+1, p4 staggered action, analytic continuation, μ_s=μ_u=μ_d, N_t=4

3. Bonati et al., Nf=2+1,stout staggered action, analytic continuation, μ_s=0, continuum extrapolated

4. Bellwied et al. (WB), Nf=2+1, 4stout staggered action, analytic continuation, <n_s>=0, cont. extrap.

5. Cea et al., Nf=2+1, HISQ staggered action, analytic continuation, μ_s=μ_u=μ_d , cont. extrapolated

Evolution of a Heavy Ion Collision

- Chemical freeze-out: inelastic reactions cease: the chemical composition of the system is fixed (particle yields and fluctuations)

- Kinetic freeze-out: elastic reactions cease: spectra and correlations are frozen (free streaming of hadrons)

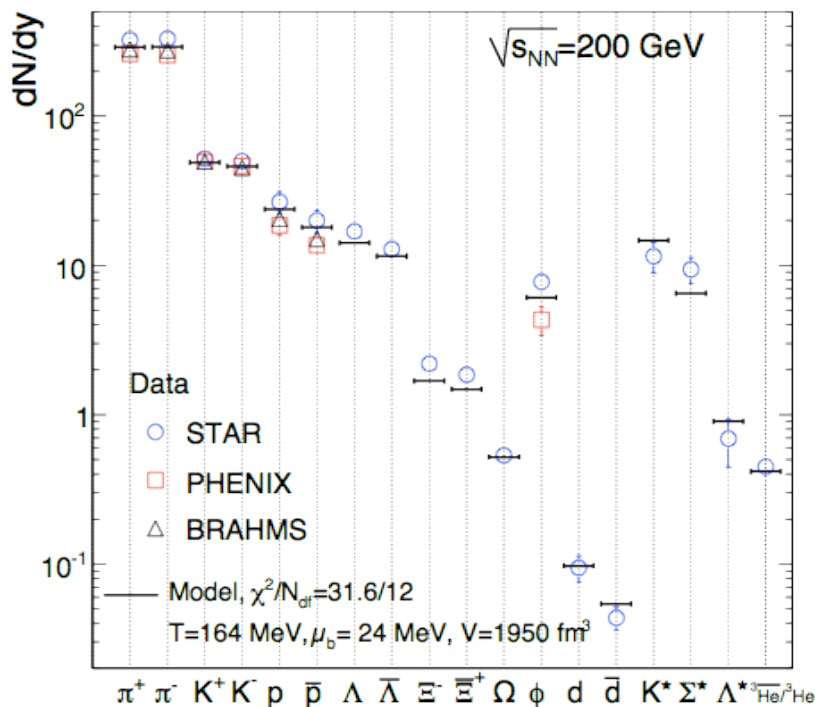- Hadrons reach the detector

Hadron yields

- $E=mc^2$: lots of particles are created

- Particle counting (average over many events)

- Take into account:
  - detector inefficiency
  - missing particles at low p_T
  - decays

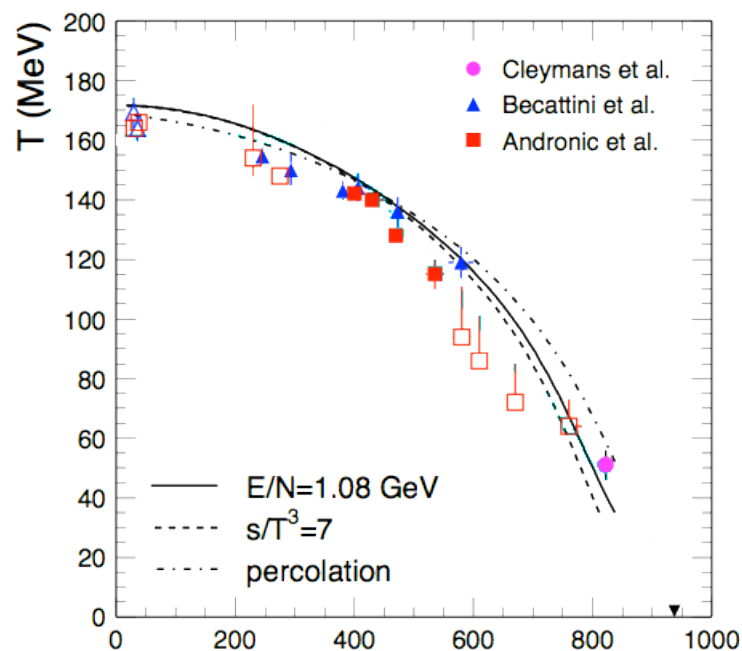- HRG model: test hypothesis of hadron abundancies in equilibrium

$$N_i = -T\frac{\partial \ln Z_i}{\partial \mu} = \frac{g_i V}{2\pi^2}\int_0^\infty \frac{p^2 \mathrm{d}p}{\exp[(E_i - \mu_i)/T] \pm 1}$$

The thermal fits

- Fit is performed minimizing the χ^2

- **Fit to yields**: parameters T, µB, V

- **Fit to ratios**: the volume V cancels out

- Changing the collision energy, it is possible to draw the freeze-out line in the T, µB plane

Fluctuations of conserved charges

- Definition:

$$\chi_{lmn}^{BSQ} = \frac{\partial^{l+m+n} p/T^4}{\partial(\mu_B/T)^l \partial(\mu_S/T)^m \partial(\mu_Q/T)^n}.$$

- Relationship between chemical potentials:

$$\mu_u = \frac{1}{3}\mu_B + \frac{2}{3}\mu_Q;$$

$$\mu_d = \frac{1}{3}\mu_B - \frac{1}{3}\mu_Q;$$

$$\mu_s = \frac{1}{3}\mu_B - \frac{1}{3}\mu_Q - \mu_S.$$

- They can be calculated on the lattice and compared to experiment

Connection to experiment

- Fluctuations of conserved charges are the cumulants of their event-by-event distribution

$$\text{mean}: M = \chi_1 \qquad\qquad \text{variance}: \sigma^2 = \chi_2$$

$$\text{skewness}: S = \chi_3/\chi_2^{3/2} \qquad\qquad \text{kurtosis}: \kappa = \chi_4/\chi_2^2$$

$$S\sigma = \chi_3/\chi_2 \qquad\qquad \kappa\sigma^2 = \chi_4/\chi_2$$

$$M/\sigma^2 = \chi_1/\chi_2 \qquad\qquad S\sigma^3/M = \chi_3/\chi_1$$

- Lattice QCD results are functions of temperature and chemical potential
  - By comparing lattice results and experimental measurement we can extract the freeze-out parameters from first principles

F. Karsch: Centr. Eur. J. Phys. (2012)

Things to keep in mind

- Effects due to volume variation because of finite centrality bin width
  - Experimentally corrected by centrality-bin-width correction method
    V. Skokov et al., PRC (2013)
- Finite reconstruction efficiency
  - Experimentally corrected based on binomial distribution A.Bzdak,V.Koch, PRC (2012)
- Spallation protons
  - Experimentally removed with proper cuts in p_T
- Canonical vs Gran Canonical ensemble
  - Experimental cuts in the kinematics and acceptance V. Koch, S. Jeon, PRL (2000)
- Proton multiplicity distributions vs baryon number fluctuations
  - Recipes for treating proton fluctuations
    M. Asakawa and M. Kitazawa, PRC(2012), M. Nahrgang et al., 1402.1238
- Final-state interactions in the hadronic phase
  - Consistency between different charges = fundamental test

    J.Steinheimer et al., PRL (2013)

"Baryometer and Thermometer"

Let us look at the Taylor expansion of R^B_{31}

$$R^B_{31}(T,\mu_B) = \frac{\chi^B_3(T,\mu_B)}{\chi^B_1(T,\mu_B)} = \frac{\chi^B_4(T,0) + \chi^{BQ}_{31}(T,0)q_1(T) + \chi^{BS}_{31}(T,0)s_1(T)}{\chi^B_2(T,0) + \chi^{BQ}_{11}(T,0)q_1(T) + \chi^{BS}_{11}(T,0)s_1(T)} + \mathcal{O}(\mu^2_B)$$

- To order μ^2_B it is independent of μ_B: it can be used as a thermometer
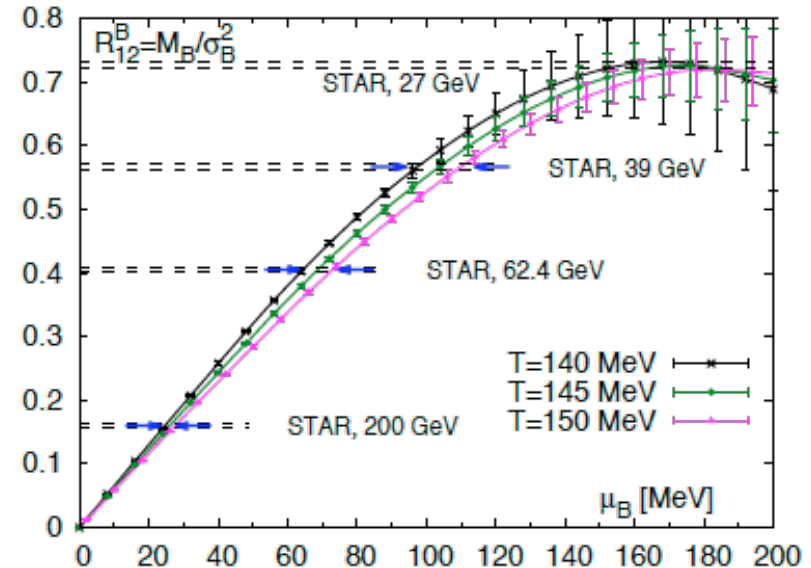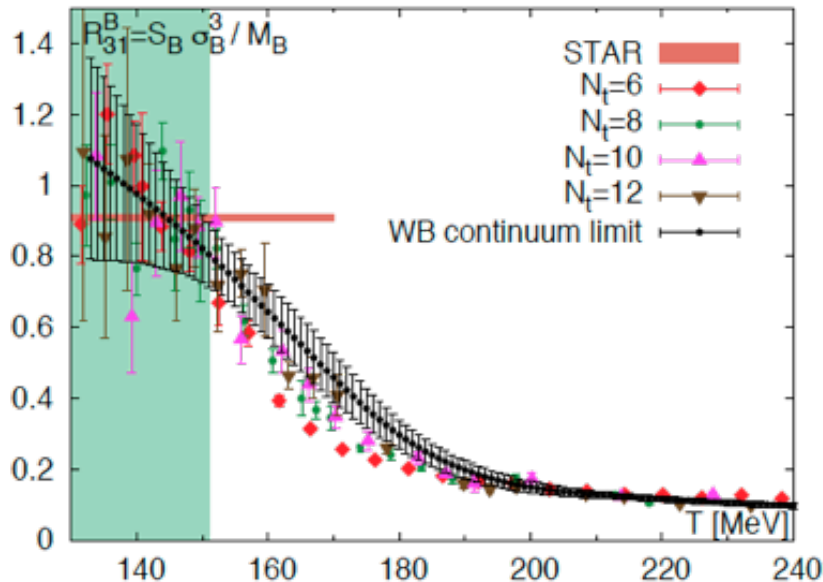
- Let us look at the Taylor expansion of R^B_{12}

$$R^B_{12}(T,\mu_B) = \frac{\chi^B_1(T,\mu_B)}{\chi^B_2(T,\mu_B)} = \frac{\chi^B_2(T,0) + \chi^{BQ}_{11}(T,0)q_1(T) + \chi^{BS}_{11}(T,0)s_1(T)}{\chi^B_2(T,0)} \frac{\mu_B}{T} + \mathcal{O}(\mu^3_B)$$

- Once we extract T from R^B_{31}, we can use R^B_{12} to extract μ_B

Freeze-out parameters from B fluctuations

Thermometer: $\dfrac{\chi_3^B(T,\mu_B)}{\chi_1^B(T,\mu_B)}=S_B\sigma_B^3/M_B$ Baryometer: $\dfrac{\chi_1^B(T,\mu_B)}{\chi_2^B(T,\mu_B)}=\sigma_B^2/M_B$

WB: S. Borsanyi et al., PRL (2014)
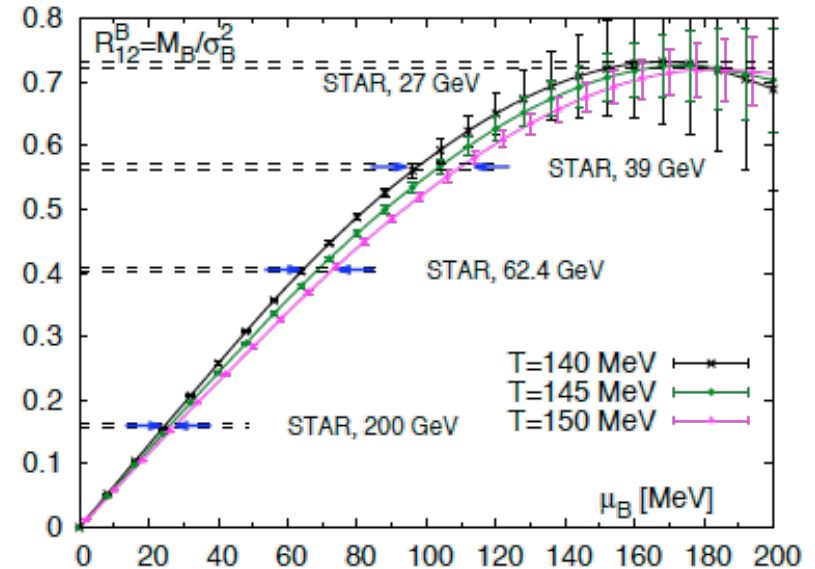STAR collaboration, PRL (2014)

- Upper limit: $T_f \le 151\pm4$ MeV

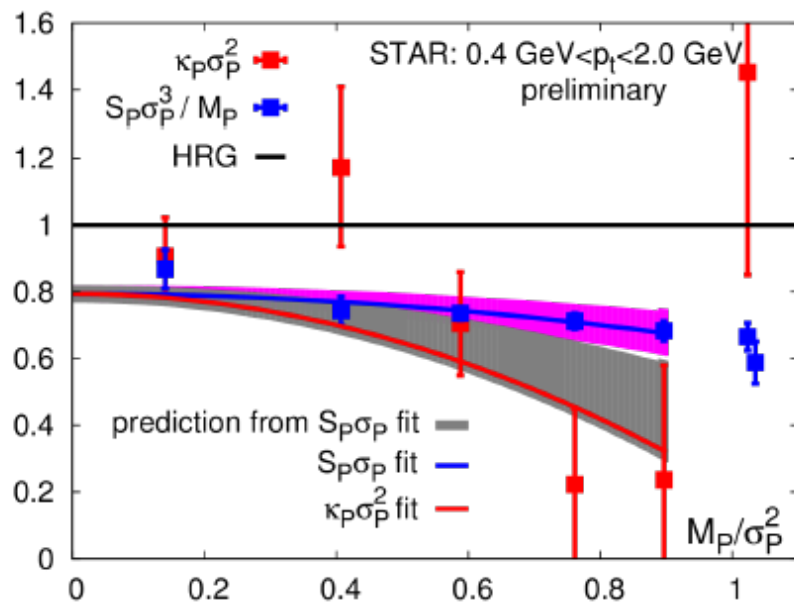- Consistency between freeze-out chemical potential from electric charge and baryon number is found.
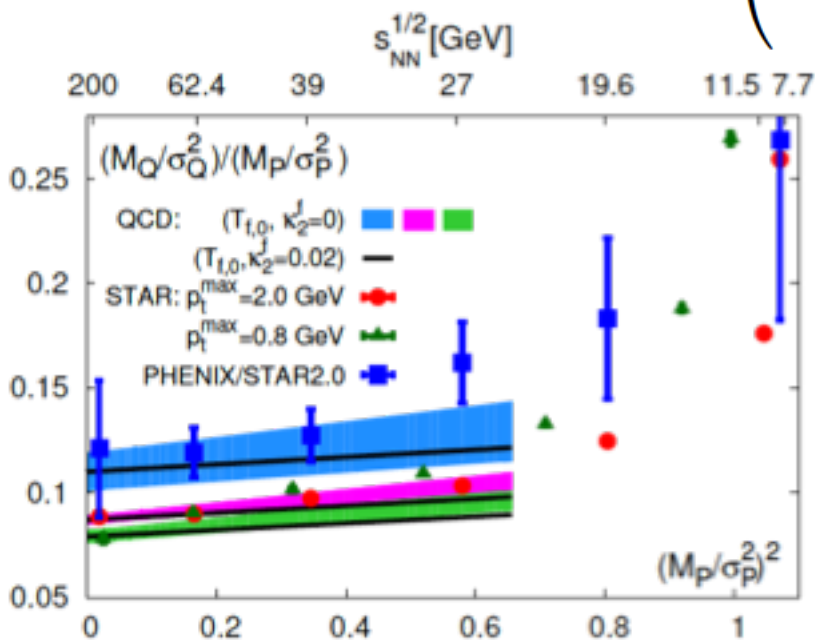
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$\sqrt{s}[GeV]$	μ_B^f [MeV] (from B)	μ_B^f [MeV] (from Q)
200	25.8±2.7	22.8±2.6
62.4	69.7±6.4	66.6±7.9
39	105±11	101±10
27	-	136±13.8

WB: S. Borsanyi et al., PRL (2014)
STAR collaboration, PRL (2014)

☐ Upper limit: $T_f \leq$ 151±4 MeV

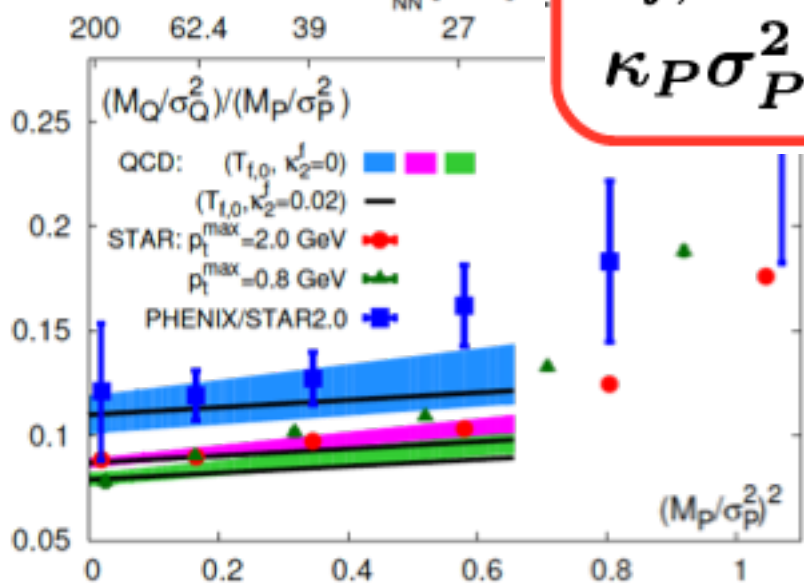☐ Consistency between freeze-out chemical potential from electric charge and baryon number is found.

Curvature of the freeze-out line

☐ Parametrization of the freeze-out line:

$$T_f(\mu_B) = T_{f,0}\left(1 - \kappa_2^f \bar{\mu}_B^2 - \kappa_4^f \bar{\mu}_B^4\right)$$

☐ Taylor expansion of the "ratio of ratios" $R_{12}^{QB} = [M_Q/\sigma_Q^2]/[M_B/\sigma_B^2]$

$$R_{12}^{QB} = R_{12}^{QB,0} + \left(R_{12}^{QB,2} - \kappa_2^f T_{f,0} \frac{dR_{12}^{QB,0}}{dT}\bigg|_{T_{f,0}}\right)\hat{\mu}_B^2$$

A. Bazavov et al., 1509.05786
STAR0.8: PRL (2013)
STAR2.0: X. Luo, PoS CPOD 2014
PHENIX: 1506.07834

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- Taylor expansion of the "ratio of ratios" $R_{12}^{QB} = [M_Q/\sigma_Q^2]/[M_B/\sigma_B^2]$

$$R_{12}^{QB} = \left(\quad \right) \hat{\mu}_B^2$$

$$\kappa_2^f < 0.011$$
$$T_{f,0} = (147 \pm 2)\ \text{MeV}$$
$$\kappa_P \sigma_P^2 < S_P \sigma_P$$

A. Bazavov et al., 1509.05786
STAR0.8: PRL (2013)
STAR2.0: X. Luo, PoS CPOD 2014
PHENIX: 1506.07834

Freeze-out line from first principles

☐ Use T- and μ_B-dependence of R_{12}^Q and R_{12}^B for a combined fit:

$$R_{12}^Q(T,\mu_B) = \frac{\chi_1^Q(T,\mu_B)}{\chi_2^Q(T,\mu_B)} = \frac{\chi_{11}^{QB}(T,0) + \chi_2^Q(T,0)q_1(T) + \chi_{11}^{QS}(T,0)s_1(T)}{\chi_2^Q(T,0)} \frac{\mu_B}{T} + \mathcal{O}(\mu_B^3).$$

$$R_{12}^B(T,\mu_B) = \frac{\chi_1^B(T,\mu_B)}{\chi_2^B(T,\mu_B)} = \frac{\chi_2^B(T,0) + \chi_{11}^{BQ}(T,0)q_1(T) + \chi_{11}^{BS}(T,0)s_1(T)}{\chi_2^B(T,0)} \frac{\mu_B}{T} + \mathcal{O}(\mu_B^3)$$

WB: S. Borsanyi et al., in preparation

What about strangeness freeze-out?

□ Yield fits seem to hint at a higher temperature for strange particles

R. Preghenella for ALICE, SQM 2012

M. Floris, QM 2014.

Initial analysis of LHC data

☐ Fluctuation data not yet available

☐ Assuming Skellam distribution, can use yields: $\hat{\chi}_N = \frac{1}{VT^3}\left(\langle N_q \rangle + \langle N_{-q} \rangle\right)$
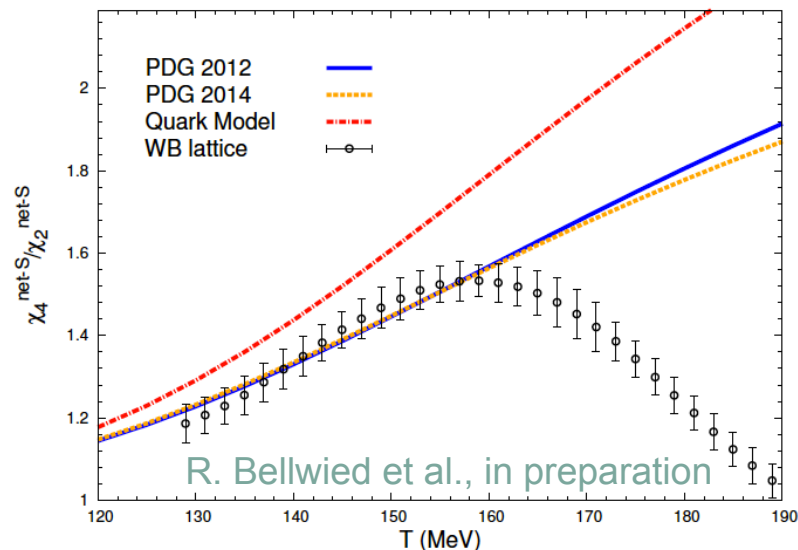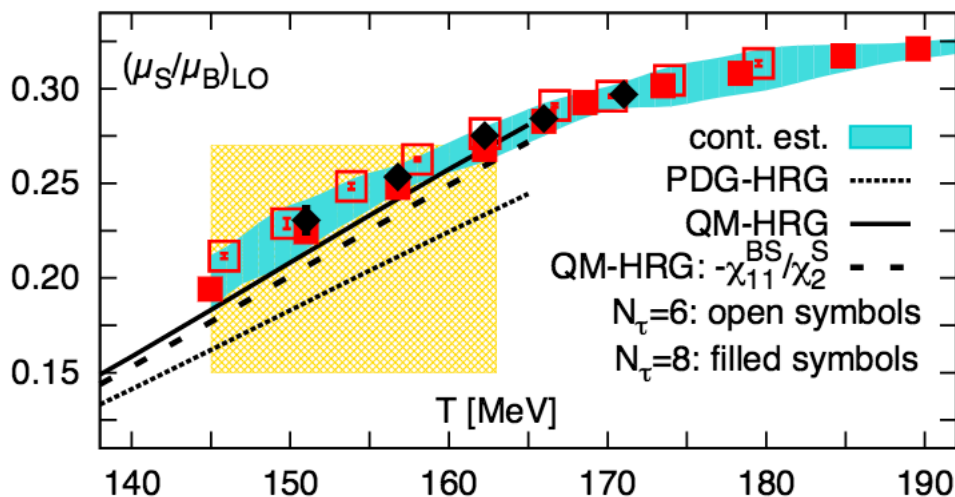

P. Braun-Munzinger et al., PLB (2015)

☐ Slightly higher temperature than at RHIC: $(150 < T_f < 163)$ MeV

☐ Looking forward to fluctuation measurements at the LHC

Missing strange states?

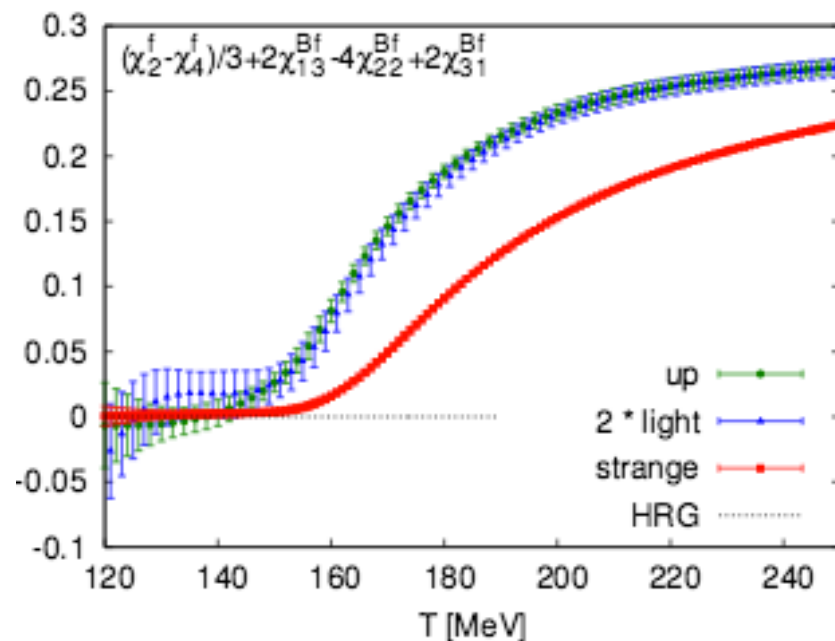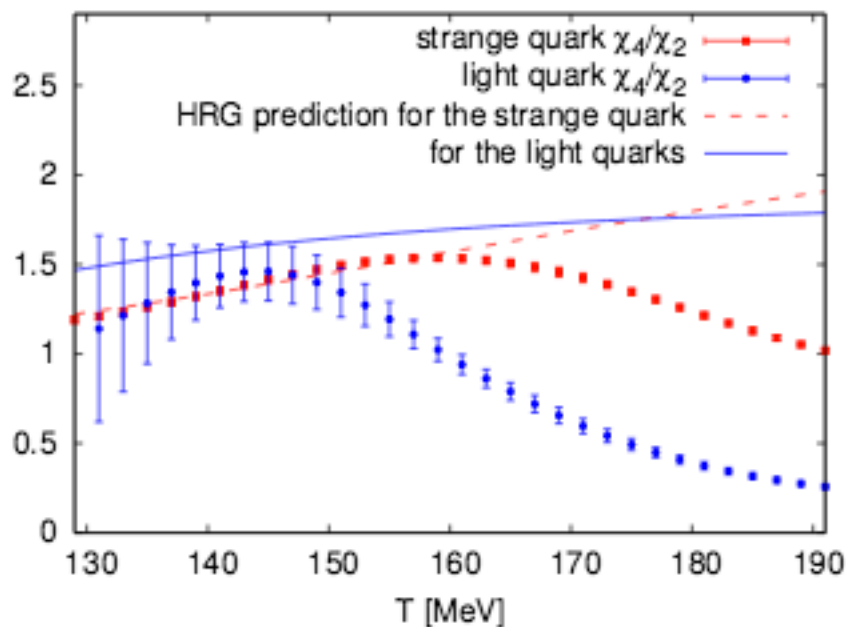- Quark Model predicts not-yet-detected (multi-)strange hadrons

- QM-HRG improves the agreement with lattice results for some observables but it worsens it for some other ones

- The effect is only relevant at finite μ_B

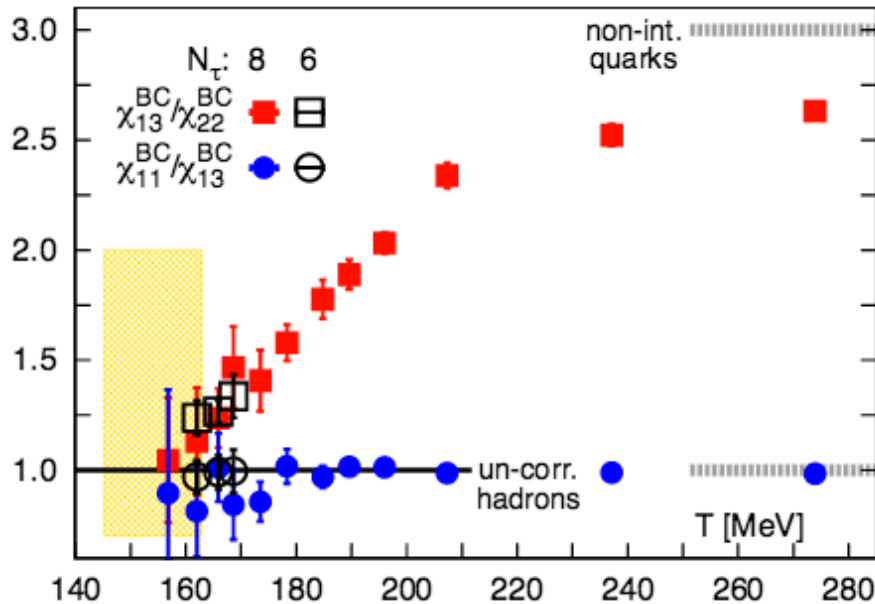- Feed-down from resonance decays neglected

A. Bazavov et al., PRL (2014)

Flavor-dependent freeze-out?

□ Lattice data hint at possible flavor-dependence in transition temperature

□ Possibility of strange bound-states above T_c?

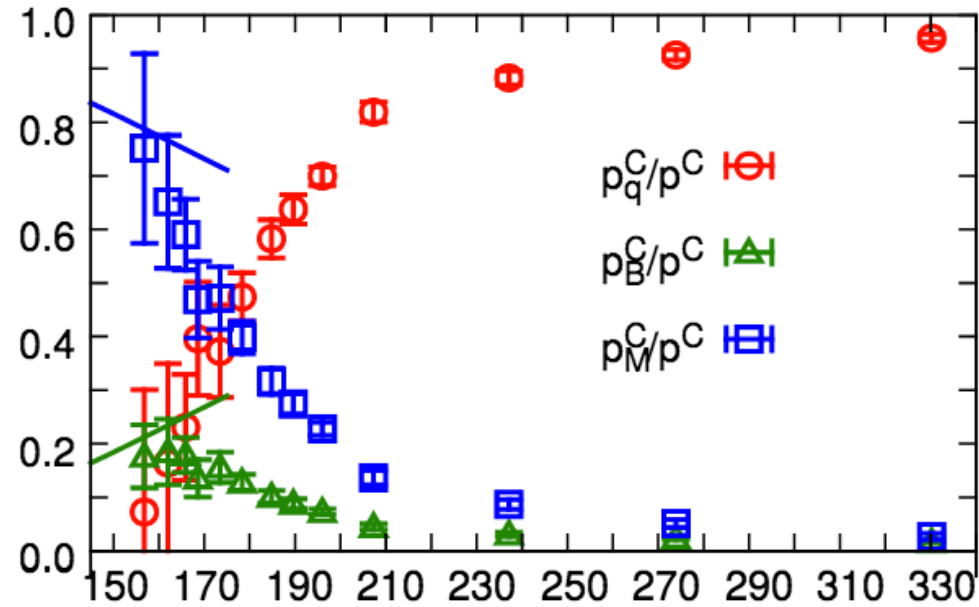# Degrees of freedom from fluctuations

- ☐ Onset of deconfinement for charm quarks:
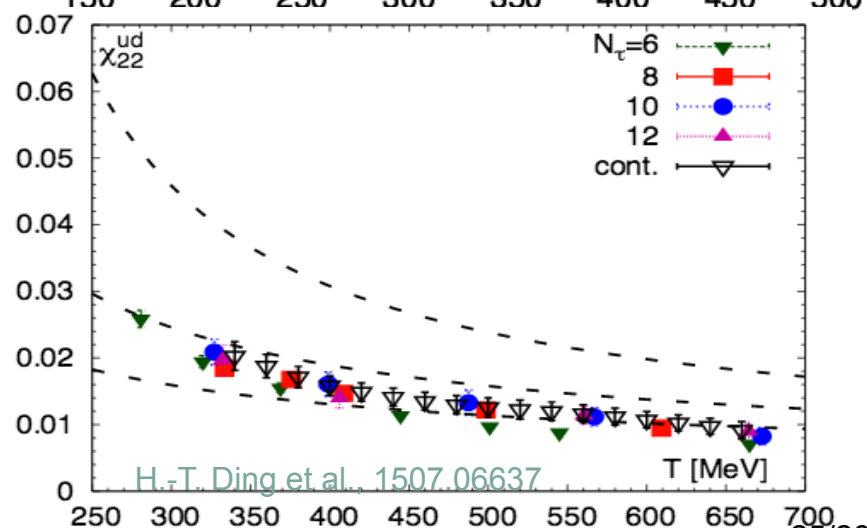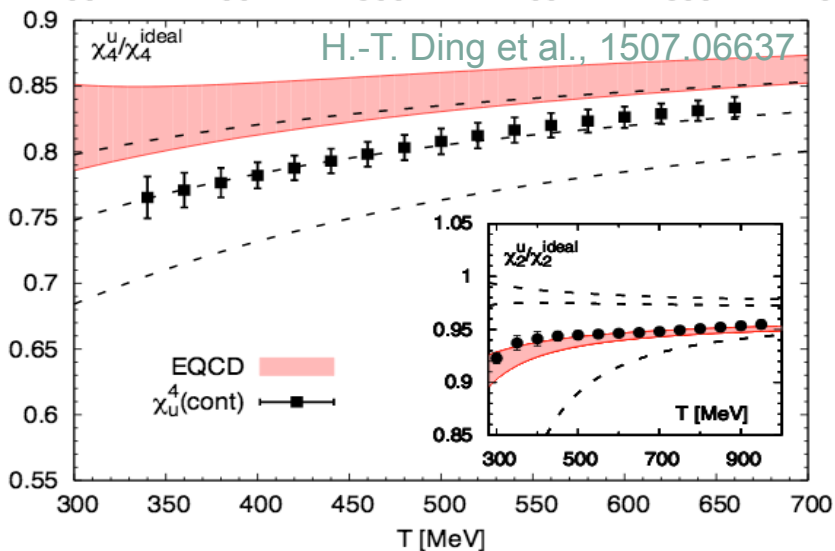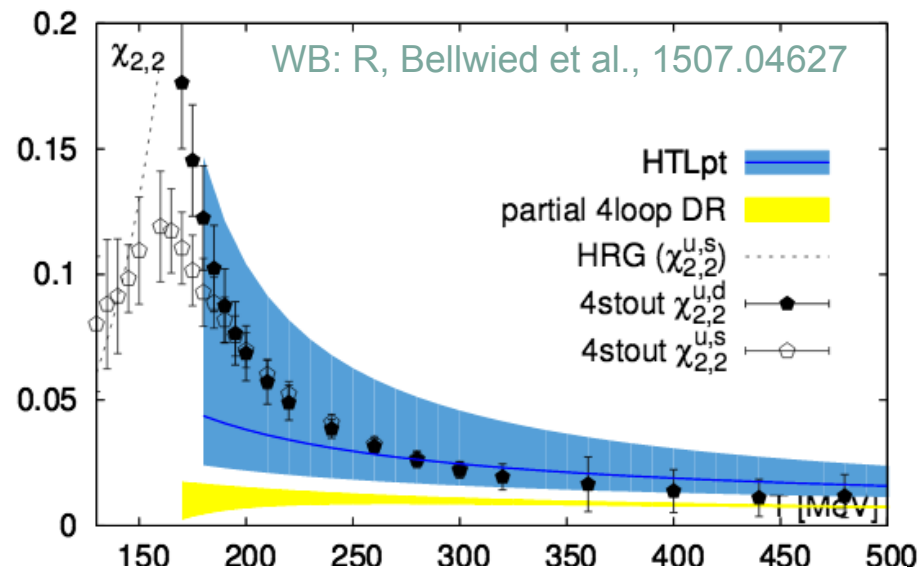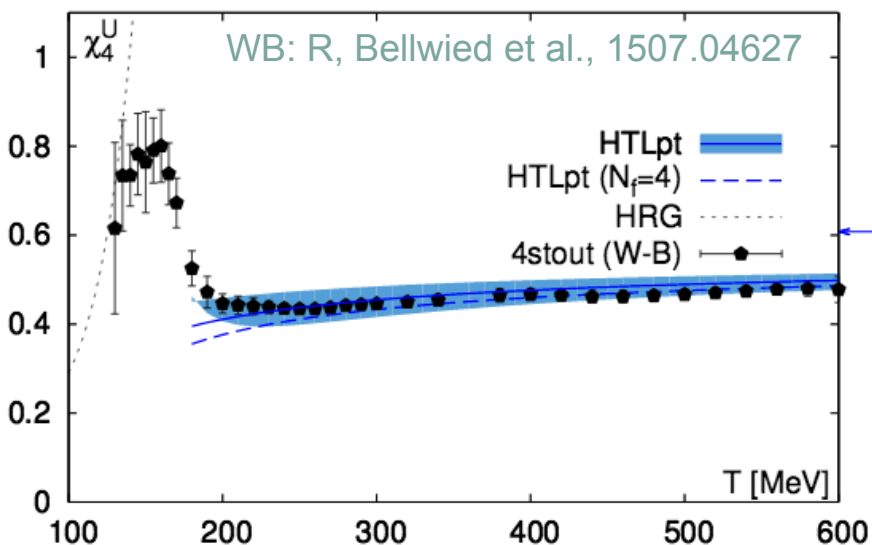

A. Bazavov et al., PLB (2014)

S. Mukherjee, P. Petreczky, S. Sharma 1509.08887

- ☐ Partial meson and baryon pressures described by HRG at T_C and dominate the charm pressure then drop gradually. Charm quark only dominant dof at T>200 MeV

Fluctuations at high temperatures

HTL: N. Haque et al., JHEP (2014); DR: S. Mogliacci et al., JHEP (2013)

Conclusions

- Unprecedented precision in lattice QCD data allows a direct comparison to experiment for the first time

- QCD thermodynamics at $\mu_B=0$ can be simulated with high accuracy

- Extensions to finite density are under control up to $O(\mu_B^6)$

- Challenges for the near future

  - Sign problem

  - Real-time dynamics

Transport properties

- Matter in the region $(1\text{-}2)T_c$ is highly non-perturbative

- Significant modifications of its transport properties

- Common problem:
  - Transport properties can be explored through the analysis of certain correlation functions:

$$G_H(\tau, \vec{p}, T) = \int_0^\infty \frac{d\omega}{2\pi} \, \rho_H(\omega, \vec{p}, T) \, \frac{\cosh(\omega(\tau - 1/2T))}{\sinh(\omega/2T)} = \int d^3x \, e^{i\mathbf{p}\mathbf{x}} \langle J^\alpha(0,0) J^{\beta\dagger}(\tau, \mathbf{x}) \rangle$$
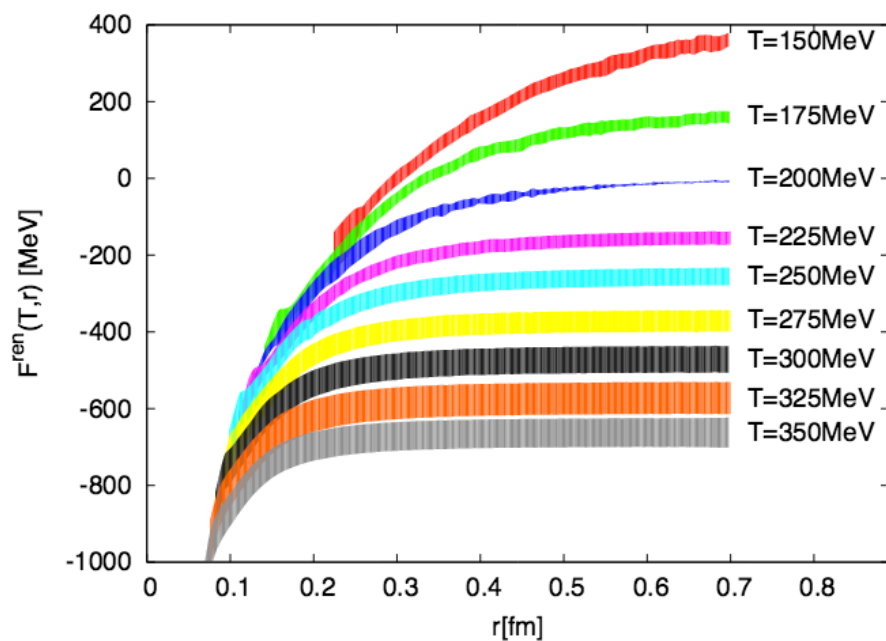
  - Challenge: integrate over discrete set of lattice points in τ direction

  - Use inversion methods like Maximum Entropy Method or modeling the spectral function at low frequencies

Quarkonia properties

- Three main approaches:

  - Potential models with heavy quark potential calculated on the lattice

    - Solve Schroedinger's equation for the bound state two-point function

  - Extract spectral functions from Euclidean temporal correlators

  - Study spatial correlation functions of quarkonia and their in-medium screening properties

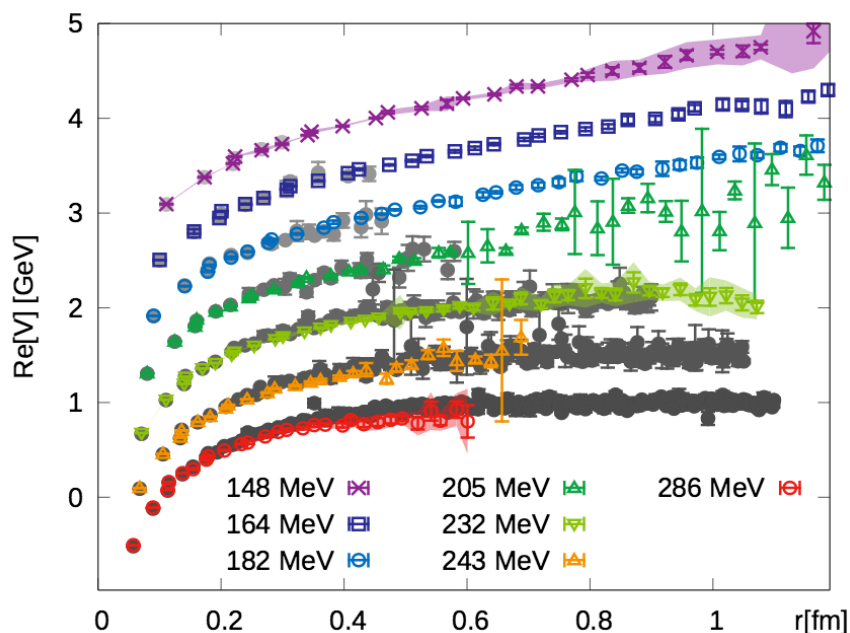# Inter-quark potential

□ Static quark-antiquark free-energy

Borsanyi et al. JHEP(2015)

□ Continuum extrapolated result with $N_f=2+1$ flavors at the physical mass
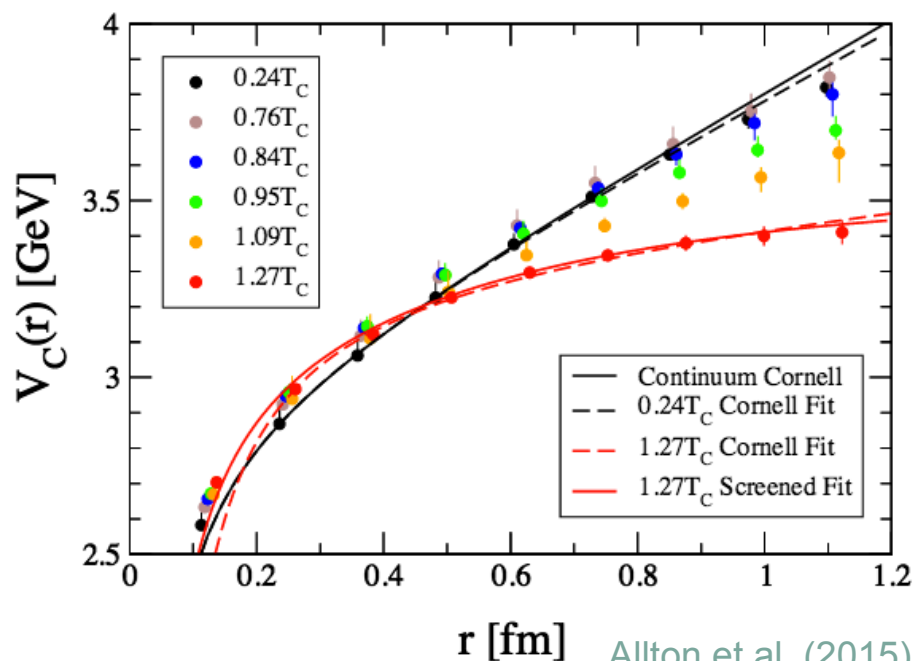
Inter-quark potential

□ Quark-antiquark potential in N_f=2+1 QCD

Burnier et al. (2014)

Allton et al. (2015)

□ Real part of the complex potential lies close to the color singlet free energy

□ Central potential: combination of pseudoscalar and vector potentials:

$$V_C(\mathbf{r}) = \frac{1}{4}V_{PS} + \frac{3}{4}V_V$$
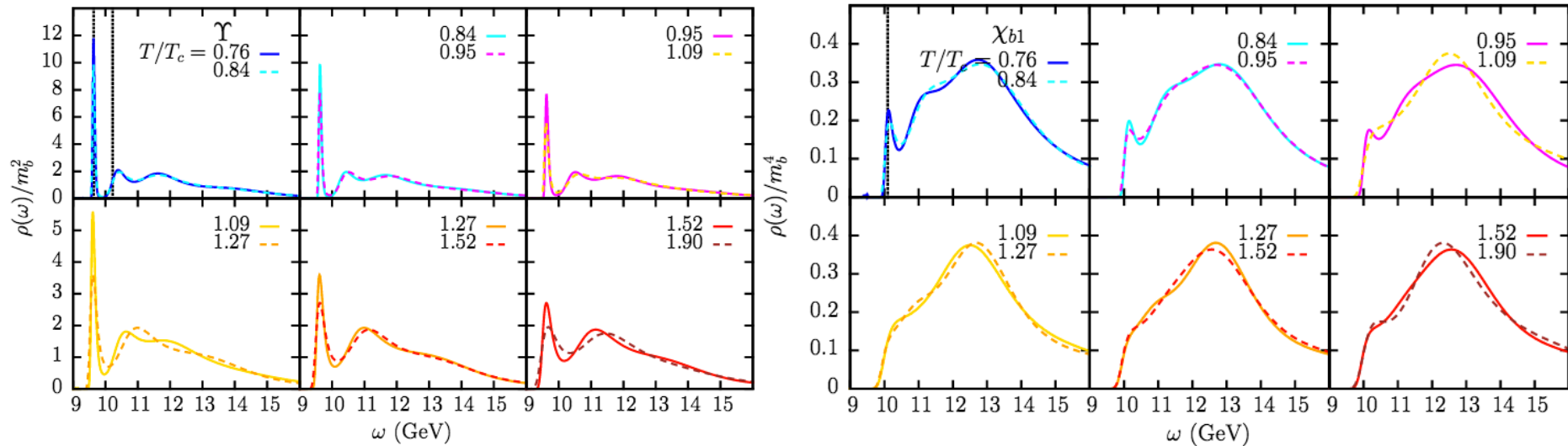
Quarkonia spectral functions

☐ Charmonium spectral functions in quenched approximation and preliminary studies with dynamical quarks yield consistent results: all charmonium states are dissociated for $T \gtrsim 1.5T_c$

H. Ding et al., PRD (2012)
G. Aarts et al., PRD (2007)
WB: S. Borsanyi et al., JHEP (2014)
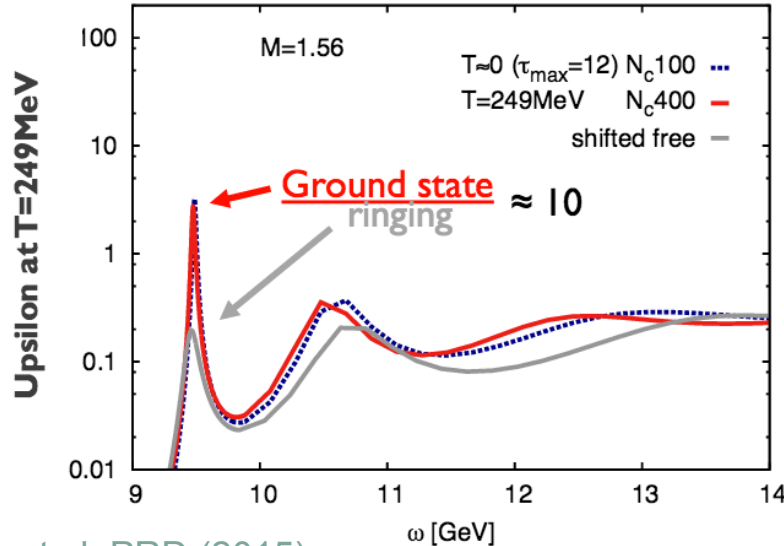
☐ Bottomonium (N_f=2+1, m_π=400 MeV), MEM:

G. Aarts et al. JHEP (2014)

☐ S-wave ground state survives up to 1.9 T_c, P-wave ground state melts just above T_c
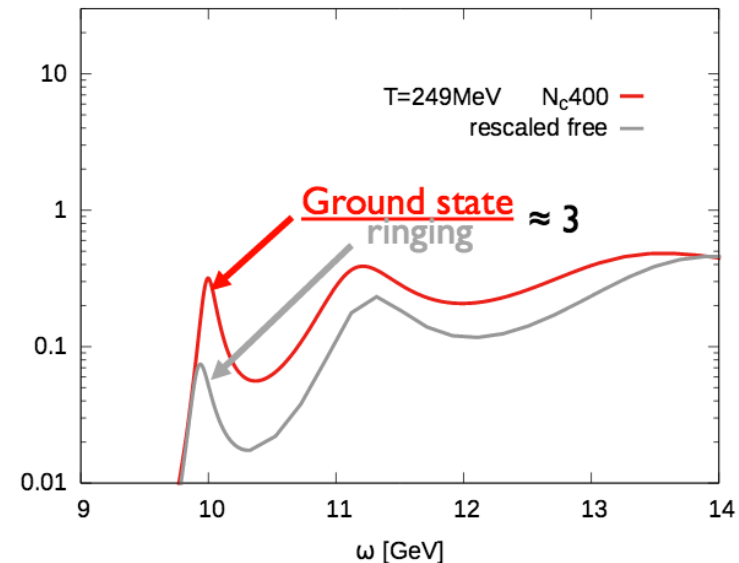
Quarkonia spectral functions

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H. Ding et al., PRD (2012)
G. Aarts et al., PRD (2007)
WB: S. Borsanyi et al., JHEP (2014)

□ Bottomonium (N_f=2+1, m_π=160 MeV), Bayesian method:

S. Kim et al. PRD (2015)
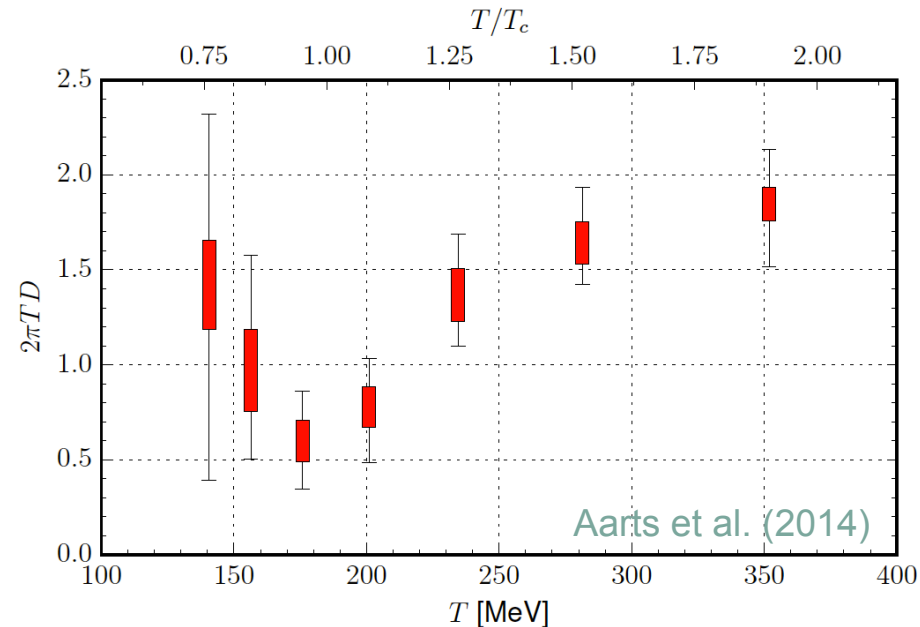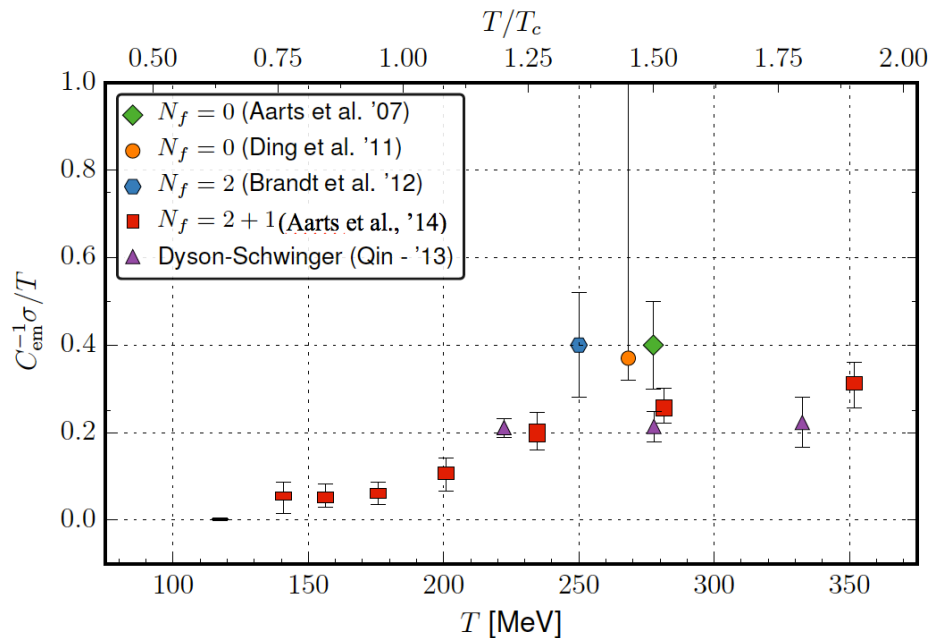
□ S-wave ground state and P-wave ground state survive up to T~250 MeV

Electric conductivity and charge diffusion

- Definitions:

$$\sigma = \frac{C_{em}}{6} \lim_{\omega \to 0} \lim_{\mathbf{p} \to 0} \sum_{i=1}^{3} \frac{\rho^{ii}(\omega, \mathbf{p}, T)}{\omega} \qquad\qquad D_Q = \sigma / \chi_2^Q$$
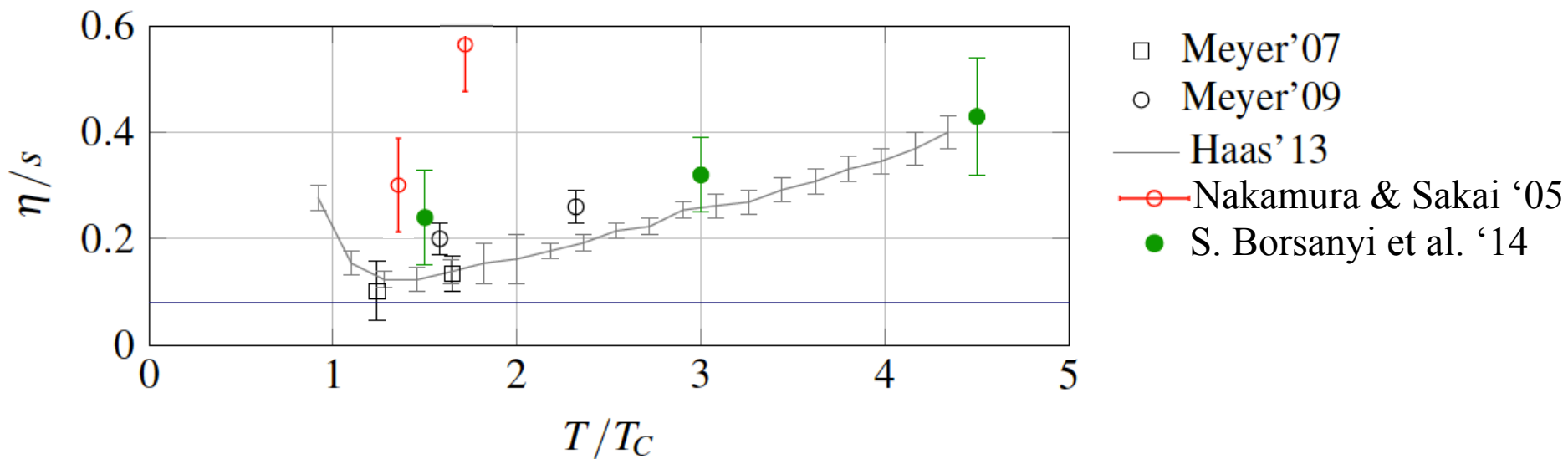

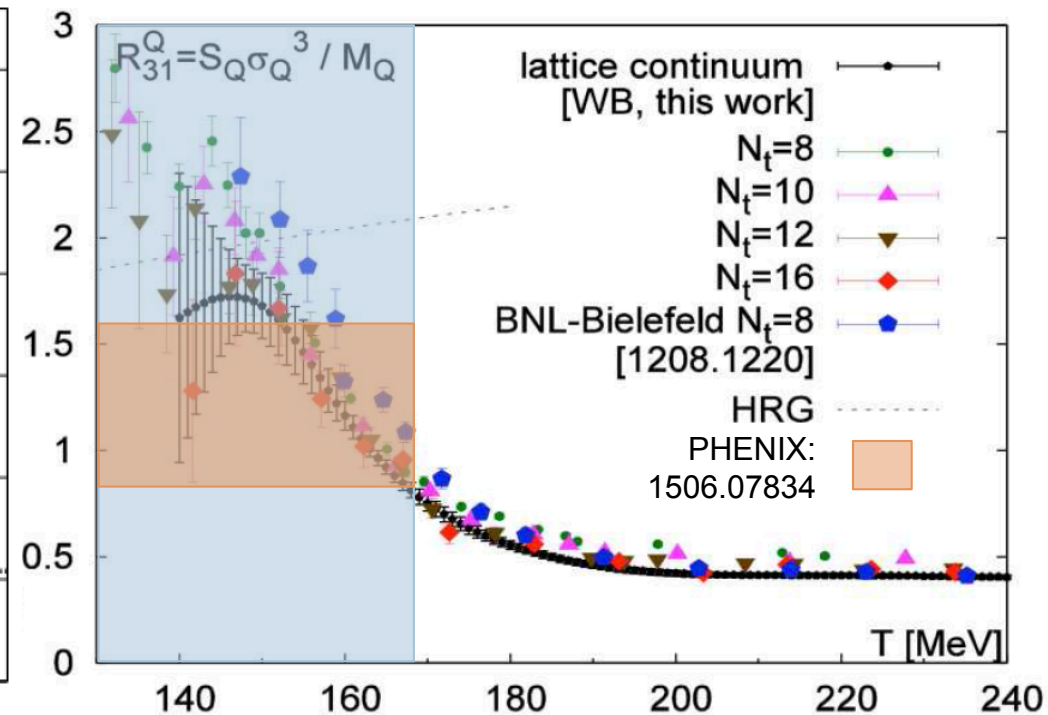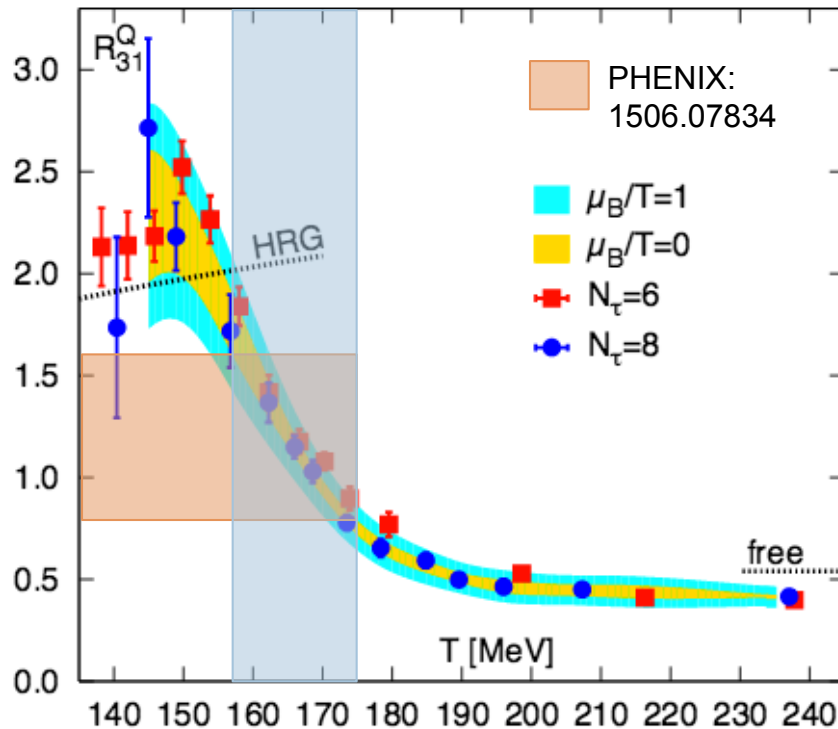- Electric conductivity measures the response of the medium to small perturbations induced by an electromagnetic field

Viscosity

□ Shear viscosity in the pure gauge sector of QCD

□ Challenge: very low signal-to noise ratio for the Euclidean energy-momentum correlator

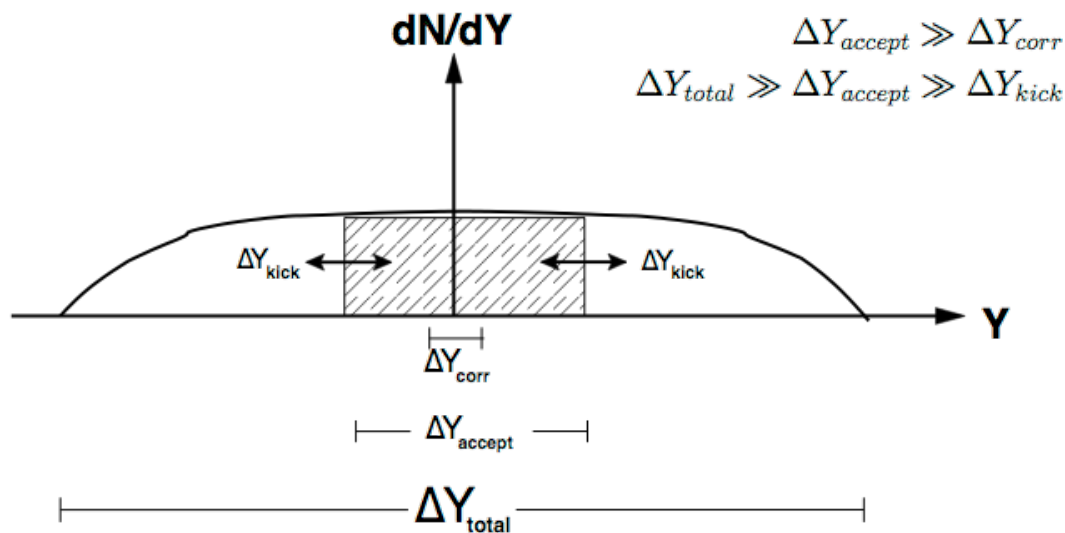# Freeze-out parameters from Q fluctuations

A. Bazavov et al. (2014)
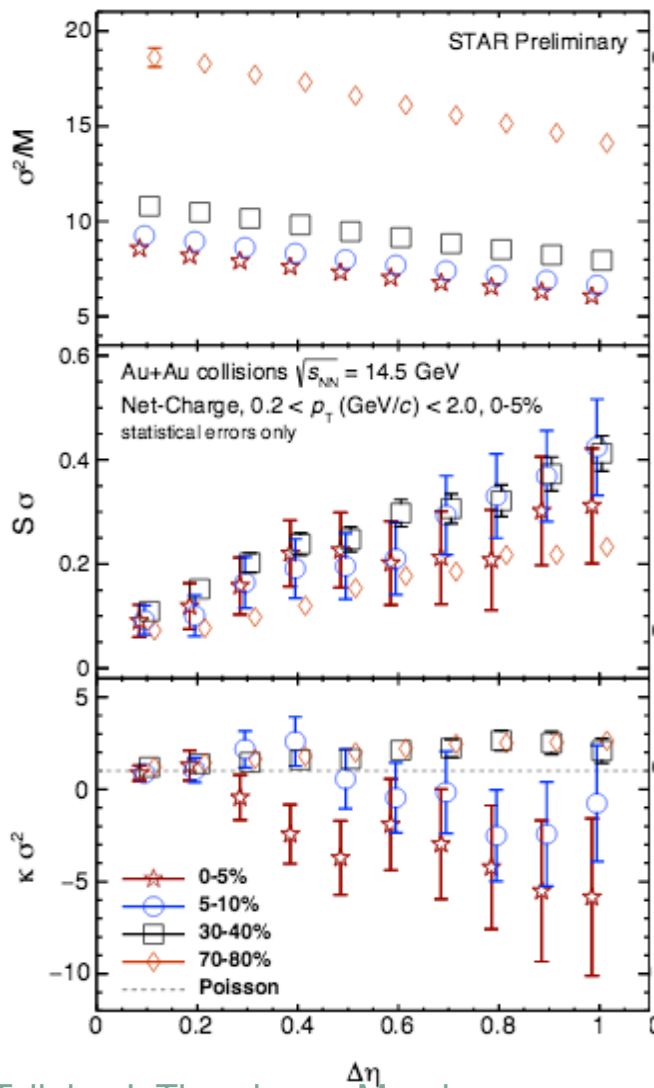
WB: Borsanyi et al. PRL (2013)

- ☐ Studies in HRG model: the different momentum cuts between STAR and PHENIX are responsible for more than 30% of their difference

  F. Karsch et al., 1508.02614

- ☐ Using continuum extrapolated lattice data, lower values for T_f are found

Effects of kinematic cuts

STAR Preliminary

σ^2/M

Au+Au collisions $\sqrt{s_{NN}} = 14.5$ GeV
Net-Charge, $0.2 < p_T$ (GeV/c) < 2.0, 0-5%
statistical errors only

$S\sigma$

$\kappa\sigma^2$

- 0-5%
- 5-10%
- 30-40%
- 70-80%
- Poisson

$\Delta\eta$

$$\frac{dN}{dY}$$

$\Delta Y_{accept} \gg \Delta Y_{corr}$

$\Delta Y_{total} \gg \Delta Y_{accept} \gg \Delta Y_{kick}$

ΔY_{kick}

ΔY_{kick}

ΔY_{corr}

ΔY_{accept}

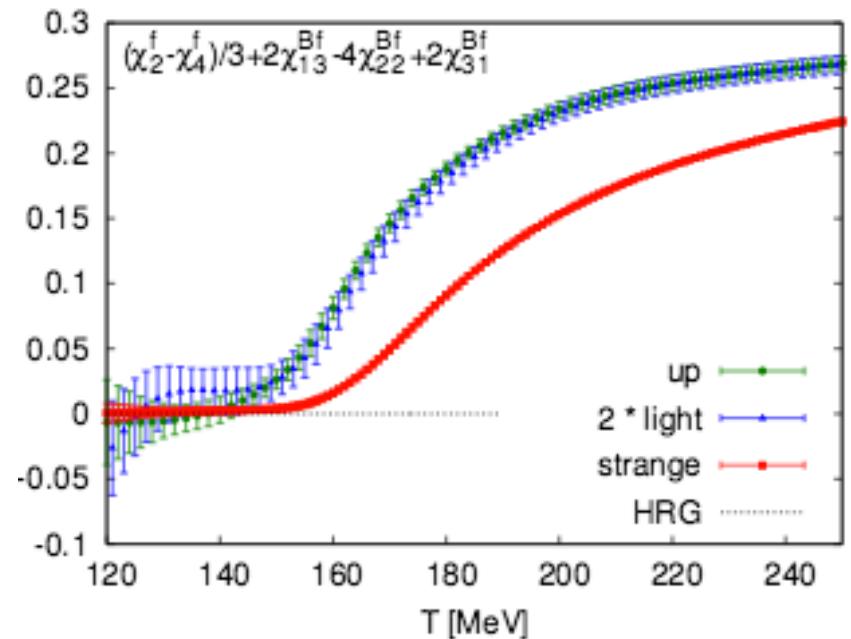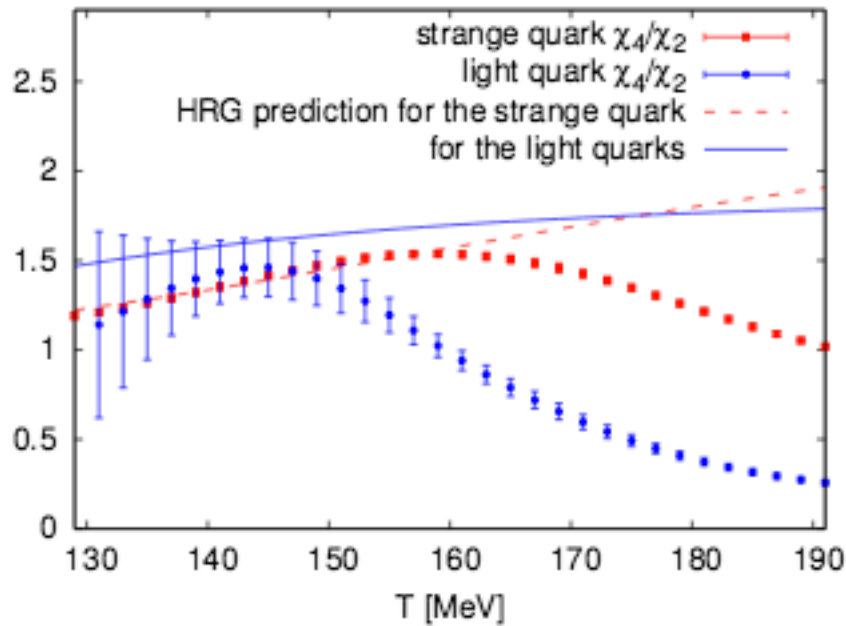ΔY_{total}

V. Koch, 0810.2520

- Rapidity dependence of moments needs to be studied for $1 < \Delta\eta < 2$

- Difference in kinematic cuts between STAR and PHENIX leads to a 5% difference in T_f

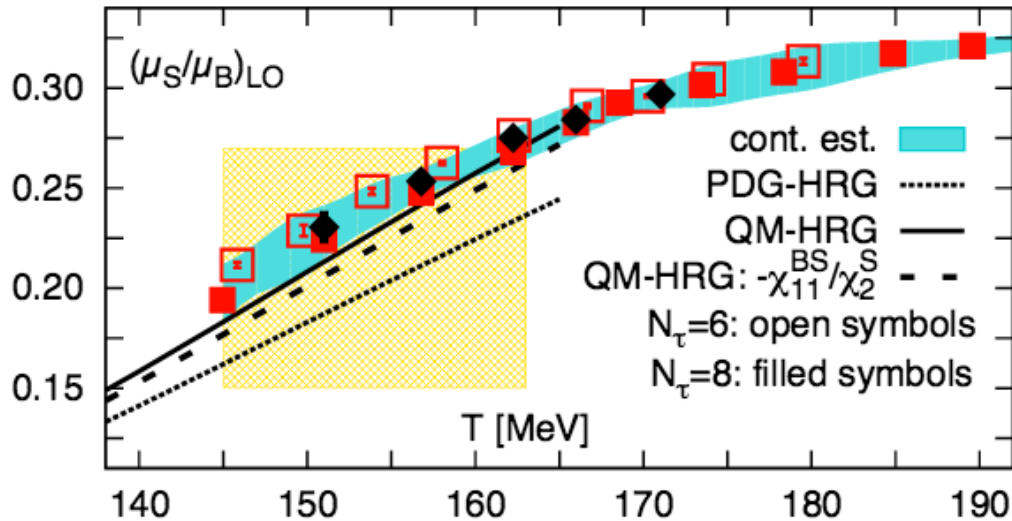Talk by F. Karsch on Monday

Talk by J. Thaeder on Monday
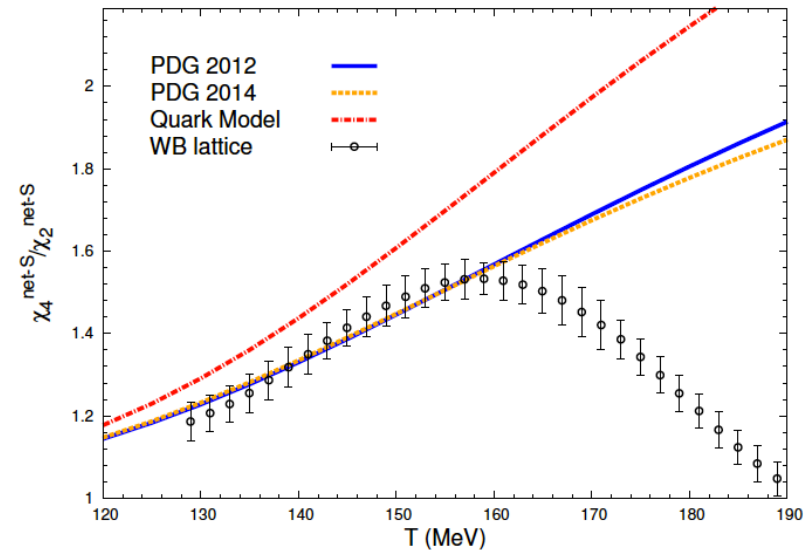
Strangeness fluctuations

□ Lattice data hint at possible flavor-dependence in transition temperature

□ Possibility of strange bound-states above T_c?
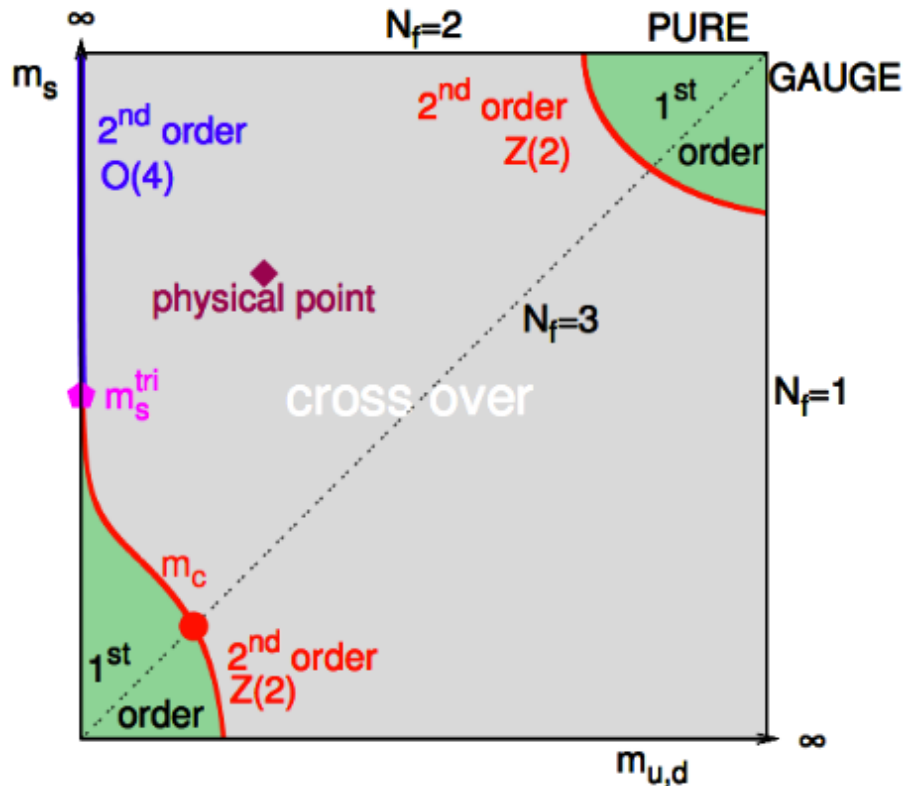
Additional strange hadrons

A. Bazavov et al., PRL (2014)

Poster by P. Alba

- ☐ Discrepancy between lattice and HRG for μ_S/μ_B can be understood by introducing higher mass states predicted by the Quark Model

- ☐ Discrepancy between QM predictions and lattice data for χ_4^S/χ_2^S needs to be understood

- ☐ Their effect on freeze-out conditions needs to be investigated taking into account their decay feed-down into stable states

P. M. Lo et al., 1507.06398

Columbia plot

- Pure gauge theory: T_c=294(2) MeV

  Francis et al., 1503.05652

- N_f=2 QCD at $m_\pi > m_\pi^{phys}$:
  - O(a) improved Wilson, N_t=16
    - m_π=295 MeV T_c=211(5) MeV
    - m_π=220 MeV T_c=193(7) MeV

      Brandt et al., 1310.8326

  - Twisted-mass QCD
  - m_π=333 MeV T_c=180(12) MeV

    Burger et al., 1412.6748

- N_f=2+1 O(a) improved Wilson
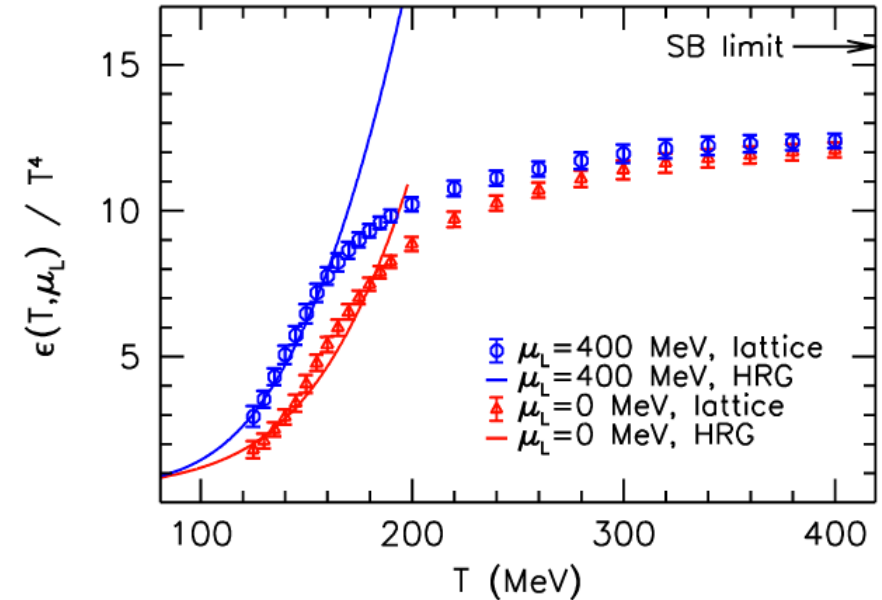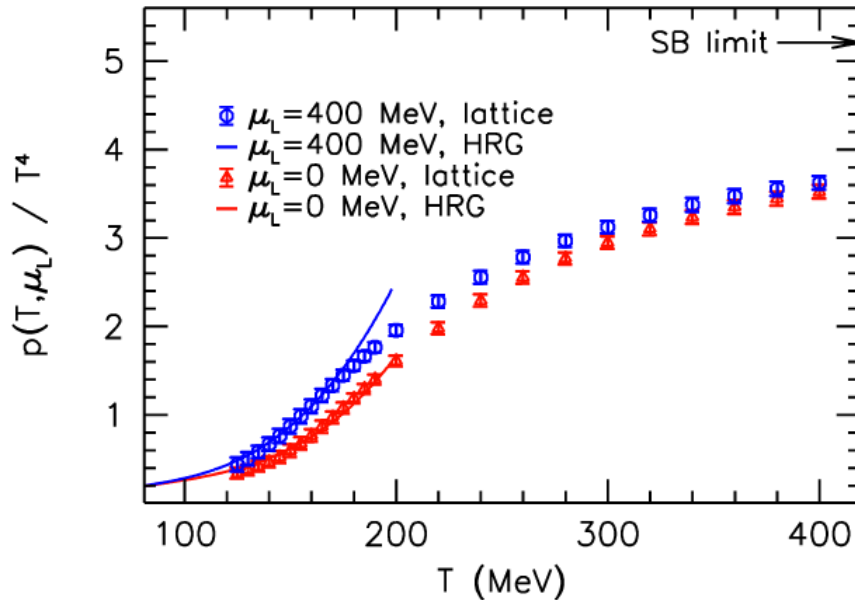 - Continuum results

    Borsanyi et al., 1504.03676

- HISQ action, N_t=6, no sign of 1st order phase transition at m_π=80 MeV

  HotQCD, 1312.0119, 1302.5740

Equation of state at $\mu_B > 0$

- Expand the pressure in powers of μ_B (or $\mu_L = 3/2(\mu_u + \mu_d)$)
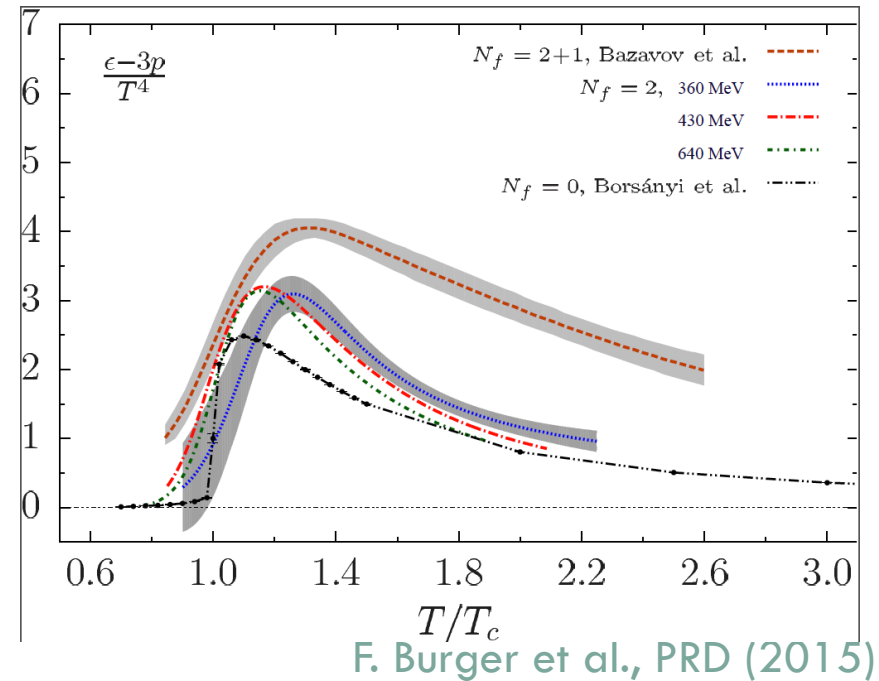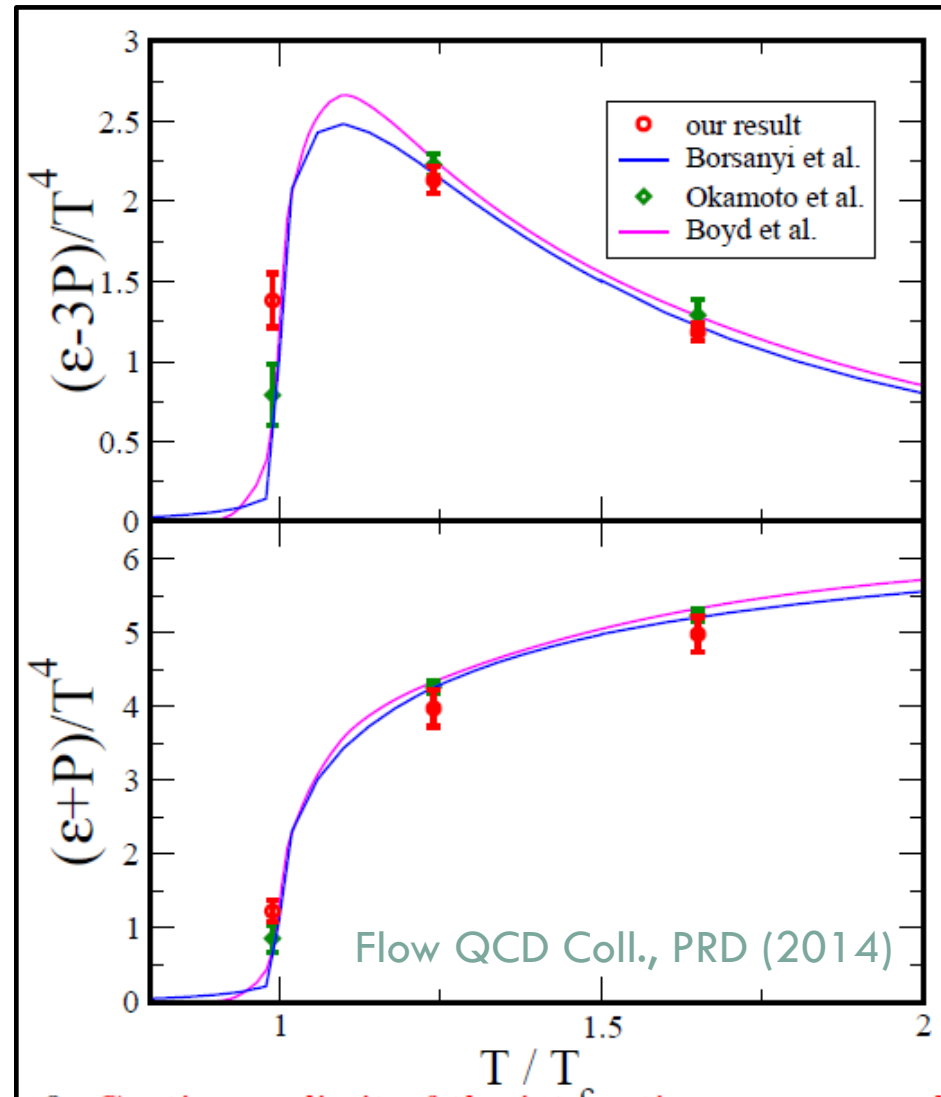
$$\frac{p(T, \{\mu_i\})}{T^4} = \frac{p(T, \{0\})}{T^4} + \frac{1}{2} \sum_{i,j} \frac{\mu_i \mu_j}{T^2} \chi_2^{ij} \quad \text{with} \quad \chi_2^{ij} \equiv \frac{T}{V} \frac{1}{T^2} \frac{\partial^2 \log \mathcal{Z}}{\partial \mu_i \partial \mu_j}\Big|_{\mu_i = \mu_j = 0}$$

S. Borsanyi et al., JHEP (2012)

- Continuum extrapolated results at the physical mass

Alternative methods for thermodynamics

Flow QCD Coll., PRD (2014)

F. Burger et al., PRD (2015)

- □ Gradient flow: EoS in the quenched approximation

- □ Twisted mass Wilson fermions: EoS available so far for heavier-than-physical quark masses and N_f=2