Rapidity Correlation Structure from Causal Hydrodynamics

George Moschelli and Rajendra Pokharel — Lawrence Technological University
Sean Gavin — Wayne State University

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I. Motivation: impact of viscosity on fluctuations and correlations

II. Hydrodynamics modes: fluctuations and dissipation

    a. Viscous diffusion of shear modes

    b. 1^{st} and 2^{nd} order hydrodynamics

III. Contributions to correlation measurements

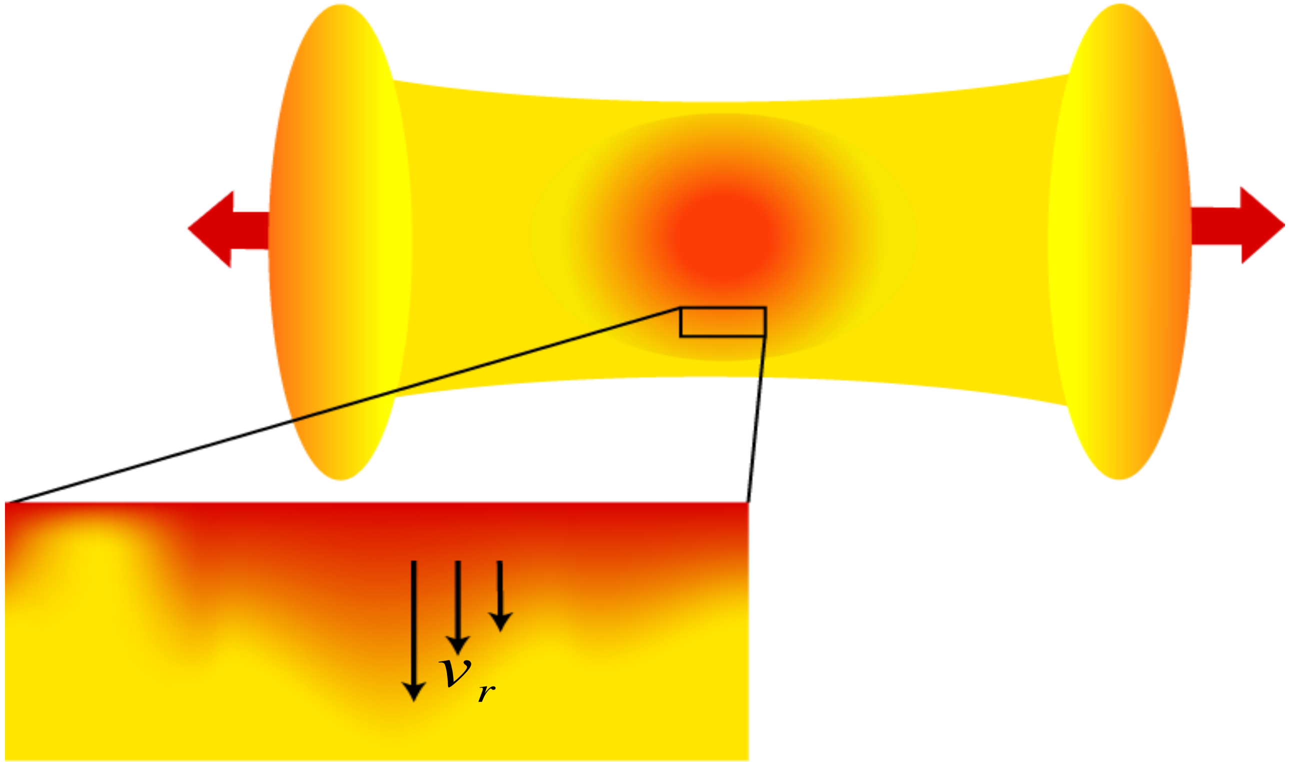
Transverse Momentum Fluctuations

small variations in transverse flow in each event

viscous friction as fluid elements flow past one another

shear viscosity drives velocity toward the average

$$T_{zr} = -\eta\, \partial v_r / \partial z$$



damping of radial flow fluctuations $\Rightarrow$ viscosity

SG & Mohamed Abdel-Aziz, Phys. Rev. Lett. 97 (2006) 162302

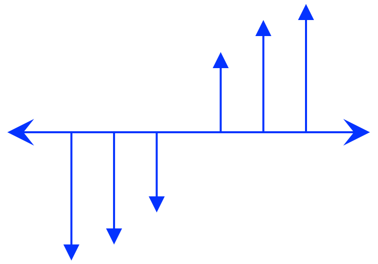
Fluctuations: Hydrodynamic Modes

Small fluctuations of the **momentum current** $M_i \equiv T_{0i} - \langle T_{0i} \rangle$

- linearized 1st order Navier Stokes
- Helmholtz decomposition: $\vec{M} \equiv \vec{g} + \vec{h}$ for $\vec{\nabla} \cdot \vec{g} = 0$ and $\vec{\nabla} \times \vec{h} = 0$

viscous diffusion of divergence-free modes:

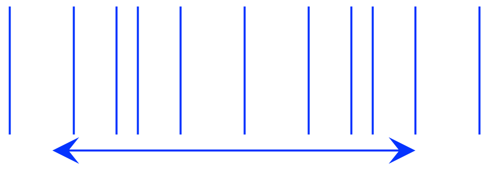
$$\partial_t \vec{g} = \nu \nabla^2 \vec{g}$$



- kinematic viscosity

$$\nu = \eta / Ts$$

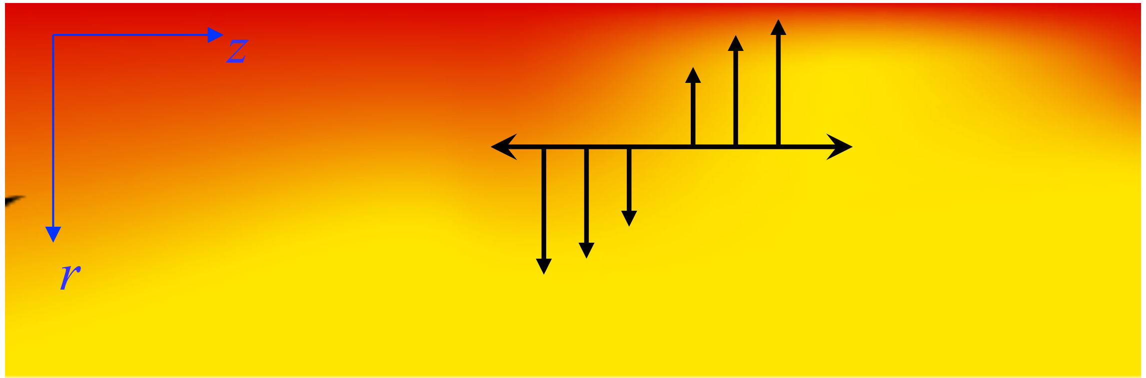
curl free modes:



sound waves – compression damped by viscosity

thermal diffusion – heat flow relative to baryons

Transverse Flow Fluctuations



transverse velocity fluctuations

➔ shear modes

$$T_{0i} - \langle T_{0i} \rangle \approx g_i$$

$$T_{ji}^{diss} \approx -\eta \nabla_j v_i = -\nu \nabla_j g_i + \text{Langevin noise}$$

diffusion equation for momentum current

$$\frac{\partial}{\partial t} g_i = \nu \nabla^2 \left(g_i + \text{noise} \right)$$

correlation function measures deviation of fluctuations from mean

$$r = \langle g_i(x_1) g_j(x_2) \rangle - \langle g_i(x_1) \rangle \langle g_j(x_2) \rangle$$

viscosity: SG & Abdel-Aziz, PRL 97 (2006) 162302
baryon diffusion: SG & Abdel-Aziz, PR C70 (2004) 034905
CME observables: Pratt, Schlichting, SG, PR C84 (2011) 024909

Noisy Diffusion

diffusion equation for momentum current

$$\frac{\partial}{\partial t} g_i = \nu \nabla^2 \left(g_i + \text{noise} \right)$$

difference equation

$$\Delta g = \nu \nabla^2 g \Delta t + \Delta W$$

noise

$$\left\langle \Delta W \right\rangle = 0 \qquad \left\langle \Delta W(x_1) \Delta W(x_2) \right\rangle = \Gamma_{12} \Delta t$$

Noise is Necessary for Equilibrium Fluctuations

noisy diffusion equation $$\partial_t \langle g_1 g_2 \rangle = \nu \left(\nabla_1^2 + \nabla_2^2 \right) \langle g_1 g_2 \rangle + \Gamma_{12}$$

noisy equilibrium state $$\nu \left(\nabla_1^2 + \nabla_2^2 \right) \langle g_1 g_2 \rangle_{eq} = \Gamma_{12}$$

correlation function $$r = \langle g_i(x_1) g_j(x_2) \rangle - \langle g_i(x_1) \rangle \langle g_j(x_2) \rangle$$

$$\rightarrow r_{eq} = (e + p) T \delta(x_1 - x_2)$$

BONUS:

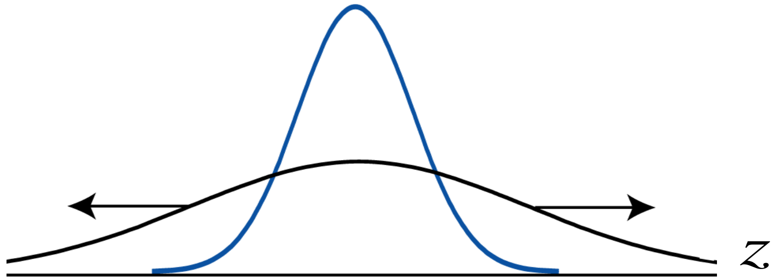
$\Delta r = r - r_{eq}$ **satisfies deterministic diffusion equation**

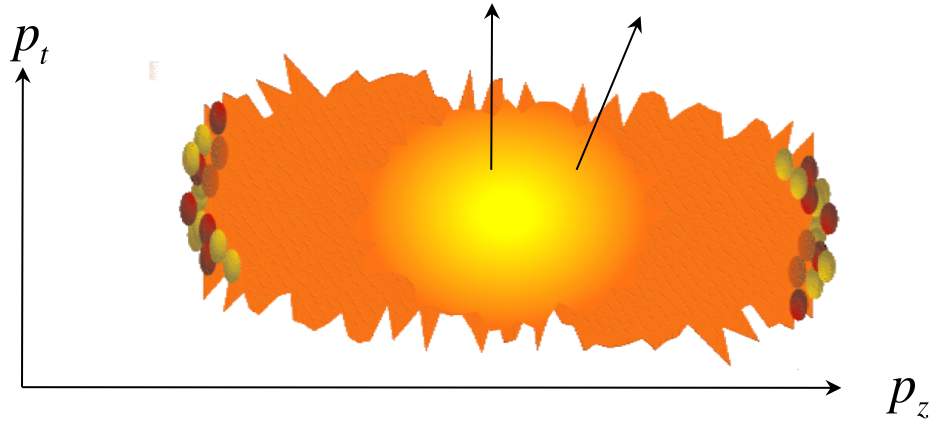
Gardiner, Handbook of Stochastic Methods, (Springer, 2002)

Measuring the Correlations

correlation function

$$r = \left\langle g_i(x_1) g_j(x_2) \right\rangle - \left\langle g_i(x_1) \right\rangle \left\langle g_j(x_2) \right\rangle$$





observable:

$$C = \frac{1}{\langle N \rangle^2} \left\langle \sum_{\text{pairs}} p_i p_j \right\rangle - \langle p_i \rangle \langle p_j \rangle \;= \frac{1}{\langle N \rangle^2} \int (r - r_{eq}) \, dx_1 \, dx_2$$

assumes: proper-time freeze out

Abdel-Aziz & S.G., PRL 97 (2006) 162302; PR C70 (2004) 034905
Pratt, Schlichting, SG, Phys. Rev. C 84 (2011) 024909

p_t Covariance Measured

$$C = \frac{1}{\langle N \rangle^2} \left\langle \sum_{a \neq b} p_t^{\,a} p_t^{\,b} \right\rangle - \langle p_t \rangle^2$$

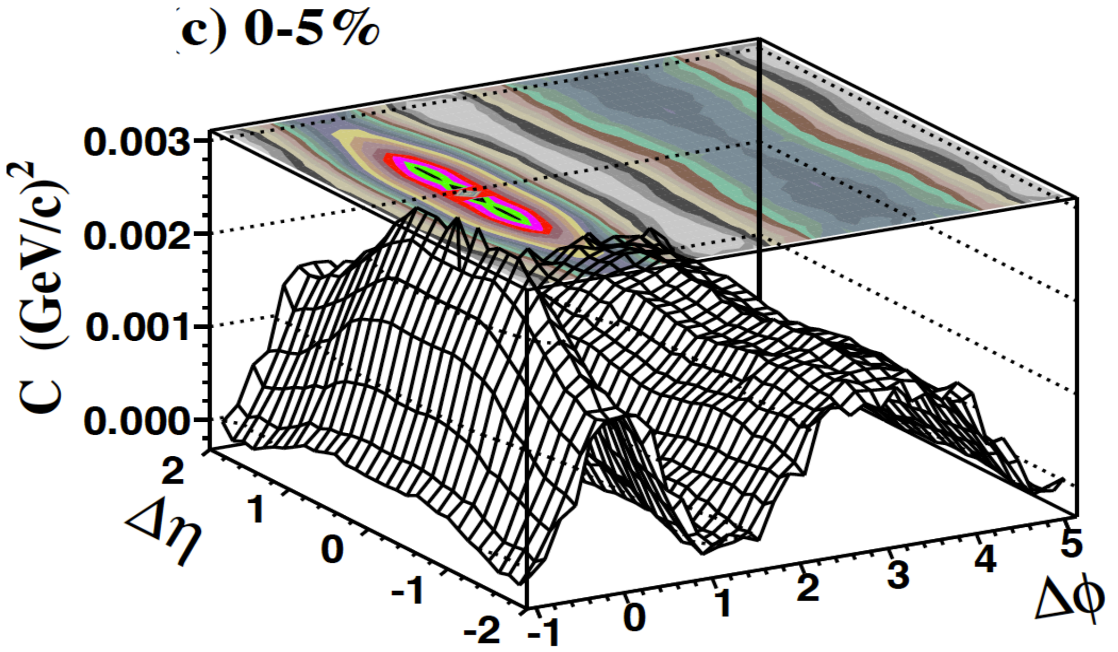
measured: rapidity width of near side peak

- fit peak + constant offset
- report rms width of the peak

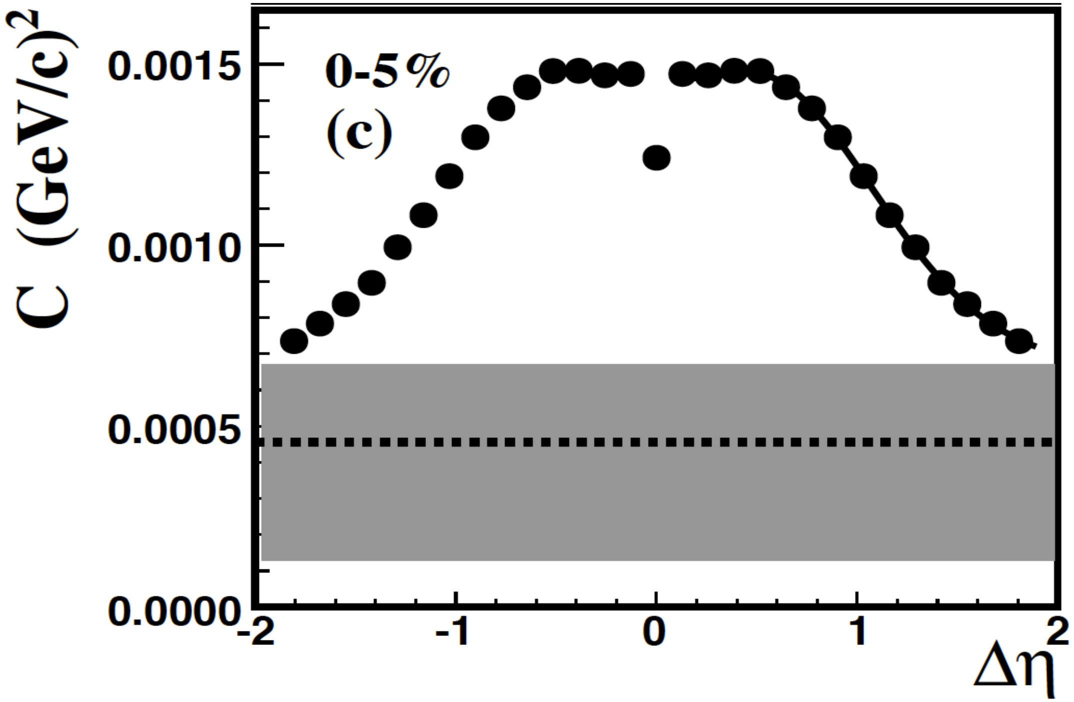
find: width increases in central collisions

$$\sigma_{central} = 1.0 \pm 0.2$$

$$\sigma_{peripheral} = 0.54 \pm 0.02$$



STAR data, Phys. Lett. B704 (2011) 467



Rapidity Width vs. Centrality

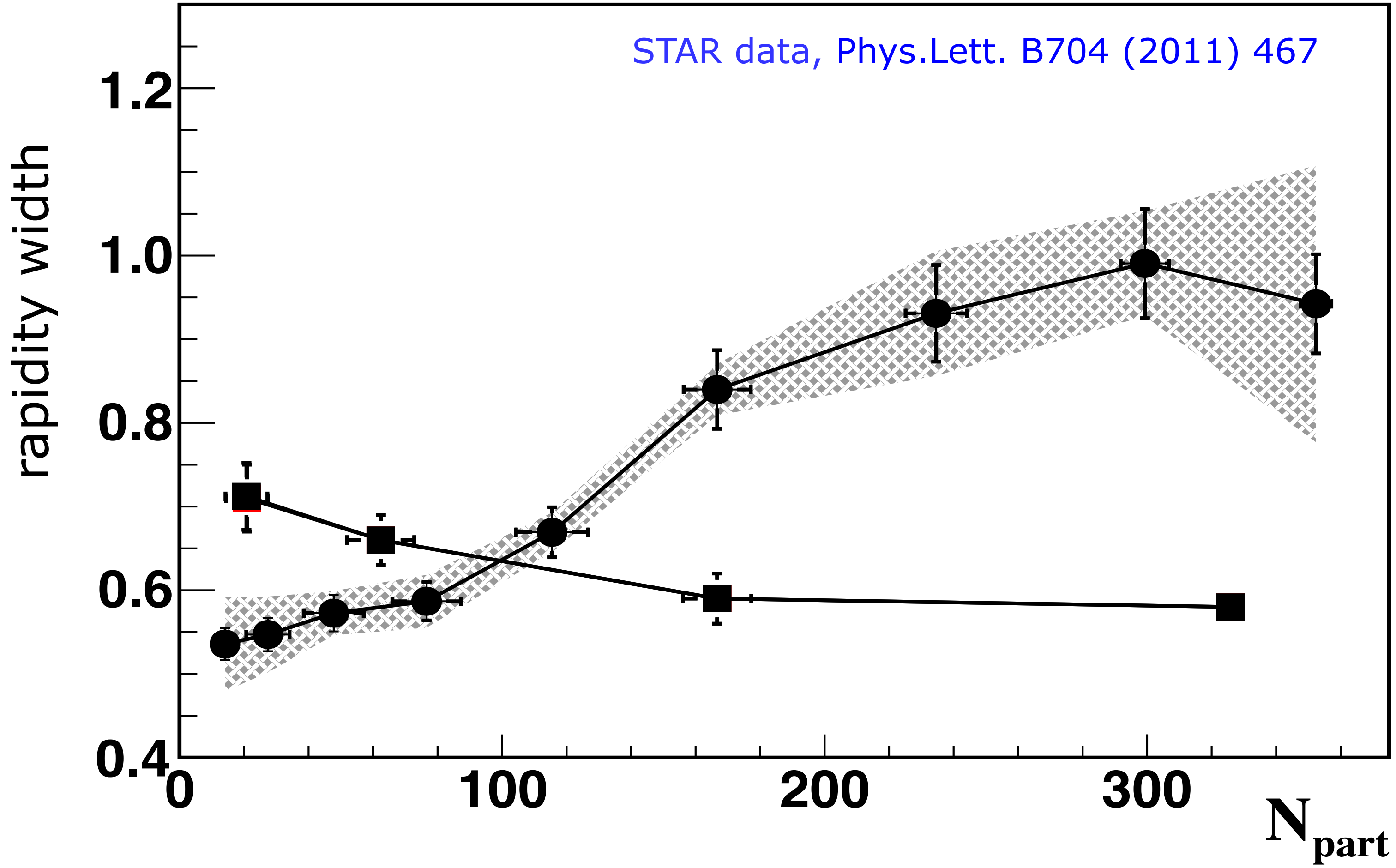
STAR: increase from peripheral to central $\Rightarrow \eta/s = 0.17 \pm 0.08$

Sharma et al., Phys. Rev. C84 (2011) 054915

NeXSPheRIO

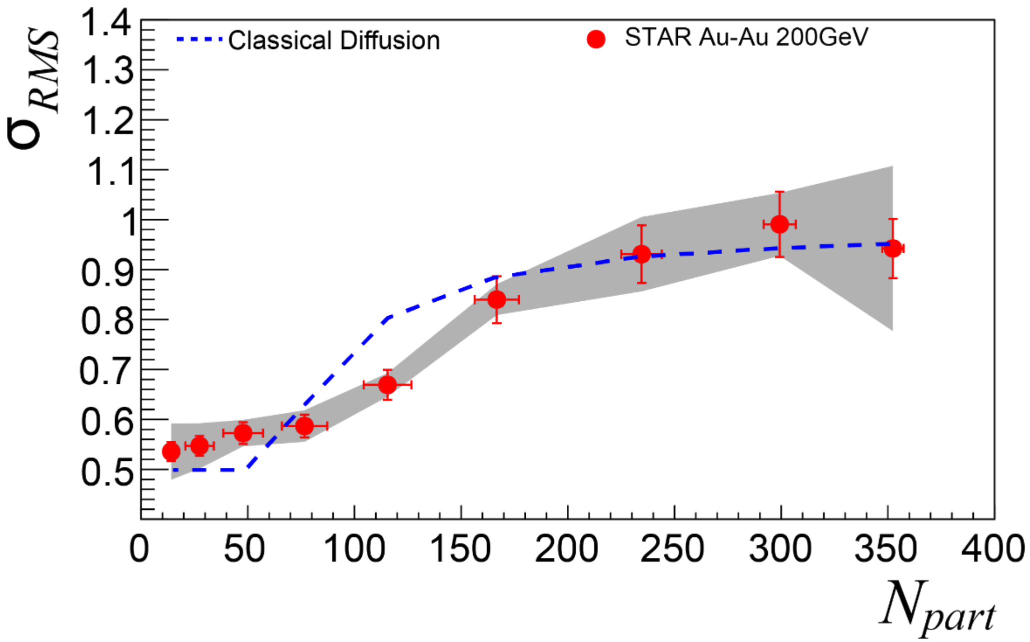
- fits most aspects of correlations (ridge, v_n, etc.)
- **not** the increased width

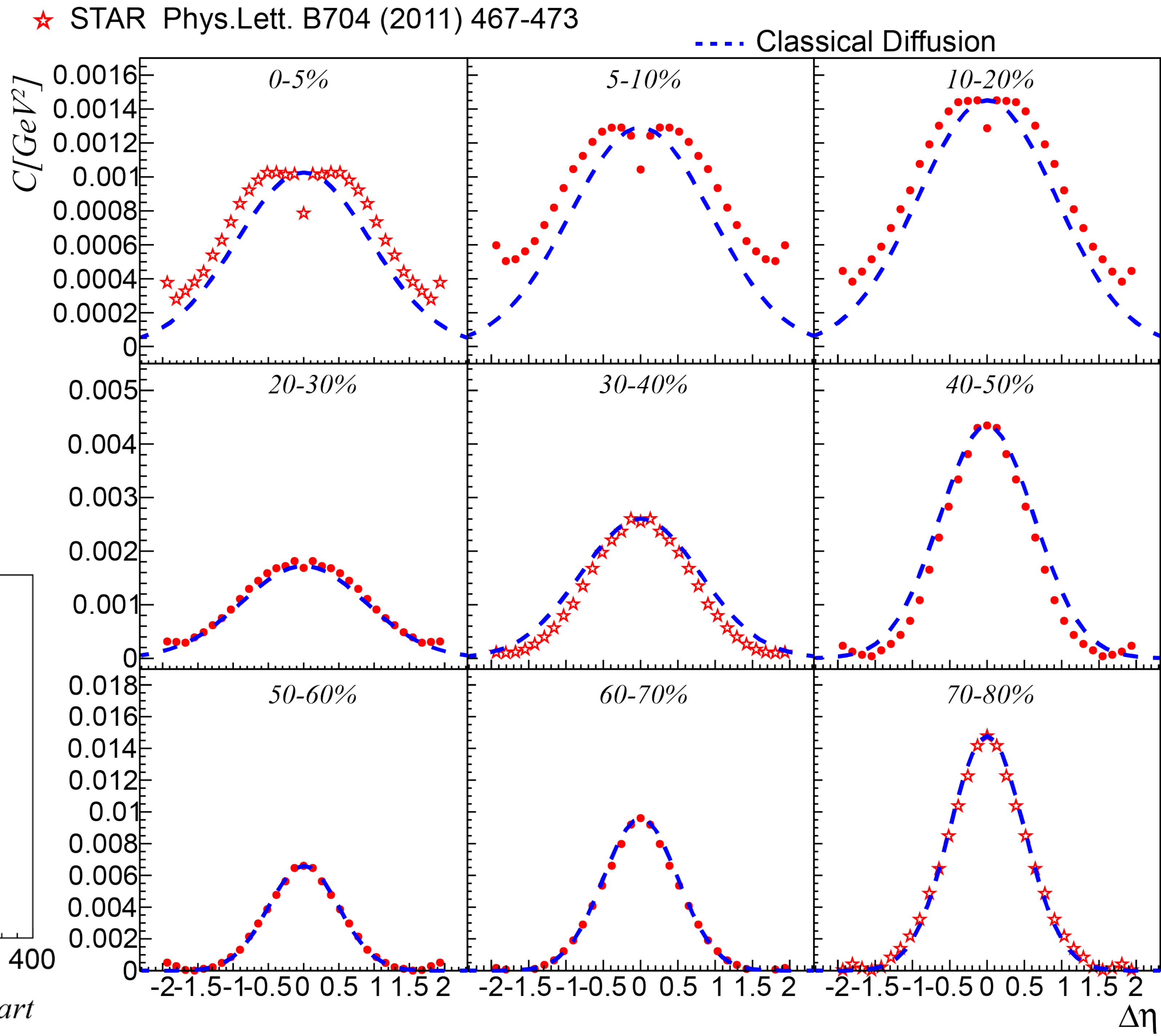
ideal fluctuating hydro doesn't explain measured growth of width



1st Order Diffusion

Fails: Gaussian profile doesn't describe shape





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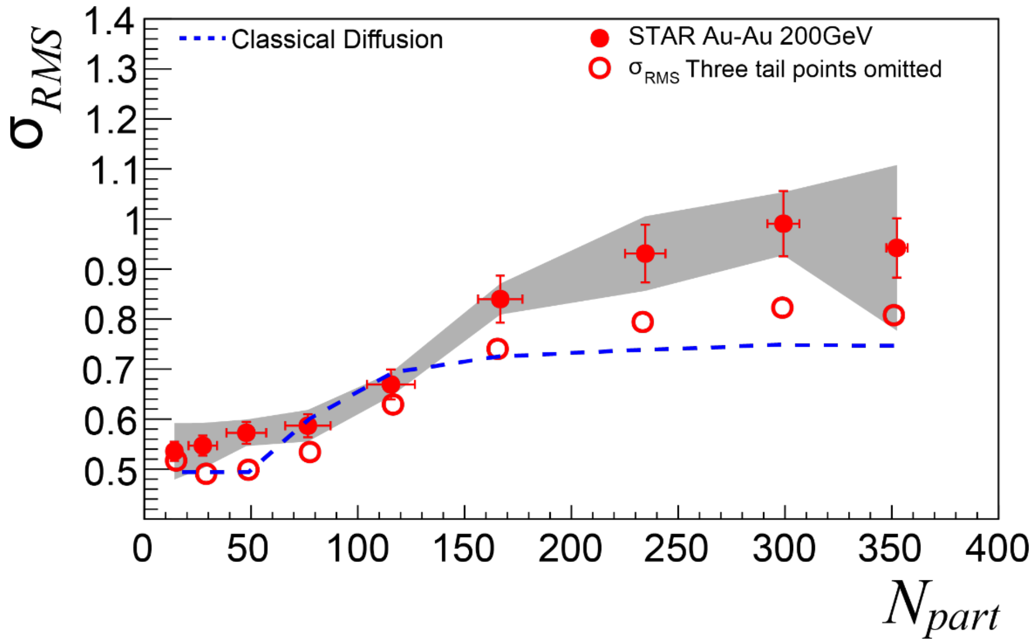
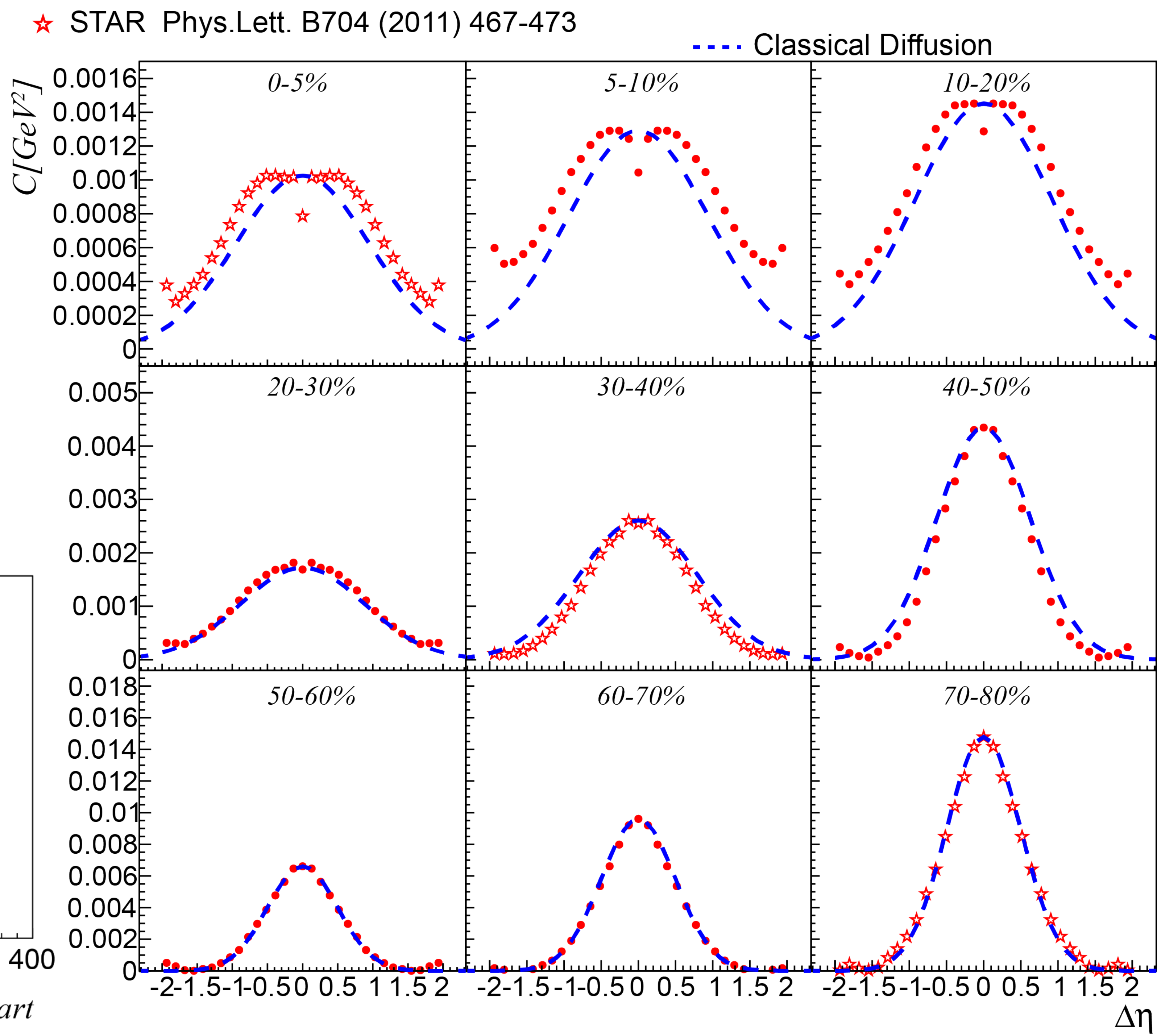
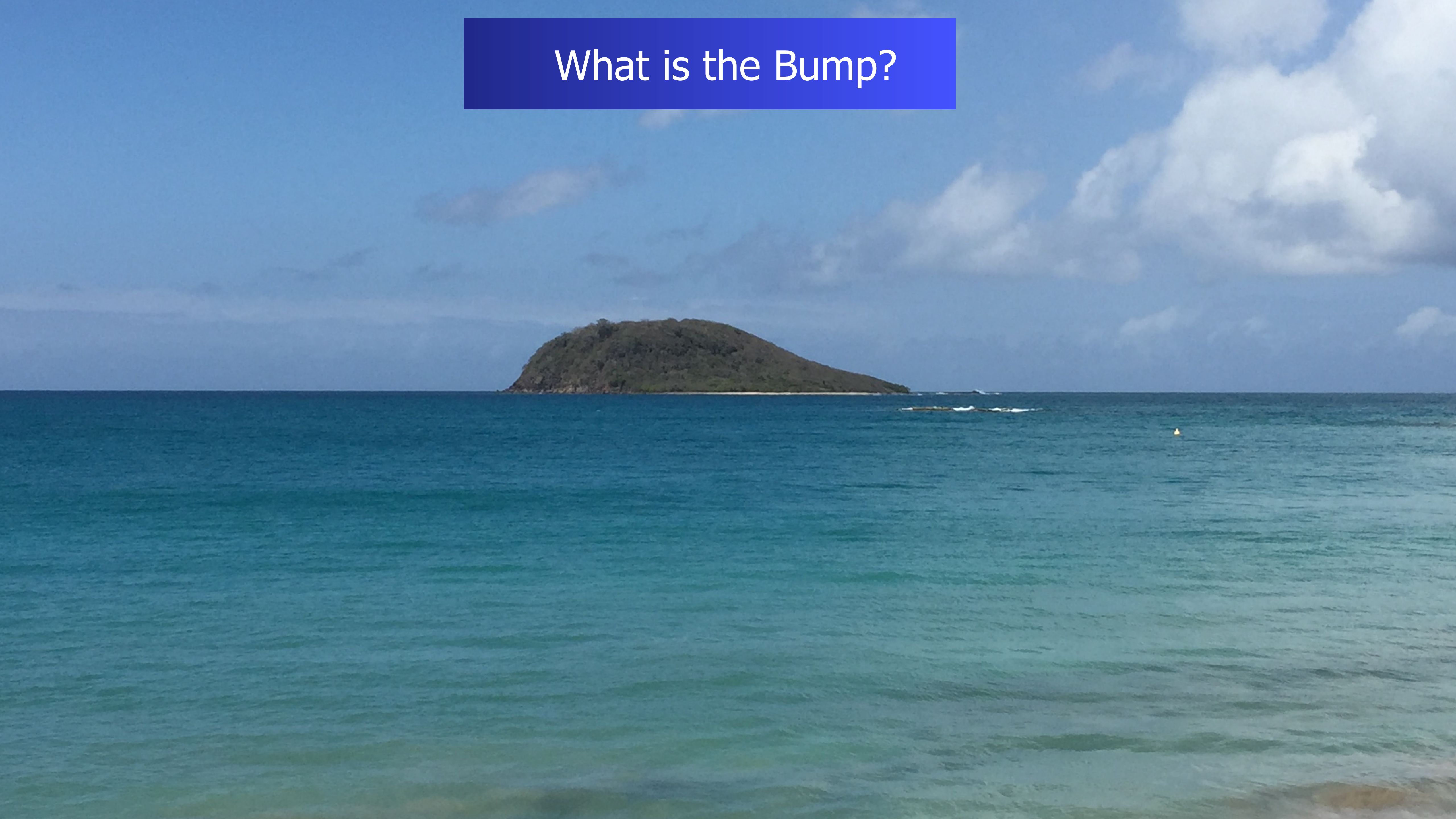




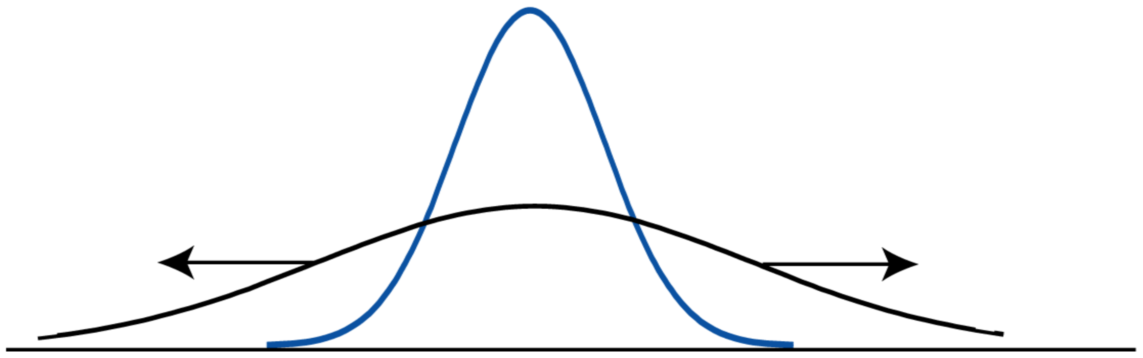
What is the Bump?



Diffusion vs. Wave Motion

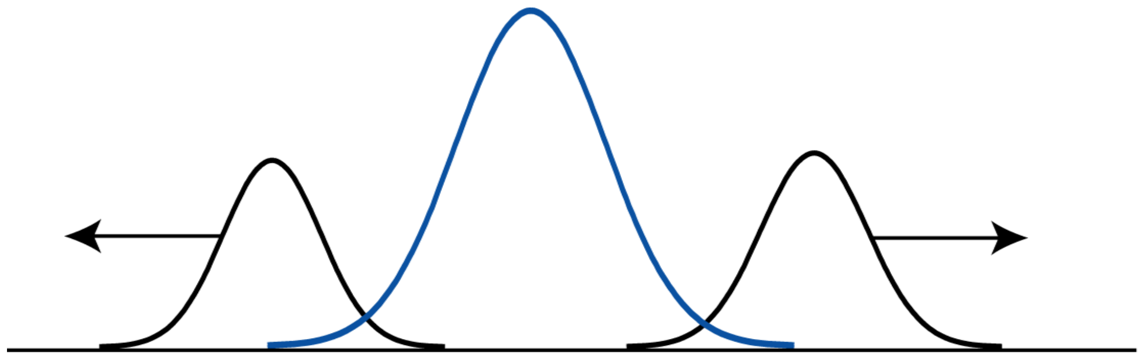
Diffusion (1st Order)

- Gaussian peak spreads
- tails violate causality



Wave propagation

- peak splits into left and right traveling pulses
- propagation speed v



2^{nd} Order Viscous Diffusion

causal transport equation:

$$\left(\frac{\tau_\pi}{2} \frac{\partial^2}{\partial t^2} + \frac{\partial}{\partial t} - \nu\left(\nabla_1^2 + \nabla_2^2\right) \right) \Delta r = 0$$

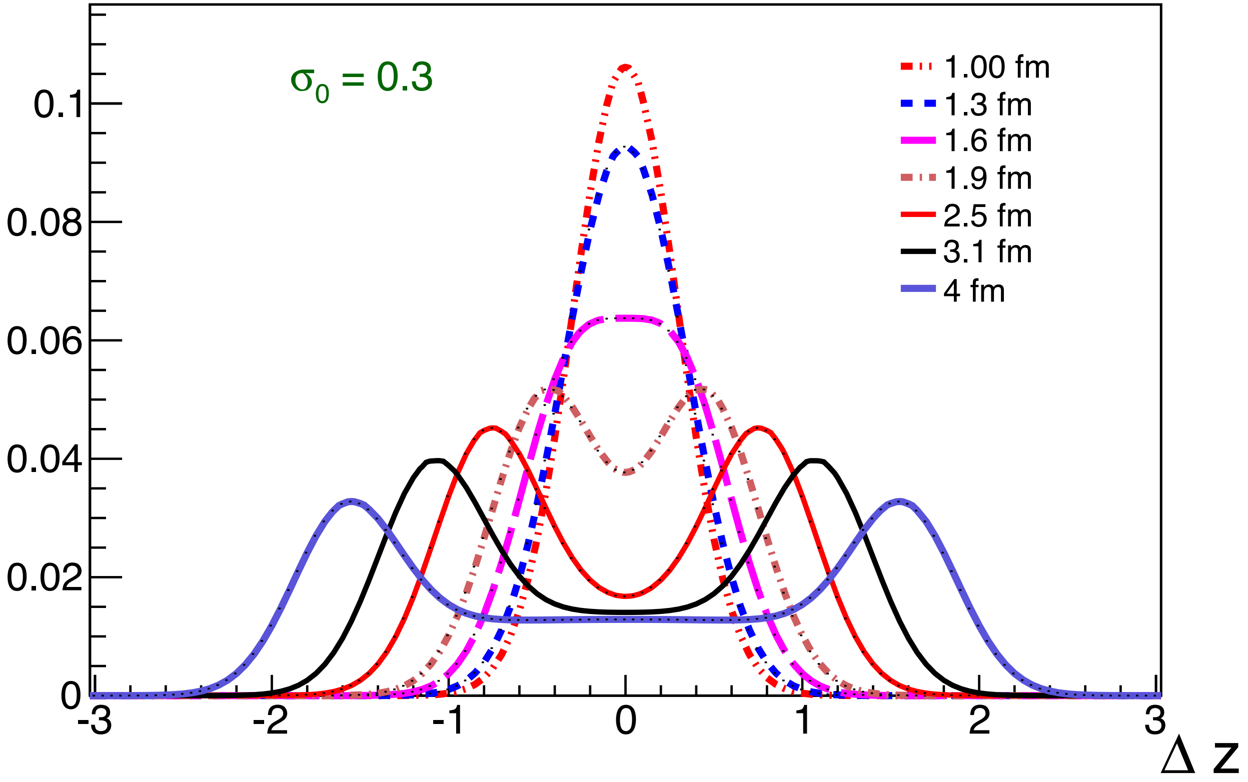
- transverse modes
- linearized Israel-Stewart

relaxation time τ_π

coordinate space:

- wave-fronts traveling at speed $= (\nu/\tau_\pi)^{1/2}$
- diffusion-like behavior in between
- no peak at $\Delta z = 0$

$\Delta r = r - r_{eq}$

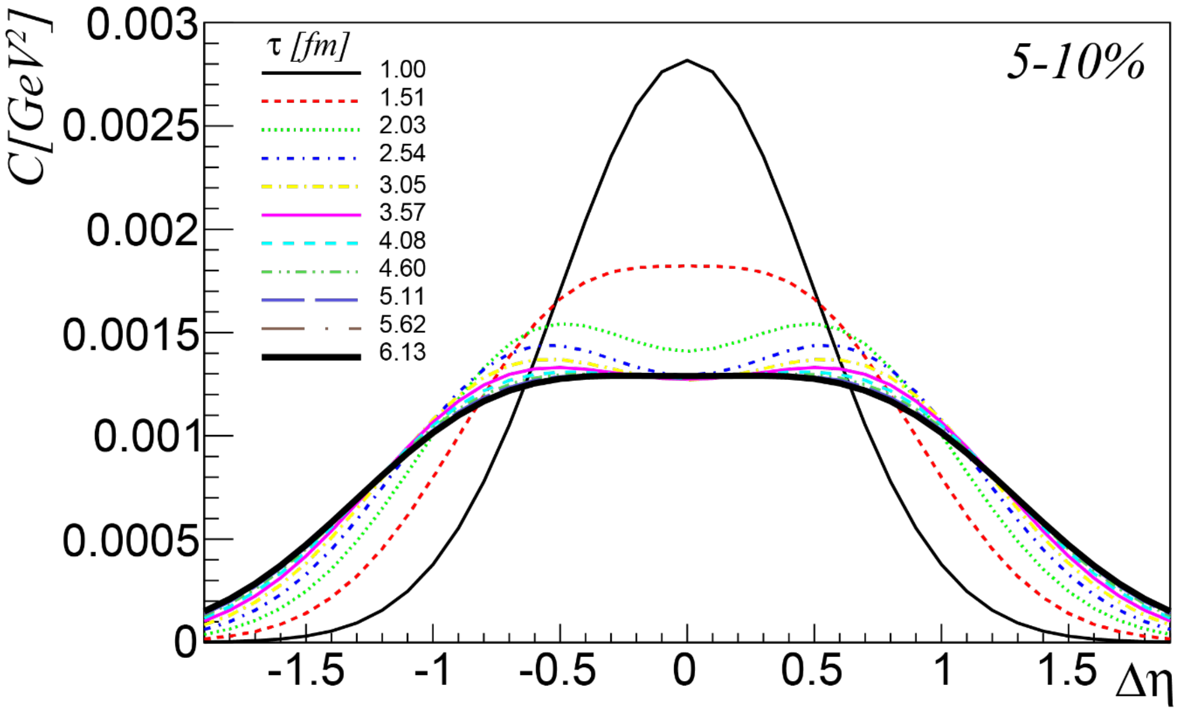


2^{nd} Order Viscous Diffusion in Rapidity

$$\left(\frac{\tau_\pi}{2}\frac{\partial^2}{\partial\tau^2}+\frac{\partial}{\partial\tau}-\frac{\nu}{\tau^2}\left(\frac{\partial^2}{\partial\eta_1^2}+\frac{\partial^2}{\partial\eta_2^2}\right)\right)\Delta r=0$$

spatial rapidity

- rapidity separation of fronts saturates
  $\Delta\eta \sim \Delta z/\tau$
- profile depends on initial width σ_0



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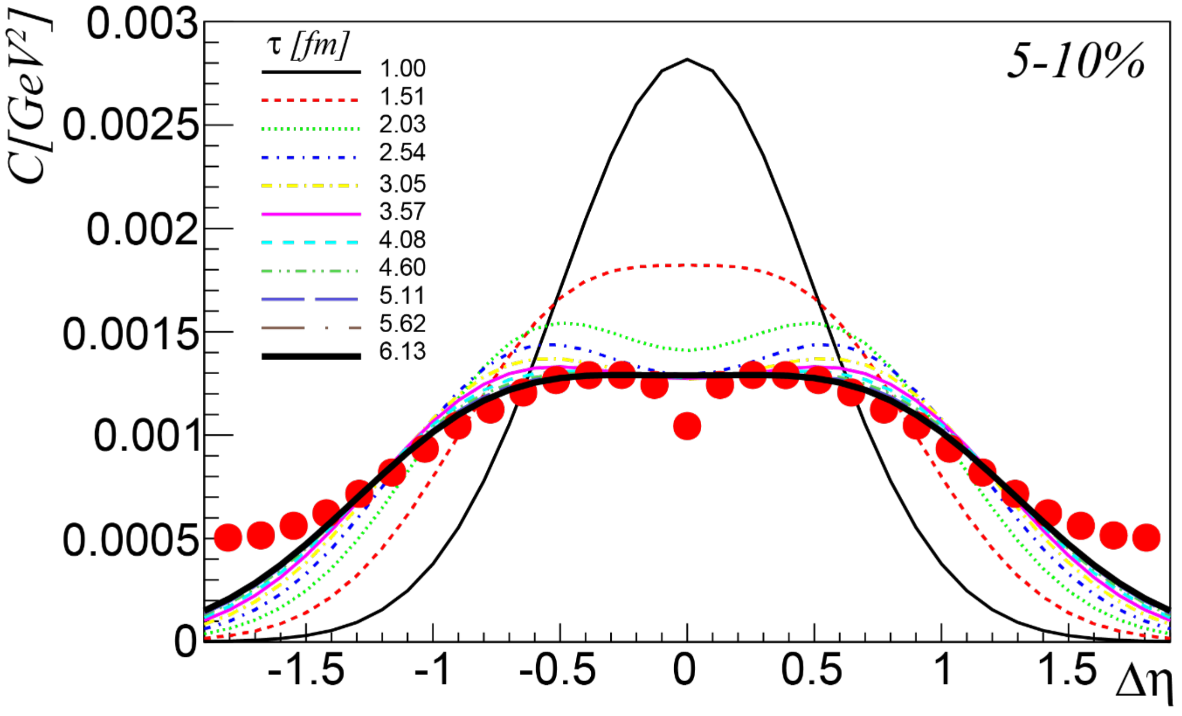
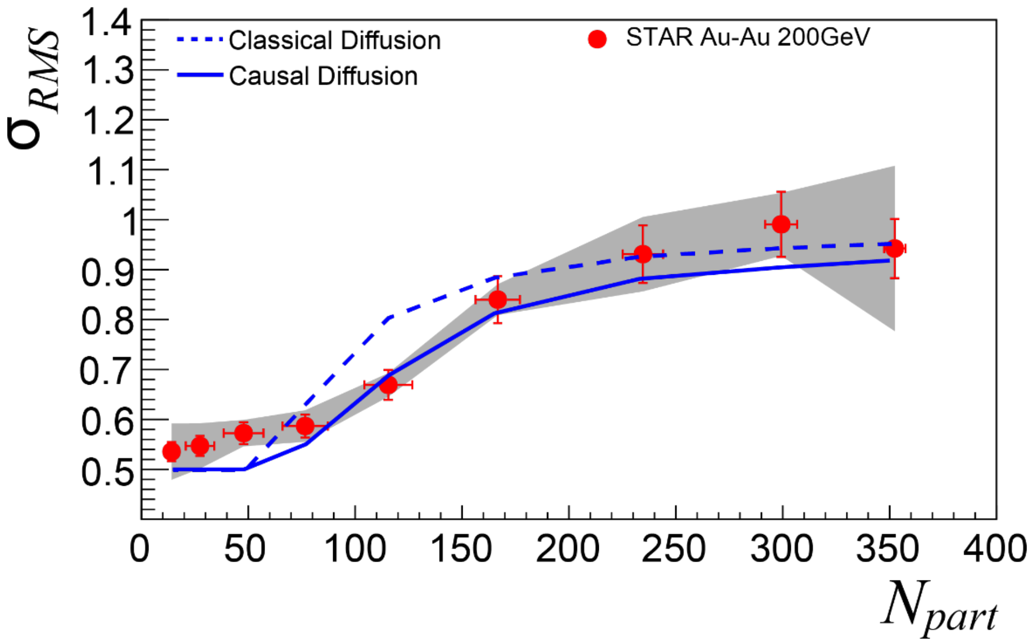
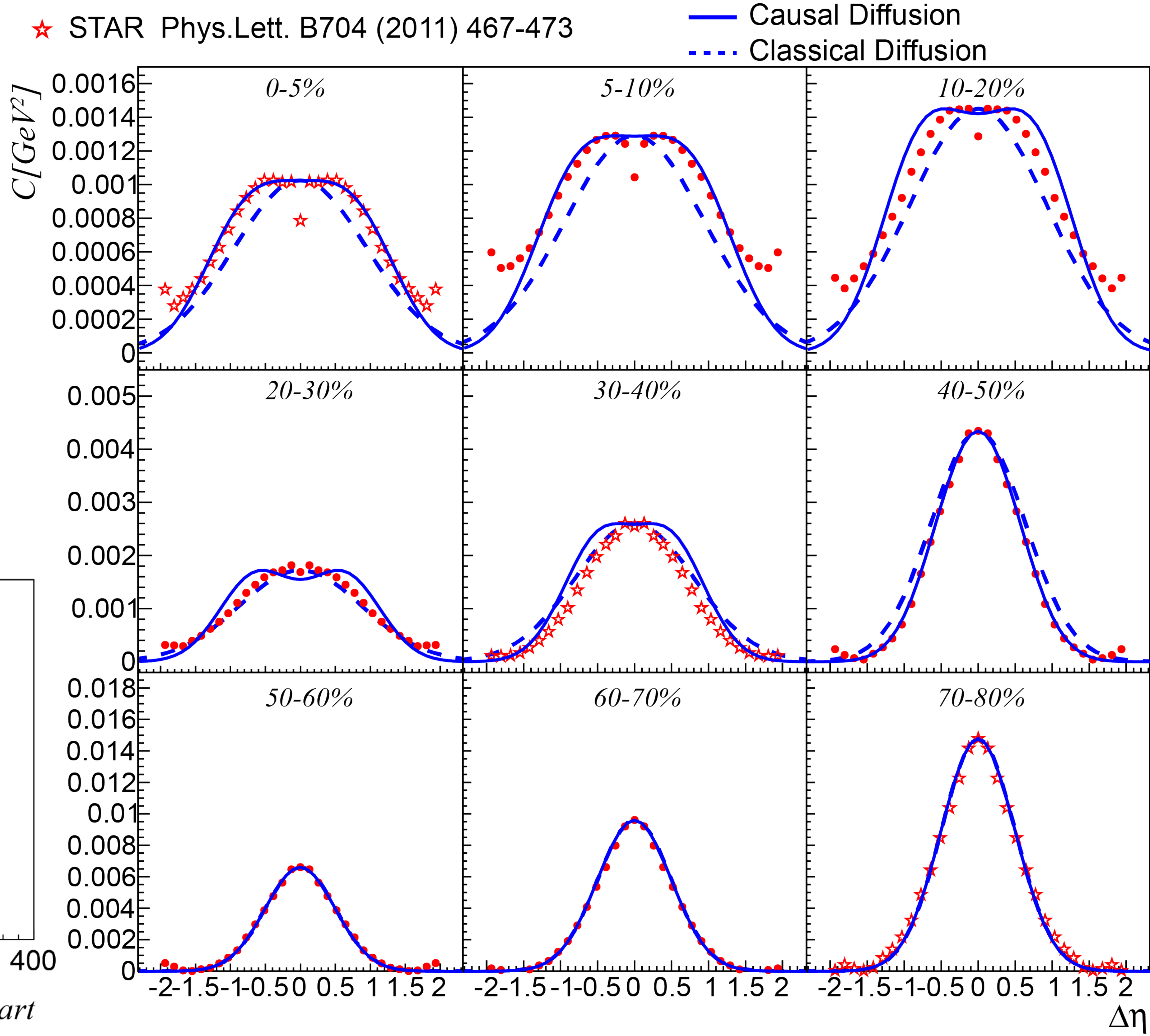


1^{st} vs. 2^{nd} Order

2^{nd} Order Works:

- Broader than Gaussian
- "Valley" appears in more central collisions





1st vs. 2nd Order

2nd Order Works:

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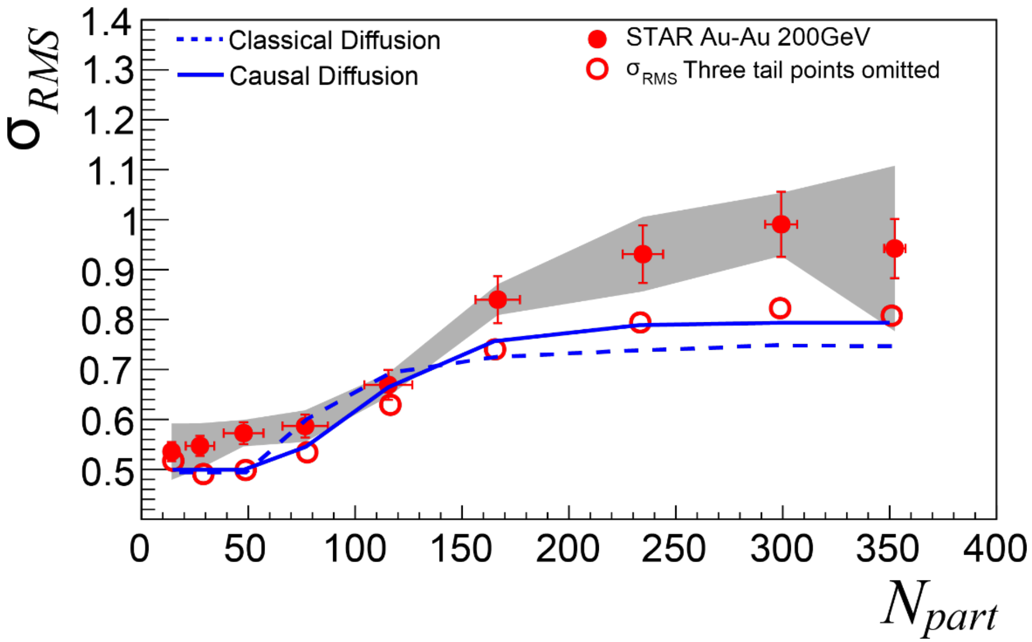
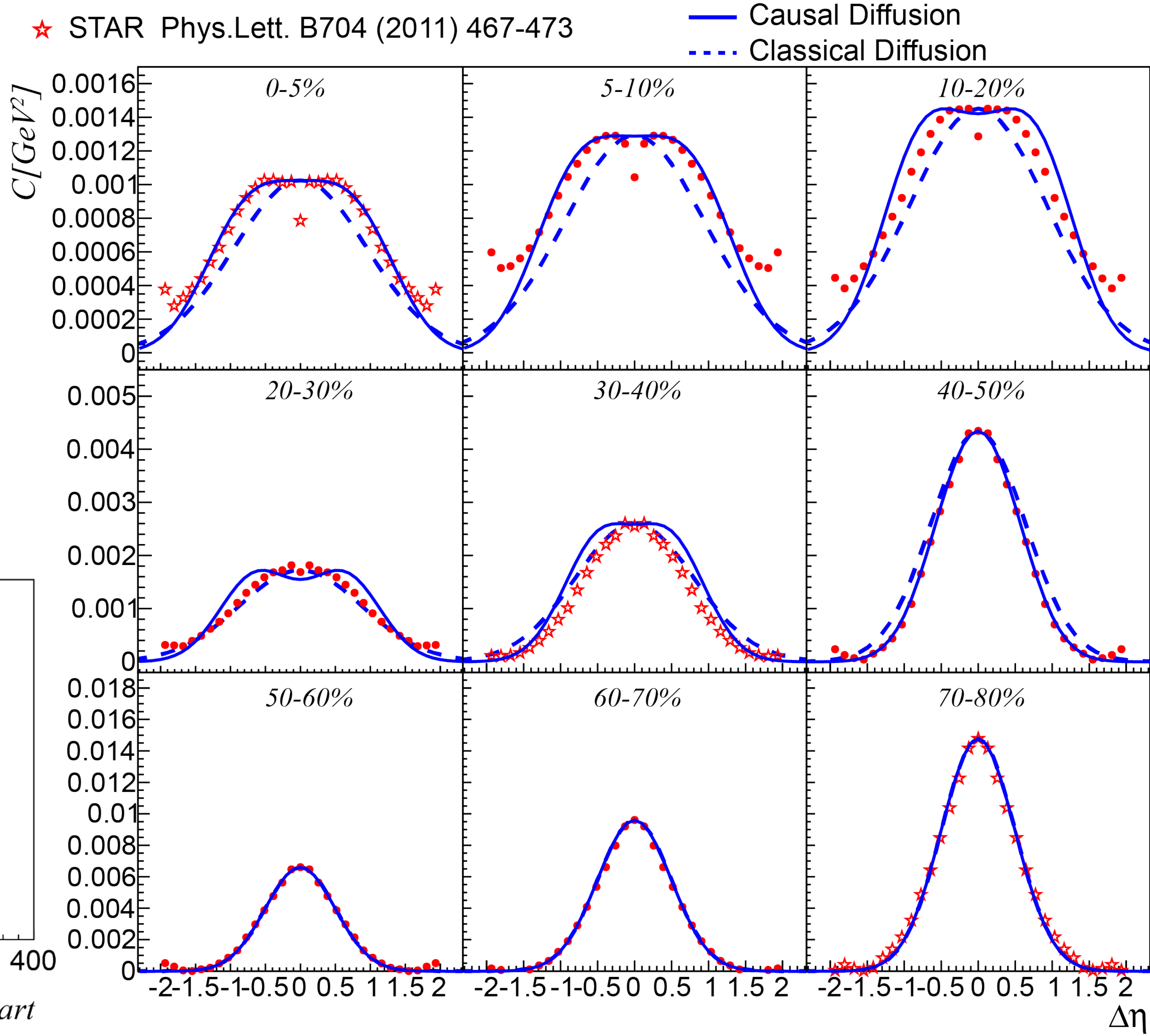




Realistic 2nd Order Viscous Diffusion

Moschelli, Pokharel, S.G., in progress

average Bjorken flow temperature vs time:

entropy production:

$$\frac{ds}{d\tau} + \frac{s}{\tau} = \frac{\Phi}{T\tau}$$

$$\tau_\pi = \beta\nu \qquad \nu = \eta / sT$$

relaxation equation:

causality delays heating

$$\frac{d\Phi}{d\tau} = -\frac{1}{\tau_\pi}\left(\Phi - \frac{4\eta}{3\tau}\right) - \frac{\kappa}{\tau}\Phi$$

$$\kappa = \frac{1}{2}\left[1 - \frac{d\ln\left(\tau_\pi / \eta T\right)}{d\ln s}\right]$$

fluctuations:

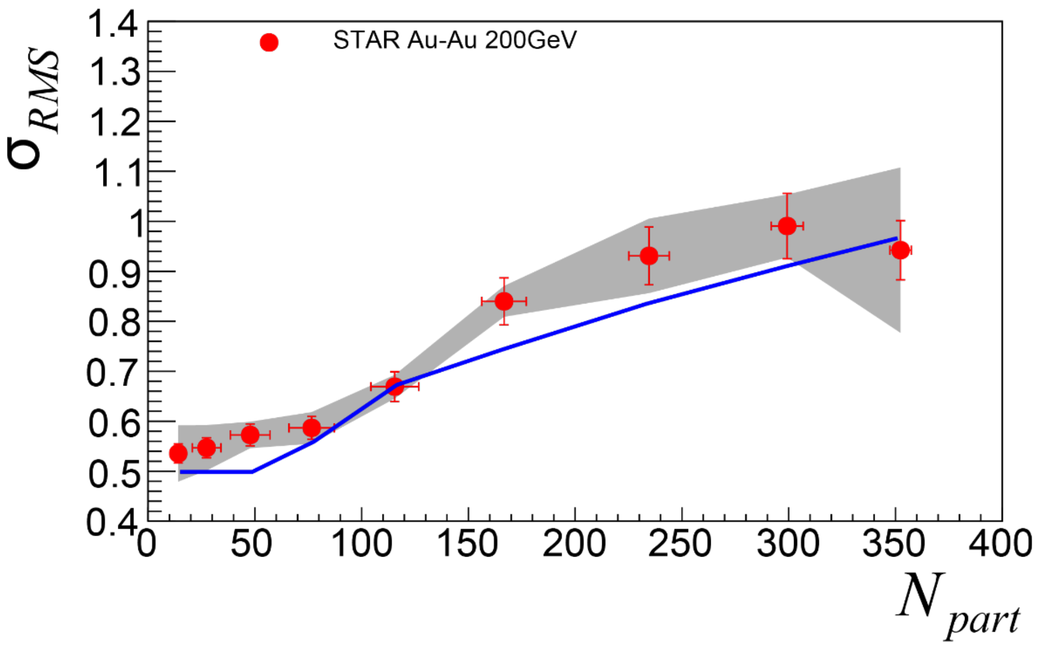
$$\left(\frac{\tau_\pi^*}{2}\frac{\partial^2}{\partial\tau^2} + \frac{\partial}{\partial\tau} - \nu^*\left(\nabla_1^2 + \nabla_2^2\right)\right)\Delta r = 0$$

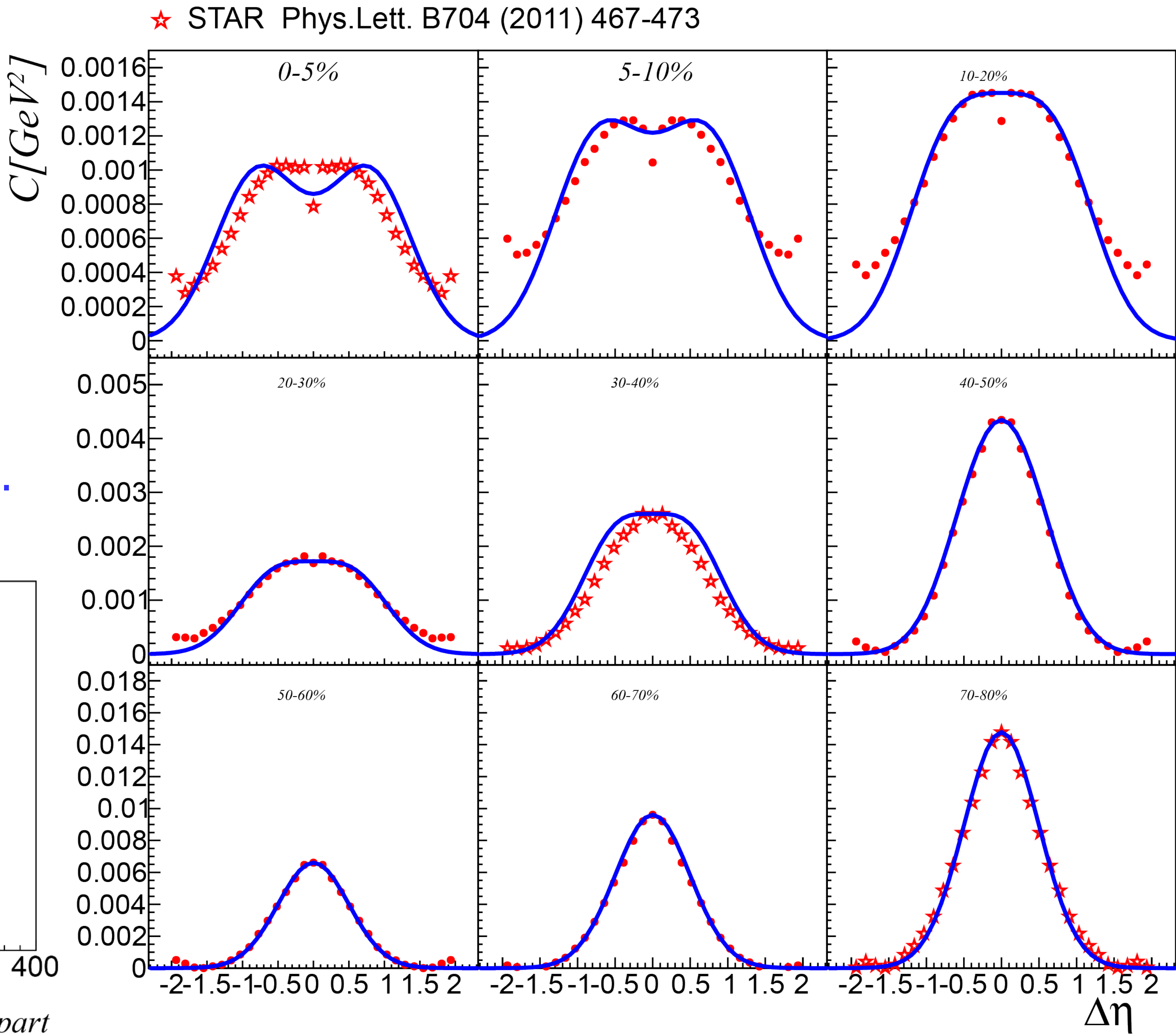
$$\tau_\pi^* = \frac{\tau_\pi}{1 + \kappa\tau_\pi / \tau} \qquad \nu^* = \frac{\nu}{1 + \kappa\tau_\pi / \tau}$$

Progress

Israel-Stewart fluctuations on Bjorken Background

- Lattice EOS – HotQCD Collaboration
- Lattice viscosity – Nakamura & Sakai
- Hagadorn HG – Noronha-Hostler et al.





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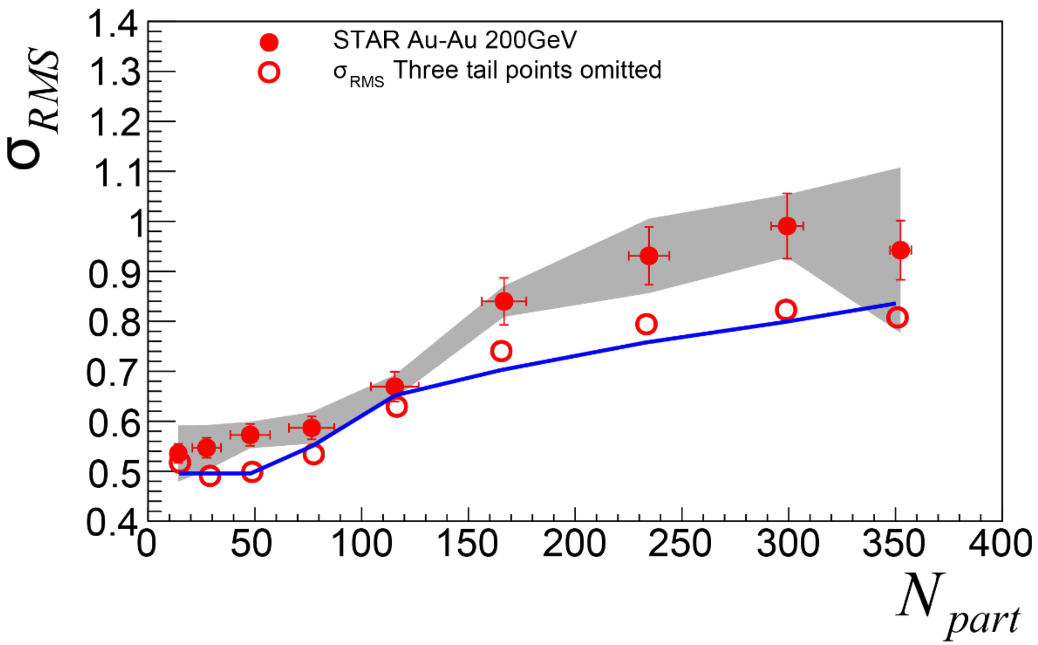
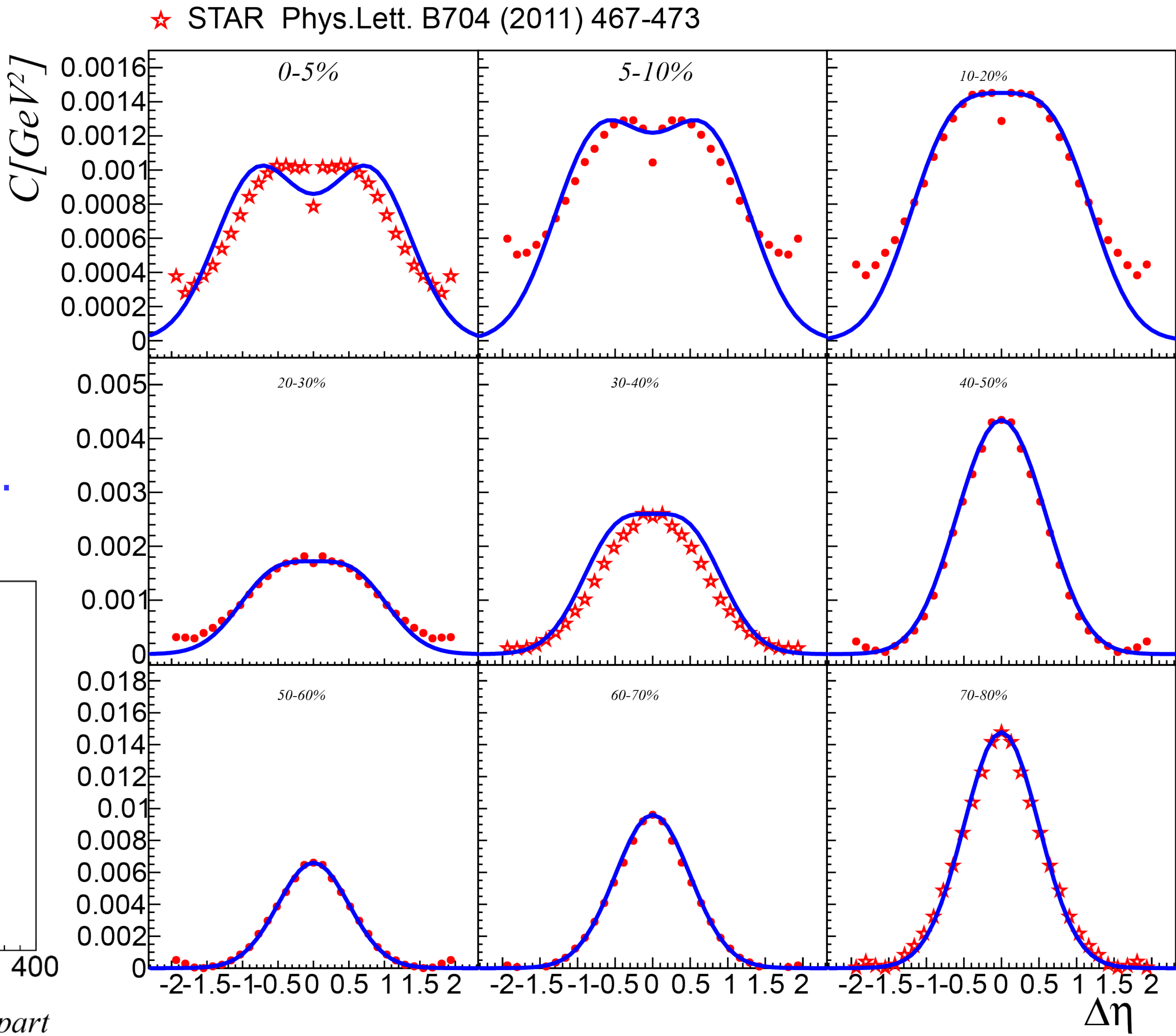
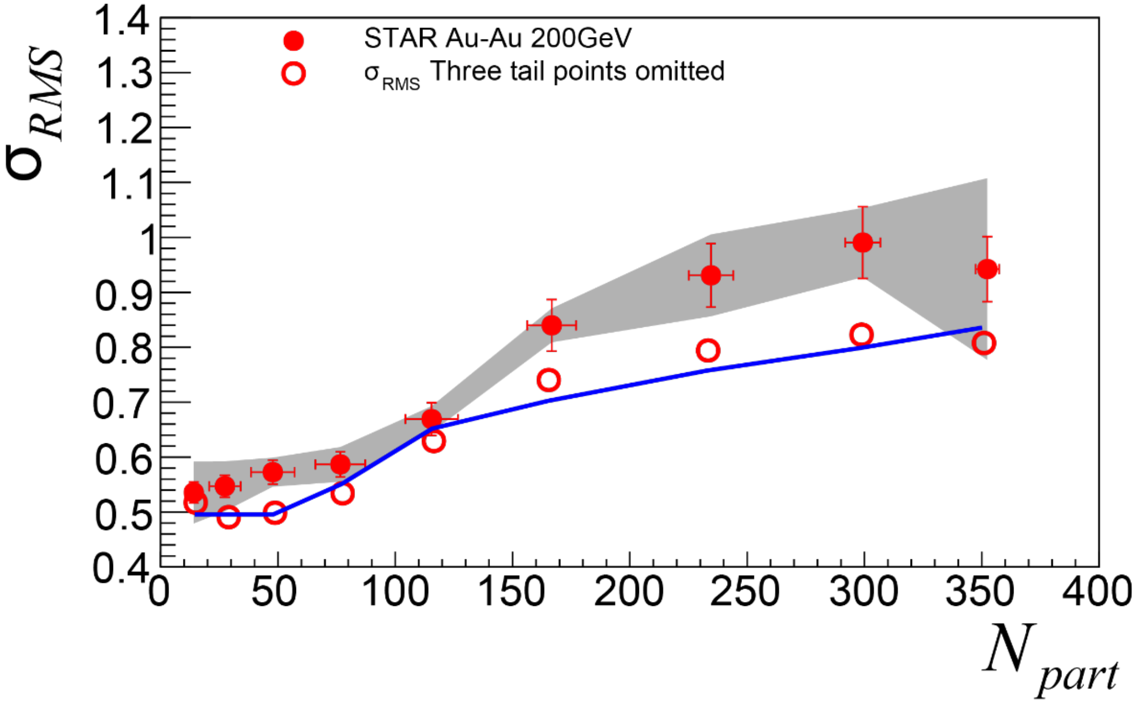




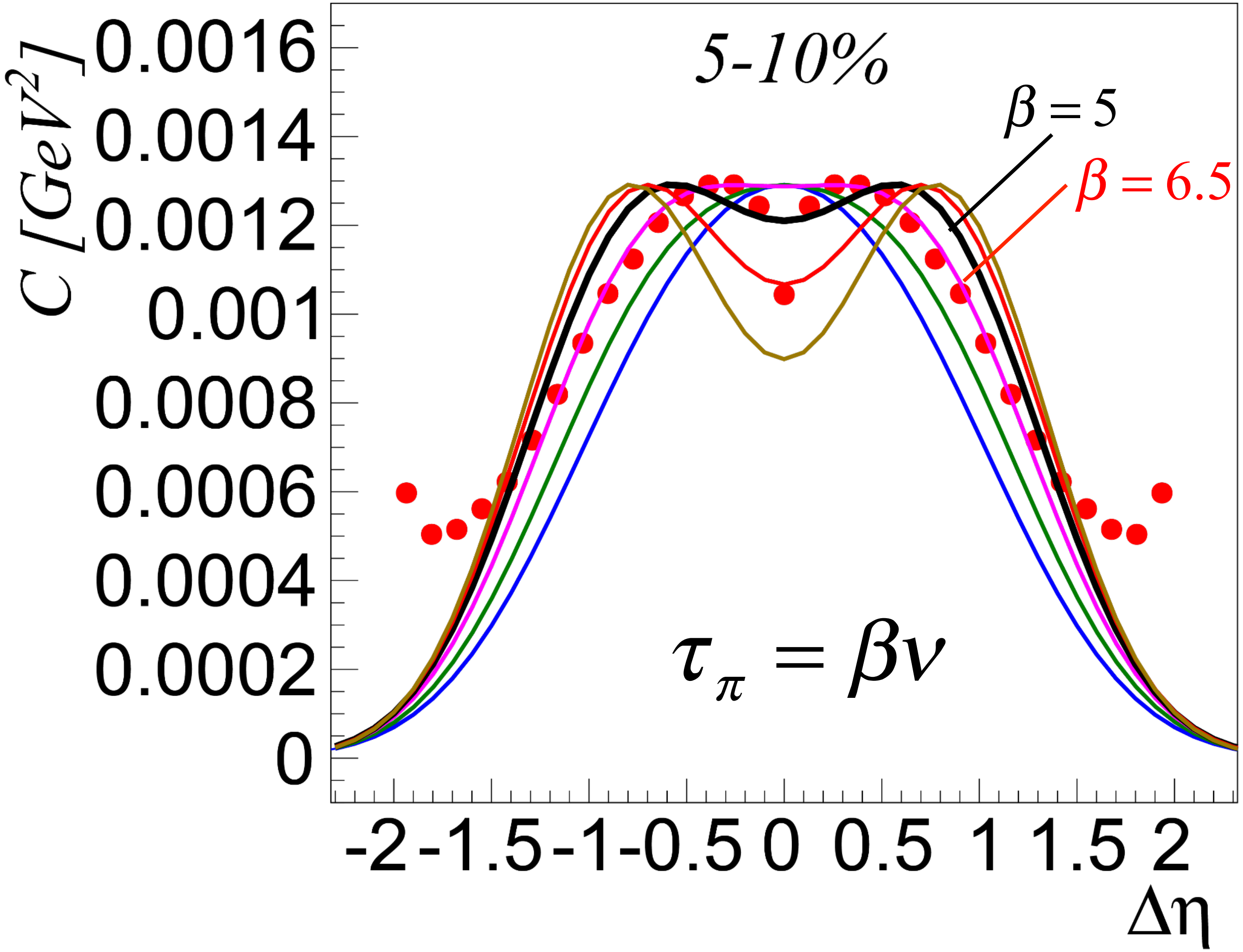
Measuring the Relaxation Time

Width $\Rightarrow$ **shear viscosity**

Valley $\Rightarrow$ **relaxation time**



Moschelli, Pokharel, S.G. in preparation



Summary: rapidity dependence of p_t correlations

Hydro formulation: longitudinal and transverse modes

- Sound waves, shear modes, and heat modes
- Diffusive transverse shear modes important for rapidity dependence of p_t correlations
- 1st and 2nd order viscous fluctuating hydro description of shear modes

Causality shapes the rapidity dependence of correlations

- Shear viscosity ➔ increase of rapidity width with centrality
- Relaxation time ➔ "valley"

Open Questions

- Influence of sound and heat modes on observables
- Charge balancing, resonances, jets, HBT


Perhaps I can help you with that hump.
What hump?

Noisy Diffusion

difference equation $$\Delta g = \nu \nabla^2 g \Delta t + \Delta W$$

noise $$\langle \Delta W \rangle = 0 \qquad \langle \Delta W(x_1) \Delta W(x_2) \rangle = \Gamma_{12} \Delta t$$

variance $$\Delta \langle g_1 g_2 \rangle = \langle g_1 \Delta g_2 \rangle + \langle \Delta g_1 g_2 \rangle + \langle \Delta g_1 \Delta g_2 \rangle$$

$$= \nu \left(\nabla_1^2 + \nabla_2^2 \right) \langle g_1 g_2 \rangle \Delta t + \Gamma_{12} \Delta t$$

Noisy Diffusion

difference equation

$$\Delta g = \nu \nabla^2 g \Delta t + \Delta W$$

noise

$$\langle \Delta W \rangle = 0 \qquad \langle \Delta W(x_1) \Delta W(x_2) \rangle = \Gamma_{12} \Delta t$$

variance

$$\Delta \langle g_1 g_2 \rangle == \left(\nu \left(\nabla_1^2 + \nabla_2^2 \right) \langle g_1 g_2 \rangle + \Gamma_{12} \right) \Delta t$$

diffusion equation for correlation function:

$$\frac{\partial}{\partial t} \langle g_1 g_2 \rangle = \nu \left(\nabla_1^2 + \nabla_2^2 \right) \langle g_1 g_2 \rangle + \Gamma_{12}$$

$$\Gamma_{12} = 2 \nabla_1 \cdot \nabla_2 \eta T \delta(x_1 - x_2)$$

Covariance Measures Momentum Flux

covariance $$C = \frac{1}{\langle N \rangle^2}\left\langle \sum_{\text{pairs } i \neq j} p_{ti} p_{tj} \right\rangle - \langle p_t \rangle^2$$

unrestricted sum: $$\sum_{\text{all} i,j} p_{ti} p_{tj} = \int p_{t1} p_{t2} dn_1 dn_2$$

$$dn = f(x,p)\, dp\, dx$$

$$g_t(x) = \int dp\; p_t \Delta f(x,p)$$

$$= \int dx_1 dx_2 \left(\int dp_1\, p_{t1} f_1 \right)\left(\int dp_2 p_{t2} f_2 \right)$$

$$= \langle N \rangle^2 \langle p_t \rangle^2 + \int g(x_1)\, g(x_2)\, dx_1\, dx_2$$

correlation function: $$r_g = \langle g_t(x_1) g_t(x_2) \rangle - \langle g_t(x_1) \rangle \langle g_t(x_2) \rangle$$

$$\int r_g\, dx_1\, dx_2 = \left\langle \sum p_{ti} p_{tj} \right\rangle - \langle N \rangle^2 \langle p_t \rangle^2 = \left\langle \sum p_{ti}{}^2 \right\rangle + \langle N \rangle^2 C$$

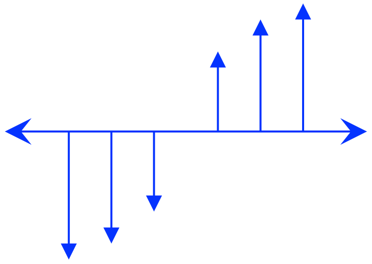
$C = 0$ in equilibrium $$C = \frac{1}{\langle N \rangle^2} \int (r_g - r_{g,eq})\, dx_1\, dx_2$$

Hydrodynamic Modes

"transverse" modes $\vec{\nabla} \cdot \vec{g} = 0$

viscous diffusion

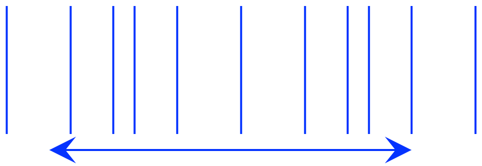
$$\partial_t \vec{g} = \nu \nabla^2 \vec{g}$$



- no transverse 'sound waves'
- kinematic viscosity $\nu = \eta / Ts$
- vorticity $\vec{\omega} \propto \vec{\nabla} \times \vec{g}$

longitudinal modes $\vec{\nabla} \times \vec{g}_L = 0$

$$\partial_t \vec{g}_L + \vec{\nabla} p = \frac{\frac{4}{3}\eta + \zeta}{sT} \vec{\nabla}(\vec{\nabla} \cdot \vec{g}_L)$$



longitudinal modes + energy and baryon conservation imply:

sound waves – compression waves, damped by viscosity

thermal diffusion – heat flow relative to baryons

Momentum in Fluctuating Hydrodynamics

momentum current – small fluctuations

$$M_i \equiv T_{0i} - \langle T_{0i} \rangle \approx (e+p)v_i \approx sTv_i$$

momentum conservation – linearized Navier-Stokes

$$\partial_t M_i + \nabla_i p = \frac{\eta/3+\zeta}{sT} \nabla_i (\vec{\nabla} \cdot \vec{M}) + \frac{\eta}{sT} \nabla^2 M_i$$

Helmholtz decomposition: $\vec{M} \equiv \vec{g}_L + \vec{g}$

"longitudinal" mode: $\vec{\nabla} \times \vec{g}_L = 0$

"transverse" modes: $\vec{\nabla} \cdot \vec{g} = 0$

Hydrodynamic Momentum Correlations

momentum flux density correlation function

$$r = \langle g_r(x_1) g_r(x_2) \rangle - \langle g_r(x_1) \rangle \langle g_r(x_2) \rangle$$

$\Delta r = r - r_{eq}$ **satisfies deterministic diffusion equation**

Gardiner, Handbook of Stochastic Methods, (Springer, 2002)

fluctuations diffuse through volume, driving r ➔ r_{eq}

width in relative spatial rapidity grows
from initial value σ_0

$$\sigma^2 = \sigma_0^{\ 2} + 4\frac{\eta}{Ts}\left(\frac{1}{\tau_0} - \frac{1}{\tau}\right)$$