

# Measurement of Transfer Matrix

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# OUTLINE



- *Matrix elements in MC & Data*
- *How well does the method work?*

- *Calculating the Transfer Matrix from data using least Squares Fitting*

**Motivations & Method**

**Analysis**

**Conclusions**

- *What needs to be improved?*



*Introduction*

Motivations & Method



# Transfer Matrix

- \* With knowledge of the upstream tracker co-ords and downstream co-ords we can work backwards to find the transfer matrix:

$$x^d = M_{00} + M_{01}x^u + M_{02}Px^u + M_{03}y^u + M_{04}Py^u, \quad (1)$$

$$Px^d = M_{10} + M_{11}x^u + M_{12}px^u + M_{13}y^u + M_{14}Py^u, \quad (2)$$

$$y^d = M_{20} + M_{21}x^u + M_{22}px^u + M_{23}y^u + M_{24}Py^u \quad (3)$$

$$Py^d = M_{30} + M_{31}x^u + M_{32}px^u + M_{33}y^u + M_{34}Py^u \quad (4)$$

- \* Use linear least squares fit



# Linear Least Squares

- \* Take a set of  $n$  data points  $(x_i, y_i)$  then

$$y_i = \sum_j a_j f_j(x_i)$$

or

$$y_i = a_0 + a_1 x_i$$

- \* The residual for the  $i$ th point is then defined as the difference in the  $y_i$  and  $f(x_i)$ :

$$r_i = (y_i - f(x_i)) = y_i - (a_0 + a_1 x_i)$$

$$S = \sum_i r_i^2 = \sum_i [y_i - (a_0 + a_1 x_i)]^2 = \sum_i (y_i - \sum_j a_j f_j(x_i))^2$$

- \* Aim is to minimize the sum of the squares ( $S$ ) of these residuals such that:

$$\frac{\partial S}{\partial a_0} = -2 \sum_i^n [y_i - (a_0 + a_1 x_i)] = 0$$

$$\frac{\partial S}{\partial a_1} = -2 \sum_i^n [y_i - (a_0 + a_1 x_i)] x_i = 0$$

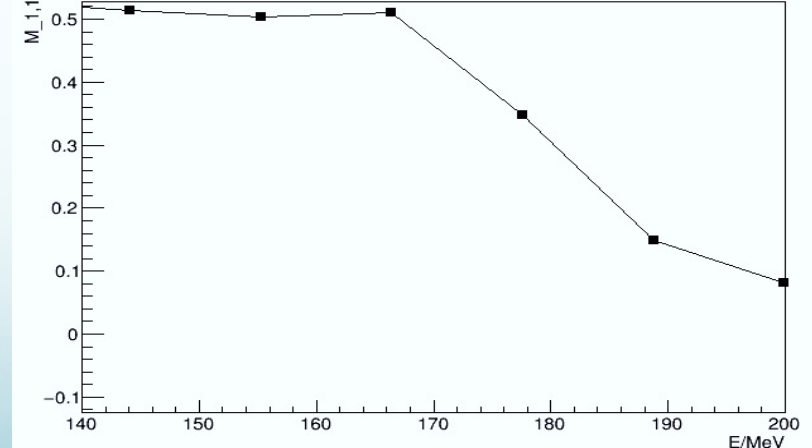
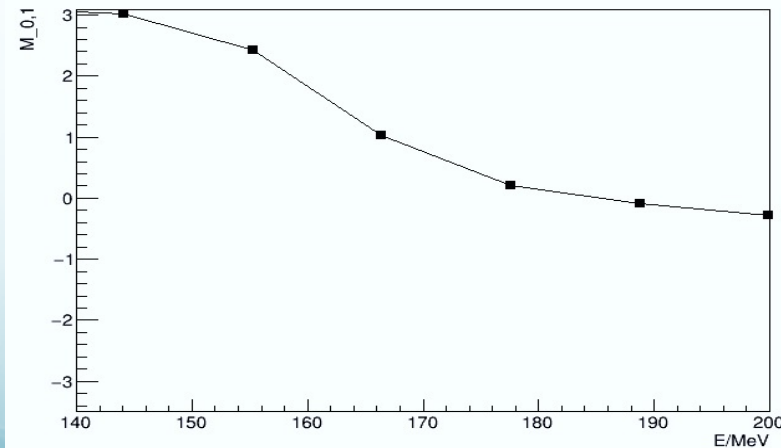
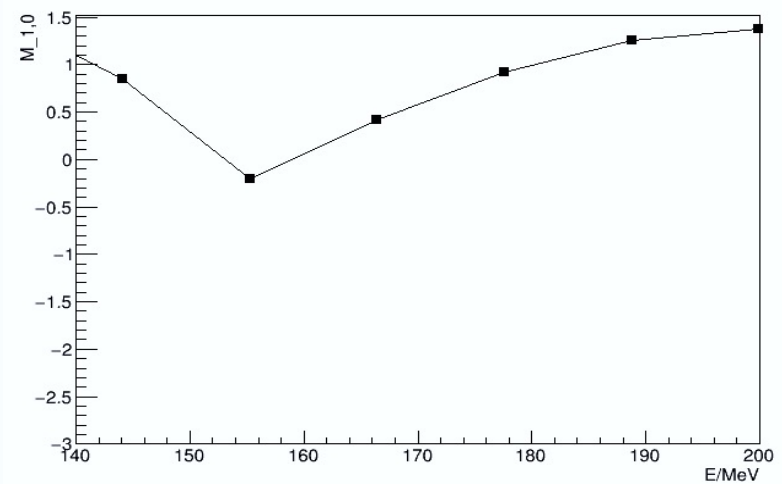
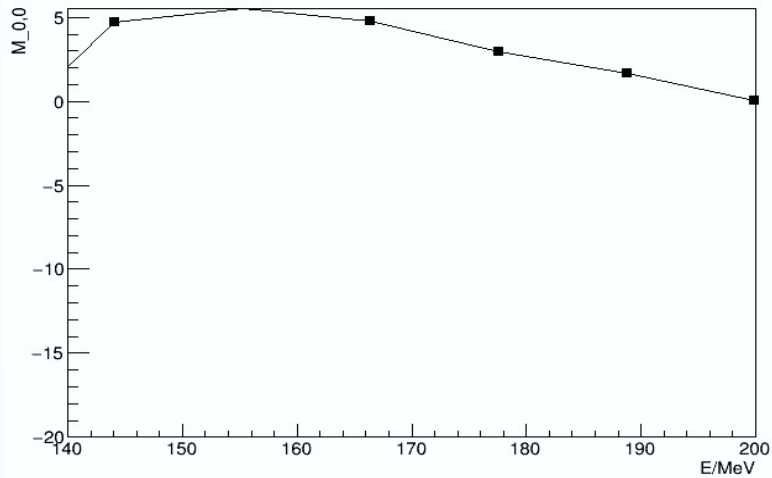


*Analysis & Results*  
Plots & Discussions



# Results : MC Truth

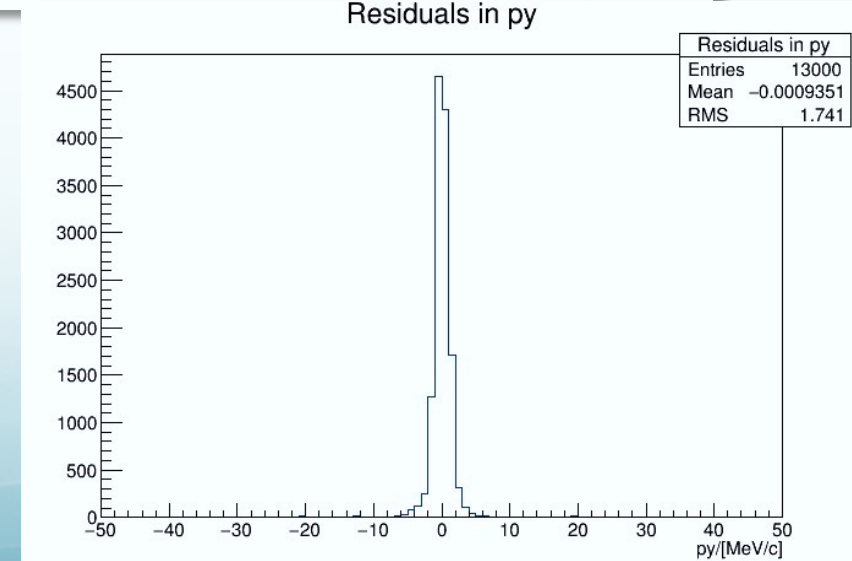
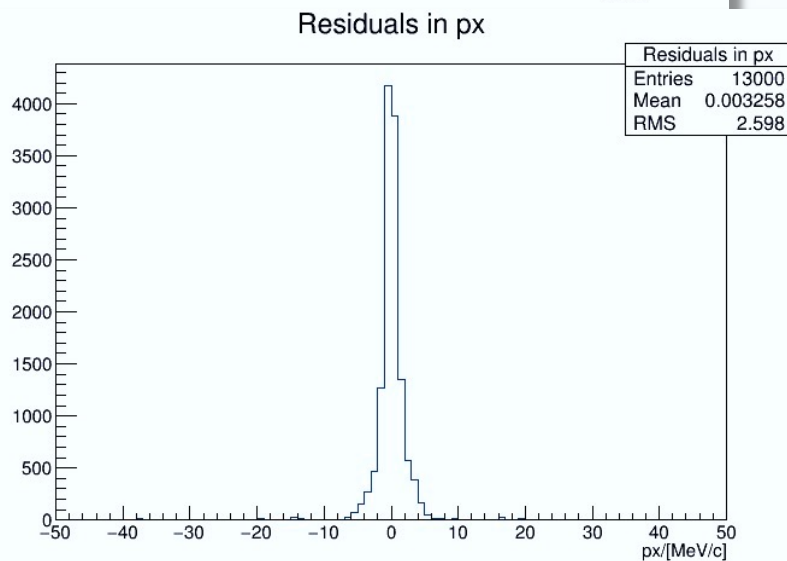
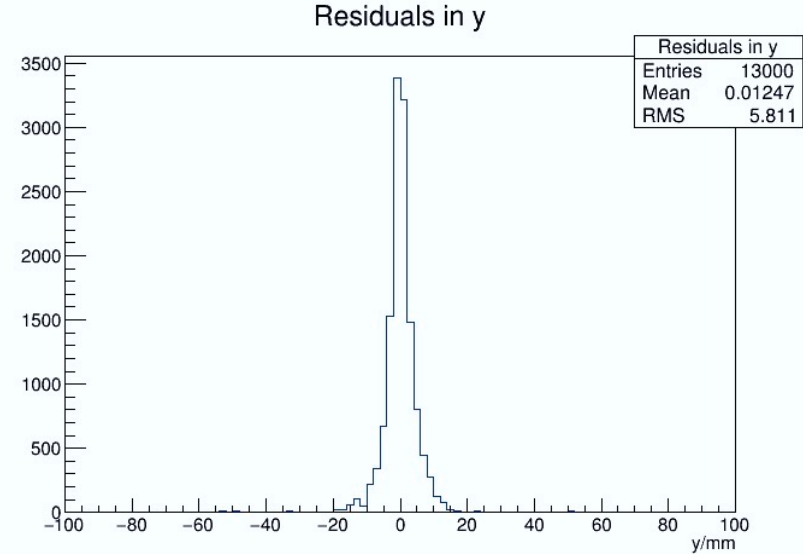
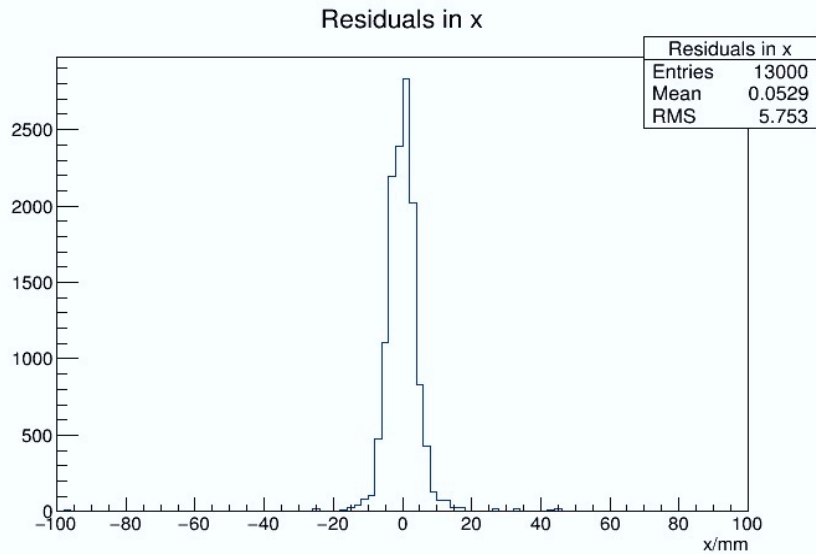
\* For MC the elements are plotted as a function of energy:



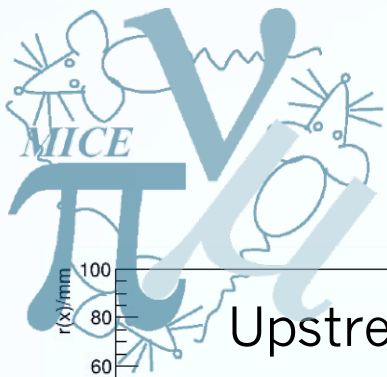


# Residuals: MC Truth

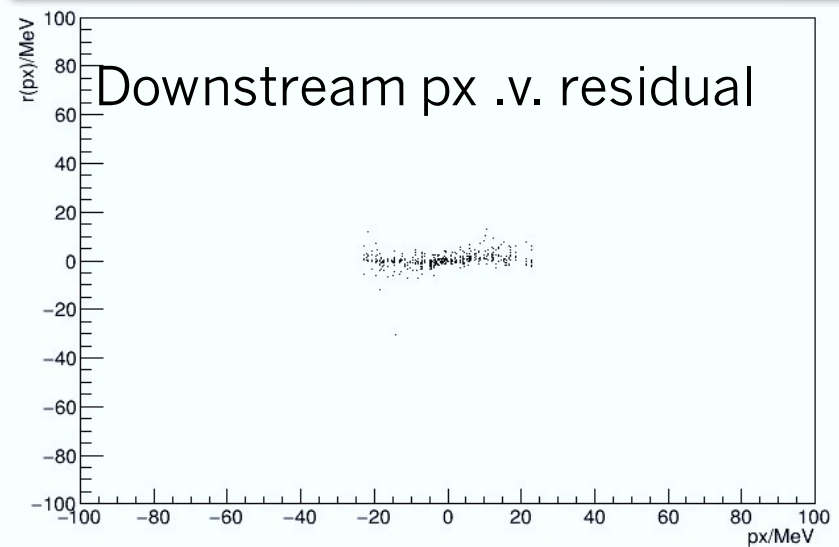
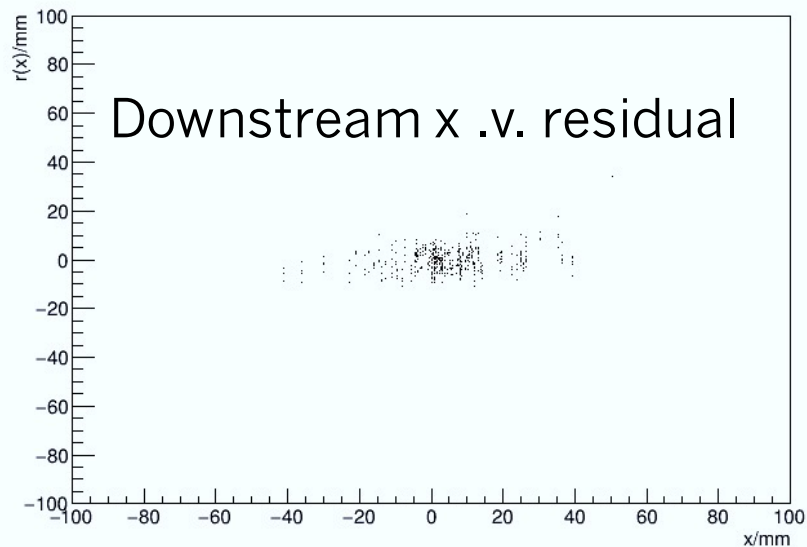
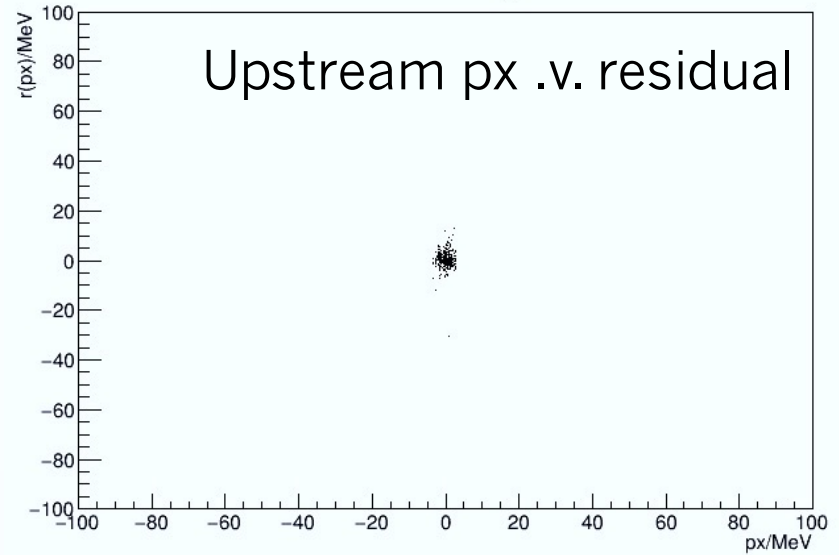
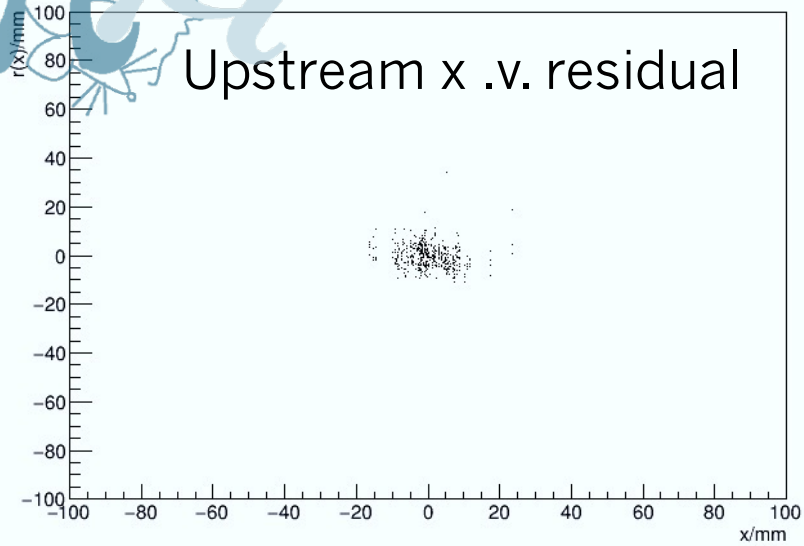
\* The Residuals are the difference in the real downstream data and that projected from the equations on slide 5

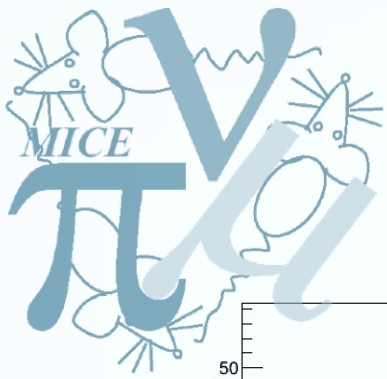






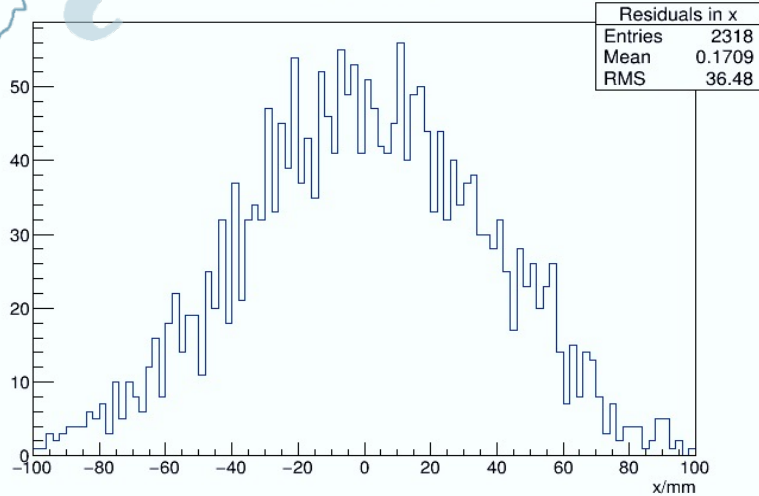
# Results : MC Truth



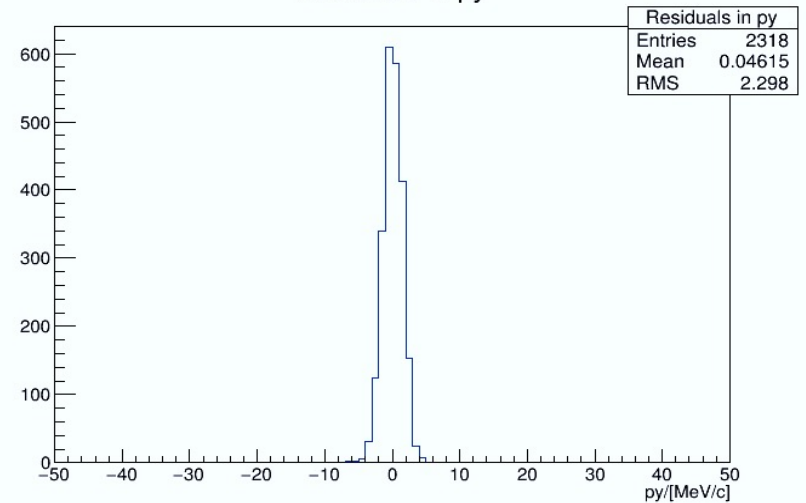


# Residuals: MC Recon

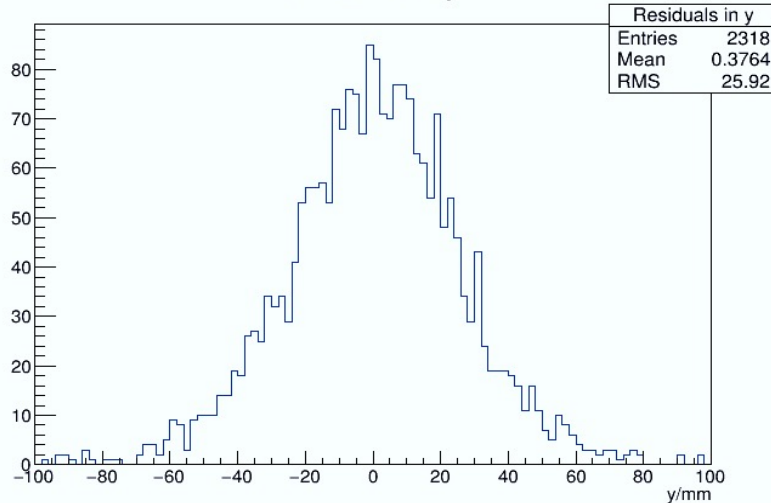
Residuals in x



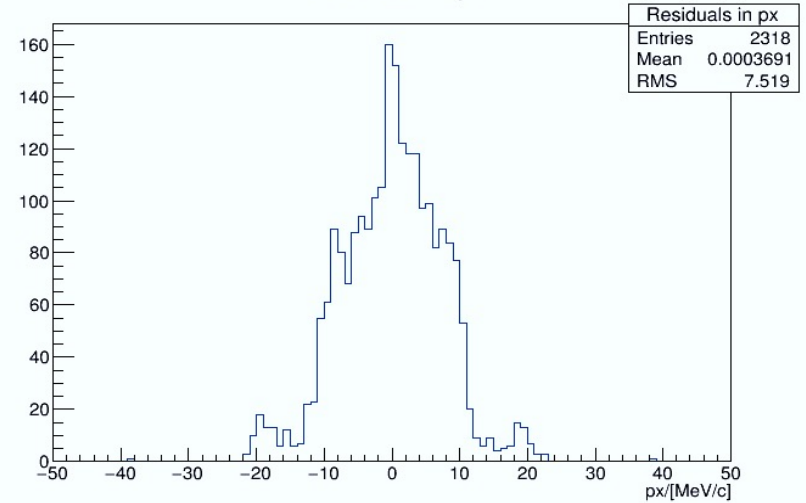
Residuals in py

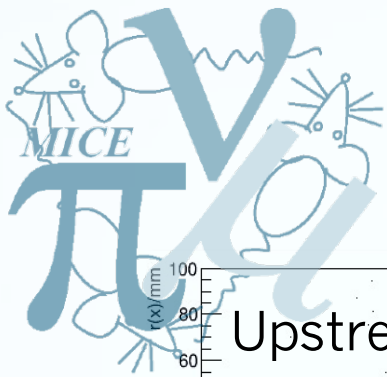


Residuals in y

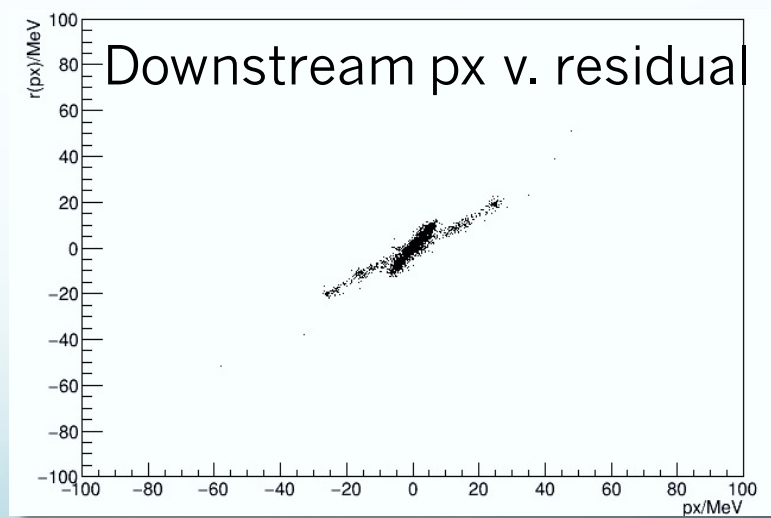
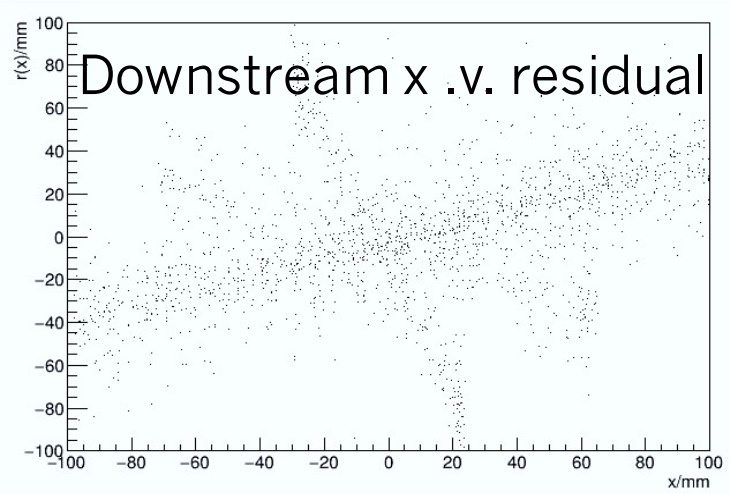
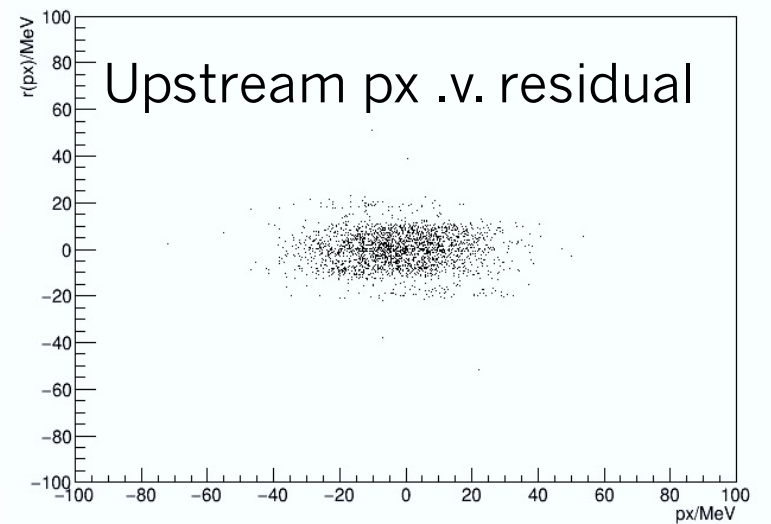
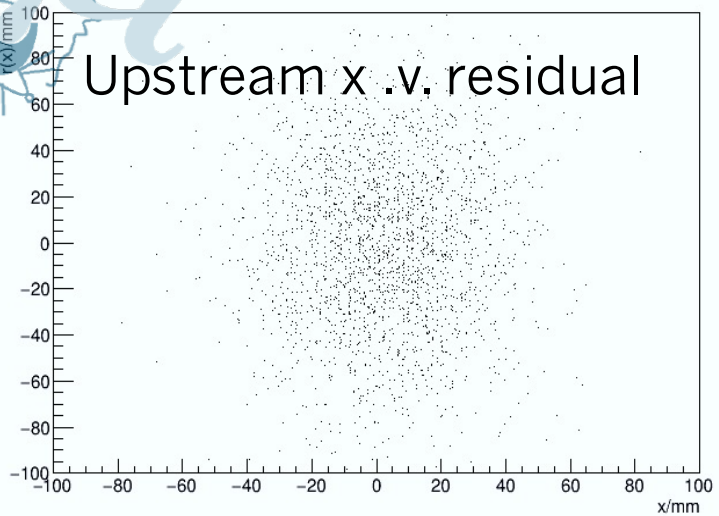


Residuals in px





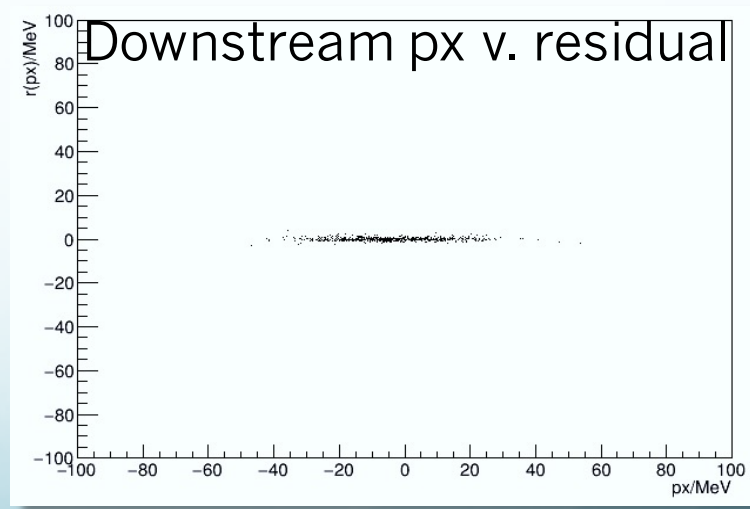
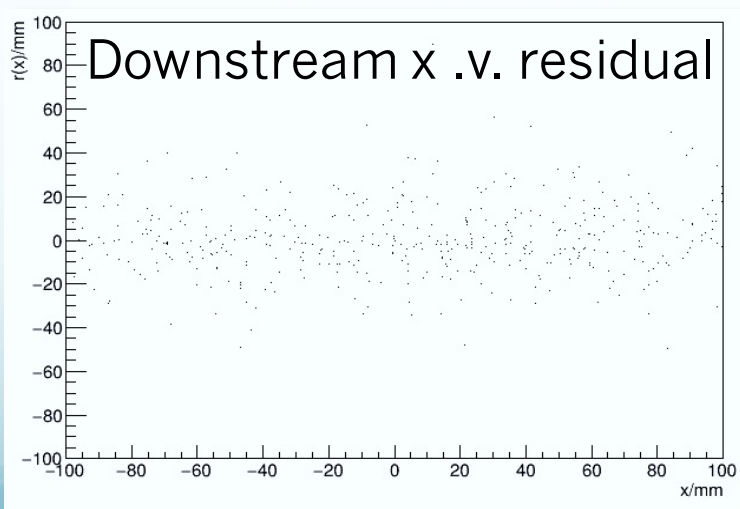
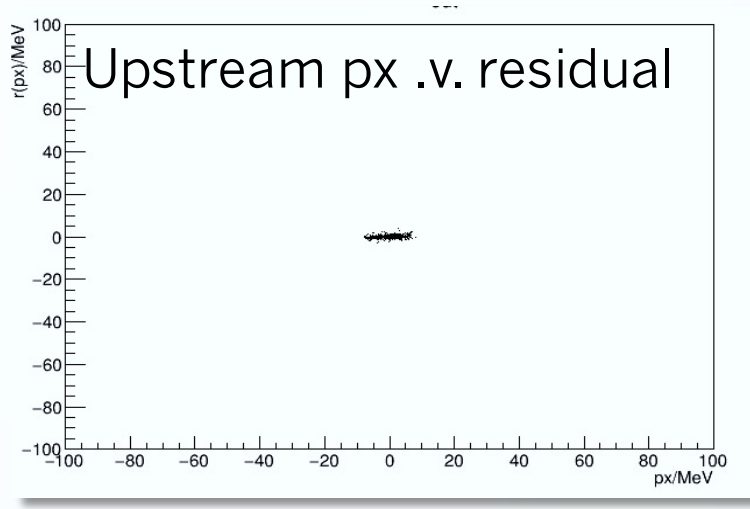
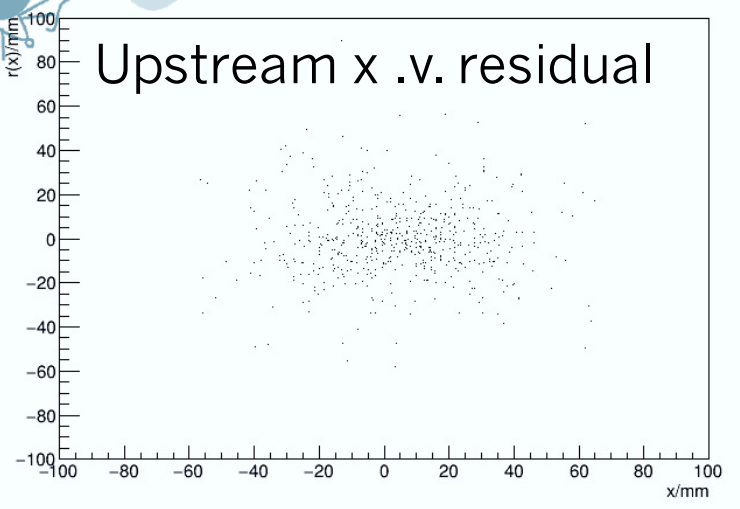
# Results: MC Recon





# VMC Recon: P Value Cut

\* I added a P-Values cut at  $P > 0.5$  .... the results look better.....





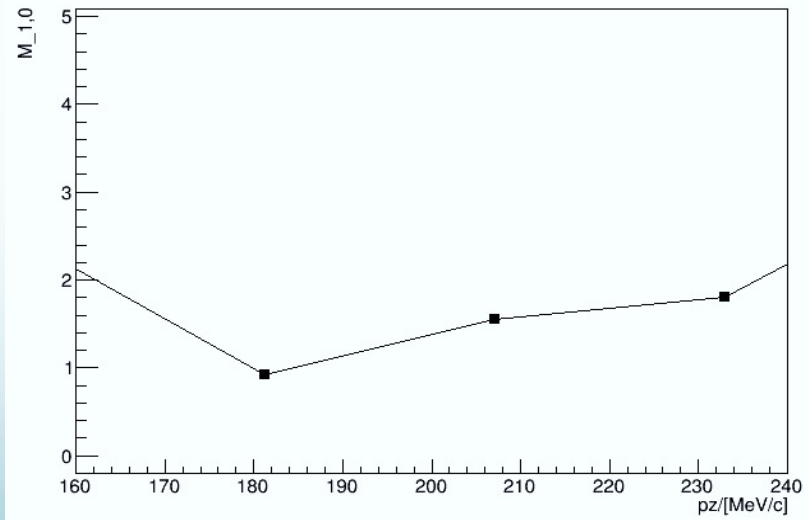
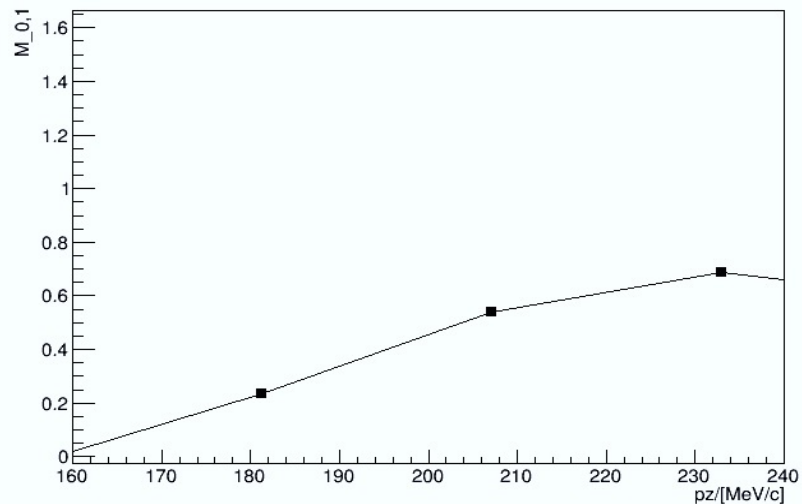
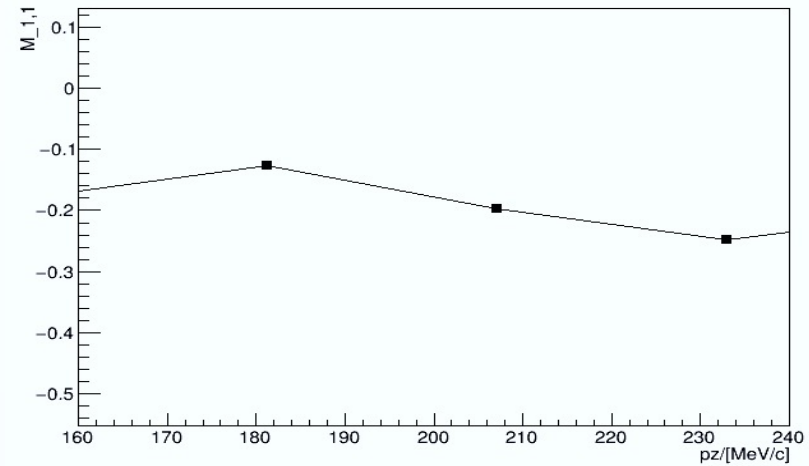
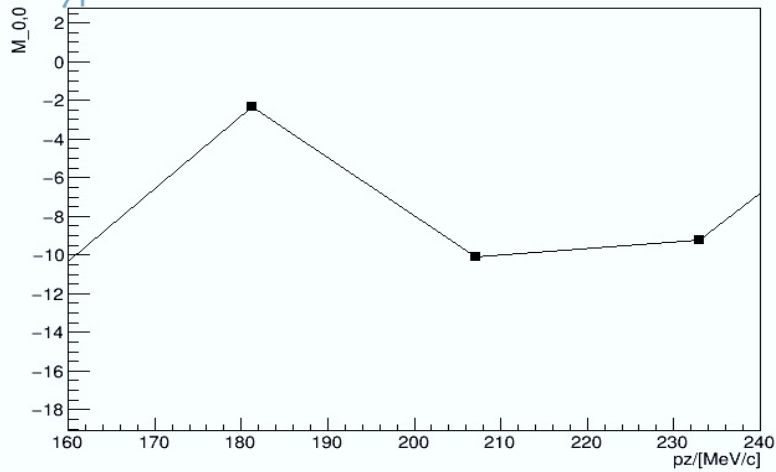
*Analysis & Results*

Data Analysis



# Results : Data

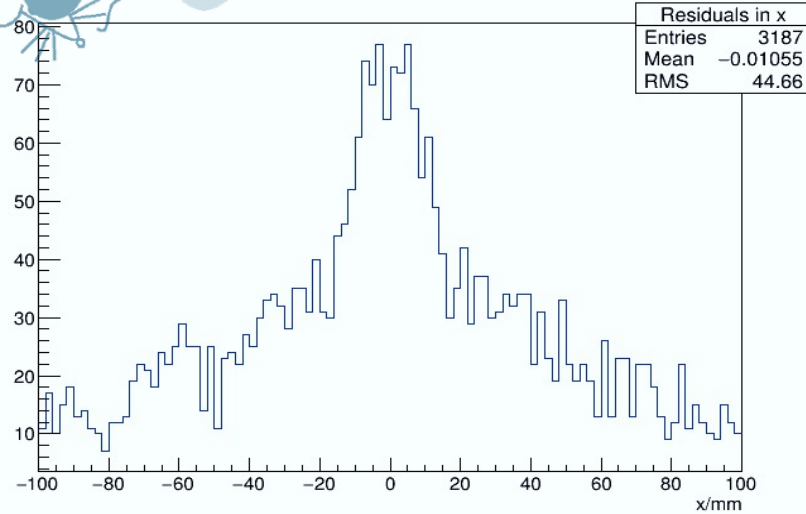
For data set 07475 the the matrix elements have been plotted as functions of  $p_z$ .



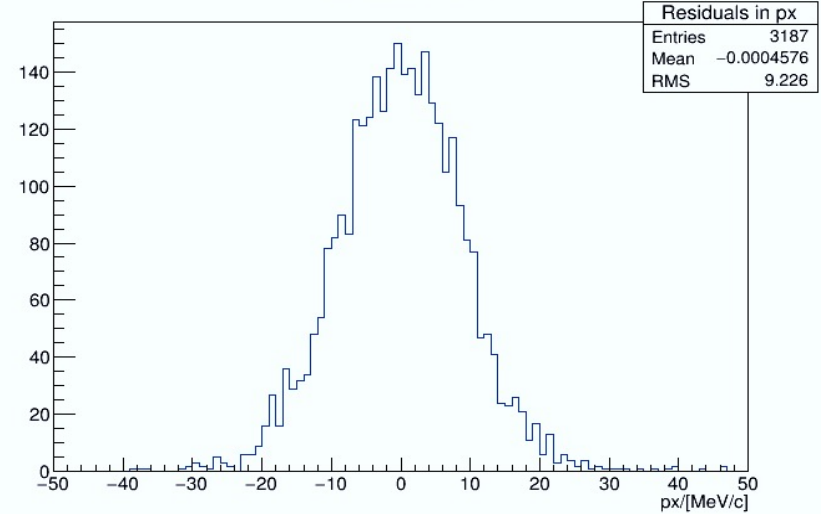


# Residuals: Data

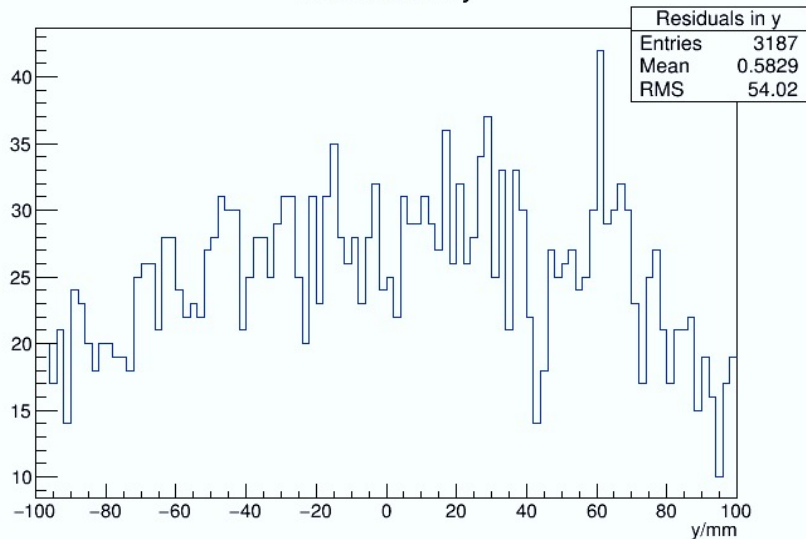
Residuals in x



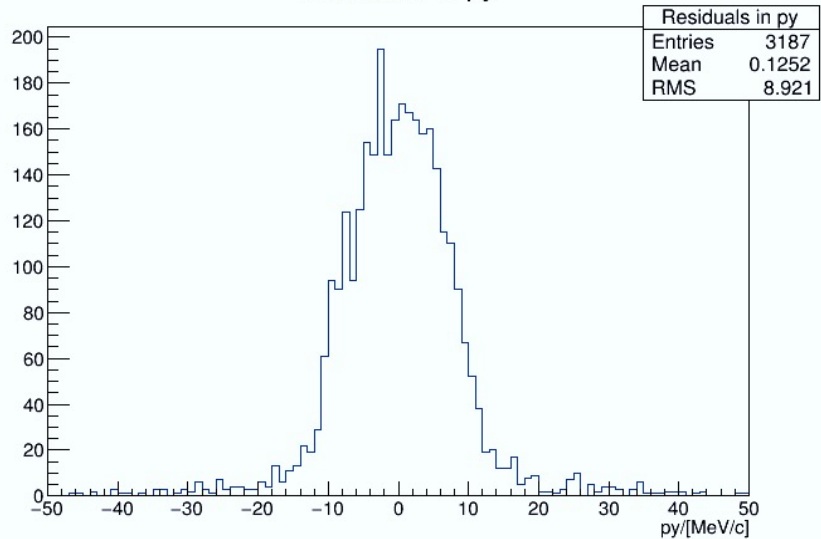
Residuals in px

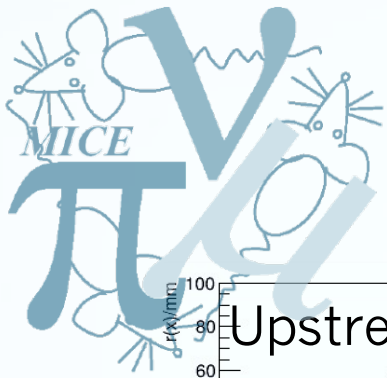


Residuals in y



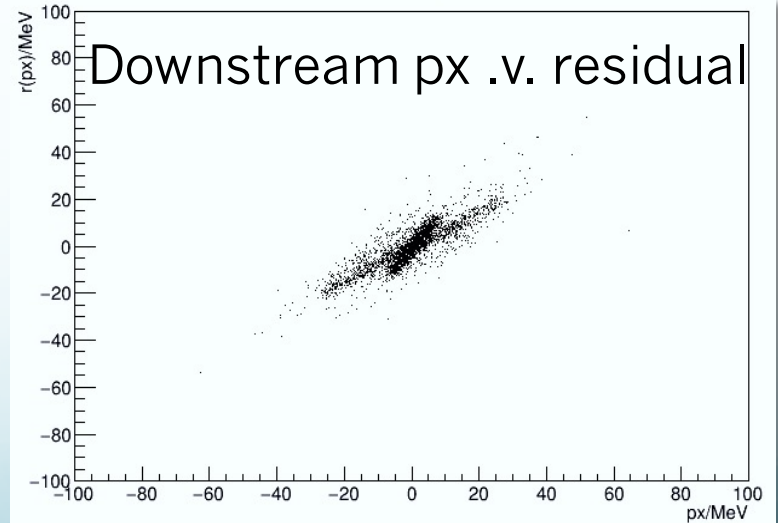
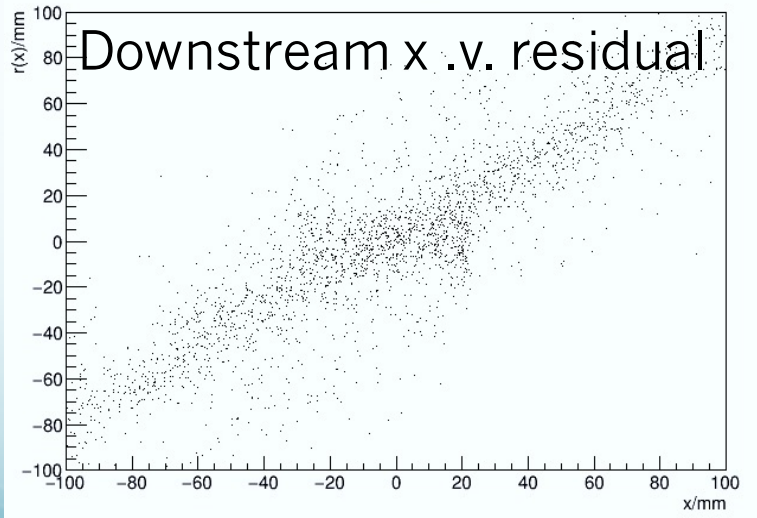
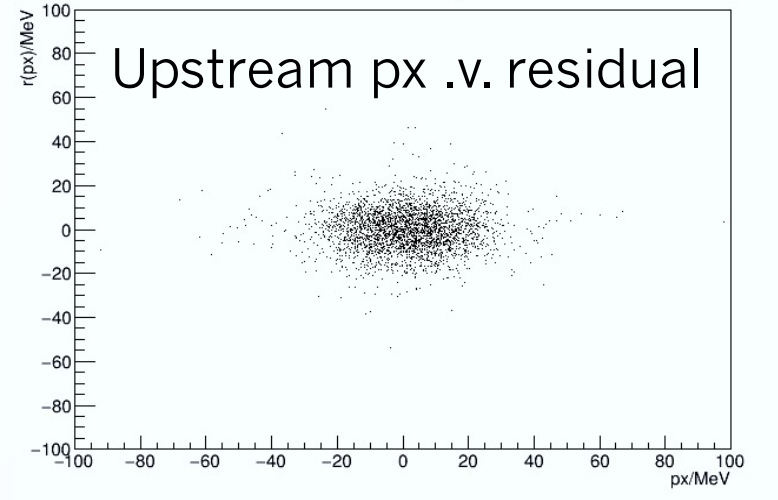
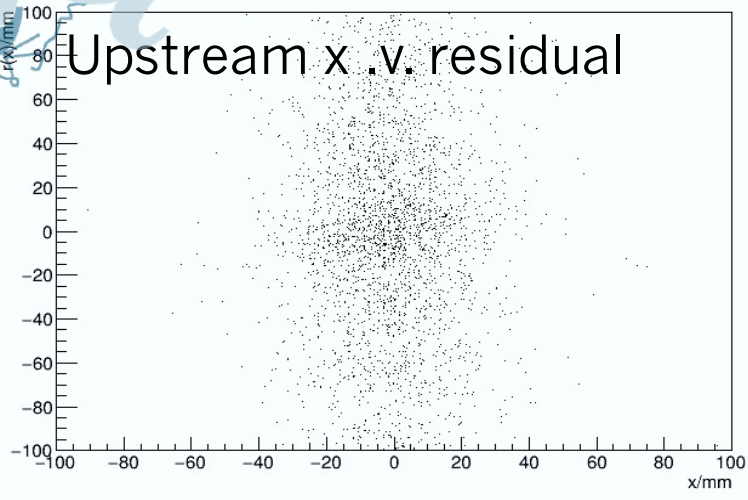
Residuals in py





Some issues with  
tracker  
reconstruction  
(...maybe ?)

# Results: Data







*Conclusions*

Summary & Future Plans



# Summary

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- \* Presented an measurement of the transfer matrix from tracker data using the least squares fitting method
- \* Residuals have acceptable RMS for MC truth
- \* MC recon and data are less nice....need to think why and fix!
- \* Matrix elements dont seem consistent with MC but need to quantify errors and use some higher stats-Work in progress here!



*Thank You For Listening*

Any Questions?