The quest for the direct CP violation parameter, ε'/ε

Amarjit Soni
BNL-HET

CERN, Oct 28, 2015

Outline

- Introduction and motivation
- Early lattice attempts: chiral symmetry is essential for this physics ...nevertheless lot of good came out esp reg UT.
- Early era with domain wall fermions.....More challenges.....
- Adopt Lellouch-Luscher method for direct K=> pi pi
- @ Threshold
- Physical kinematics requires addressing new challenges: 1st complete results @physical kinematics
- Some implications
- Summary and Outlook

Challenges aglore!

- Mixing with ldo
- Dim-6 op mixing: fine –tuning problem
- (Exact] Chiral symm on the lattice is essential!
- Quench pathologies for Q6: full QCD is essential!
- Strange quark too heavy for ChPT to work in K=> pi pi with controllable errors
- Disconnected diagrams contribute=>Huge computational challenge

[formally wrong to neglect these...i,.e. wrong time dependence]

- Even with exact methods, in the end significant double

cancellations putting severe additional non-trivial demand
on the final accuracy

Introduction: Recall some basics & perspectives

What’s the hooplah

- $\Gamma[K_s \Rightarrow \pi^+ \ \pi^-] / \Gamma[K^+ \Rightarrow \pi^+ \ \pi^0] = 670$!

- Often referred to as the Delta I=1/2 Rule [or Puzzle…why is it not O(1)?] in textbooks with us since late 50’s!
- And commonly attributed to QCD [some invoke BSM]

→ Nice test of calculational method

- While it is useful to understand the origin of that large enhancement the more important target is the related quantity, ε’/ε

Sad story of ε'/ε
[ACTUALLY all DIRECT CP]

- **~15% measurement obtained with heroic efforts (spanning over 20 years!) on both sides of the Atlantic at a cost very iikely well over $200M:**

$$\mathrm{Re}(\epsilon'/\epsilon) = 1.65(26) \times 10^{-3}$$

- **Its been WORTHLESS FOR a long time, i.e. It has 0 impact on theory**
- **ONLY LATTICE METHODS CAN CHANGE THIS FACT**
- **My entry into lattice methods ~1982 was motivated by wanting to reliably calculate ε' ...experimental measurements did not come about till much later**

Reminder

$$|A_2| = 1.573(56)\times 10^{-8} GeV,$$

$$|A_0| = 3.3197(19)\times 10^{-7} GeV,$$

$$|A_0/A_2| = 21.13(77).$$

$\Delta I = 3/2$

$I=2$

$I=0,\ 1/2,\ 3/2$

$\Rightarrow \sqrt{450}$

ε' / ε: Direct CPV

$$\eta_{+-} = |\eta_{+-}| e^{i\phi_{+-}} = \frac{A(K_L \to \pi^+\pi^-)}{A(K_S \to \pi^+\pi^-)}$$

$$\eta_{00} = |\eta_{00}| e^{i\phi_{00}} = \frac{A(K_L \to \pi^0\pi^0)}{A(K_S \to \pi^0\pi^0)}$$

$$\eta_{+-} = \epsilon + \epsilon', \qquad \eta_{00} = \epsilon - 2\epsilon'$$

$$\epsilon' = \frac{i e^{i(\delta_2 - \delta_0)}}{\sqrt{2}} \frac{\mathrm{Re}A_2}{\mathrm{Re}A_0} \left[\frac{\mathrm{Im}A_2}{\mathrm{Re}A_2} - \frac{\mathrm{Im}A_0}{\mathrm{Re}A_0} \right].$$

$$\mathrm{Re}\left(\frac{\epsilon'}{\epsilon}\right) = \frac{\omega}{\sqrt{2}|\epsilon|}\left[\frac{\mathrm{Im}(A_2)}{\mathrm{Re}(A_2)} - \frac{\mathrm{Im}(A_0)}{\mathrm{Re}(A_0)}\right]$$

Indirect CP̸

BNL '64
CRONIN + FITCH N.P.

$$|\epsilon| = 2.228(11) \times 10^{-3},$$

$$\mathrm{Re}(\epsilon'/\epsilon) = 1.65(26) \times 10^{-3}.$$

DIRECT CP̸

$\epsilon' \sim O(10^{-6})$!

Its presumed importance:

- lies in its very small size => Perhaps new phenomena has a better chance of showing up
- Exceedingly important monitor of flavor –alignment
- Simple arguments strongly suggest most BSM will have very different predictions than the SM; SUSY, warped extra dim, extra Higgs models, models with heavier quarks all have new CP-odd phases and therefore new sources for contributing to eps'
- Therefore, in many ways eps' is rather analogous to nedm…….both being very sensitive to BSM
- Understanding eps', nedm is important for learning how naturalness really works in nature

QCD presents a most difficult challenge

- Kaon is only ~500 MeV...highly susceptible to non-perturbative effects
- Making accurate predictions exceedingly difficult

- QCD is an integral part of any underlying theory: SM or BSM

- W/O taming QCD, i.e. w/o precise calculations including effects of QCD,

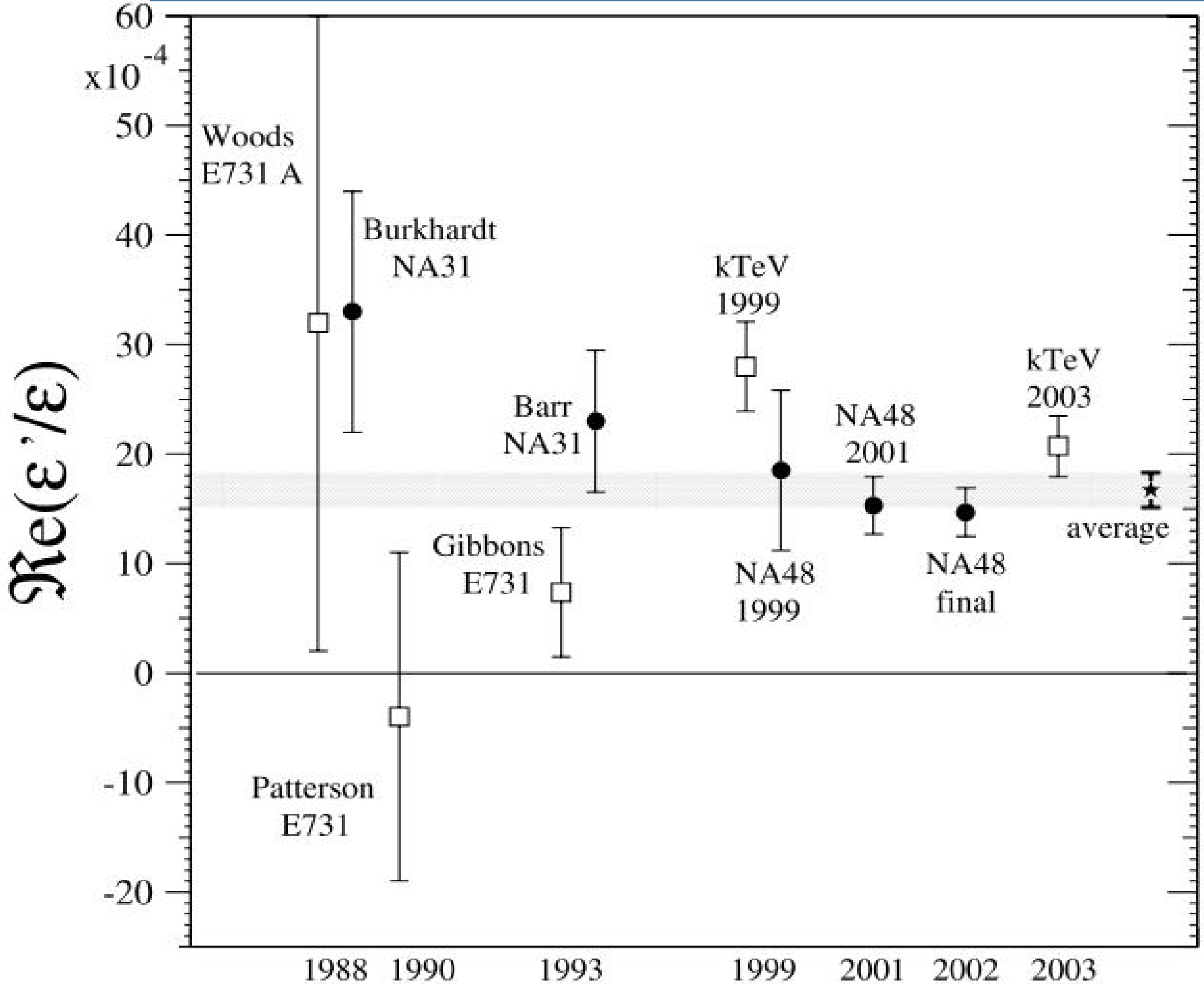
experimental measurements often attained at enormous cost cannot be used economically and effectively; eps', g-2 of muon, anomalies in B=K(*) l+ l- and countless others serve as examples

A.S. in Proceedings of Lattice '85 (FSU)..1st Lattice meeting ever attended

The matrix elements of some penguin operators control in the standard model another CP violation parameter, namely ϵ'/ϵ.[6,8] Indeed efforts are now underway for an improved measurement of this important parameter.[10] In the absence of a reliable calculation for these parameters, the experimental measurements, often achieved at tremendous effort, cannot be used effectively for constraining the theory. It is therefore clearly important to see how far one can go with MC techniques in alleviating this old but very difficult

With C. Bernard [UCLA]

A monumental experimental achievement!



Konrad Kleinknecht "Uncovering CPV"

$16.6(2.3) \times 10^{-4}$
PDG 2014

Some early works that shaped my thinking

LEFT-HANDED CURRENTS AND *CP* VIOLATION

John ELLIS, Mary K. GAILLARD * and D.V. NANOPOULOS **
CERN, Geneva

Received 9 February 1976
(Revised 26 April 1976)

An analysis is presented of a model for *CP* violation due to Kobayashi and Maskawa, which has six quarks and purely left-handed currents. The model can reproduce approximately results of the Glashow, Iliopoulos and Maiani four-quark model in *CP* conserving processes. It gives a close approximation to superweak predictions for *CP* violation in common K decays, whatever the masses of the fifth and sixth quarks. In principle there are substantial deviations from superweak predictions in rare K decays, but these seem difficult to observe in practice. There are also deviations from superweak results for *CP* violation in charmed particle decays. The neutron electric dipole moment is very sensitive to the masses of the b and t quarks, lying in a range containing the superweak prediction.

CP Noninvariance in the Decays of Heavy Charged Quark Systems

Myron Bander, D. Silverman, and A. Soni
Department of Physics, University of California, Irvine, California 92717

(Received 9 May 1979)

Within the context of a six-quark model combined with quantum chromodynamics we stud the asymmetry in the decay of heavy charged mesons into a definite final state as compare with the charge-conjugated mode. We find that, in decays of mesons involving the b quark, measurable asymmetries may arise. This would present the first evidence for *CP* noninvariance in charged systems.

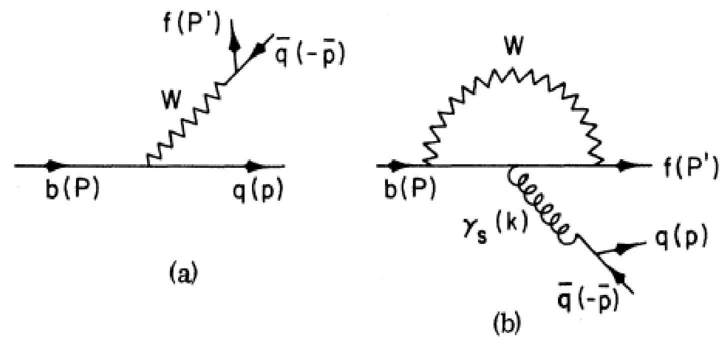


FIG. 1. Diagrams for the reaction $b(P) \to f(P') + q(p) + \bar{q}(-\bar{p})$, where $f = d$ or s. (a) The usual charged-current process, (b) contribution to the same reaction via gluon emission. (b) is the source for the absorptive part necessary for *CP* asymmetry. Note that gluon momentum is timelike unlike the case in "penguin diagrams" of Refs. 8 and 9.

BABAR+BELLE ~ '07: $a_{cp}[B^0 \to K^+\pi^-] \sim 10\%$, Finally establis
$\sim 10^5 * \epsilon'$

WoW! B is the place to be for (large) dir CP....need formula sin Xsin...

2006 BABAR and then BELLE .....CP-odd TN-even

Strong Effects in Weak Nonleptonic Decays

Mark B. Wise (SLAC). Apr 1980. 123 pp.

Many important related works

- Altarelli and Maiani
- Gaillard and Lee
- Shifman, Vainshtein and Zhakrov
- Gilman and Wise
- Buras & Co
- Martinelli et al
- Pich and Rafael
- Bijnens et al

MOTHER of all (lattice) calculations to date: A Personal Perspective

- ~1/3 of a century
- 9 PhD thesis: Terry Draper (UCLA), George Hockney(UCLA), Cristian Calin (Columbia=CU), Jack Laiho(Princeton), Sam Li(CU), Matthew Lightman(CU), Elaine Goode(Southampton), Qi Liu(CU), Daiqian Zhang(CU)
- Post-docs & such: Tom Blum (U Conn), Chris Dawson(google), Chris Kelly (RIKEN-BNL-RC); Christoph Lehner (HET; BNL).

I. Wilson Fermions with Bernard ~'82 **See also Martinelli et al [WF]** **Giusti et al [WF]** **Sharpe et al [Stag F]**	**Lattice χ S is a pre-requisite for this physics** **Off-shoot B-physics important observables identified & studied=> evolved into UT**		
II (a) DWF with Blum ~ '95 II(b) DWF with RBC[with Blum, Christ and Mawhinney became "flagship" project of RBC] ~'97.	LOχPT; Quenched approx.[QA] Same QA is disastrous for this physics [Golterman-Pallante] pathologies; NPR of full $\Delta S=1$ accomplished for the 1st time used since then.	CRAY @ NERSC QCDSP ~ 1 TF	
III. DWF with full QCD RBC, ~ '02	Used LOχPT + full QCD Large chiral corrections	QCDSP ~ 1TF	
IV. DWF with full QCD RBC + UKQCD, ~ '06	Direct K=>ππ, [Lellouch-Luscher method] @ threshold	QCDOC ~ 10 TF	
V. DWF with full QCD, RBC + UKQCD ~ '11	Direct K=>ππ, [Lellouch-Luscher method] ; physical kinematics	BG/Q ~ 100TF@BNL; RBRC;ANL; Edinburgh	
Vi. Same ~now	Same	Seeking new hardware ~1.5PF;NERSC;ANL;UK	

UNIVERSITY OF CALIFORNIA

Los Angeles

Lattice Evaluation of Strong Corrections

to Weak Matrix Elements —

The Delta-I Equals One-Half Rule

A dissertation submitted in partial satisfaction of the

requirements for the degree Doctor of Philosophy

in Physics

by

Terrence Arthur James Draper

1984

UKQCD Collaboration

- Edinburgh
  - Peter Boyle
  - Luigi Del Debbio
  - Julien Frison
  - Jamie Hudspith
  - Richard Kenway
  - Ava Khamseh
  - Brian Pendleton
  - Karthee Sivalingam
  - Oliver Witzel
  - Azusa Yamaguchi
- Plymouth
  - Nicolas Garron
- York (Toronto)
  - Renwick Hudspith
- Southampton
  - Jonathan Flynn
  - Tadeusz Janowski
  - Andreas Juttner
  - Andrew Lawson
  - Edwin Lizarazo
  - Antonin Portelli
  - Chris Sachrajda
  - Francesco Sanfilippo
  - Matthew Spraggs
  - Tobias Tsang
- CERN
  - Marina Marinkovic

m λc l KITP

RBC Collaboration

- BNL
  - Chulwoo Jung
  - Taku Izubuchi (RBRC)
  - Christoph Lehner
  - Meifeng Lin
  - Amarjit Soni
- RBRC
  - **Chris Kelly**
  - Tomomi Ishikawa
  - Taichi Kawanai
  - Shigemi Ohta (KEK)
  - Sergey Syritsyn
- Columbia
  - Ziyuan Bai
  - Xu Feng
  - Norman Christ
  - Luchang Jin
  - Robert Mawhinney
  - Greg McGlynn
  - David Murphy
  - **Daiqian Zhang**
- Connecticut
  - **Tom Blum**

Many lattice technicalities in the backups

Δ S=1 H_W

W C to NLO

Buchalla, Buras, Lautenbacher

RMP /96; Ciuchini et al 95

$$H_W = \frac{G_F}{\sqrt{2}} V_{us}^* V_{ud} \sum_{i=1}^{10} \left[z_i(\mu) + \tau y_i(\mu) \right] Q_i(\mu).$$

$$\tau = -V_{ts}^* V_{td} / V_{us}^* V_{ud}.$$

Tree

$$Q_1 = (\bar{s}_\alpha d_\alpha)_L (\bar{u}_\beta u_\beta)_L,$$

$$Q_2 = (\bar{s}_\alpha d_\beta)_L (\bar{u}_\beta u_\alpha)_L,$$

QCDP

$$Q_3 = (\bar{s}_\alpha d_\alpha)_L \sum_{q=u,d,s} (\bar{q}_\beta q_\beta)_L,$$

$$Q_4 = (\bar{s}_\alpha d_\beta)_L \sum_{q=u,d,s} (\bar{q}_\beta q_\alpha)_L,$$

$$Q_5 = (\bar{s}_\alpha d_\alpha)_L \sum_{q=u,d,s} (\bar{q}_\beta q_\beta)_R,$$

$$Q_6 = (\bar{s}_\alpha d_\beta)_L \sum_{q=u,d,s} (\bar{q}_\beta q_\alpha)_R,$$

$\longrightarrow 0$

$m_q \to 0$

$$Q_7 = \frac{3}{2}(\bar{s}_\alpha d_\alpha)_L \sum_{q=u,d,s} e_q (\bar{q}_\beta q_\beta)_R,$$

$$Q_8 = \frac{3}{2}(\bar{s}_\alpha d_\beta)_L \sum_{q=u,d,s} e_q (\bar{q}_\beta q_\alpha)_R,$$

$$Q_9 = \frac{3}{2}(\bar{s}_\alpha d_\alpha)_L \sum_{q=u,d,s} e_q (\bar{q}_\beta q_\beta)_L,$$

$$Q_{10} = \frac{3}{2}(\bar{s}_\alpha d_\beta)_L \sum_{q=u,d,s} e_q (\bar{q}_\beta q_\alpha)_L,$$

EWP

$\longrightarrow$ const

$m \to 0$

QCDP

EWP

Operators in linearly independent (chiral) basis, classified by SU(3)LXSU(3)R irreps

$$Q_1' = 3Q_1 + 2Q_2 - Q_3, \qquad (172)$$

(27,1)

$$Q_2' = \frac{1}{5}(2Q_1 - 2Q_2 + Q_3), \qquad (173)$$

$$Q_3' = \frac{1}{5}(-3Q_1 + 3Q_2 + Q_3), \qquad (174)$$

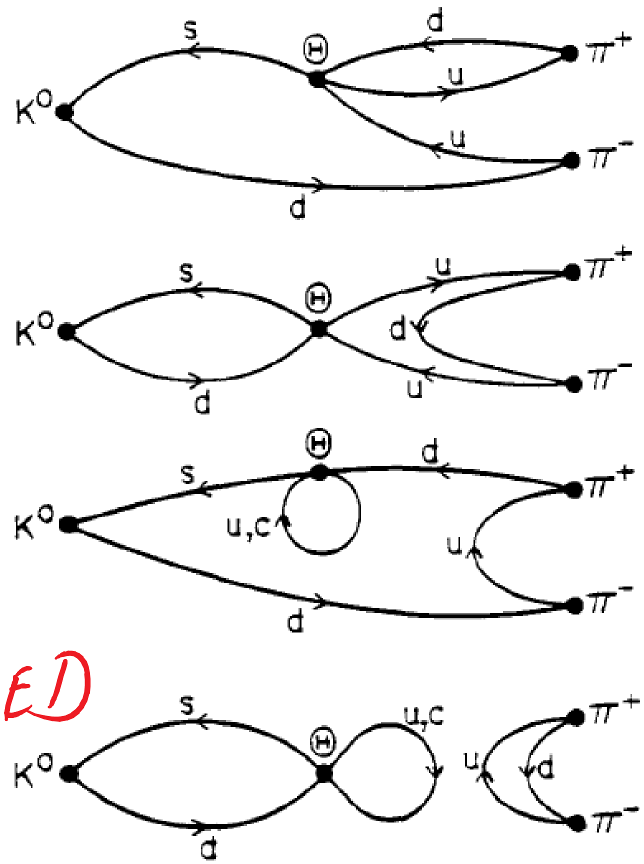
(8,1)

$$Q_i' = Q_i; \quad i \in \{5,6,7,8\}. \qquad (175)$$

(8,1) (8,8)

RBC 2001 PRD

Distinct types of quark flow diagrams



4-point functions

EYE-diagram

VERY HARD

DISCONNECTED DIAGRAM

SUPER HARD

PHYSICAL REVIEW D VOLUME 32, NUMBER 9 1 NOVEMBER 1985

Application of chiral perturbation theory to $K \to 2\pi$ decays

Claude Bernard, Terrence Draper,* and A. Soni
Department of Physics, University of California, Los Angeles, California 90024

H. David Politzer and Mark B. Wise
Department of Physics, California Institute of Technology, Pasadena, California 91125

BDSPW

(Received 3 December 1984)

Chiral perturbation theory is applied to the decay $K \to 2\pi$. It is shown that, to quadratic order in meson masses, the amplitude for $K \to 2\pi$ can be written in terms of the unphysical amplitudes $K \to \pi$ and $K \to 0$, where 0 is the vacuum. One may then hope to calculate these two simpler amplitudes with lattice Monte Carlo techniques, and thereby gain understanding of the $\Delta I = \frac{1}{2}$ rule in K decay. The reason for the presence of the $K \to 0$ amplitude is explained: it serves to cancel off unwanted renormalization contributions to $K \to \pi$. We make a rough test of the practicability of these ideas in Monte Carlo studies. We also describe a method for evaluating meson decay constants which does not require a determination of the quark masses.

LO ChPT

J.LAIHO & AS ~2004 NLO

BDSPW relations

$$\langle 0 | \Theta^{(8,1)} | K^0 \rangle = \frac{16iv}{f^3}(m_s' - m_d')\alpha_2^{(8,1)}$$

$$= \frac{4i}{f}[(m'_{K^+})^2 - (m'_{\pi^+})^2]\alpha_2^{(8,1)} ,$$

$$\langle 0 | \Theta^{(27,1)} | K^0 \rangle = 0 ,$$

$$\langle \pi^+ | \Theta^{(8,1)} | K^+ \rangle = \frac{4m_M{}^2}{f^2}(\alpha_1^{(8,1)} - \alpha_2^{(8,1)}) , \qquad (14)$$

$$\langle \pi^+ | \Theta^{(27,1)} | K^+ \rangle = -\frac{4m_M{}^2}{f^2}\alpha^{(27,1)} ,$$

$$\langle \pi^+\pi^- | \Theta^{(8,1)} | K^0 \rangle = \frac{4i}{f^3}(m_{K^0}{}^2 - m_{\pi^+}{}^2)\alpha_1^{(8,1)} ,$$

$$\langle \pi^+\pi^- | \Theta^{(27,1)} | K^0 \rangle = -\frac{4i}{f^3}(m_{K^0}{}^2 - m_{\pi^+}{}^2)\alpha^{(27,1)} ,$$

Following BDSPW '84.

LO χPT allows huge simplification: 3 point functions

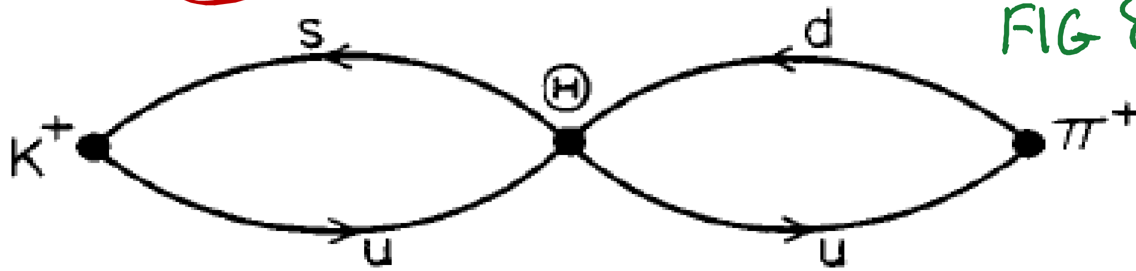
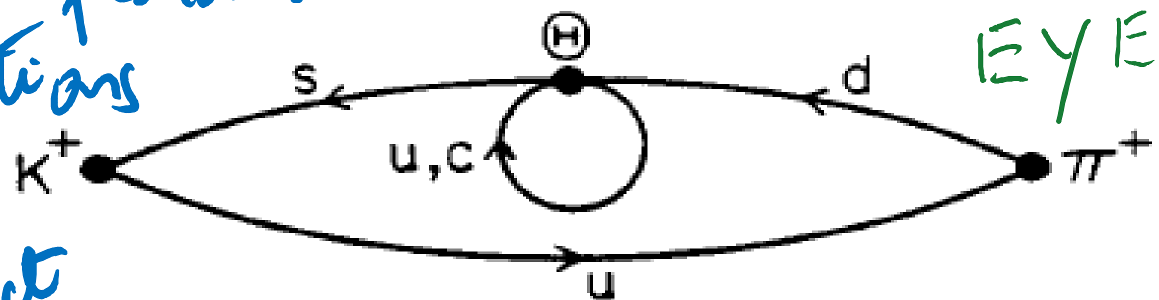
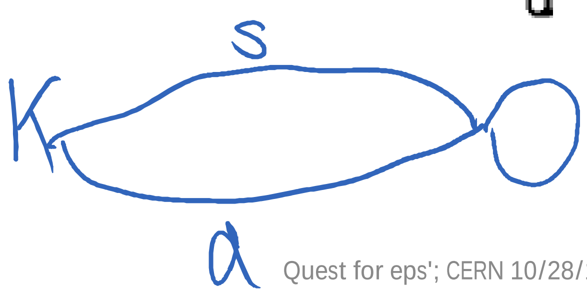


FIG 8



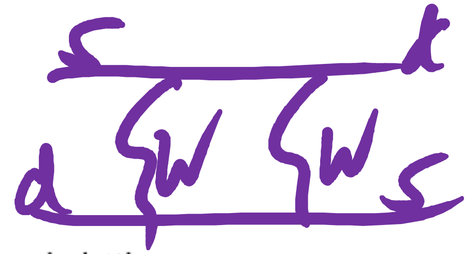
EYE

No disconnect diagrams

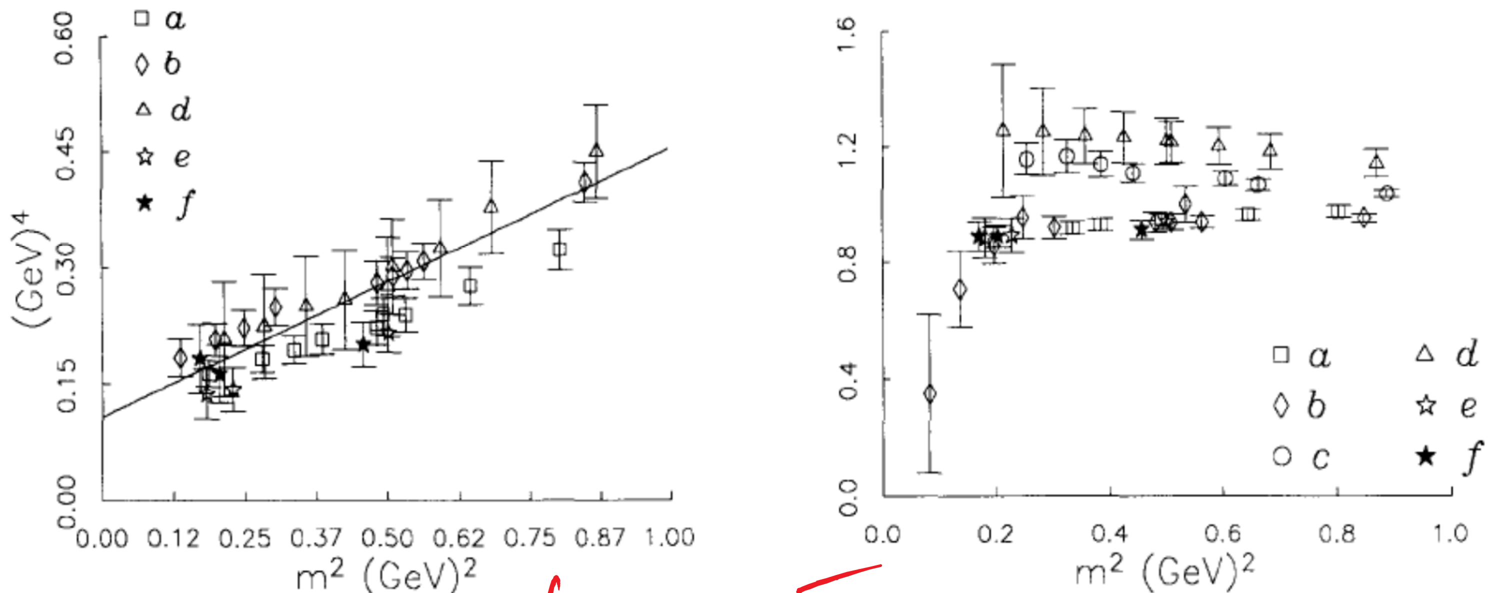


Vacuum

$\langle K | (\bar{s}\gamma_{\mu L} d)^2 | \bar{K} \rangle$



162 C. Bernard, A. Soni / Weak matrix elements on the lattice



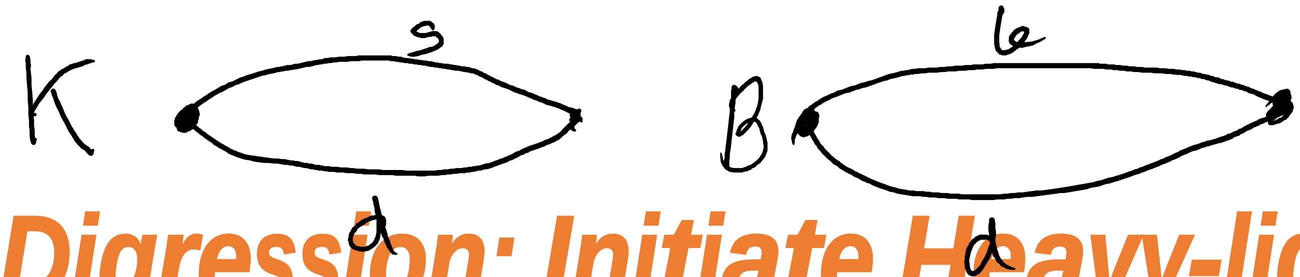
χS Violation by K-K̄ ⇒ FINE TUNING PROBLEM

Additional key difficulties from BK to ReA0 in absence of chiral symmetry

- More diagrams
- Additonal diagrams are a lot harder
- Ops 1 => 10
- Deadily operator mixing esp with lower dimensional ops.
- Lack of chiral symmetry <=> fine-tuning problem
- 4-pt function and not 3
- “Disconnected” diagrams need to be tackled

…………..

Neccessity is the mother……



Digression: Initiate Heavy-light WME program-> important (B) application to UT constraints

RATIONALE: Chiral symmetry less of a concern, utilize in a profitable way stored away light quark propagators towards exploratory application to important physical observables

Lattice calculation of weak amplitudes of D and B mesons

C. Bernard*
Department of Physics, Indiana University, Bloomington, Indiana 47405

T. Draper
TRIUMF, 4004 Wesbrook Mall, Vancouver, British Columbia, Canada V6T 2A3

G. Hockney† and A. Soni
Department of Physics, University of California, Los Angeles, California 90024
(Received 27 June 1988)

A lattice calculation of the pseudoscalar decay constants and the [Δ(flavor)=2] mixing matrix elements for D and B mesons is reported. Calculations are done (in the quenched approximation) with $\beta=6.1$ on a $12^3\times 33$ lattice; results from $\beta=5.7$ on a $16^3\times 25$ lattice contribute to our estimate of the systematic errors. An extrapolation to large meson mass is required in order to treat the B meson. We find $f_{bd}=105\pm17\pm30$ MeV, $f_{bs}=155\pm31\pm48$ MeV, $f_{cd}=174\pm26\pm46$ MeV, $f_{cs}=234\pm46\pm55$ MeV (with normalization such that $f_\pi=132$ MeV). The ratios of these quantities have considerably smaller errors: $f_{bd}/f_{cd}=0.60\pm0.01\pm0.03$, $f_{bs}/f_{cs}=0.66\pm0.004\pm0.09$, $f_{bs}/f_{bd}=1.47\ \pm0.07\pm0.30$, and $f_{cs}/f_{cd}=1.35\pm0.07\pm0.21$. For the lattice "$B$ parameters" we find $B_{LL}^{\text{latt}}=1.01\pm0.06\pm0.18$ and $B_{LR}^{\text{latt}}=1.16\pm0.01\pm0.11$ for the bd system, with quite similar values for the cu and bs systems. These B parameters are defined slightly differently than in the continuum and are effectively renormalization-group invariant. The first error in each of our results is statistical; the second is an estimate of the systematic errors due to scale-breaking, finite-size, extrapolation and operator-renormalization effects.

1st application to B-physics
COARSE Lattice; Large syst. error

Lattice computation of the decay constants of B and D mesons

Claude W. Bernard
Department of Physics, Washington University, St. Louis, Missouri 63130

James N. Labrenz
Department of Physics FM-15, University of Washington, Seattle, Washington 98195

Amarjit Soni
Department of Physics, Brookhaven National Laboratory, Upton, New York 11973
(Received 1 July 1993)

Semileptonic decays on the lattice: The exclusive 0^- to 0^- case

Claude W. Bernard*
Institute for Theoretical Physics, University of California, Santa Barbara, California 93106

Aida X. El-Khadra
Theory Group, Fermi National Accelerator Laboratory, P. O. Box 500, Batavia, Illinois 60510

Amarjit Soni
Institute for Theoretical Physics, University of California, Santa Barbara, California 93106 and Department of Physics, Brookhaven National Laboratory, Upton, New York 11973[†]
(Received 21 December 1990)

PHYSICAL REVIEW D, VOLUME 58, 014501

SU(3) flavor breaking in hadronic matrix elements for B-$\bar{B}$ oscillations

Later Δm_s
CDF, DØ

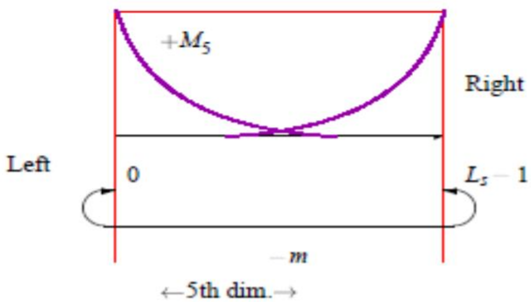
C. Bernard
Department of Physics, Washington University, St. Louis, Missouri 63130

T. Blum and A. Soni
Department of Physics, Brookhaven National Laboratory, Upton, New York 11973
(Received 28 January 1998; published 5 May 1998)

EXACT CHIRAL SYMMETRY ON THE LATTICE

Conventional fermions do not preserve chiral-flavor symmetry on the lattice (Nielsen - Ninomiya Theorem) $\Rightarrow \Delta S = 1, \Delta I = 1/2$ case mixing with lower dim. (power-divergent) operators & or mixing of 4-quark operators with wrong chirality ones makes lattice study of $K - \pi$ physics virtually impossible.

Domain Wall Fermions (Kaplan, Shamir, Narayanan and Neuberger)



Shamir & Furman, NPB 439, 54,1995

Practical viability of DWF for QCD demonstrated (96-97) Tom Blum & A. S.
Chiral symmetry on the lattice, $a \neq 0$! Huge improvement

$\Rightarrow$ Now widespread use at BNL and elsewhere

PHYSICAL REVIEW D VOLUME 56, NUMBER 1 1 JULY 1997

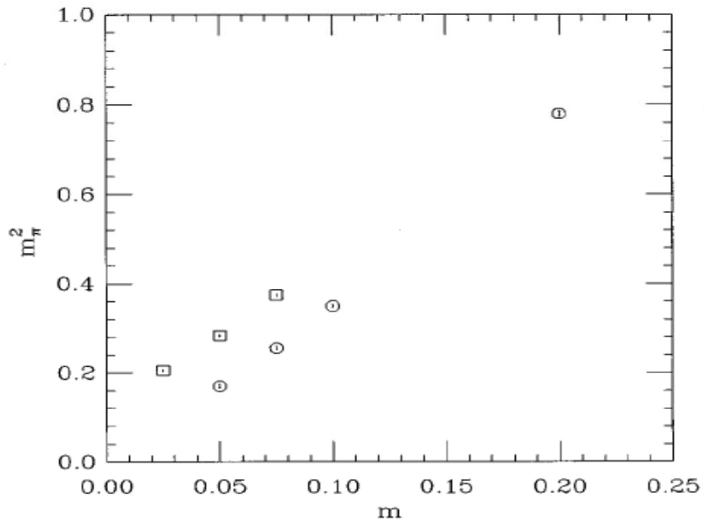
QCD with domain wall quarks

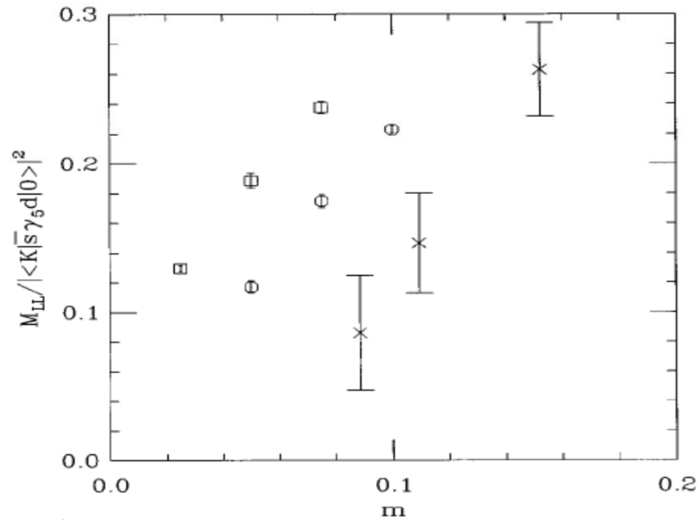
1st Simulation with DWQ ~'97

T. Blum* and A. Soni†
Department of Physics, Brookhaven National Laboratory, Upton, New York 11973

(Received 27 November 1996)

We present lattice calculations in QCD using Shamir's variant of Kaplan fermions which retain the continuum $SU(N)_L \times SU(N)_R$ chiral symmetry on the lattice in the limit of an infinite extra dimension. In particular, we show that the pion mass and the four quark matrix element related to K_0-$\overline{K_0}$ mixing have the expected behavior in the chiral limit, even on lattices with modest extent in the extra dimension, e.g., $N_s = 10$. [S0556-2821(97)00113-6]





K→2π via ChPT with DWQ in Quench Approx

PHYSICAL REVIEW D **68**, 114506 (2003)

Kaon matrix elements and *CP* violation from quenched lattice QCD: The 3-flavor case

T. Blum,[1] P. Chen,[2] N. Christ,[2] C. Cristian,[2] C. Dawson,[3] G. Fleming,[2,*] R. Mawhinney,[2] S. Ohta,[4,1] G. Siegert,[2] A. Soni,[3] P. Vranas,[5] M. Wingate,[1,*] L. Wu,[2] and Y. Zhestkov[2]

[1]*RIKEN-BNL Research Center, Brookhaven National Laboratory, Upton, New York 11973, USA*

[2]*Physics Department, Columbia University, New York, New York 10027, USA*

[3]*Physics Department, Brookhaven National Laboratory, Upton, New York 11973, USA*

[4]*Institute for Particle and Nuclear Studies, KEK, Tsukuba, Ibaraki, 305-0801, Japan*

[5]*IBM Research, Yorktown Heights, New York 10598, USA*

(Received 19 July 2002; published 30 December 2003)

C ALSO CP-PACS

We report the results of a calculation of the $K\to\pi\pi$ matrix elements relevant for the $\Delta I=1/2$ rule and ϵ'/ϵ in quenched lattice QCD using domain wall fermions at a fixed lattice spacing $a^{-1}\sim 2$ GeV. Working in the three-quark effective theory, where only the u, d, and s quarks enter and which is known perturbatively to next-to-leading order, we calculate the lattice $K\to\pi$ and $K\to|0\rangle$ matrix elements of dimension six, four-fermion operators. Through lowest order chiral perturbation theory these yield $K\to\pi\pi$ matrix elements, which we then normalize to continuum values through a nonperturbative renormalization technique. For the ratio of isospin amplitudes $|A_0|/|A_2|$ we find a value of 25.3 ± 1.8 (statistical error only) compared to the experimental value of 22.2, with individual isospin amplitudes 10%–20% below the experimental values. For ϵ'/ϵ, using known central values for standard model parameters, we calculate $(-4.0\pm2.3)\times10^{-4}$ (statistical error only) compared to the current experimental average of $(17.2\pm1.8)\times10^{-4}$. Because we find a large cancellation between the $I=0$ and $I=2$ contributions to ϵ'/ϵ, the result may be very sensitive to the approximations employed. Among these are the use of quenched QCD, lowest order chiral perturbation theory, and continuum perturbation theory below 1.3 GeV. We also calculate the kaon B parameter B_K and find $B_{K,\overline{\rm MS}}(2\ {\rm GeV})=0.532(11)$. Although currently unable to give a reliable systematic error, we have control over statistical errors and more simulations will yield information about the effects of the approximations on this first-principles determination of these important quantities.

QCDSP

RBC Collaboration

~98→~'05 1 TF

hep-lat/010075

TABLE XXXIX. The contribution in GeV from the renormalized continuum operator $Q_{i,\mathrm{cont}}$ to the real parts of $\langle(\pi\pi)_I|-i\mathcal{H}^{(\Delta S=1)}|K^0\rangle$ for $\mu=1.51$ GeV. The central values for the standard model parameters given in Table XXXVII have been used.

	Real A_0		Real A_2	
i	choice 1	choice 2	choice 1	choice 2
1	$3.02(68)\times10^{-8}$	$4.28(97)\times10^{-8}$	$-4.11(18)\times10^{-9}$	$-4.82(22)\times10^{-9}$
2	$2.00(18)\times10^{-7}$	$2.83(25)\times10^{-7}$	$1.392(62)\times10^{-8}$	$1.635(73)\times10^{-8}$
3	$1.4(29)\times10^{-10}$	$2.0(41)\times10^{-10}$	0.0	0.0
4	$-3.80(84)\times10^{-9}$	$-5.4(12)\times10^{-9}$	0.0	0.0
5	$-6.9(29)\times10^{-10}$	$-9.8(41)\times10^{-10}$	0.0	0.0
6	$4.99(77)\times10^{-9}$	$7.1(11)\times10^{-9}$	0.0	0.0
7	$4.04(21)\times10^{-11}$	$8.00(42)\times10^{-11}$	$2.86(15)\times10^{-11}$	$3.63(19)\times10^{-11}$
8	$-5.74(32)\times10^{-11}$	$-1.137(63)\times10^{-10}$	$-4.06(22)\times10^{-11}$	$-5.15(28)\times10^{-11}$
9	$-3.91(39)\times10^{-12}$	$-5.54(56)\times10^{-12}$	$4.69(21)\times10^{-13}$	$5.51(25)\times10^{-13}$
10	$2.27(41)\times10^{-12}$	$3.23(59)\times10^{-12}$	$3.70(17)\times10^{-13}$	$4.35(20)\times10^{-13}$

RBC

QA ; ChPT

PRD ~ 02

TABLE XLIX. Our final values for physical quantities using one-loop full QCD extrapolations to the physical kaon mass (choice 2) and a value of $\mu = 2.13$ GeV for the matching between the lattice and continuum. The errors for our calculation are statistical only. ←

Quantity	Experiment	This calculation (statistical errors only)
Re A_0(GeV)	3.33×10^{-7}	$(2.96 \pm 0.17) \times 10^{-7}$
Re A_2(GeV)	1.50×10^{-8}	$(1.172 \pm 0.053) \times 10^{-8}$
ω^{-1}	22.2	(25.3 ± 1.8)
Re(ϵ'/ϵ)	$(15.3 \pm 2.6) \times 10^{-4}$(NA 48)	$(-4.0 \pm 2.3) \times 10^{-4}$
	$(20.7 \pm 2.8) \times 10^{-4}$(KTEV)	

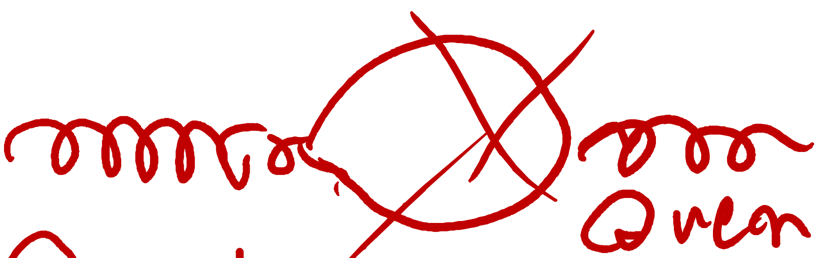
RBC = RBRC + BNL + CU

See Golterman & Pallante '01; '04; Aubin et al (RBC) '06

Extremely serious quench patholgy

- Most important for Q6 as it LR=> (S+P)(S-P); AND it makes the most important contribution to ε'

Source of problem is that H_eff for ΔS=1 has operators such as Q6 with Quark content $(\bar{s} d)(\bar{u} u)$ → quark loop from weak interaction



Quench approx

Q_6 gets unphysical contribution for Q_8
(8,1) (8,8)

Full QCD But ChPT a la BDSPW

(Sam)Shu Li, PhD thesis, Columbia '08

Conclusion

Quantity	This analysis	Quenched	Experiment
$\mathrm{Re}A_0$ (GeV)	$4.5(11)(53) \times 10^{-7}$	$2.96(17) \times 10^{-7}$	3.33×10^{-7}
$\mathrm{Re}A_2$ (GeV)	$8.57(99)(300) \times 10^{-9}$	$1.172(53) \times 10^{-8}$	1.50×10^{-8}
$\mathrm{Im}A_0$ (GeV)	$-6.5(18)(77) \times 10^{-11}$	$-2.35(40) \times 10^{-11}$	
$\mathrm{Im}A_2$ (GeV)	$-7.9(16)(39) \times 10^{-13}$	$-1.264(72) \times 10^{-12}$	
$1/\omega$	$50(13)(62)$	$25.3(1.8)$	**22.2**
$\mathrm{Re}(\epsilon'/\epsilon)$	$7.6(68)(256) \times 10^{-4}$	$-4.0(2.3) \times 10^{-4}$	1.65×10^{-3}

$\Rightarrow$

- ChPT approach to $K \to \pi\pi$ faces severe difficulties.
- RBC/UKQCD studying physical $\pi\pi$ final states.
- DWF on coarse lattices and large volumes: $4 \to 5$ fm?
- Vranas auxiliary determinant (Renfrew talk on Wed.)

LARGE SYSTEMATIC
errors DUE ChPT

Lattice

N. Christ @LAT08

CMP/01. A Breakthrough paper

Direct K-> $\pi\pi$ (a la Lellouch-Luscher), using finite volume correlation* functions, [i.e. w/o ChPT] RBC initiates around 2006

CONTINUED BY RBC-UKQCD {(mostly) Edinburgh – Southampton}

* Allows to bypass Maini-Testa theorem

Relating lattice ME to physical amplitudes

$$A_{2/0} = F \frac{G_F}{\sqrt{2}} V_{ud} V_{us} \sum_{i=1}^{10} \sum_{j=1}^{7} \left[\Big(z_i(\mu) + \tau y_i(\mu) \Big) Z_{ij}^{\mathrm{lat} \to \overline{\mathrm{MS}}} M_j^{\frac{3}{2}/\frac{1}{2}, \mathrm{lat}} \right]$$

F is the Lellouch-Luscher factor which relates finite volume ME to the infinite volume

$$A = \frac{1}{\pi q} \sqrt{\frac{\partial \phi}{\partial q} + \frac{\partial \delta}{\partial q}} \sqrt{m_K} E_{\pi\pi} L^{2/3} M$$

→ Phase shift

A/M is LL factor F

$q = \frac{pL}{2\pi}$;

$\hookrightarrow \sim \frac{\delta}{q}$ for small p

φ is a somewhat complicated function of q and boundary Conditions [See Daiqian Zhang thesis]

Relating bare LME => MS-bar ME

M_i^{lat} (Bare M_i on finite volume Lattice)	
$\Downarrow$	LL Factor[19]
M_i in infinite volume, Minkowski space.	
$\Downarrow$	Lat$\rightarrow$RI/SMOM NPR matrix
M_i in RI/SMOM scheme	
$\Downarrow$	RI/SMOM$\rightarrow$ $\overline{\text{MS}}$ matching matrix
M_i in $\overline{\text{MS}}$ scheme	

Table 3.2: Work flow from bare lattice matrix elements to $\overline{\text{MS}}$ decay matrix elements.

ROME-Southampton
MARTINELLI et al
'94

Sturm et al '09

DAIQIAN
Zhang
thesis.

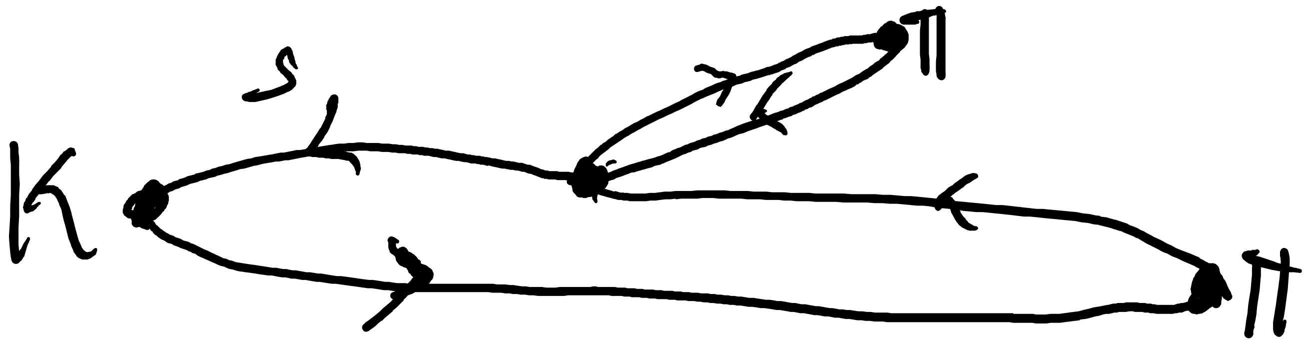
i=2 ,(much³) simpler one; only 3 operators etc

$$Q^{3/2}_{(27,1)} = (\bar{s}^i d^i)_L\{(\bar{u}^j u^j)_L - (\bar{d}^j d^j)_L\} + (\bar{s}^i u^i)_L(\bar{u}^j d^j)_L,$$

$$O^{3/2}_{(8,8)} = (\bar{s}^i d^i)_L\{(\bar{u}^j u^j)_R - (\bar{d}^j d^j)_R\} + (\bar{s}^i u^i)_L(\bar{u}^j d^j)_R,$$

$$O^{3/2}_{(8,8)\mathrm{mix}} = (\bar{s}^i d^j)_L\{(\bar{u}^j u^i)_R - (\bar{d}^j d^i)_R\} + (\bar{s}^i u^j)_L(\bar{u}^j d^i)_R,$$

For $I = 2$, $\Delta I = 3/2$ only a single type of quark flow diagram



No disconnected contribution

More on delta i=3/2 continuum 2012-2015

- Original physical measurement [Phys.Rev.Lett. 108 (2012) 141601]

$$\mathrm{Re}(A_2) = 1.38(5)_{\mathrm{stat}}(26)_{\mathrm{sys}} \times 10^{-8}\ \mathrm{GeV}$$

$$\mathrm{Im}(A_2) = -6.54(46)_{\mathrm{stat}}(120)_{\mathrm{sys}} \times 10^{-13}\ \mathrm{GeV}$$

20% sys error dominated by 15% discretization error

PRD '12

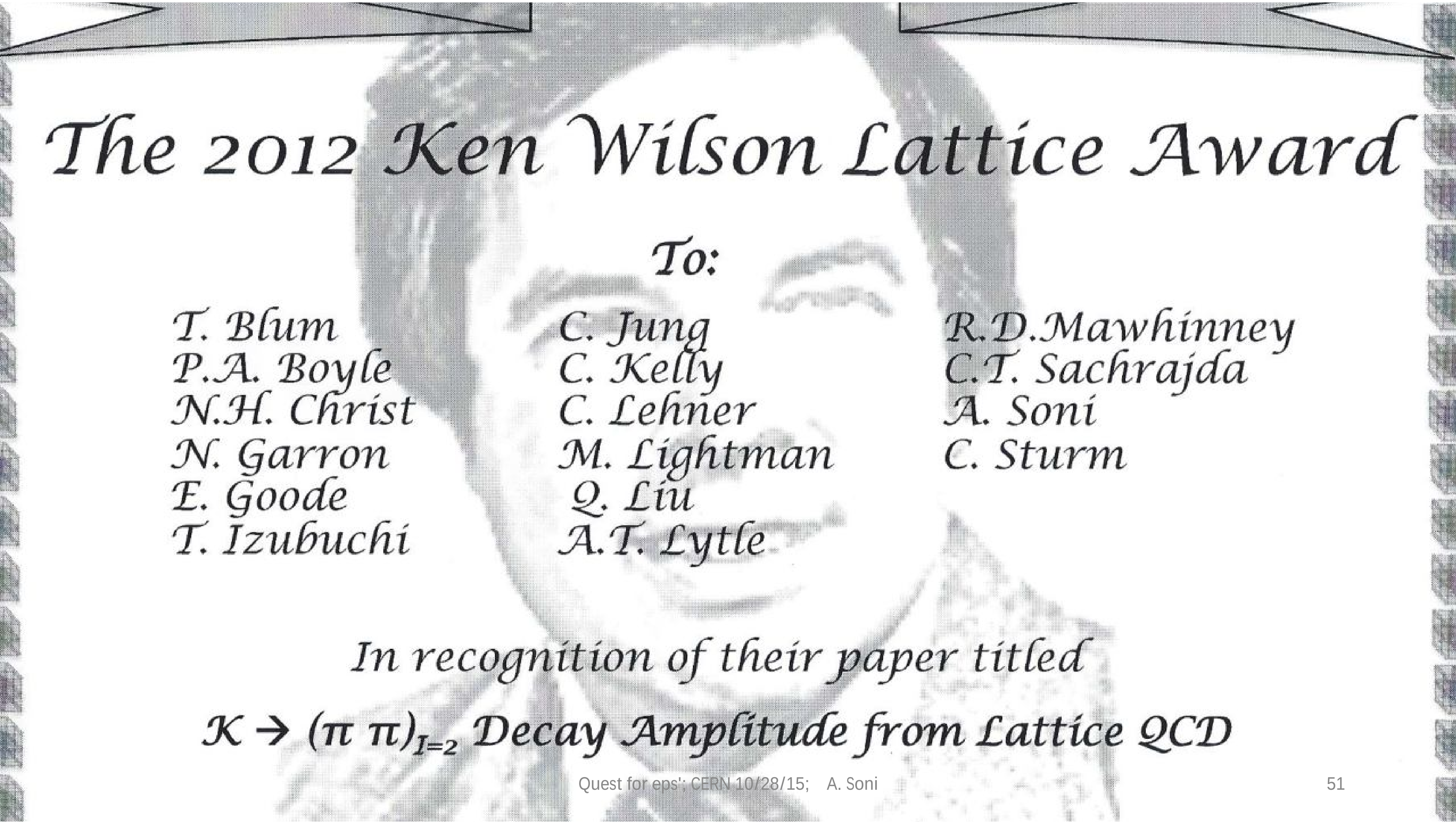
$32^3 \times 64 \times 16$; $a^{-1} = 1.364$ GeV

146 Configs; $m_\pi = 142.1$ MeV

$m_K = 505.5$ MeV

TABLE IX. Systematic error budget for ReA_2 and ImA_2.

	ReA_2	ImA_2
Lattice artifacts	15%	15%
Finite-volume corrections	6.0%	6.5%
Partial quenching	3.5%	1.7%
Renormalization	1.8%	5.6%
Unphysical kinematics	0.4%	0.8%
Derivative of the phase shift	0.97%	0.97%
Wilson coefficients	6.6%	6.6%
Total	18%	19%



The 2012 Ken Wilson Lattice Award

To:

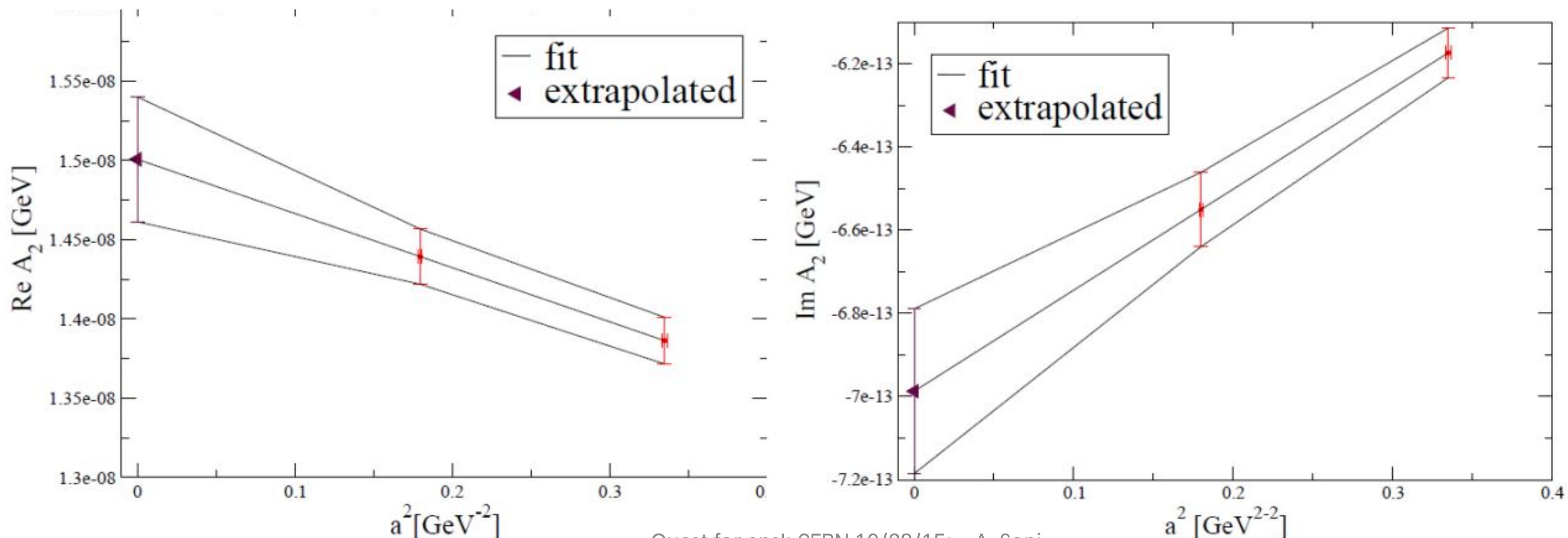
T. Blum
P.A. Boyle
N.H. Christ
N. Garron
E. Goode
T. Izubuchi
C. Jung
C. Kelly
C. Lehner
M. Lightman
Q. Liu
A.T. Lytle
R.D.Mawhinney
C.T. Sachrajda
A. Soni
C. Sturm

In recognition of their paper titled

$K \rightarrow (\pi\ \pi)_{I=2}$ Decay Amplitude from Lattice QCD

@ the Continuum

- Calculation has now been repeated on RBC & UKQCD 48^3x96 and 64^3x128 Mobius DWF ensembles with (5 fm)^3 volumes and a=0.114 fm, a=0.084 fm.



C Kelly

Expt 1.48×10^{-8} GeV

$$\mathrm{Re}(A_2) = 1.50(4)_{\mathrm{stat}}(14)_{\mathrm{sys}} \times 10^{-8}\ \mathrm{GeV}$$
$$\mathrm{Im}(A_2) = -6.99(20)_{\mathrm{stat}}(84)_{\mathrm{sys}} \times 10^{-13}\ \mathrm{GeV}$$

10%, 12% total errors on Re, Im!

- Systematic error completely dominated by perturbative error on NPR and Wilson coefficients.
- Future considerations:
    - Higher order PT calculation of NPR and Wilson coeffs.
    - Step-scaling NPR to higher energy scale.

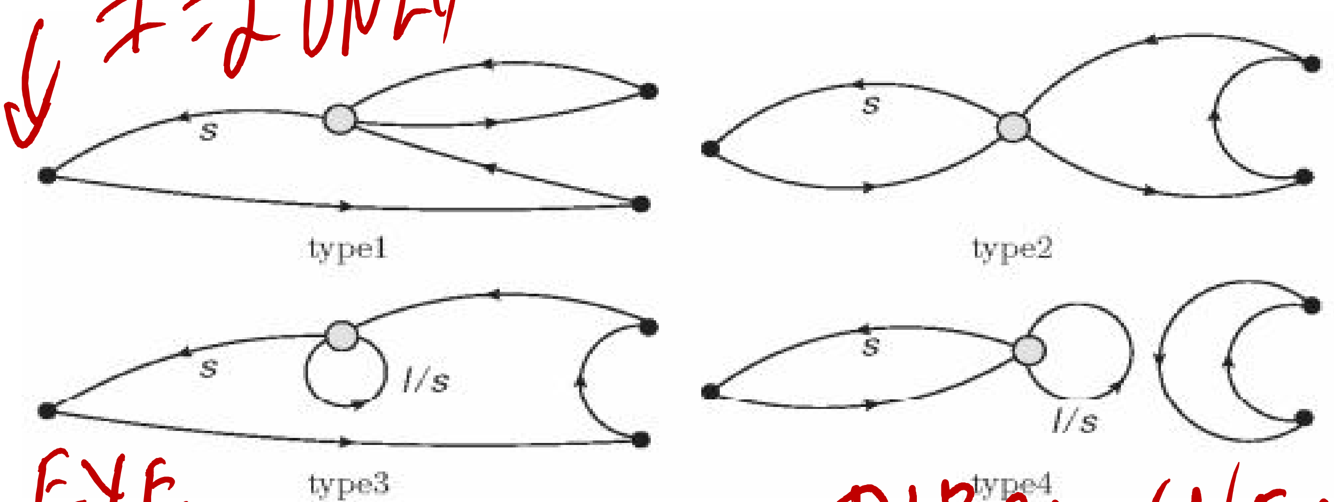
Systematic errors in $\mathrm{Im}A_2/\mathrm{Re}A_2$	48^3	64^3	cont
NPR (nonperturbative)	0.1%	0.1%	0.1%
NPR (perturbative)	7.6 %	6.7 %	7.6 %
Finite volume corrections	3.5 %	3.5 %	3.5 %
Unphysical kinematics	1.8 %	4.6%	4.6%
Wilson coefficients	12.0 %	10.5 %	12.0%
Derivative of the phase shift	0	0	0
Total	14.7%	13.7%	15.3%

TABLE XIII: Systematic error breakdown for $\mathrm{Im}A_2/\mathrm{Re}A_2$.

K=> pi pi quark flow diagrams types on the lattice

I=2 ONLY

I=0 All



EYE

DISCONNECTED

FIG. 1. Examples of the four types of diagram contributing to the $\Delta I = 1/2$, $K \to \pi\pi$ decay. Lines labeled ℓ or s represent light or strange quarks. Unlabeled lines are light quarks.

Ensemble

- 32^3x64 Mobius DWF ensemble with IDSDR gauge action at β=1.75. Coarse lattice spacing (a^{-1}=1.378(7) GeV) but large, (4.6 fm)^3 box.
- Using Mobius params (b+c)=32/12 and L_s=12 obtain same explicit χSB as the L_s=32 Shamir DWF + IDSDR ens. used for ΔI=3/2 but at reduced cost.
- Utilized USQCD 512-node BG/Q machine at BNL, the DOE "Mira" BG/Q machines at ANL and the STFC BG/Q "DiRAC" machines at Edinburgh, UK.
- Performed 216 independent measurements (4 MDTU sep.).
- Cost is ~1 BG/Q rack-day per complete measurement (4 configs generated + 1 set of contractions).
- G-parity BCs in 3 spatial directions results in close matching of kaon and $\pi\pi$ energies:

PHYSICAL MASSES & Kinematics!

m_K=490.6(2.4) MeV

$E_{\pi\pi}$(I=0) = 498(11) MeV

$E_{\pi\pi}$(I=2) = 573.0(2.9) MeV

E_π=274.6(1.4) MeV (m$_\pi$ = 143.1(2.0) MeV)

75%

15%

$Q_2 =$ 

large cancellation

dominant

<1%

i	Re(A_0)(GeV)	Im(A_0)(GeV)
1	$1.02(0.20)(0.07) \times 10^{-7}$	0
2	$3.63(0.91)(0.28) \times 10^{-7}$	0
3	$-1.19(1.58)(1.12) \times 10^{-10}$	$1.54(2.04)(1.45) \times 10^{-12}$
4	$-1.86(0.63)(0.33) \times 10^{-9}$	$1.82(0.62)(0.32) \times 10^{-11}$
5	$-8.72(2.17)(1.80) \times 10^{-10}$	$1.57(0.39)(0.32) \times 10^{-12}$
6	$3.33(0.85)(0.22) \times 10^{-9}$	$-3.57(0.91)(0.24) \times 10^{-11}$
7	$2.40(0.41)(0.00) \times 10^{-11}$	$8.55(1.45)(0.00) \times 10^{-14}$
8	$-1.33(0.04)(0.00) \times 10^{-10}$	$-1.71(0.05)(0.00) \times 10^{-12}$
9	$-7.12(1.90)(0.46) \times 10^{-12}$	$-2.43(0.65)(0.16) \times 10^{-12}$
10	$7.57(2.72)(0.71) \times 10^{-12}$	$-4.74(1.70)(0.44) \times 10^{-13}$
Tot	$4.66(0.96)(0.27) \times 10^{-7}$	$-1.90(1.19)(0.32) \times 10^{-11}$

TABLE I. Contributions to A_0 from the ten continuum, $\overline{\rm MS}$ operators $Q_i(\mu)$, for $\mu = 1.53$ GeV. Two statistical errors are shown: one from the lattice matrix element (left) and one from the lattice to $\overline{\rm MS}$ conversion (right).

$$\mathrm{Re}(A_0) = 4.66(1.00)(1.21) \times 10^{-7}\ \mathrm{GeV}$$

3.32×10^{-7} GeV expt

$$\mathrm{Im}(A_0) = -1.90(1.23)(1.04) \times 10^{-11}\ \mathrm{GeV}$$

$$\boxed{\begin{aligned}\mathrm{Re}A_2 &= 1.381(46)_{\mathrm{stat}}(258)_{\mathrm{syst}}10^{-8}\ \mathrm{GeV},\\ \mathrm{Im}A_2 &= -6.54(46)_{\mathrm{stat}}(120)_{\mathrm{syst}}10^{-13}\ \mathrm{GeV}.\end{aligned}}$$

2012 PRD

$a^{-1} = 1.364$ GeV

$32^3 \times 64 \times 32$

$$\boxed{\begin{aligned}\mathrm{Re}(A_2) &= 1.50(4)_{\mathrm{stat}}(14)_{\mathrm{syst}} \times 10^{-8}\ \mathrm{GeV};\\ \mathrm{Im}(A_2) &= -6.99(20)_{\mathrm{stat}}(84)_{\mathrm{syst}} \times 10^{-13}\ \mathrm{GeV}.\end{aligned}}$$

$\rightarrow$ 2015 PRD

1.48×10^{-8} Expt

$a^{-1} = 1.728$ GeV $48^3 \times 96 \times 24$

$= 2.358$ GeV $64^3 \times 128 \times 12$

Continuum

$$\mathrm{Re}(\epsilon'/\epsilon)_{\mathrm{EWP}} = -(6.6 \pm 1.0) \times 10^{-4}$$

For A2 error is now completely dominated by perturbation theory calculation of Wilson coeffs!

Digression: Understanding the Delta I=1/2 enhancement

Vacuum saturation

The discussion of direct calculations of the nonleptonic amplitudes is beyond the scope of this book. Suffice it to say that no treatment is presently adequate. Let us give the simplest estimate, called *vacuum saturation*, as a convenient benchmark with which to compare the theory. For simplicity we consider only O_1 (the largest $\Delta I = 1/2$ operator) and O_4 (the $\Delta I = 3/2$ operator),

$$\mathcal{H}_W \simeq \frac{G_F}{2\sqrt{2}} V_{ud}^* V_{us} (c_1 O_1 + c_4 O_4) \quad , \tag{4.16}$$

with $c_1 \simeq 1.9$ and $c_4 \simeq 0.5$. The vacuum saturation approximation consists of inserting the vacuum intermediate state between the two currents in any way possible, *e.g.*

$$\begin{aligned}
&\langle \pi^+(\mathbf{p}_+)\pi^-(\mathbf{p}_-)|\bar{d}\gamma^\mu(1+\gamma_5)u\bar{u}\gamma_\mu(1+\gamma_5)s|\bar{K}^0(\mathbf{k})\rangle \\
&\quad = \langle \pi^-(\mathbf{p}_-)|\bar{d}\gamma^\mu\gamma_5 u|0\rangle\langle\pi^+(\mathbf{p}_+)|\bar{u}\gamma_\mu s|\bar{K}^0(\mathbf{k})\rangle \\
&\qquad + \langle\pi^+(\mathbf{p}_+)\pi^-(\mathbf{p}_-)|\bar{u}_\beta\gamma^\mu u_\alpha|0\rangle\langle 0|\bar{d}_\alpha\gamma_\mu\gamma_5 s_\beta|\bar{K}^0(\mathbf{k})\rangle \\
&\quad = -i\sqrt{2}F_\pi f_+ p_-^\mu (k+p_+)_\mu - \frac{i}{3}\sqrt{2}F_K f_+ k_\mu (p_- - p_+)^\mu \quad .
\end{aligned} \tag{4.17}$$

In obtaining this result the Fierz rearrangement property

$$\bar{d}_\alpha\gamma^\mu(1+\gamma_5)u_\alpha\bar{u}_\beta\gamma_\mu(1+\gamma_5)s_\beta = \bar{d}_\alpha\gamma^\mu(1+\gamma_5)s_\beta\bar{u}_\beta\gamma_\mu(1+\gamma_5)u_\alpha$$

has been used, where α, β are color indices which are summed over. In addition, the color singlet property of currents is employed,

$$\langle 0|\bar{d}_\alpha\gamma_\mu\gamma_5 s_\beta|\bar{K}^0(\mathbf{k})\rangle = i\sqrt{2}F_K k_\mu \frac{\delta_{\alpha\beta}}{3} \quad . \tag{4.18}$$

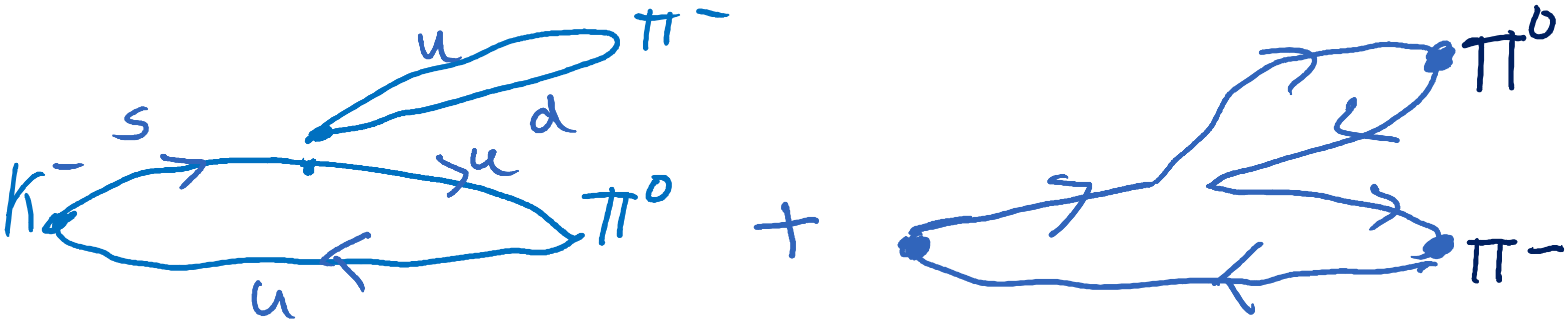
Within the vacuum saturation approximation, we see that the amplitudes are given completely by known semileptonic decay matrix elements. Putting in all of the constants, we find that

$$\begin{aligned}
A_0 &= \frac{G_F}{3} V_{ud}^* V_{us} F_\pi (m_K^2 - m_\pi^2) c_1 = 0.84 \times 10^{-7} m_K \ , \\
A_2 &= \frac{2\sqrt{2}G_F}{3} V_{ud}^* V_{us} F_\pi (m_K^2 - m_\pi^2) c_4 = 0.42 \times 10^{-7} m_K \quad .
\end{aligned} \tag{4.19}$$

We see that the above estimate of A_2 works reasonably well, but that A_0 falls considerably short of the observed $\Delta I = 1/2$ amplitude. This demonstrates that vacuum saturation is not a realistic approximation. However, it does serve to indicate how much additional $\Delta I = 1/2$ en-

Donoghue,,G,H
“Dynamics of
The SM” ‘92

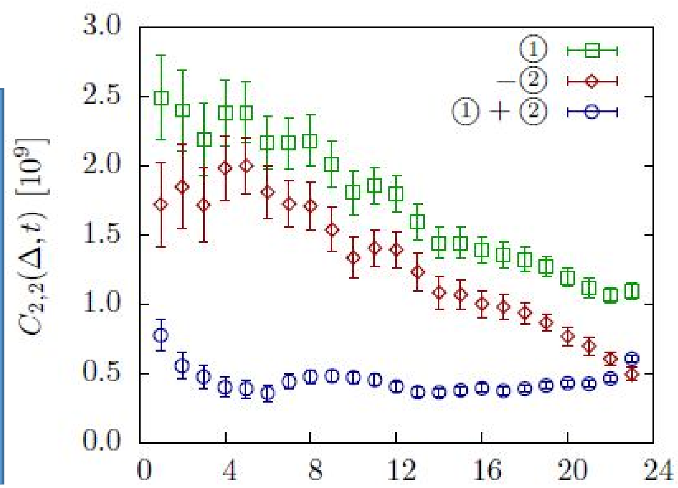
Dissecting 3/2 Amp on the lattice



Tr × Tr (N^2)

Tr (N)

Simplest basic step is Significantly different from phenomenological expectations



DRAMATIC CANCELLATION!

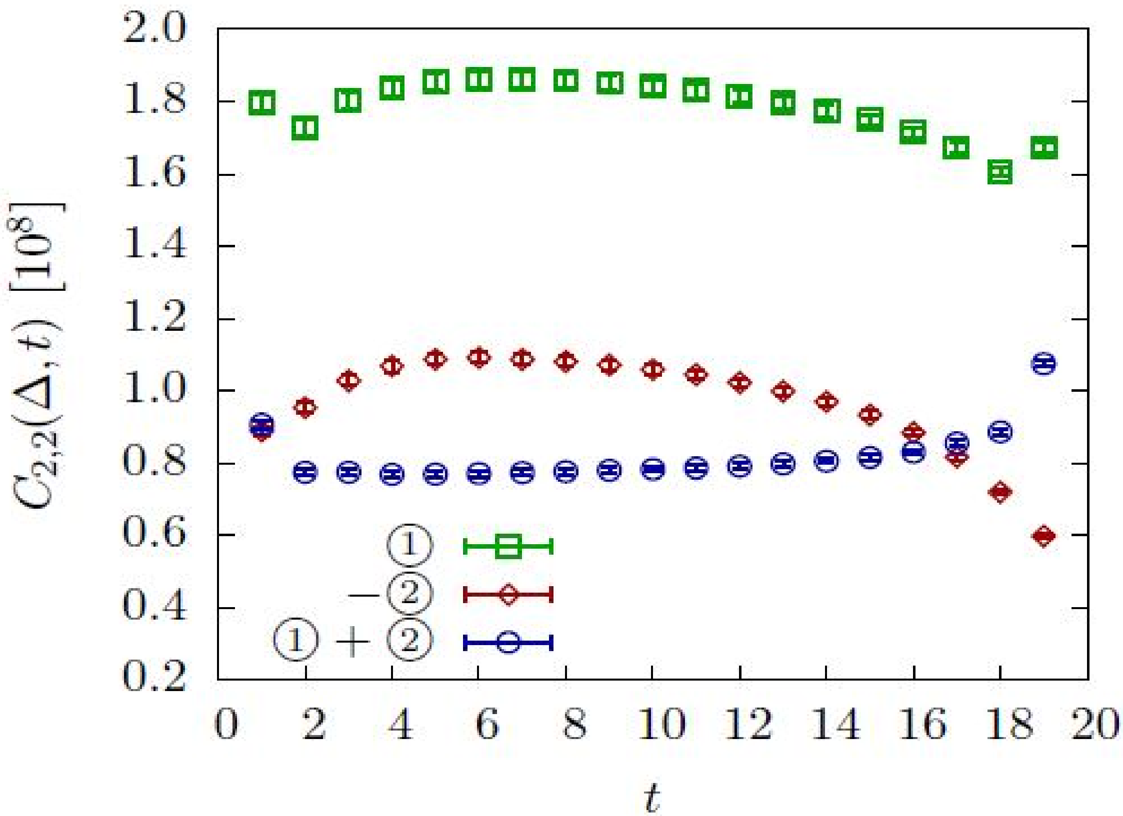
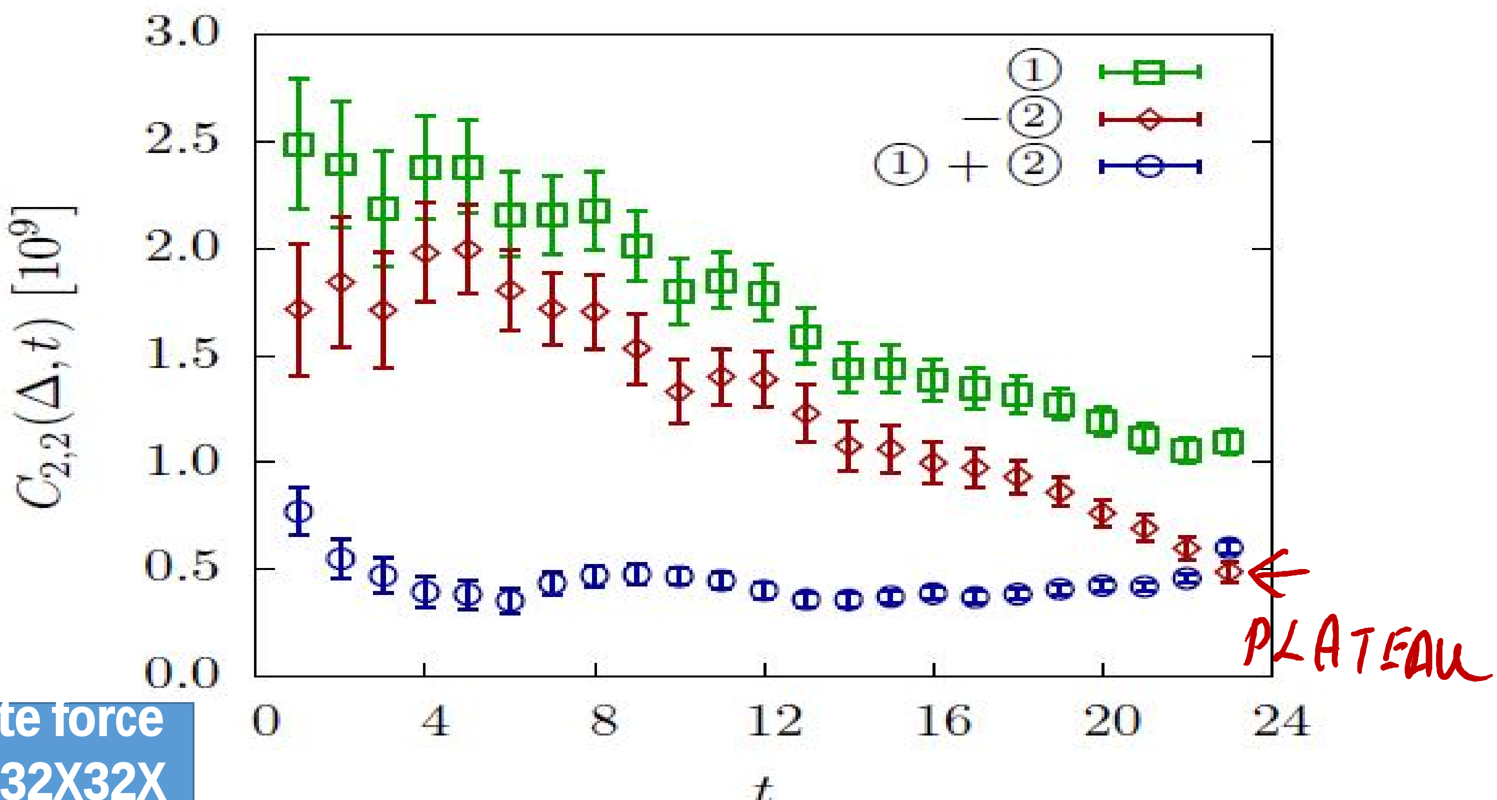


FIG. 3: Contractions ①, -② and ① + ② as functions of t from the simulation at threshold with $m_\pi \simeq 330\,\mathrm{MeV}$ and $\Delta = 20$.



Brute force 32X32X32X 64X16

FIG. 2: Contractions ①, -② and ① + ② as functions of t from the simulation at physical kinematics and with $\Delta = 24$.

QCDOC 10 Tf

Mass depends of ReA2, A0

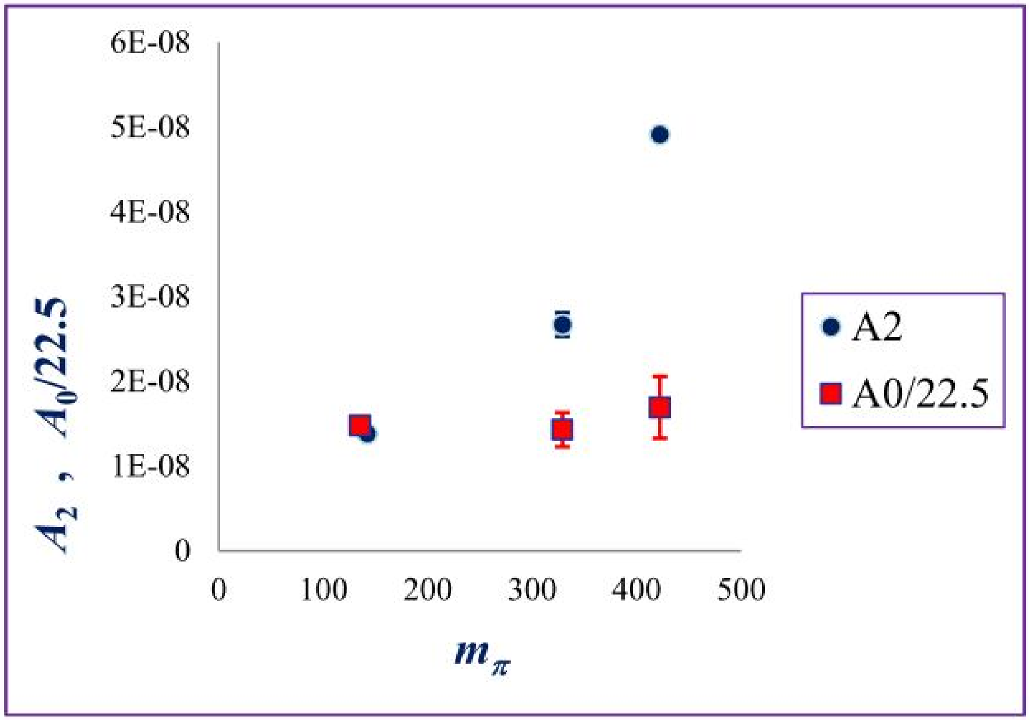
PRL 2013

	a^{-1} [GeV]	m_π [MeV]	m_K [MeV]	ReA_2 [10^{-8} GeV]	ReA_0 [10^{-8} GeV]	$\frac{\mathrm{Re}A_0}{\mathrm{Re}A_2}$	notes
16^3 Iwasaki	1.73(3)	422(7)	878(15)	4.911(31)	45(10)	9.1(2.1)	threshold calculation
24^3 Iwasaki	1.73(3)	329(6)	662(11)	2.668(14)	32.1(4.6)	12.0(1.7)	threshold calculation
IDSDR	1.36(1)	142.9(1.1)	511.3(3.9)	1.38(5)(26)	-	-	physical kinematics
Experiment	–	135 - 140	494 - 498	1.479(4)	33.2(2)	22.45(6)	

TABLE I: Summary of simulation parameters and results obtained on three DWF ensembles.

Due to the cancellation, 3/2 amplitude decreases significantly as the pion mass is lowered towards its physical value

Compare A_2 and $A_0/22.5$



NHCC
KITP
Aug '15

Repercussions for B_K

THE $\Delta S = 2$ MATRIX ELEMENT FOR K^0–$\bar{K}^0$ MIXING ☆

John F. DONOGHUE, Eugene GOLOWICH and Barry R. HOLSTEIN
Department of Physics and Astronomy, University of Massachusetts, Amherst, MA 01103, USA

Received 29 July 1982

We use SU(3) and PCAC to relate the $\Delta S = 2$ matrix element $\mathcal{M} = \langle K^0 | \bar{d}\gamma_\mu(1+\gamma_5)s\,\bar{d}\gamma^\mu(1+\gamma_5)s | \bar{K}^0\rangle$ to experimental information on the $\Delta I = 3/2$ contribution to $K \to 2\pi$. Our result is $\mathcal{M} = 0.10\, m_K^4$ which is about 33% of the vacuum insertion value.

Contiuum expectations for M_{LL}

K-K Mixing

To get an order of magnitude of the size of $\langle \mathrm{K}|\mathcal{O}_{\mathrm{JJ}}|\bar{\mathrm{K}}\rangle$, we make the 'vacuum saturation' approximation

$\leftarrow 2[1+1/3]$

$$\langle \mathrm{K}|[\bar{\mathrm{d}}\gamma^{\mu}(1-\gamma_5)\mathrm{s}][\bar{\mathrm{d}}\gamma_{\mu}(1-\gamma_5)\mathrm{s}]|\bar{\mathrm{K}}\rangle = \frac{8}{3}\langle \mathrm{K}|\bar{\mathrm{d}}\gamma^{\mu}\gamma_5\mathrm{s}|0\rangle\langle 0|\bar{\mathrm{d}}\gamma_{\mu}\gamma_5\mathrm{s}|\bar{\mathrm{K}}\rangle$$

Cheng & Li, Gauge Field Theory

$$= \frac{8}{3}\frac{f_{\mathrm{K}}^2 m_{\mathrm{K}}^2}{2m_{\mathrm{K}}} \qquad (12.93)$$

where $f_{\mathrm{K}} \simeq 1.23\, f_{\pi}$ is the kaon decay constant; the factor $(2m_{\mathrm{K}})^{-1}$ arises from the normalization of the state. The factor 8/3 corresponds to the four ways of Wick contraction times a colour factor 2/3. The hope in making such

To make contact with phenomenology, one must evaluate the matrix element of $O^{\Delta S=2}$ between K^0 and $\bar{K}^0$ states. It is conventional to express the results in terms of the so-called *B-parameter*,

DGH

$$\langle K^0 | O^{\Delta S=2} | \bar{K}^0 \rangle \equiv \frac{16}{3} F_K^2 m_K^2 B \ , \qquad (1.19)$$

where $B = 1$ corresponds to the simple vacuum saturation approximation would be too large. Such a statement requires some estimate of the matrix element of the $\Delta S = 2$ operator. Gaillard and Lee used a version of the vacuum insertion approximation [see 9.4.7 and 9.4.8]. If we insert a vacuum state between all possible pairs of quark fields in $O_{+}^{d\bar{s}}$, we get

$2[1+\frac{1}{3}] = 8/3$

Weak Interactions & Modern Particle Theory
Howard Georgi

Breakdown of LgN for N=3 @ $m_\pi \sim 140$ MeV

$$\langle K^0 | O_{+}^{d\bar{s}} | \overline{K^0} \rangle \simeq \frac{8}{3} f_K^2 m_K^2 \quad (10.2.7)$$

Lattice $2[1 \rightarrow .7]$ @ m_π
~ 0.6

Results for ε'

- Using $\mathrm{Re}(A_0)$ and $\mathrm{Re}(A_2)$ from experiment and our lattice values for $\mathrm{Im}(A_0)$ and $\mathrm{Im}(A_2)$ and the phase shifts,

EWP

QCDP

PARTIAL Cancellation

$$\mathrm{Re}\left(\frac{\varepsilon'}{\varepsilon}\right) = \mathrm{Re}\left\{\frac{i\omega e^{i(\delta_2-\delta_0)}}{\sqrt{2}\varepsilon}\left[\frac{\mathrm{Im}A_2}{\mathrm{Re}A_2} - \frac{\mathrm{Im}A_0}{\mathrm{Re}A_0}\right]\right\}$$

$$\boxed{\begin{array}{ll} = 1.38(5.15)(4.43)\times 10^{-4}, & \text{(this work)} \\ 16.6(2.3)\times 10^{-4} & \text{(experiment)} \end{array}}$$

RBC-UKQCD
arXiv:1505.07863 to appear in PRL

Bearing in mind the largish errors in this first calculation, we interpret that our result are consistent with experiment at ~2σ level

$$\omega = \frac{\mathrm{Re}\,A_2}{\mathrm{Re}\,A_0} \sim 0.045$$

Re A_0, A_2 from expt ba now.

Description	Error	Description	Error
Finite lattice spacing	8%	Finite volume	7%
Wilson coefficients	12%	Excited states	$\lesssim 5\%$
Parametric errors	5%	Operator renormalization	15%
Unphysical kinematics	$\lesssim 3\%$	Lellouch-Lüscher factor	11%
Total (added in quadrature)			26%

TABLE II. Representative, fractional systematic errors for the individual operator contributions to $\mathrm{Re}(A_0)$ and $\mathrm{Im}(A_0)$.

A possible difficulty: strong phases

- The continuum and our lattice determinations of strong phase difference differs at the ~2σ level:

Not directly accessible to expt

-

$$\phi_{\varepsilon'} = \delta_2 - \delta_0 + \frac{\pi}{2} = \begin{cases} (42.3 \pm 1.5)^{\text{o}} & \text{PDG [2]} \\ (54.6 \pm 5.8)^{\text{o}} & \cancel{\text{RBC}}\ \text{[47, 48]} \end{cases}.$$

RBC-UKQCD

$\phi_\varepsilon \sim 43.5 \pm 0.5^{\circ}$

Fortunately, due to the central value of the combination $\delta_2 - \delta_0 + \pi/2 - \phi_\varepsilon$ and to the large uncertainties in the determination of the various matrix elements, these two choices yield almost identical results; ~~for definiteness, we~~

Lehner, Lunghi + AS, 1508.01801

Underlying method is systematically improvable: Proof of the pudding

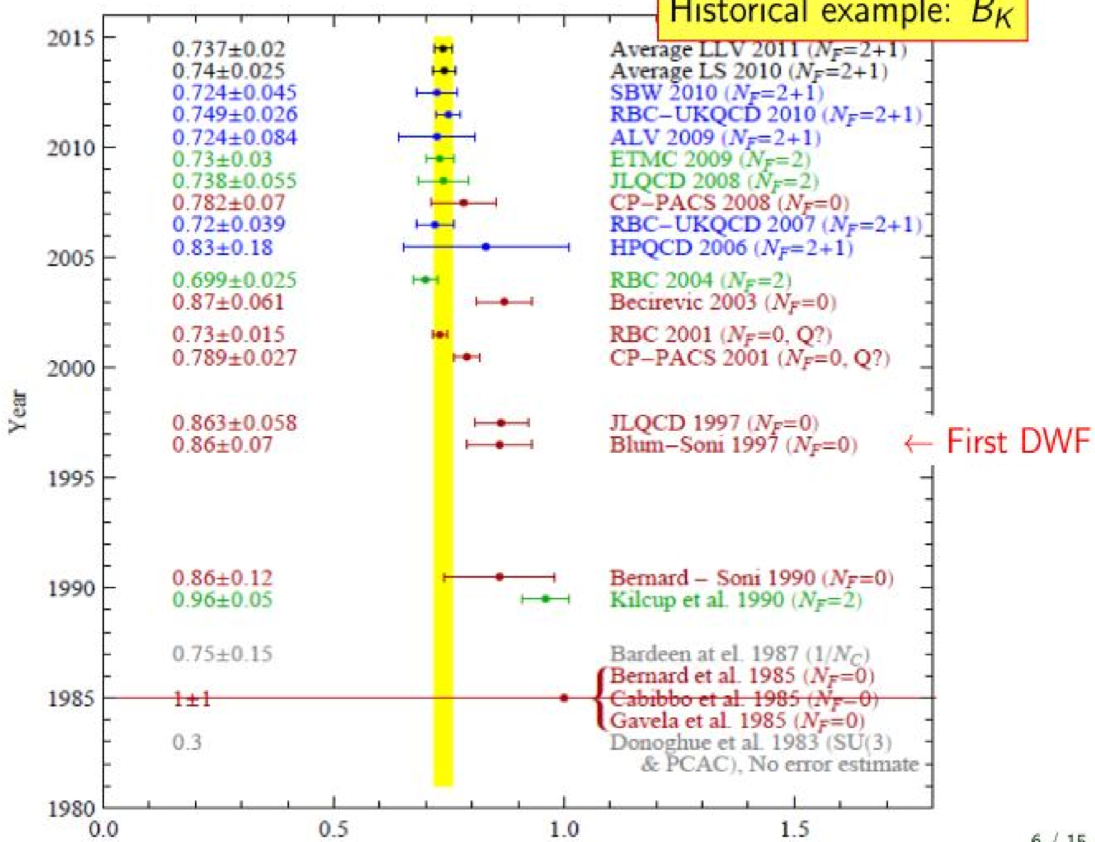
- BK in full QCD with DWF '07 error O(7%)
- ~2012 many discretizations , WA error O(1-2%)
- Similarly, Kl3 O(1/2%), A2 O(10%) , fB's O(few %) , BB's O(few%)........

- For sure A0, A2 for ε' will also go that way for quite sometime to come.........to ~10% total

After that EM & isospin effects will have to be ascertained quantitatively.

Power of the lattice: Only method to systematically reduce the NP error!

Historical example: B_K



6 / 15

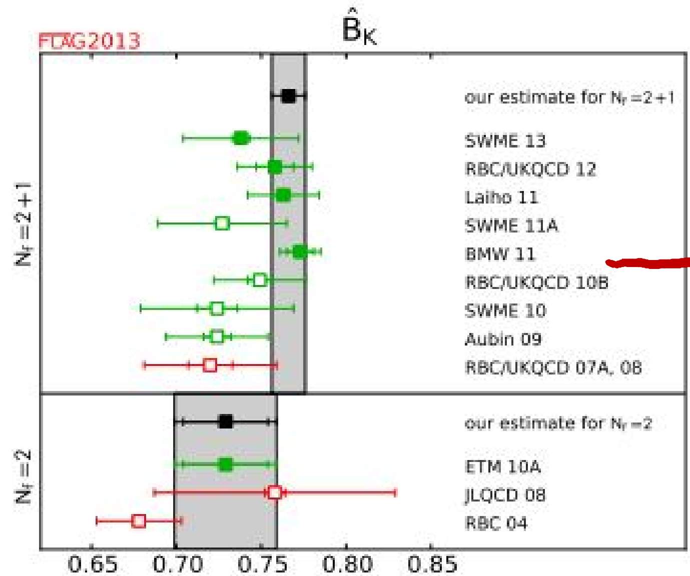
AB-initio Calculat

$$B_K \equiv \frac{\langle K | (\bar{s}\gamma_\mu d)^2 | \bar{K}\rangle}{8/3\, f_K^2\, m_K^2}$$

Status before lattice 2014

Garron LAT14

FLAG [Aoki et al., '13-14]



FLAG 2013

$$N_f = 2+1: \qquad \hat{B}_K = 0.7661(99),$$

~1.3%

Near future plans ~ O(1) year

- More configs are being generated
- Codes are being developed for running on new hardwares that will soon become available
- Intense effort is being made to reduce some major source of systematic errors [due op. renorm]
- Longer paper in ~ 6 months

It'd be good for others on the lattice to get into.

Recently, Ishizuka et al **arXiv:1505.05289, exploit CPS symmetry with Wilson fermions along the lines proposed by Bernard, Draper, Hockney & AS ('87) and Dawson et al ('98) to study K=> pi pi and eps' AT THRESHOLD……a good start**

Consistent within large errors with RBC-UKQCD
@threshold done in 2012

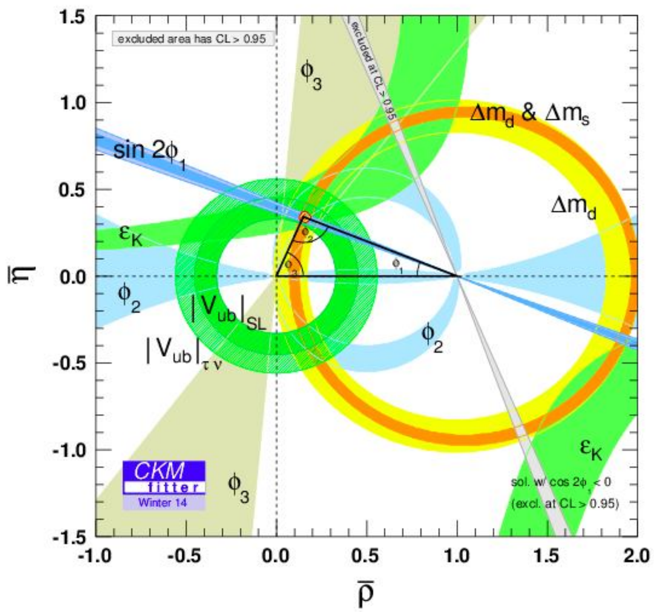
Some Implications

Emerging lattice approach to K-Unitarity Triangle

1508.01801 Lehner, Lunghi, AS

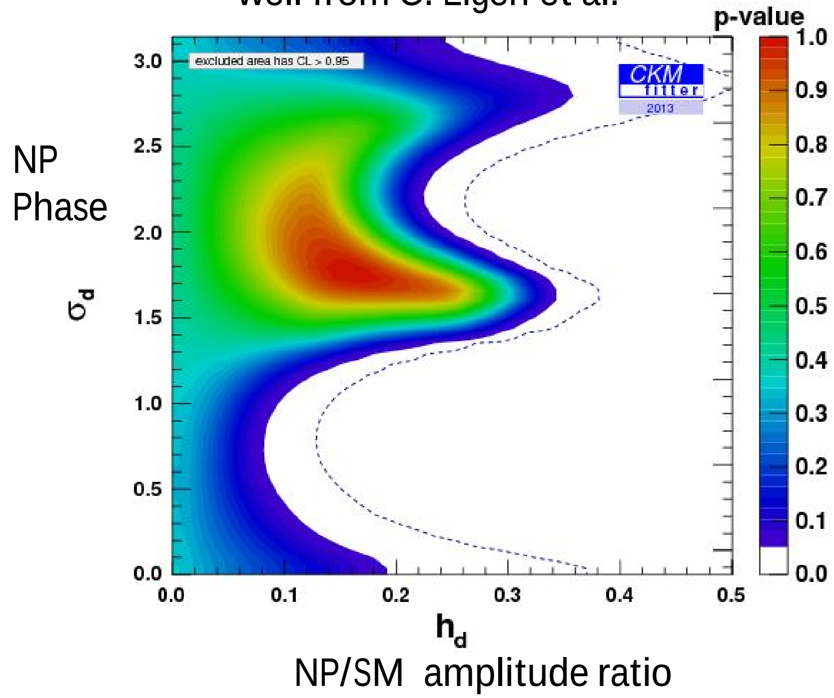
Results from Global Fits to Data (CKMFitter Group)

Great progress on φ_3 or γ (first from B factories and now in the last two years from LHCb (several new results at ICHEP2014). <u>These measure the phase of V_{ub}</u>



Looks good
(except for an issue with $|V_{ub}|$)

ICHEP2014: Similar results from UTFIT (D. Derkach) as well from G. Eigen et al.



But a 10-20% NP amplitude in B_d mixing is perfectly compatible with all current data.

A lesson from history (I)

"A special search at Dubna was carried out by E. Okonov and his group. They did not find a single $K_L \rightarrow \pi^+ \pi^-$ event among 600 decays into charged particles [12] (Anikira et al., JETP 1962). At that stage the search was terminated by the administration of the Lab. The group was unlucky."

-Lev Okun, "The Vacuum as Seen from Moscow"

1964: BF= 2×10^{-3}

A failure of imagination ? Lack of patience ?

Christenson, Cronin, FITCH + TURLAY BNL '64

=> **Precision! Precision! Precision! Need of the day.**
=> **Also, since we are searching for small effects, using different probes may be valuable**

=

- **In B's, in conjunction with experiments, Lattice WME helped in attaining a milestone in our understanding of CP**
- **Analogously can lattice sharpen tests now via K's?**
- **Since m_K is ~10 times lighter, the non-perturbative effects are much more difficult and quantitatively a lot bigger, can the lattice meet this long-standing challenge and render K-tests become precise?**

Promising developments on the lattice in K-decays.........RBC-UKQCD

- In the process of taming ε' also
- Long-distance (non-local) effects; most interesting & important in

ΔmK because of extreme sensitivity to chiral structure of Heff see Beall, Bander + AS, PRL '82 ....δO(40%) Brod & Gorban

See N.Christ et al PRD'13; PRL'14... Look forward to

ΔmK from lattice as a useful observable for constraining NP.

Buras, Guadagnoli, Isidori '10

- εK LD .....δO(7%).....N.Christ talk @LAT'15 & many more
- K+ => π ν ν......δO(few%).......Xu Feng talk @ lat'15
- K => π e e..........A. Lawson talk @ Lat'15; [A.Portelli]; C. Sachrajda @LAT'14
- => Pathways to K-UT

A dream for some

Blucher, Winstein and Yamanaka '09

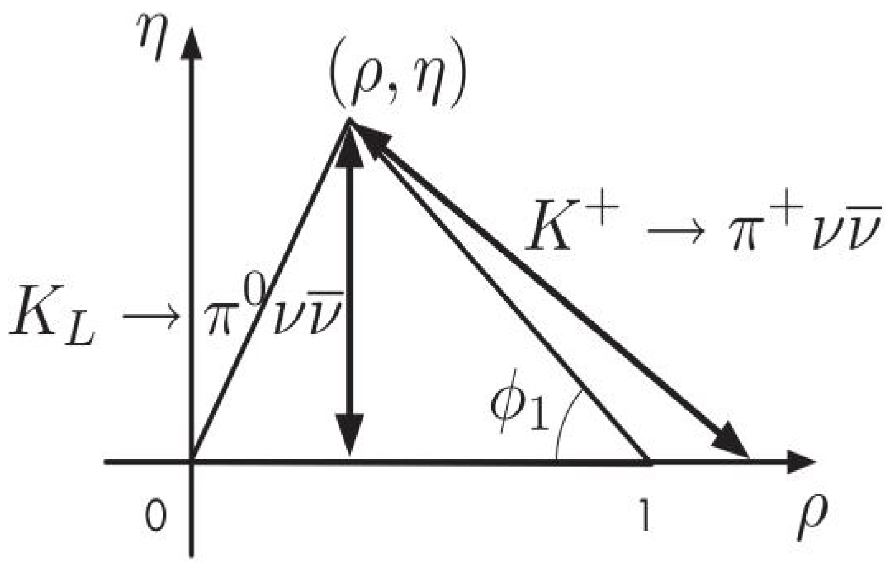


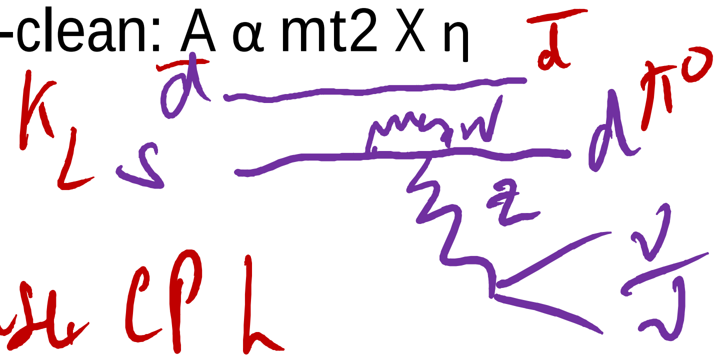
Fig. 14. Unitarity triangle.

A Faster way in the offing?

More on K-decays=>rare K's

Taku Yamanaka @ CKM 2014

- KL => π0 ν ν ..."Gold-plated," i.e Theory super-clean: A α mt2 X η



LITTENBERG PRD '89

Nothing → Nothing

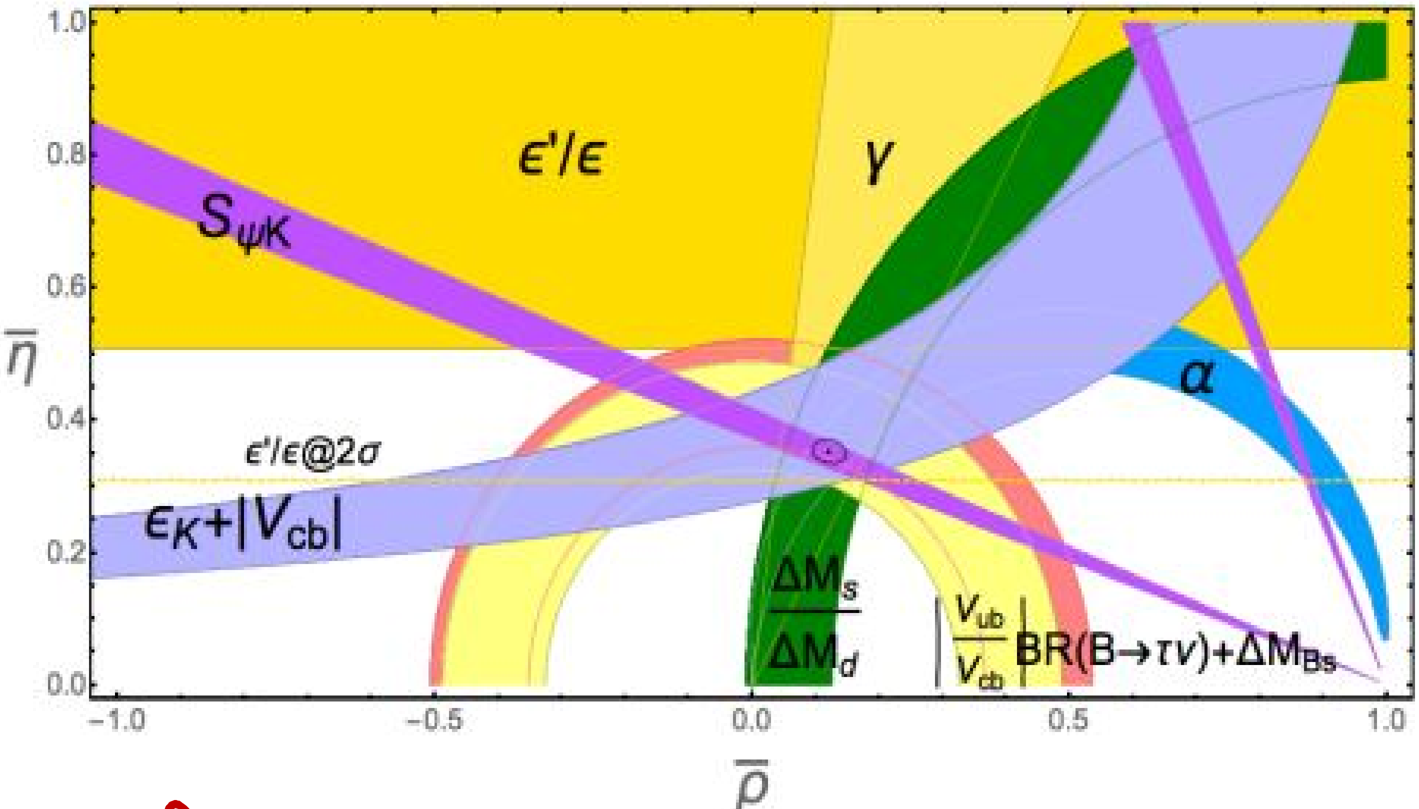
use "CP L"

- **Observe: The above expt is exceedingly**

challenging (esp for precision) and expensive.

- **Assertion: Once the (exptal) community realizes we mean business by reducing errors on Im A0 to around ~ 20% they will get the message loud and clear: It is much more cost effective to invest in better lattice calculation(s) of eps' ......**

Lattice ϵ'/ϵ & SUT ≡ The UT.

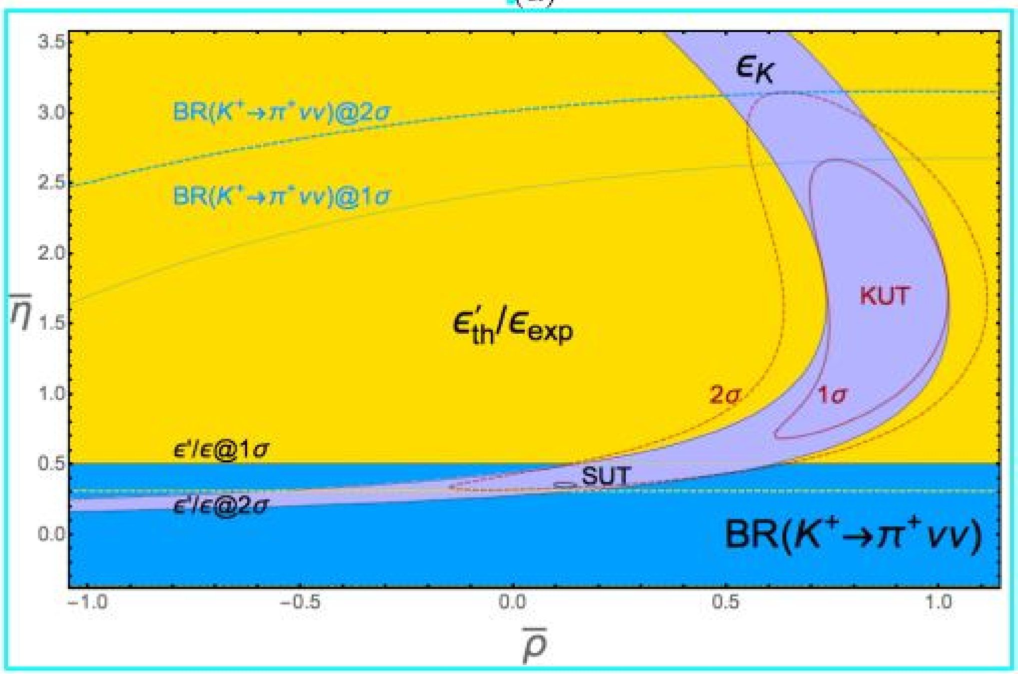


LLS

Sketch of an emerging K-UT

$$\mathrm{BR}(K^+ \to \pi^+ \nu\bar{\nu}) = \begin{cases} (8.64 \pm 0.60) \times 10^{-11} & \text{SM} \\ (17.3^{+11.5}_{-10.5}) \times 10^{-11} & \text{E949} \end{cases}$$ BNL

(a)



$$\mathrm{Re}\left(\frac{\varepsilon'}{\varepsilon}\right)_K = \begin{cases} (16.7 \pm 1.6) \times 10^{-4} \\ (1.36 \pm 5.21_{\mathrm{stat}} \pm 4.49_{\mathrm{syst}}) \times 10^{-4} \end{cases}$$

PDG2015

RBC+UKQCD '15

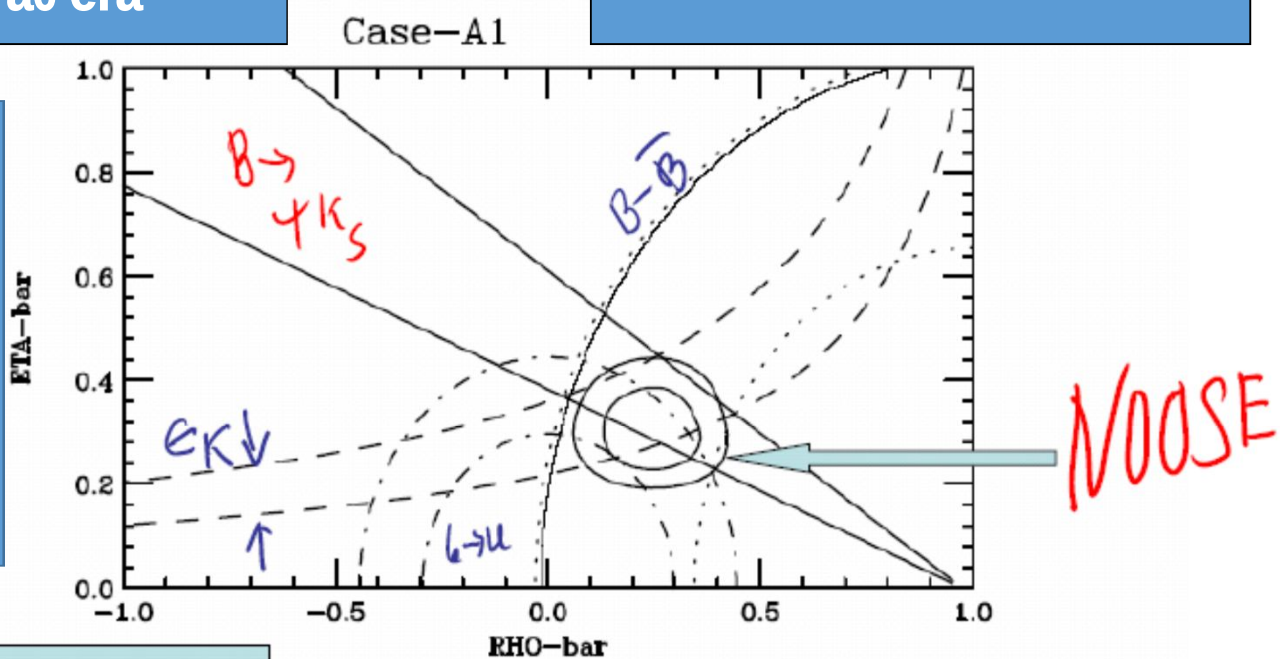
LLS

In the "beginning" "Dawn " of the asymmetric B-Fac era

Atwood &AS, hepph/0103197

B-CP Feb'01 Ise, Japan

1st Hint of confirmation of CKM CP description



Most bands due
To theory errors

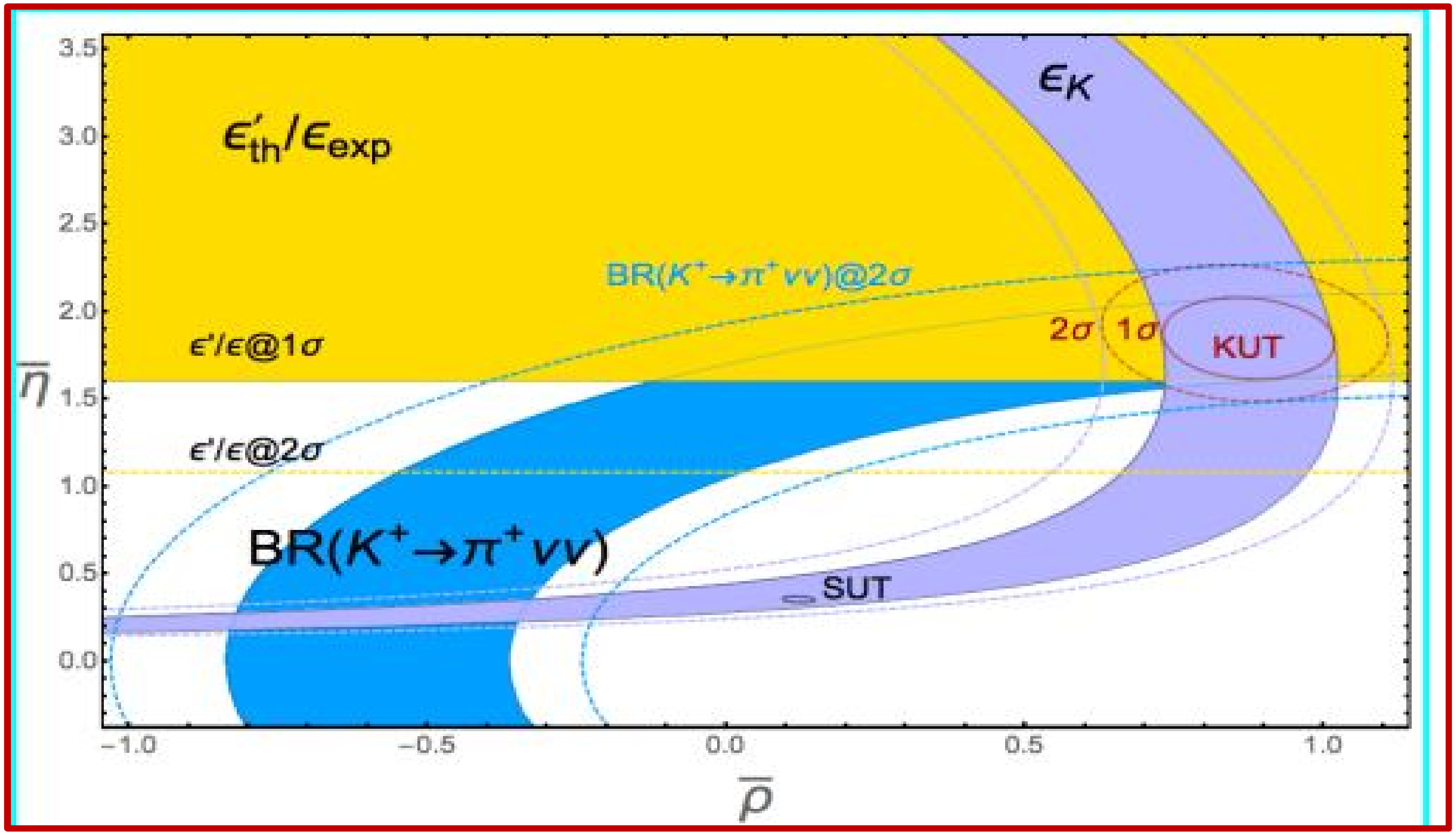
New physics will be a perturbation, important to use clean theory and lots of statistics.

POSSIBLE KUT CIRCA 2020

ILLUSTRATION
assuming
$\delta\, \mathrm{Im}\, A_0$ w/8%

NO
unique
ρ, η !

use
NA62
K^+
10%



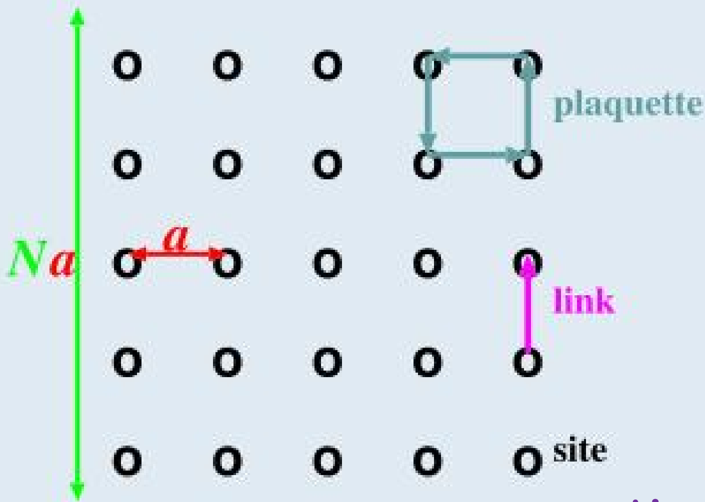
Summary + outlook

- Significant progress in K=> ππ with physical masses and kinematics
- Presented 1st computation of ε'/ε with controlled errors:
- $1.38(5.15)(4.43) \times 10^{-4}$; 16.6(2.3)X 10^{-4} expt consistente ~ 2σ
- Trying hard to reduce syst and stat errors
- Detailed paper, hopefully with some improvements, in ~ 6 months
- New (faster) hardware later this year or'16=> should have significantly reduced errors in 1-3 years
- Expect errors < ~ 10% in ~5 years; thence EM & isospin needs tackling
- Experimentalists ought to think of improved measurements of ε', error now ~15%

- Perhaps easier than precise measurement of KL=>πνν

EXTRAS

Lattice QCD



typical values:

$a^{-1} = 2\text{–}5$ GeV, $Na = 2\text{–}7$ fm

continuum limit: $a \to 0$, Na fixed

infinite volume: $Na \to \infty$

$$\langle O \rangle = \frac{1}{Z} \int [dU]\,[d\psi][d\bar{\psi}]\; O[U] e^{-S[U,\psi,\bar{\psi}]}$$

$U_\mu(n) = \exp[-igA_\mu(n)]$

"Measurement": average over a representative ensemble of gluon configurations $\{U_i\}$ with probability $P(U_i) \propto \int [d\psi][d\bar{\psi}] e^{-S[U,\psi,\bar{\psi}]}$

$$\langle O \rangle = \frac{1}{n} \sum_{i=1}^{n} O(U_i) + \Delta O \qquad \Delta O \propto \frac{1}{\sqrt{n}} \xrightarrow{n \to \infty} 0$$

G BALI eIITM

Lattice Determination of K→ππ

- On the lattice compute $M_j = \langle (\pi\pi)_I | Q_j | K \rangle$
- Operators must be renormalized into same scheme as Wilson coeffs: Use RI-(S)MOM NPR and perturbatively match to MSbar at high scale.
- Mixing under renormalization, hence Z is a matrix.

$$A_{2/0} = F \frac{G_F}{\sqrt{2}} V_{ud} V_{us} \sum_{i=1}^{10} \sum_{j=1}^{7} \left[\Big(z_i(\mu) + \tau y_i(\mu) \Big) Z_{ij}^{\mathrm{lat} \to \overline{\mathrm{MS}}} M_j^{\frac{3}{2}/\frac{1}{2},\mathrm{lat}} \right],$$

- F is finite-volume correction calculated using LL method.
- Important to calculate with physical (energy-conserving) kinematics. With physical masses:

$$2 \times m_\pi \sim 270 \text{ MeV} \qquad m_K \sim 500 \text{ MeV}$$

we require non-zero relative momentum for the pions.

C. Kelly @ Lat'15

- This is excited state of the ππ-system. Possibilities:
  - try to perform multi-state fits to very noisy data (esp. A_0 where there are disconn. diagrams) or
  - modify boundary conditions to remove the ground-state

Operator Normalization
(Rome-Southampton)

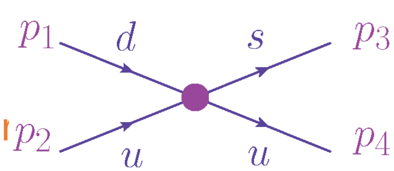
- Effective weak Hamiltonian H_W contains four-quark operators normalized in the MS scheme.
- Use non-perturbative methods to convert lattice operators to RI scheme:
  - Evaluate Landau-gauge, off-shell Green's functions:

$$\left(\Gamma(p_1, p_2, p_3, p_4)_j\right)^{\alpha\beta\gamma\delta}_{abcd} = \prod_{i=1}^{4}\left(\int d^4x_i e^{ip_i\cdot x_i}\right)\left\langle \overline{q}^{\alpha}_a(x_1)\overline{q}^{\beta}_b(x_2)O_j q^{\delta}_c(x_3)q^{\gamma}_d(x_4)\right\rangle$$

  - Impose normalization conditions:

$$\mathrm{tr}\{P_i\Gamma_j\} = F_{ij}$$

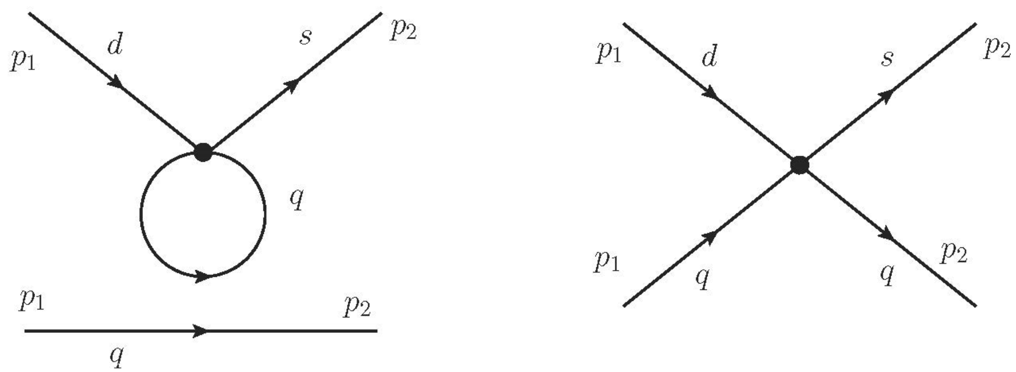
- Use continuum perturbation theory to conver



N. Christ
@KITP/15

RI/SMOM normalization of chiral operators

- For (8,1) operators must include disconnected diagrams.
- Use $p_1 = 2\pi\,(4,4,0,0)/L$ and $p_2 = 2\pi\,(0,4,4,0)/L$
- $p_1^2 = p_2^2 = (p_1 - p_2)^2 = 1.531\ \text{GeV}^2$
- Use 100 configurations



N. Christ
KITP '15

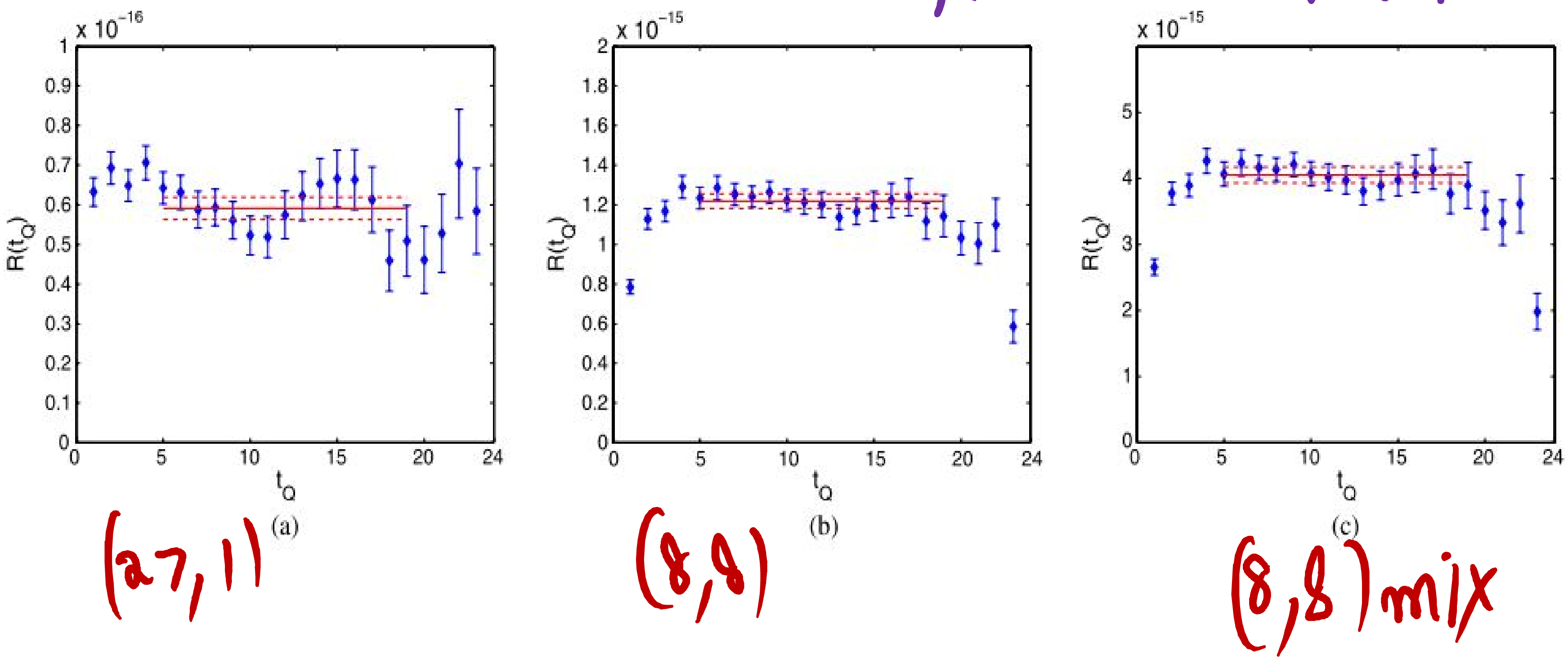
i	$\mathcal{M}'^{(i)}_{\rm SMOM}$ (GeV)^3	$\mathcal{M}^{(i)}_{\overline{\rm MS}}$ (GeV)^3
1	-0.0675(1109)(128)	-0.151(29)(36)
2	-0.156(27)(30)	0.169(42)(41)
3	0.212(52)(40)	-0.0492(652)(118)
4	—	0.271(93)(65)
5	-0.193(62)(37)	-0.191(48)(46)
6	-0.366(103)(70)	-0.379(97)(91)
7	0.225(37)(43)	0.219(37)(53)
8	1.65(5)(31)	1.72(6)(41)
9	—	-0.202(54)(49)
10	—	0.118(42)(28)

TABLE S II. The renormalized matrix elements in the RI/SMOM($\not{q}, \not{q}$) scheme and chiral basis (second column). The third column shows the matrix elements of the traditional, physical operators $Q_i^{\overline{\rm MS}}$ defined in the $\overline{\rm MS}$ scheme. The latter are obtained by applying the 10×7 scheme-change matrix factors given in Table SIV to the numbers in the second column. The left error shown is statistical and the right is systematic. For the second column the systematic error is a uniform 19% estimate obtained from Table II by omitting the errors associated with operator renormalization, input parameters and Wilson coefficients. (This estimate ignores the errors associated with our use of an incomplete set of off-shell, RI/SMOM operators, an error which we perturbatively estimate at the 1% level.) The errors in the third column are similarly obtained from Table II except only the parametric and Wilson coefficient errors are omitted, giving a uniform 24% systematic error estimate.

RBC-UKQCD PRL
Supplement

I=2 matrix elements @ physical kinematics

RBC-UKQCD PRD'12



Physical Kinematics

- A_2 calculation used APBC on d-quarks, removes stationary **charged pion** state BUT breaks isospin and doesn't work for π^0.
- Solution: Use G-parity BCs:

$$\hat{G} = \hat{C} e^{i\pi \hat{I}_y} \quad : \quad \hat{G}|\pi^{\pm}\rangle = -|\pi^{\pm}\rangle \qquad \hat{G}|\pi^0\rangle = -|\pi^0\rangle$$

- As a boundary condition: (i=+, -, 0)

$$\pi^i(x+L) = \hat{G}\pi^i(x) = -\pi^i(x) \longrightarrow |p| \in (\pi/L,\, 3\pi/L,\, 5\pi/L \ldots)$$

(moving ground state)

- At quark level:

$$\hat{G} \begin{pmatrix} u \\ d \end{pmatrix} = \begin{pmatrix} -C\bar{d}^T \\ C\bar{u}^T \end{pmatrix}$$

where C = $\gamma^2\gamma^4$ in our conventions

- Gauge invariance $\rightarrow$ gauge field must obey charge conjugation BCs; new ensembles needed.
- For stationary kaon we must introduce fictional degenerate partner to the strange quark: s'

$$|\tilde{K}^0\rangle = \left(|\bar{s}d\rangle + |\bar{u}s'\rangle\right)/\sqrt{2}$$

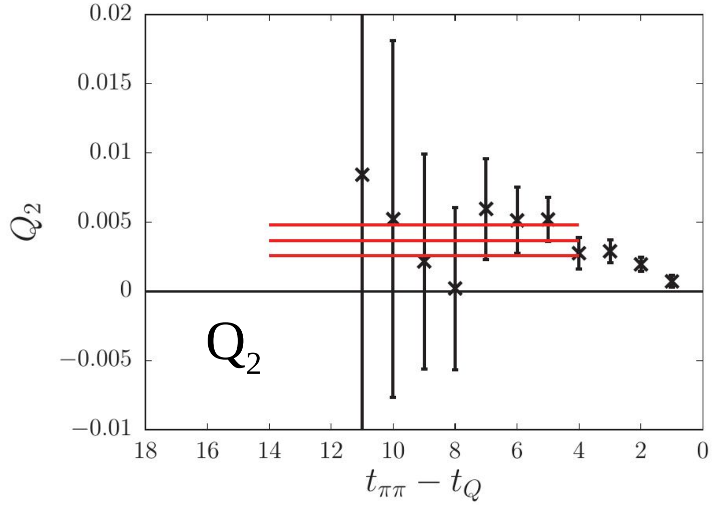
is G-parity even (p=0)

- Coupling of unphysical kaon partner to physical operators exponentially suppressed and can be neglected.

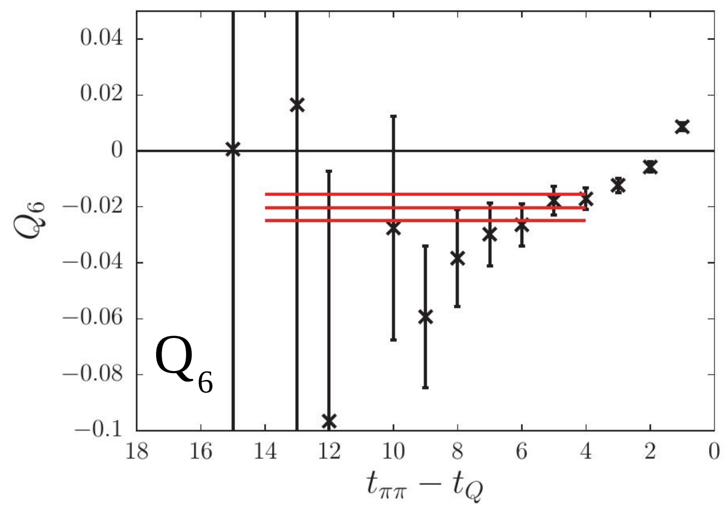
ChrisKelly 2015

Matrixelementfits

[Dominant contribution to $Re(A_0)$]



[Dominant contribution to $Im(A_0)$]



- Use $t_{min}(\pi \to Q) = 4$ here rather than 6 as signal quickly decays into noise (40% increase in stat. error with t_{min} =5!).
- However comparison to t_{min}=3 shows no statistically resolvable difference, suggesting excited state contamination small.
- Estimate 5% excited state systematic by comparing single-exp fit result for $\pi\pi(I=0)$ amplitude with t_{min}=4 to double-exp fit with t_{min}=3.

Chris Kelly Lattice 2015

Systematic errors

- Errors for each separate operator matrix element:

Description	Error	Description	Error
Finite lattice spacing	8%	Finite volume	7%
Wilson coefficients	12%	Excited states	$\leq 5\%$
Parametric errors	5%	Operator renormalization	15%
Unphysical kinematics	$\leq 3\%$	Lellouch-Lüscher factor	11%
Total (added in quadrature)			26%

- Treat as uncorrelated when combining to form A_0.
- 15% renormalization error dominant due to low, 1.53 GeV renormalization scale. Estimate by comparing two different RI/SMOM intermediate schemes and use the largest observed differences.
- 12% Wilson coefficient error large for same reason. Conservatively estimate as largest observed fractional change between using LO and NLO.

Chris K @ lattice 2015