Precise parton distributions, α_s and quark masses

Sven-Olaf Moch

Universität Hamburg

CERN Phenomenology Seminar, Geneva, Nov 13, 2015

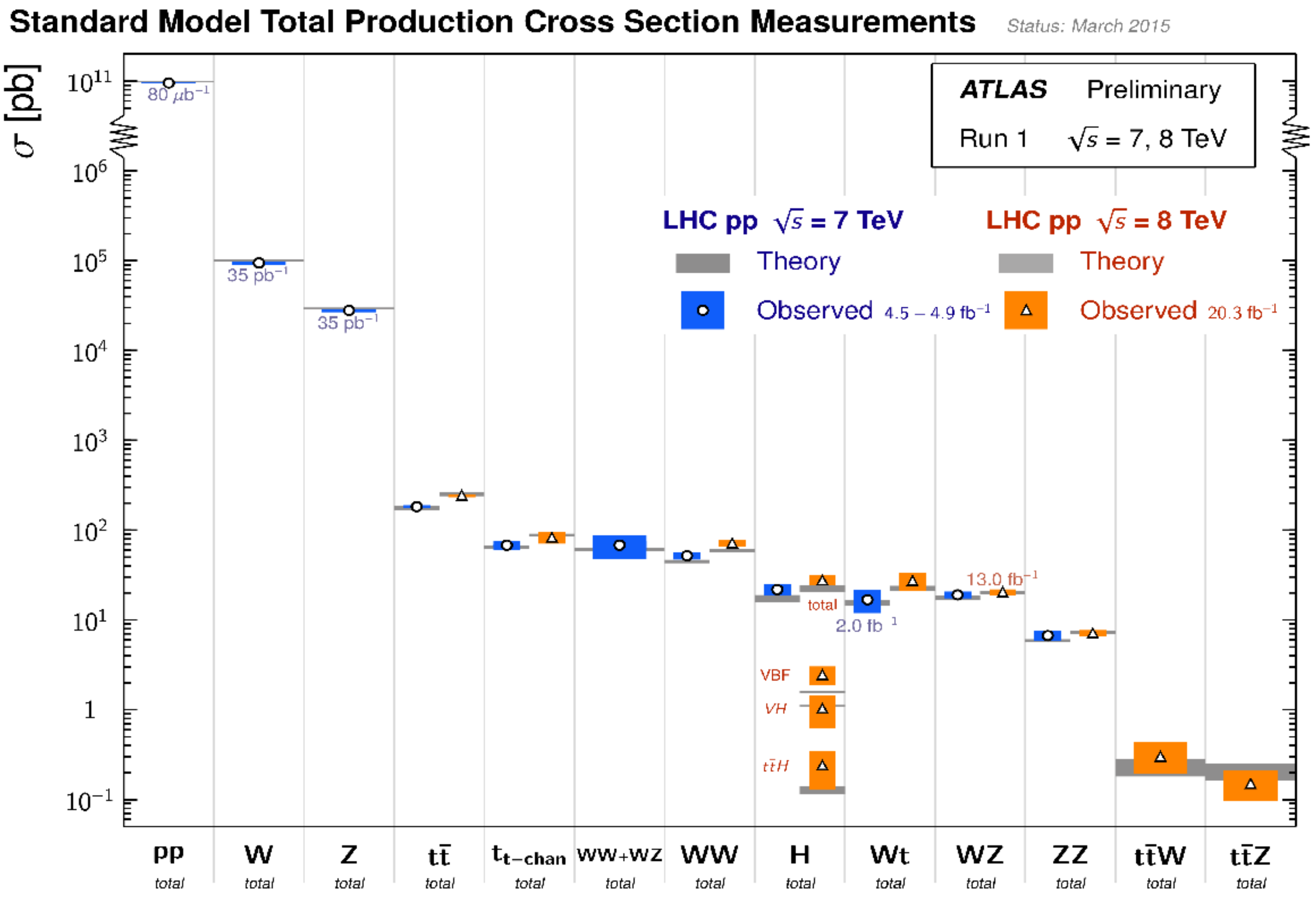
Based on work done in collaboration with:

- *Iso-spin asymmetry of quark distributions and implications for single top-quark production at the LHC*
  S. Alekhin, J. Blümlein, S. M. and R. Plačakytė `arXiv:1508.07923`
- *Determination of Strange Sea Quark Distributions from Fixed-target and Collider Data*
  S. Alekhin, J. Blümlein, L. Caminada, K. Lipka, K. Lohwasser, S. M., R. Petti, and R. Plačakytė `arXiv:1404.6469`
- *The ABM parton distributions tuned to LHC data*
  S. Alekhin, J. Blümlein and S. M. `arXiv:1310.3059`
- Many more papers of ABM and friends . . .
  `2008 - ...`

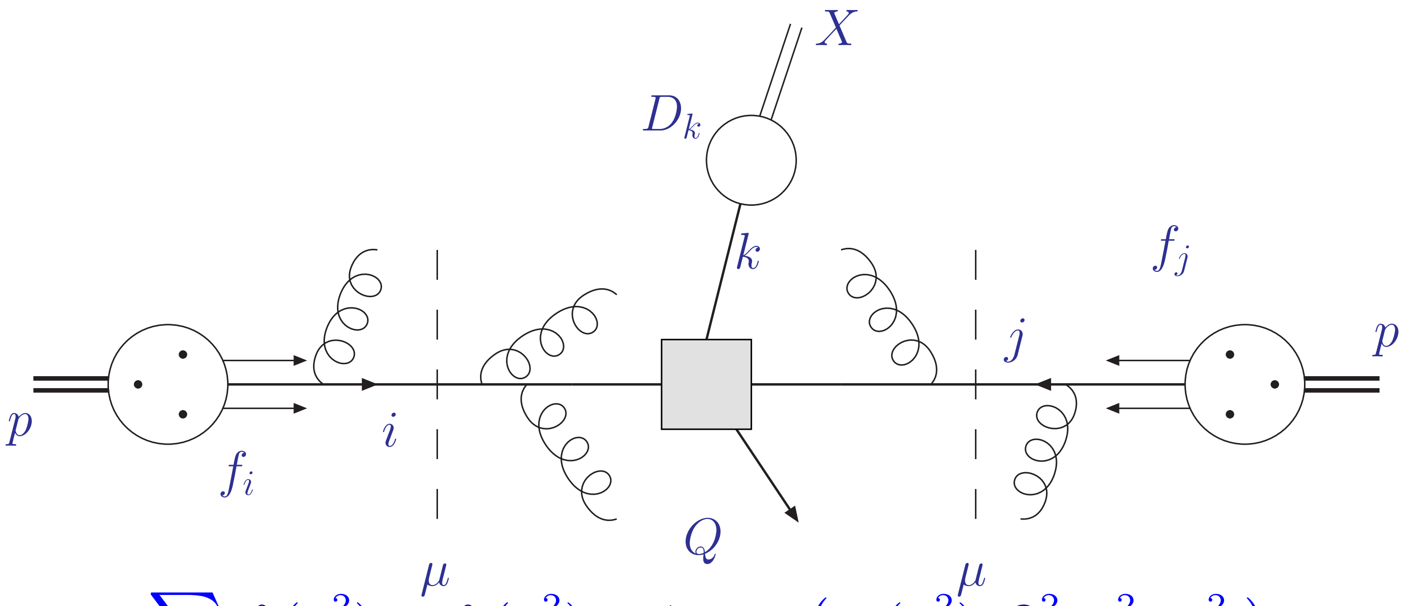
Standard Model cross sections

Cross sections for Standard Model processes at the LHC

- All dominant channels for Higgs boson production observed Atlas coll. '15



QCD factorization



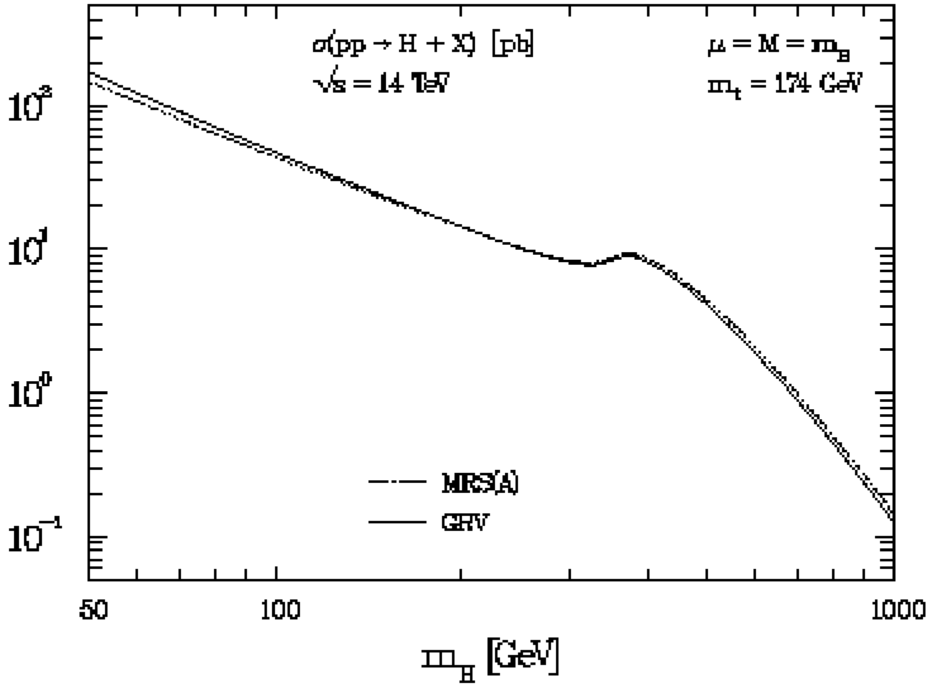
$$\sigma_{pp\to X} = \sum_{ij} f_i(\mu^2) \otimes f_j(\mu^2) \otimes \hat{\sigma}_{ij\to X}\left(\alpha_s(\mu^2), Q^2, \mu^2, m_X^2\right)$$

- Factorization at scale μ
  - separation of sensitivity to dynamics from long and short distances
- Hard parton cross section $\hat{\sigma}_{ij\to X}$ calculable in perturbation theory
  - cross section $\hat{\sigma}_{ij\to k}$ for parton types i, j and hadronic final state X
- Non-perturbative parameters: parton distribution functions f_i, strong coupling α_s, particle masses m_X
  - known from global fits to exp. data, lattice computations, . . .

Higgs boson production

Higgs cross section (1995)

NLO QCD corrections



MRS(A): Martin, Roberts and Stirling, Phys. Rev. D50 (1994) 6734

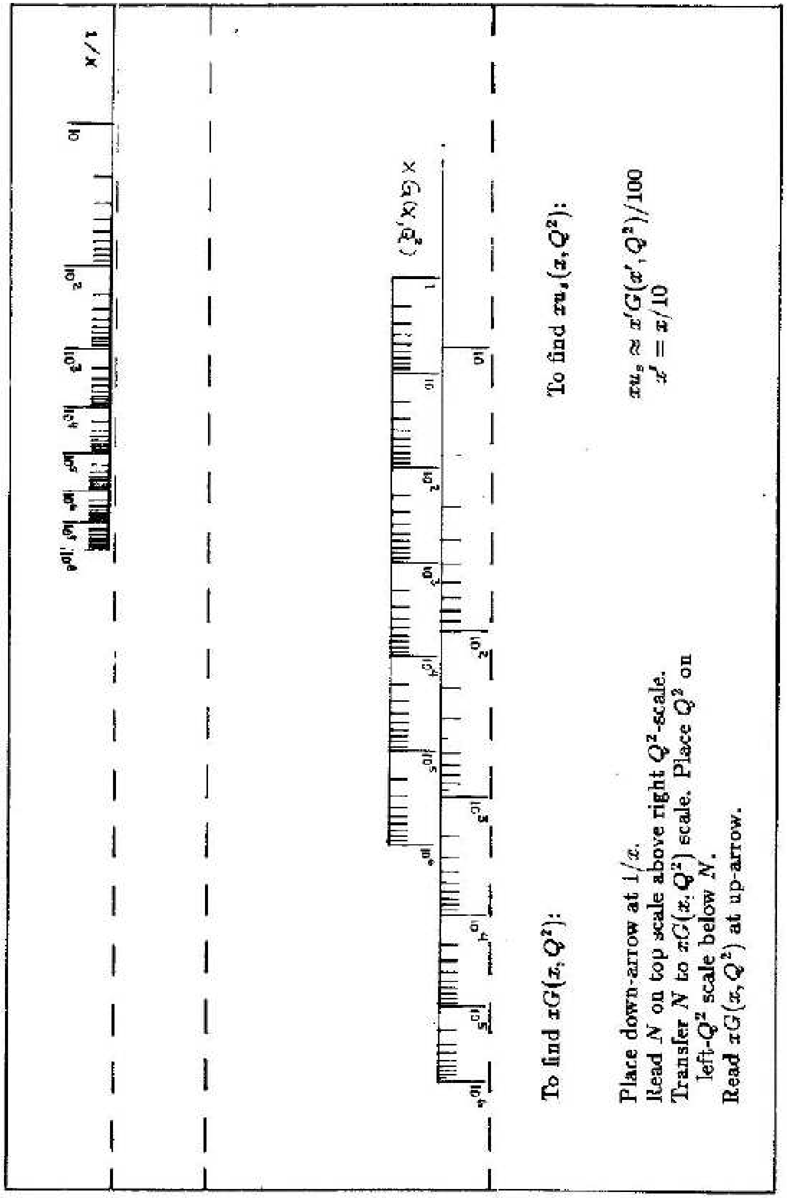
GRV: Glück, Reya and Vogt, Z. Phys. C53 (1992) 127

One of the main uncertainties *in the prediction of the Higgs production cross section is due to the* **gluon density**. *[. . .] Adopting a set of representative parton distributions [. . .], we find a* **variation of about 7%** *between the maximum and minimum values of the cross section for Higgs masses above ∼ 100 GeV.*

Spira, Djouadi, Graudenz, Zerwas (1995)
hep-ph/9504378

J. Ralston (Phys.Lett. B172 (1986) 430)

Pocket partonometer



To find $xu_s(x,Q^2)$:

$xu_s \approx x'G(x',Q^2)/100$
$x' = x/10$

To find $xG(x,Q^2)$:

Place down-arrow at $1/x$.
Read N on top scale above right Q^2-scale.
Transfer N to $xG(x,Q^2)$ scale. Place Q^2 on left-Q^2 scale below N.
Read $xG(x,Q^2)$ at up-arrow.

for t- or heavier particle distributions one must model thresholds numerically such as done in ref. [4] [‡4]. However, departures from a symmetrically distributed sea, which complicate the boundary conditions, can be reproduced by the ratios $u_s \approx d_s \approx s_s \approx 2c_s \approx 2b_s$.

The analytic gluon solution (3), boundary conditions included, is calculated by the partonometer (fig. 2). The scales automate the logarithms of certain functions of $1/x$ and Q^2 left to the reader. In systematic testing the accuracy of the gizmo is at the 10–20% level depending on the operator's ability to read logarithmic scales. It is much better than interpolating between graphs such as fig. 1a. The speed is even faster than adding a new card [‡4] to an existing program that runs.

Gluon distributions are read off directly; see the example below. Quark sea distributions can be evaluated using the identity

$$xu_s(x,Q^2) = (2/b)\partial xG(x,Q^2)/\partial y, \qquad (7)$$

and evaluating the derivative numerically. But wait! To minimize reading errors, one finds that the derivative above and the normalization change are roughly represented by

$$xu_s(x,Q^2) \approx x'G(x',Q^2)/100, \quad x' = x/10. \qquad (8)$$

This estimate is actually quite close to the re-scaled $xu_s(x,Q^2)$ of ref. [5] and is not too bad a match to

---

[‡4] Private communication with well known phenomenologist.

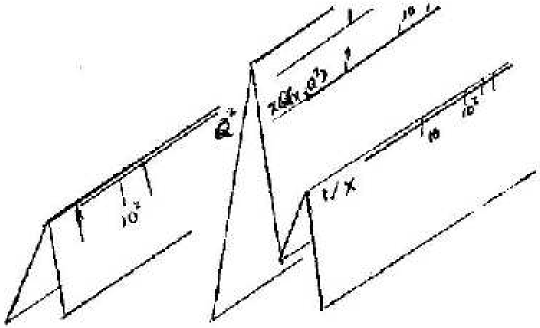
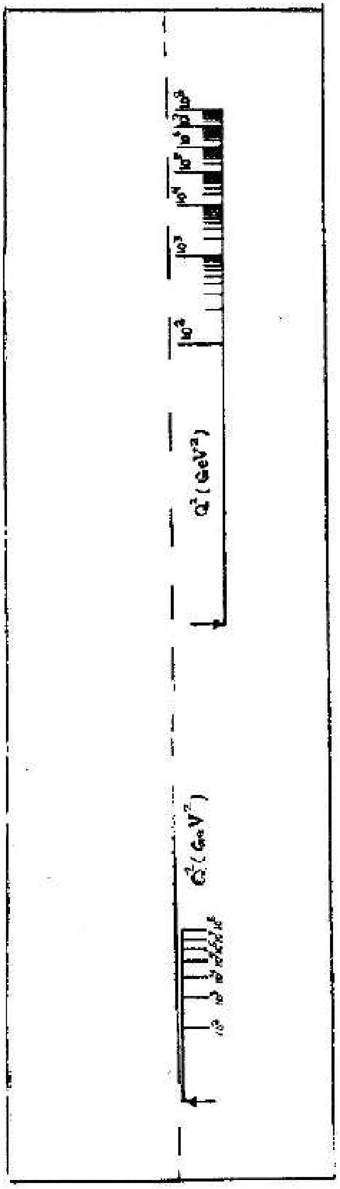
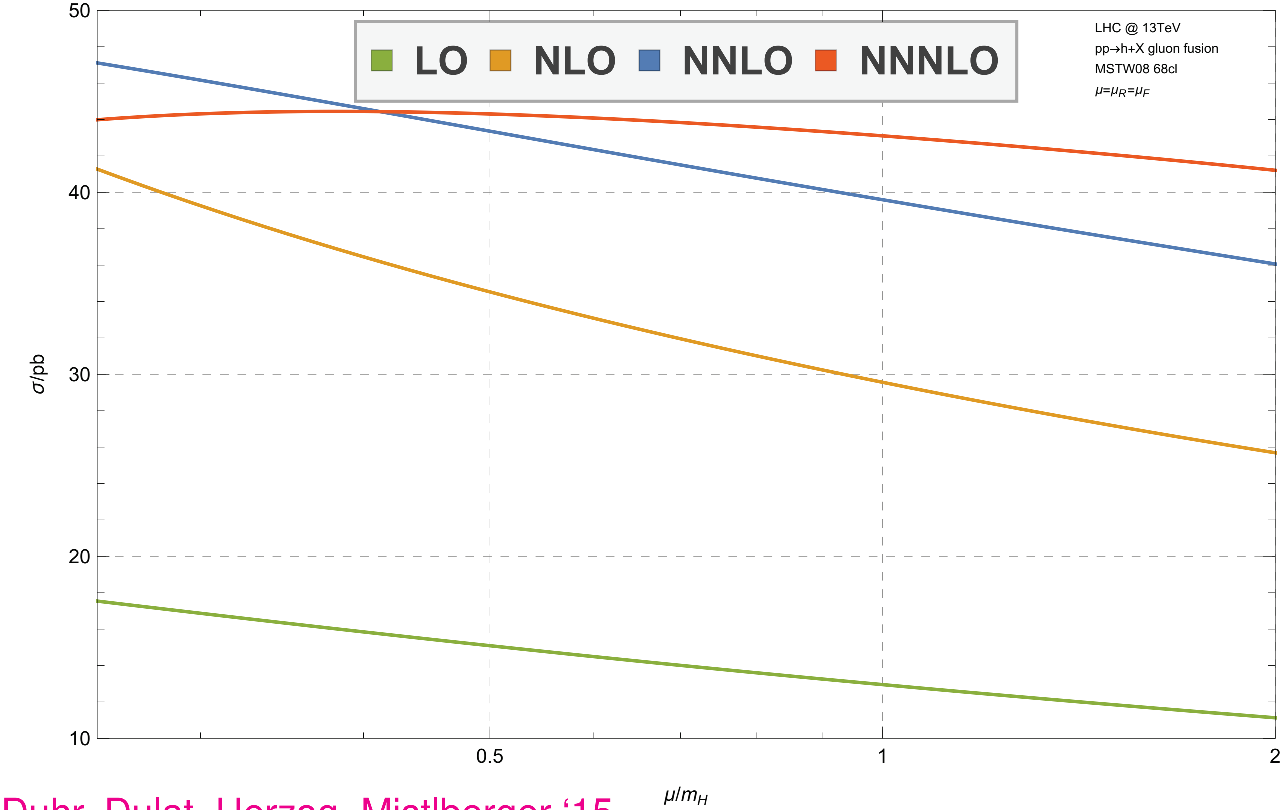


Fig. 2. The partonometer. To assemble: cut on solid lines, fold on dotted lines.



Higgs cross section (2015)

Exact N^{3}LO QCD corrections



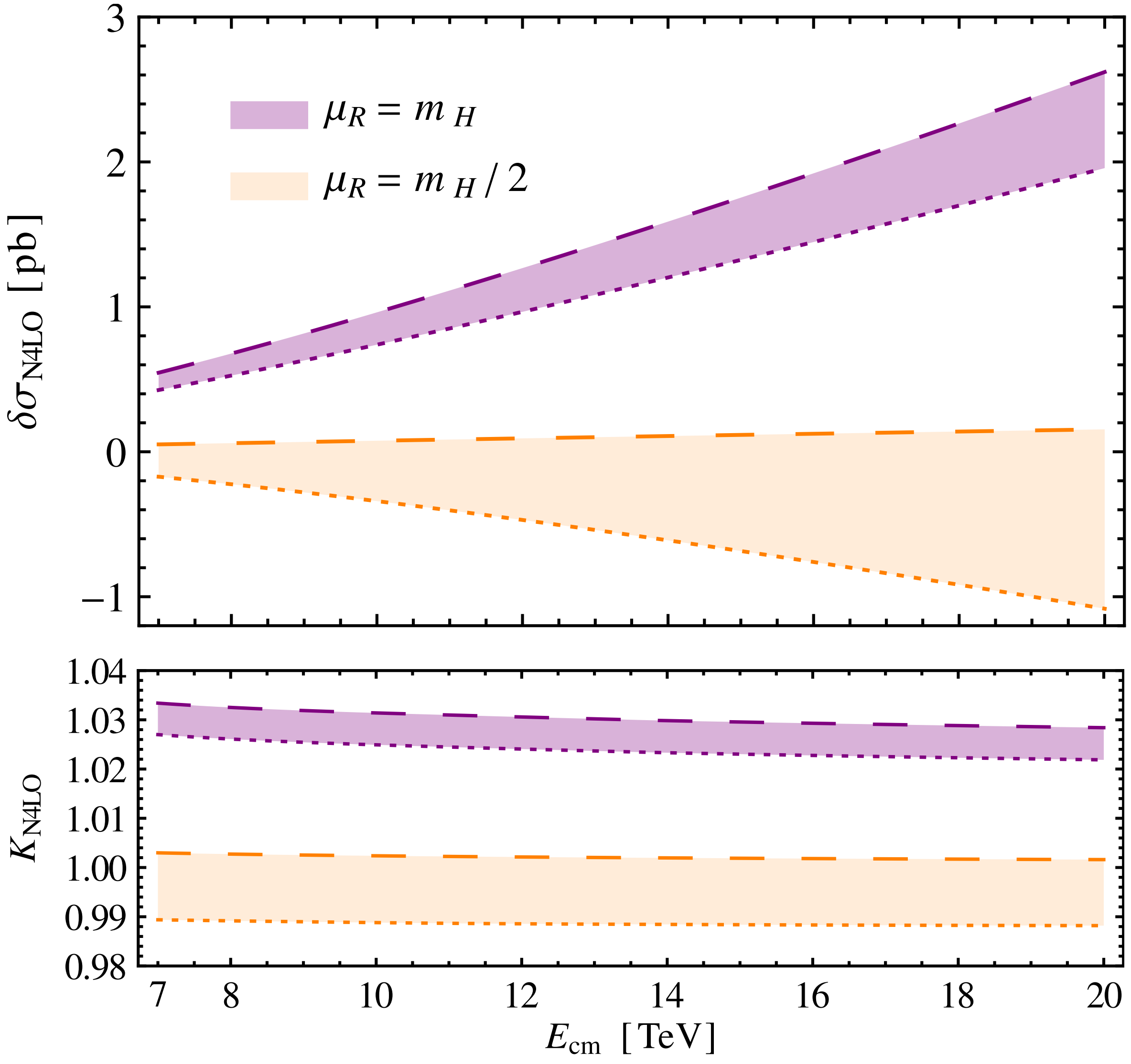
Anastasiou, Duhr, Dulat, Herzog, Mistlberger ‘15

- Apparent convergence of perturbative expansion
- Scale dependence of exact N^{3}LO prediction with residual uncertainty 3%
- Minimal sensitivity at scale $\mu = m_H/2$

Approximate N^{4}LO QCD corrections

- Consistency check with approximate N^{4}LO corrections at two scales $\mu = m_H$ and $\mu = m_H/2$
- K-factor $\simeq 1\%$ for $\mu = m_H/2$ with at $\sqrt{s} = 13$ TeV

de Florian, Mazzitelli, S.M., Vogt '14



Parton content of the proton

Parton distributions

PDF dependence of cross section

- Cross section $\sigma(H)$ at NNLO with uncertainties: $\sigma(H) + \Delta\sigma(\mathrm{PDF} + \alpha_{\rm s})$ at $\sqrt{s} = 13$ TeV for $m_H = 125.0$ GeV
- Comparison for PDF sets at NNLO

ABM12 Alekhin, Blümlein, S.M. '13	39.80 ± 0.84
CT14 Dulat et al. '15	$42.33\ ^{+\ 1.43}_{-\ 1.68}$
MMHT14 Martin, Motylinski, Harland-Lang, Thorne '14	$42.36\ ^{+\ 0.56}_{-\ 0.78}$
NNPDF3.0 Ball et al. '14	42.59 ± 0.80

- Large spread for predictions from different PDFs $\sigma(H) = 39.8 \ldots 42.6$
- PDF and α_s differences between sets amount to up to 7%
    - significantly larger than residual theory uncertainty due to N^{3}LO QCD corrections

How to explain the differences ?

Data in global PDF fits

Data sets considered in ABM12 analysis

- Analysis of world data for deep-inelastic scattering and fixed-target data for Drell-Yan process
  - inclusive DIS data HERA, BCDMS, NMC, SLAC $(NDP = 2699)$
  - semi-inclusive DIS charm production data HERA $(NDP = 52)$
  - Drell-Yan data (fixed target) E-605, E-866 $(NDP = 158)$
  - neutrino-nucleon DIS (di-muon data) CCFR/NuTeV $(NDP = 178)$
  - LHC data for $W^{\pm}$- and Z-boson production ATLAS, CMS, LHCb $(NDP = 60)$

Iterative cycle of PDF fits

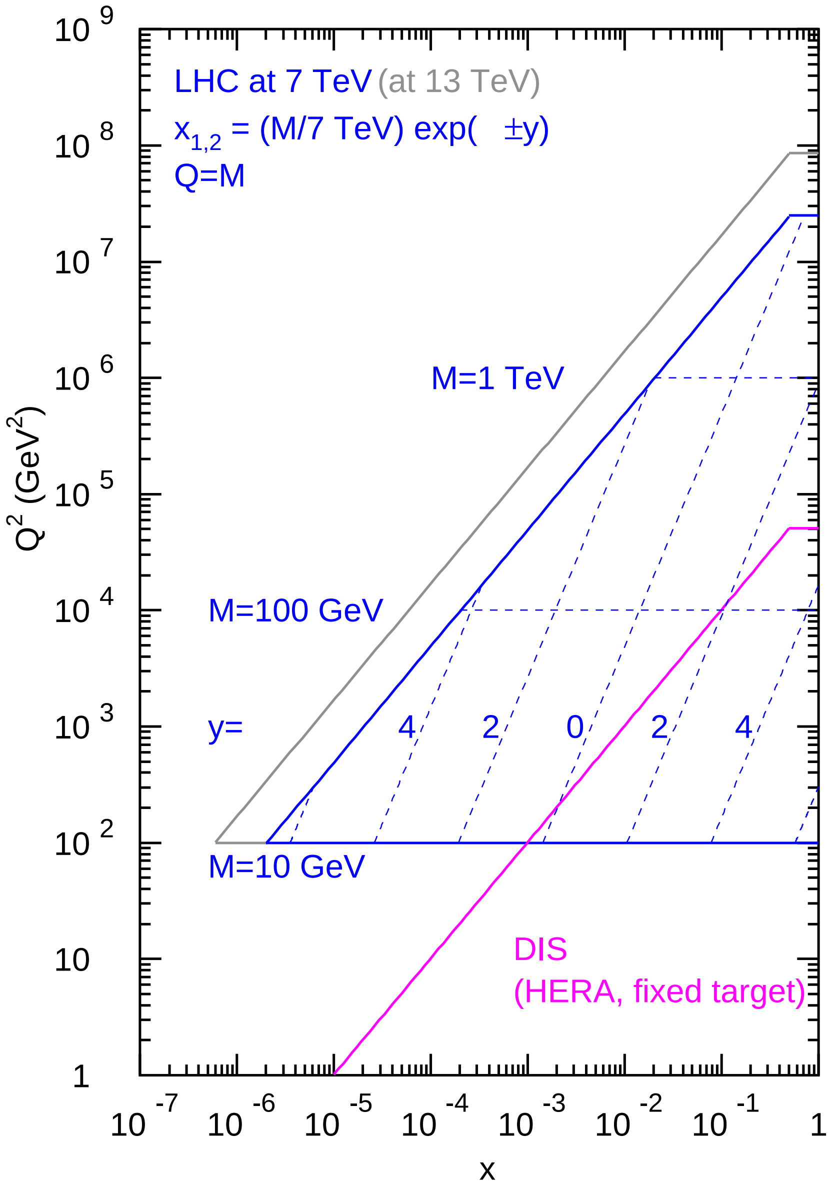
i) check of compatibility of new data set with available world data

ii) study of potential constraints due to addition of new data set to fit

iii) perform high precision measurement of the non-perturbative parameters
  - parton distributions
  - strong coupling $\alpha_s(M_Z)$
  - heavy quark masses

Parton kinematics at LHC

- Information on proton structure depends on kinematic coverage



- LHC run at $\sqrt{s}$ = 7/8 TeV
  - parton kinematics well covered by HERA and fixed target experiments
- Parton kinematics with $x_{1,2} = M/\sqrt{S} e^{\pm y}$
  - forward rapidities sensitive to small-x
- Cross section depends on convolution of parton distributions
  - small-x part of f_i and large-x PDFs f_j

$$\sigma_{pp\to X} = \sum_{ij} f_i(\mu^2) \otimes f_j(\mu^2) \otimes \left[\ldots\right]$$

Theory considerations in PDF fits

Theory considerations

- Strictly NNLO QCD for determination of PDFs and α_s
- Consistent scheme for treatment of heavy quarks
  - $\overline{\mathrm{MS}}$-scheme for quark masses and α_s
  - fixed-flavor number scheme for $n_f = 3, 4, 5$
- Consistent theory description for consistent data sets
  - low scale DIS data with account of higher twist
- Full account of error correlations

Interplay with perturbation theory

- Accuracy of determination driven by precision of theory predictions
- Non-perturbative parameters sensitive to
  - radiative corrections at higher orders
  - chosen scheme (e.g. ($\overline{MS}$ scheme)
  - renormalization and factorization scales μ_R, μ_F
  - . . .

Benchmark measurements

DIS

- Structure functions for neutral and charged current known to $\mathcal{O}(\alpha_s^3)$
  S.M, Vermaseren, Vogt '04–'08
  - F_2, F_3, known N^{3}LO, F_L known NNLO
- Heavy-quark structure functions
  - asymptotic NNLO terms at large $Q^2 \gg m^2$ Bierenbaum, Blümlein, Klein '09; Behring, Bierenbaum, Blümlein, De Freitas, Klein, Wissbrock '14
  - approximate NNLO expressions for neutral and charged current
    Lo Presti, Kawamura, S.M., Vogt '12, Blümlein, A. Hasselhuhn, and T. Pfoh '14

LHC

- Complete NNLO QCD corrections available for
  - $W^\pm$- and Z-boson production
    Hamberg, van Neerven, Matsuura '91; Harlander, Kilgore '02
  - hadro-production of top-quark pairs Czakon, Fiedler, Mitov '13
  - single top-quark production (t-channel) Brucherseifer, Caola, Melnikov '14
- Hadroproduction of jets only known at NLO
  - ongoing effort towards NNLO (corrections expected to be as big as $\mathcal{O}(15-20\%)$) Gehrmann-De Ridder, Gehrmann, Glover, Pires '13

ABM PDF ansatz

- PDFs parameterization at scale $\mu = 3 \mathrm{GeV}$ in scheme with $n_f = 3$
  Alekhin, Blümlein, S.M. '12
  - ansatz for valence-/sea-quarks, gluon with polynomial $P(x)$
  - strange quark is taken in charge-symmetric form
  - 24 parameters in polynomials $P(x)$
  - 4 additional fit parameters: $\alpha_s^{(n_f=3)}(\mu = 3$ GeV), m_c, m_b and deuteron correction
  - simultaneous fit of higher twist parameters (twist-4)

$$
\begin{aligned}
xq_v(x,Q_0^2) &= \frac{2\delta_{qu} + \delta_{qd}}{N_q^v} x^{a_q} (1-x)^{b_q} x^{P_{qv}(x)} \\
xu_s(x,Q_0^2) = x\bar{u}_s(x,Q_0^2) &= A_{us} x^{a_{us}} (1-x)^{b_{us}} x^{a_{us}\, P_{us}(x)} \\
x\Delta(x,Q_0^2) = xd_s(x,Q_0^2) - xu_s(x,Q_0^2) &= A_\Delta x^{a_\Delta} (1-x)^{b_\Delta} x^{P_\Delta(x)} \\
xs(x,Q_0^2) = x\bar{s}(x,Q_0^2) &= A_s x^{a_s} (1-x)^{b_s} \,, \\
xg(x,Q_0^2) &= A_g x^{a_g} (1-x)^{b_g} x^{a_g\, P_g(x)}
\end{aligned}
$$

- Ansatz provides sufficient flexibility; no additional terms required to improve the quality of fit

Quality of fit

Statistical tests

- Goodness-of-fit estimator
  - χ^2 values compared to number of data points (typically a few thousand in global fit)

Covariance matrix

- Positive-definite covariance matrix
  - correlations for ABM11 PDF fit parameters (I)

    Alekhin, Blümlein, S.M. '12

	a_u	b_u	$\gamma_{1,u}$	$\gamma_{2,u}$	a_d	b_d	A_d	b_Δ	A_u	a_{us}	b_{us}	a_G	b_G
a_u	1.0000	0.9256	0.9638	-0.2527	0.3382	0.2922	0.1143	-0.4267	0.4706	0.3117	0.1422	0.0982	0.1127
b_u		1.0000	0.9574	-0.5608	0.1933	0.1200	0.1058	-0.3666	0.3712	0.2674	0.1537	0.0453	0.1878
$\gamma_{1,u}$			1.0000	-0.4504	0.2328	0.2329	0.0906	-0.3379	0.4106	0.2876	0.0812	0.0491	0.1627
$\gamma_{2,u}$				1.0000	0.3007	0.3119	-0.0242	-0.0118	0.0587	0.0026	-0.0305	0.0949	-0.1876
a_d					1.0000	0.8349	-0.2010	-0.3371	0.3786	0.2592	0.1212	-0.0377	0.1305
b_d						1.0000	-0.2669	-0.0599	0.2768	0.1941	-0.0698	-0.0926	0.2088
A_d							1.0000	-0.2132	0.0549	0.0245	0.2498	-0.0523	0.0614
b_Δ								1.0000	-0.1308	-0.0729	-0.7208	-0.0124	-0.0225
A_u									1.0000	0.9240	-0.0723	0.3649	-0.1674
a_{us}										1.0000	-0.0144	0.2520	-0.1095
b_{us}											1.0000	-0.1274	0.1808
a_G												1.0000	-0.6477
b_G													1.0000

Quality of fit

Statistical tests

- Goodness-of-fit estimator
  - χ^2 values compared to number of data points (typically a few thousand in global fit)

Covariance matrix

- Positive-definite covariance matrix
  - correlations for ABM11 PDF fit parameters (II)

    Alekhin, Blümlein, S.M. '12

	$\gamma_{1,G}$	$\alpha_s(3,3\text{ GeV})$	$\gamma_{1,\Delta}$	$\gamma_{1,us}$	$\gamma_{1,d}$	$\gamma_{2,d}$	A_s	b_s	a_s	a_Δ	m_c	m_b
a_u	-0.0727	-0.0611	0.3383	0.6154	0.2320	-0.0724	-0.0681	-0.0763	-0.0935	0.0026	0.0900	-0.0053
b_u	-0.1130	-0.1725	0.2992	0.4848	0.0849	0.0720	-0.0723	-0.0618	-0.0926	0.0049	0.0349	-0.0118
$\gamma_{1,u}$	-0.1106	-0.1338	0.2753	0.5638	0.1316	-0.0535	-0.0798	-0.0854	-0.1059	-0.0060	0.0817	0.0003
$\gamma_{2,u}$	0.1174	0.2195	-0.0210	0.0822	0.3712	-0.3310	0.0339	0.0143	0.0381	-0.0098	0.0430	-0.0004
a_d	-0.1631	-0.0208	0.0319	0.4974	0.9570	-0.4636	-0.0700	-0.0996	-0.0979	-0.2121	0.1066	-0.0150
b_d	-0.2198	-0.0913	-0.1775	0.4092	0.8985	-0.8498	-0.0533	-0.0669	-0.0806	-0.2252	0.0822	-0.0068
A_d	-0.0825	0.0188	0.8558	-0.0289	-0.2624	0.2852	-0.0075	-0.0189	-0.0180	0.9602	0.0420	0.0120
b_Δ	0.0530	-0.0801	-0.6666	-0.0904	-0.1981	-0.2532	-0.0022	0.0257	0.0048	-0.0260	-0.0166	-0.0056
A_u	0.2502	-0.0157	0.1265	0.7525	0.3047	-0.0668	-0.7064	-0.6670	-0.7267	0.0345	0.2137	0.0358
a_{us}	0.1845	-0.0216	0.0683	0.5714	0.2157	-0.0554	-0.8768	-0.8081	-0.8980	0.0145	0.0430	0.0074
b_{us}	-0.1619	-0.0715	0.5343	-0.3656	0.0293	0.2430	-0.0345	-0.0132	-0.0356	0.1527	-0.0899	-0.0058
a_G	0.8291	0.2306	-0.0260	0.3692	-0.0966	0.1496	0.0087	0.0007	0.0464	-0.0541	-0.0661	0.0417
b_G	-0.9184	-0.6145	0.0538	-0.2770	0.1990	-0.2552	0.0381	0.0616	-0.0468	0.0502	0.1847	0.0861

Quality of fit

Statistical tests

- Goodness-of-fit estimator
  - χ^2 values compared to number of data points (typically a few thousand in global fit)

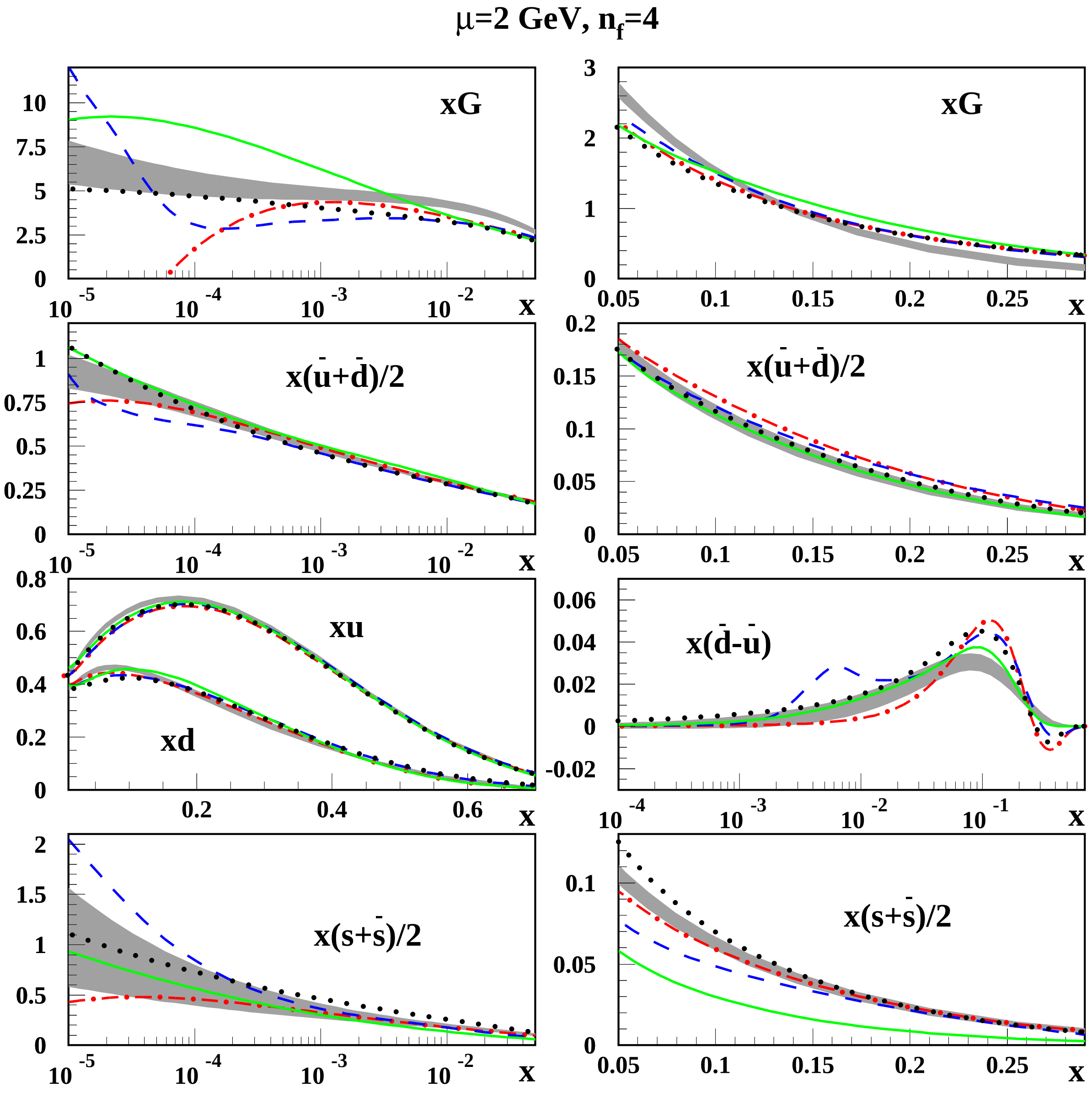
Covariance matrix

- Positive-definite covariance matrix
  - correlations for ABM11 PDF fit parameters (III)

    Alekhin, Blümlein, S.M. ‘12

	$\gamma_{1,G}$	$\alpha_s(3,3\ \text{GeV})$	$\gamma_{1,\Delta}$	$\gamma_{1,us}$	$\gamma_{1,d}$	$\gamma_{2,d}$	A_s	b_s	a_s	a_Δ	m_c	m_b
$\gamma_{1,G}$	1.0000	0.3546	-0.0876	0.2751	-0.2215	0.2410	-0.0539	-0.0634	0.0122	-0.0658	-0.1149	-0.0474
$\alpha_s(3,3\ \text{GeV})$		1.0000	0.0601	0.1127	-0.0761	0.1534	-0.0176	-0.0121	0.0883	0.0022	-0.5641	-0.0526
$\gamma_{1,\Delta}$			1.0000	0.0699	-0.1081	0.3796	-0.0050	-0.0329	-0.0175	0.7098	0.0418	0.0113
$\gamma_{1,us}$				1.0000	0.4099	-0.1547	-0.2622	-0.3181	-0.2801	-0.0785	0.1870	0.0103
$\gamma_{1,d}$					1.0000	-0.6540	-0.0688	-0.0892	-0.0974	-0.2332	0.0999	-0.0093
$\gamma_{2,d}$						1.0000	0.0212	0.0128	0.0413	0.1876	-0.0396	-0.0049
A_s							1.0000	0.8584	0.9689	-0.0109	0.0596	0.0116
b_s								1.0000	0.8826	-0.0173	-0.0777	0.0003
a_s									1.0000	-0.0204	-0.0845	-0.0145
a_Δ										1.0000	0.0385	0.0085
m_c											1.0000	0.1451
m_b												1.0000

Parton distributions tuned to LHC data



- 1σ band for ABM12 PDFs (NNLO, 4-flavors) at $\mu = 2\ \mathrm{GeV}$

  Alekhin, Blümlein, S.M.'13
- comparison with:

  JR09 (solid lines),

  MSTW (dashed dots),

  NN23 (dashes) and

  CT10 (dots)
- Some interesting observations to be made . . .

Heavy quarks in deep-inelastic scattering

Treatment of heavy-quarks

Light quarks

- Neglect “light quark” masses $m_u, m_d \ll \Lambda_{QCD}$ and $m_s < \Lambda_{QCD}$ in hard scattering process
  - scale-dependent u, d, s, g PDFs from mass singularities

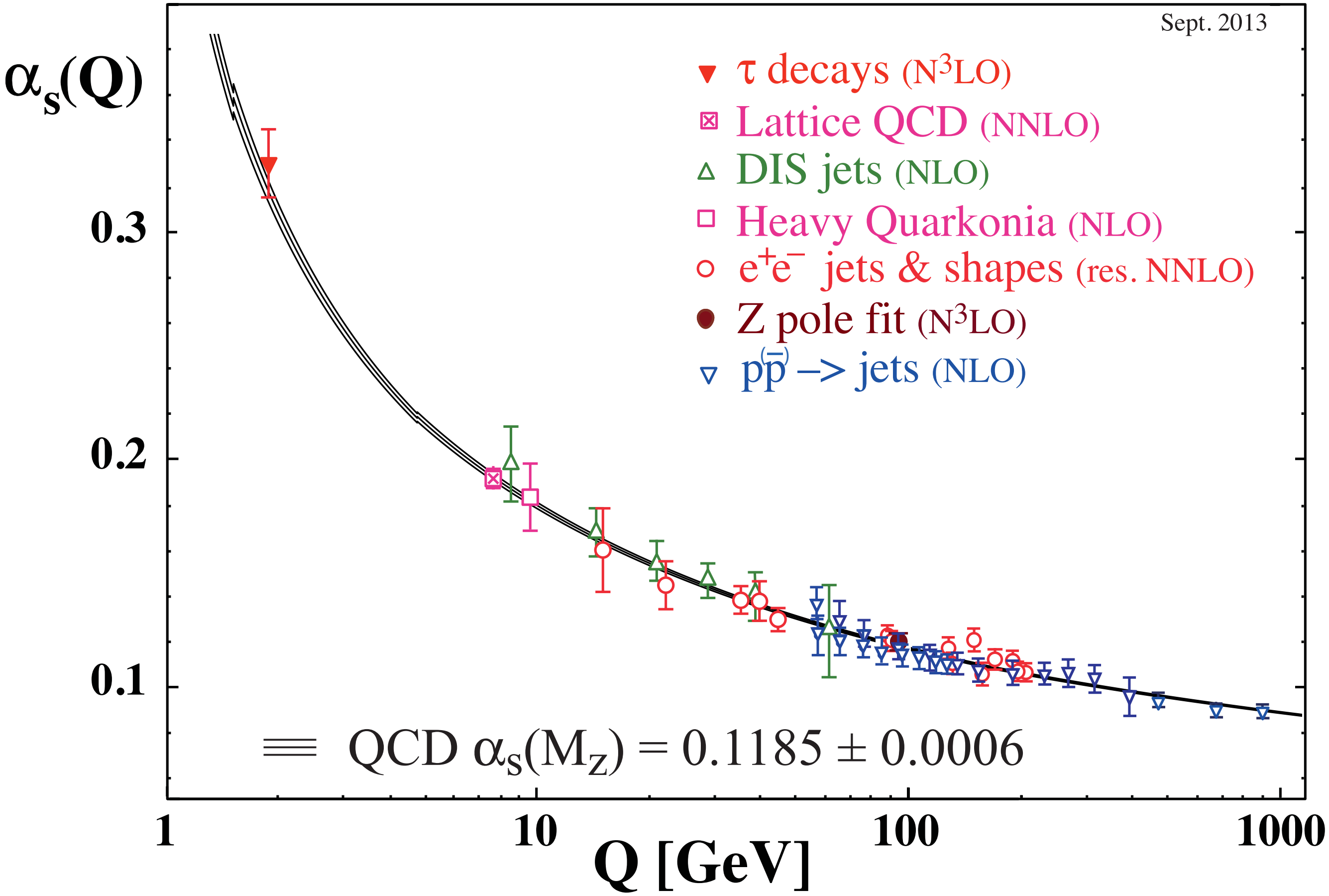
Heavy quarks

- No mass singularities for $m_c, m_b, m_t \gg \Lambda_{QCD}$, no (evolving) PDFs
  - c and b PDFs for $Q \ggg m_c, m_b$ generated perturbatively
  - matching of two distinct theories
    $\longrightarrow n_f$ light flavors + heavy quark of mass m at low scales
    $\longrightarrow n_f + 1$ light flavors at high scales

Strong coupling with flavor thresholds

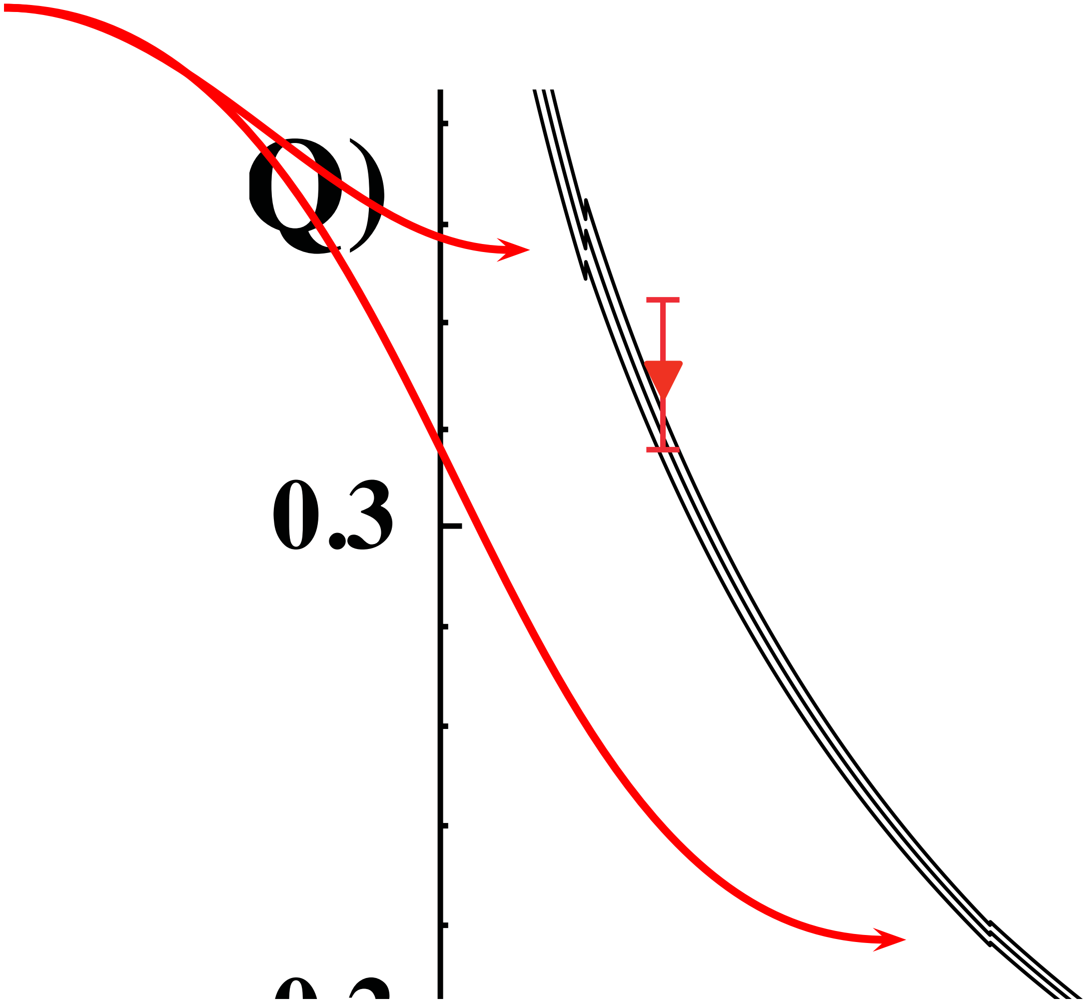
- Solution of QCD β-function for $\alpha_s^{n_l} \longrightarrow \alpha_s^{(n_l+n_h)}$
  - discontinuities for $n_f = 3 \rightarrow n_f = 4 \rightarrow n_f = 5$
- Big picture

Bethke for PDG 2014



Strong coupling with flavor thresholds

- Solution of QCD β-function for $\alpha_s^{n_l} \longrightarrow \alpha_s^{(n_l+n_h)}$
  - discontinuities for $n_f = 3 \to n_f = 4 \to n_f = 5$
- Zoom



Treatment of heavy-quarks

Charm structure function

- F_2^c at HERA (assume no “intrinsic charm”)
  - $Q \not\gg m_c$: Fixed flavor-number scheme FFNS
    u, d, s, g partons and massive charm coeff. fcts.
  - $Q \ggg m_c$: Zero-mass variable flavor-number scheme ZM-VFNS
    terms $m_c/Q \to 0$, $n_f = 4$ PDFs (matching), $m_c = 0$ coeff. fcts.
  - $Q \gg m_c$: General-mass variable flavor-number scheme GM-VFNS
    terms $m_c/Q \neq 0$, but quasi-collinear logs $\ln(Q/m_c)$ large
    $n_f = 4$ PDFs, “interpolating” coeff. fcts. (matching prescriptions)

FFNS

- Perturbative QCD predictions for F_2^c and F_L^c (neutral current)
  - complete NLO predictions Laenen, Riemersma, Smith, van Neerven '92
  - approximate expressions to NNLO
    Lo Presti, Kawamura, S.M., Vogt '12
  - asymptotic NNLO terms at large $Q^2 \gg m^2$ Bierenbaum, Blümlein, Klein '09;
    Behring, Bierenbaum, Blümlein, De Freitas, Klein, Wissbrock '14

VFNS

- Variable flavor number schemes $\longrightarrow$ matching of two distinct theories
  Aivazis, Collins, Olness, Tung '94; Thorne, Roberts '98;
  Buza, Matiounine, Smith, van Neerven '98
  $\longrightarrow$ n_f light flavors + heavy quark of mass m at low scales
  $\longrightarrow$ $n_f + 1$ light flavors at high scales
- Important aspects of variable flavor number schemes
  - mass factorization to be carried out before resummation
    $\longrightarrow$ mass factorization involves both heavy and light component of structure function
  - matching conditions required through NNLO
    Chuvakin, Smith, van Neerven '00
- Details of implementation matter in global fits

VFNS implementation

- GM-VFNS implementation using BSMN

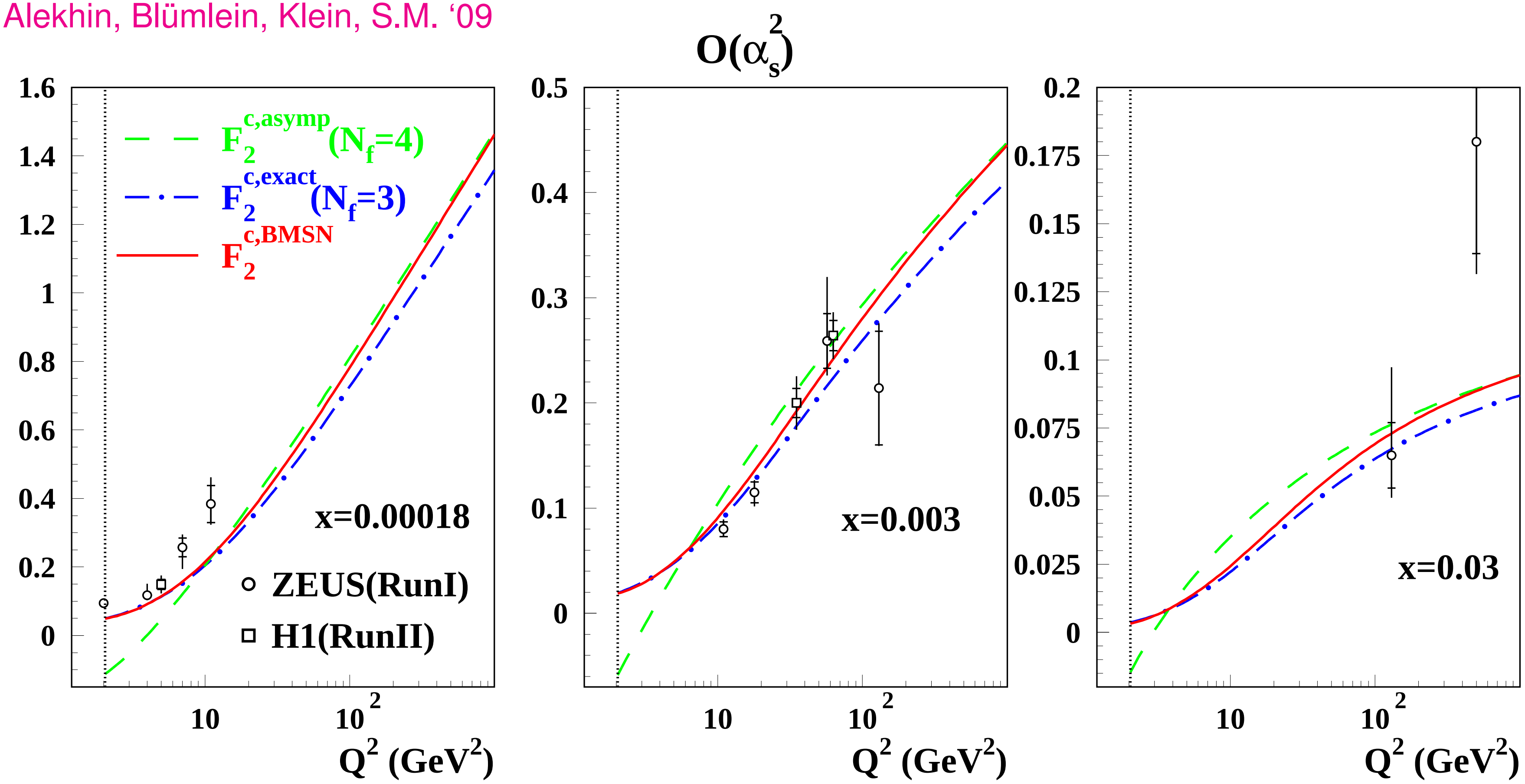
  Buza, Matiounine, Smith, van Neerven ‘98
- DIS structure function F_2^h for heavy-quark h

$$F_2^{h,\mathrm{BMSN}}(N_f+1,x,Q^2) =$$

$$= \quad F_2^{h,\mathrm{exact}}(N_f,x,Q^2) + F_2^{h,\mathrm{ZMVFN}}(N_f+1,x,Q^2) - F_2^{h,\mathrm{asymp}}(N_f,x,Q^2)$$

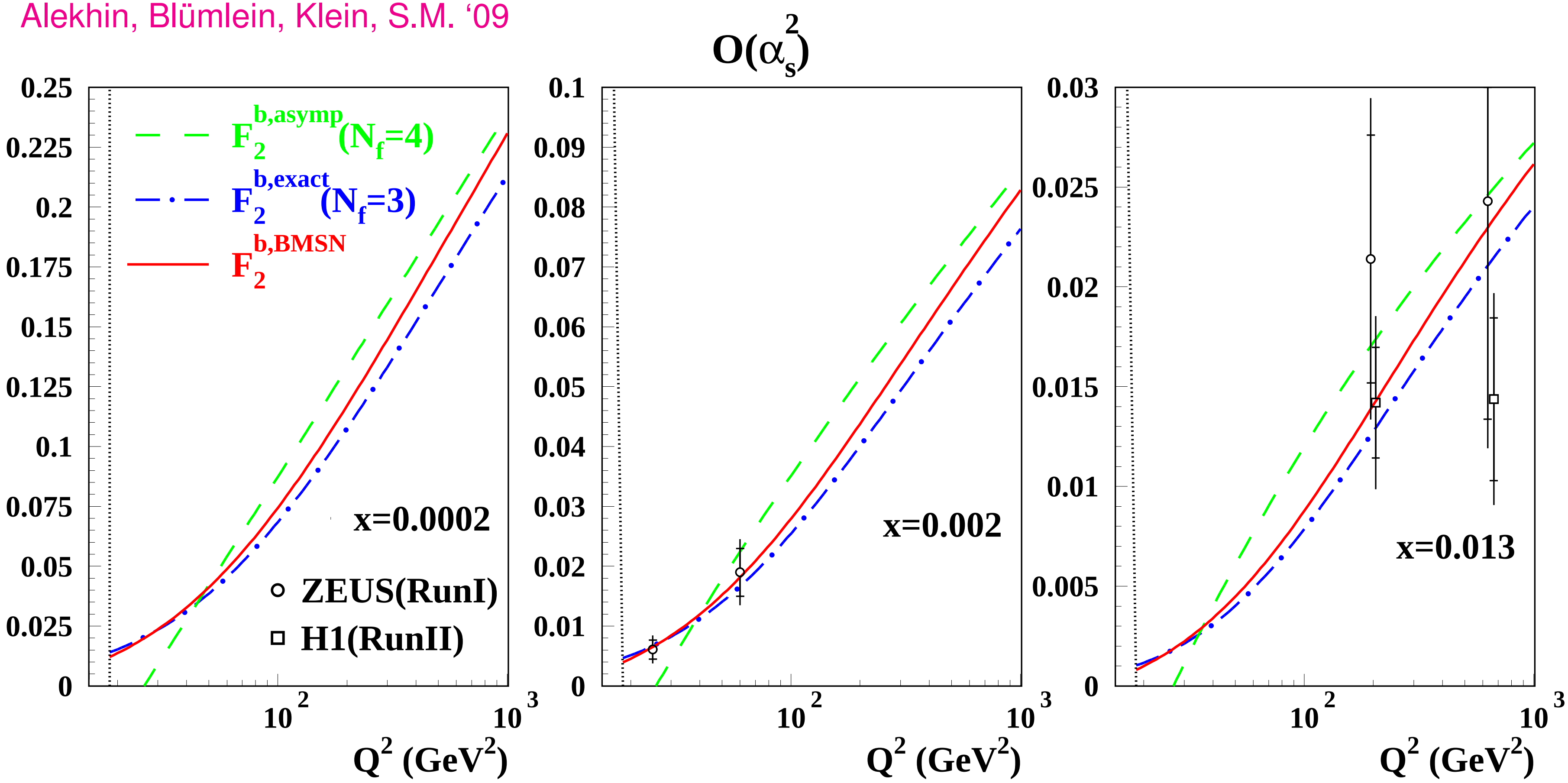
  - $F_2^{h,\mathrm{exact}}$: massive heavy-quark structure function ($m \neq 0$)
  - $F_2^{h,\mathrm{ZMVFN}}$: DIS structure function with zero mass ($m = 0$)
  - $F_2^{h,\mathrm{asymp}}$: asymptotic expansion of heavy-quark structure function (logarithms $\ln(Q^2/m^2)$)

Alekhin, Blümlein, Klein, S.M. ‘09



- F_2^c in different schemes compared to H1- and ZEUS-data
  - GMVFN scheme in BMSN prescription (solid lines)
  - 3-flavor scheme (dash-dotted lines)
  - 4-flavor scheme (dashed lines)
  - charm-quark mass $m_c = 1.43\ \mathrm{GeV}$ (vertical dotted line)

Alekhin, Blümlein, Klein, S.M. ‘09



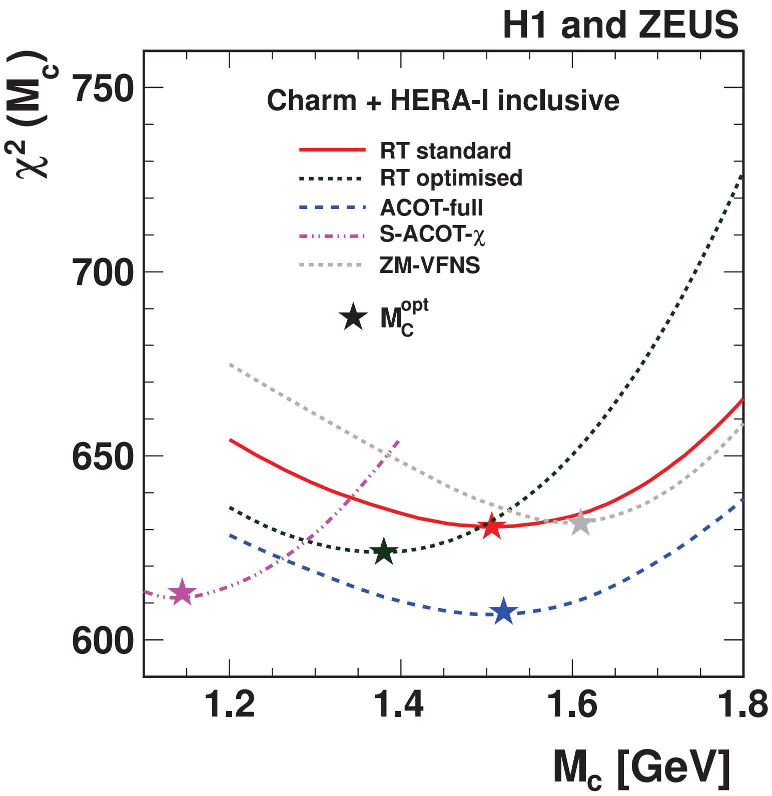
- F_2^b in different schemes compared to H1- and ZEUS-data
  - GMVFN scheme in BMSN prescription (solid lines)
  - 3-flavor scheme (dash-dotted lines)
  - 4-flavor scheme (dashed lines)
  - bottom-quark mass $m_b = 4.30\ \mathrm{GeV}$ (vertical dotted line)

Charm quark mass vs. data

- Data on F_2^c at HERA has correlation of m_c, $\alpha_S(M_Z)$, gluon PDF

  $\sigma_{c\bar{c}} \sim \alpha_s\, m_c^2\, g(x)$
- Comparison of measured data with predictions in various VFNS schemes
  - data shows very good sensitivity to value of m_c
  - fit of value of m_c strongly dependent on particular choice of VFNS

    H1 coll. arxiv:1211.1182



Charm quark mass in PDF fits

- Choice of value for heavy-quark masses part of uncertainty
- PDF fits usually assume pole mass scheme for heavy-quarks
    - PDG quotes running masses:
      charm: $m_c(m_c) = 1.27^{+0.07}_{-0.11}$ GeV, bottom: $m_b(m_b) = 4.20^{+0.17}_{-0.07}$ GeV

	m_c (GeV)	m_c scheme	χ^2/NDP (HERA data)	F_2^c scheme	NNLO Wilson coeff.
ABM12 arXiv:1310.3059	$1.24^{+0.05}_{-0.03}$	$m^{\overline{MS}}$	62/52	FFNS($n_f = 3$)	**yes**
CT14 arXiv:1506.07443	1.3 (assumed)	$m^{\rm pole}$	59/47	S-ACOT-χ	**no**
MMHT arXiv:1510.02332	1.25	$m^{\rm pole}$	75/52	RT optimal	**no**
NNPDF3.0 arXiv:1410.8849	1.275 (assumed)	$m^{\rm pole}$	60/47	FONLL-C	**no**

- Charm mass m_c "tuning" parameter effects the Higgs cross section
    - linear rise in $\sigma(H) = 40.6 \dots 43.8$ for $m_c = 1.05 \dots 1.75$ with MSTW PDFs Martin, Stirling, Thorne, Watt '10

$W^{\pm}$- and Z-boson production

$W^{\pm}$- and Z-boson production

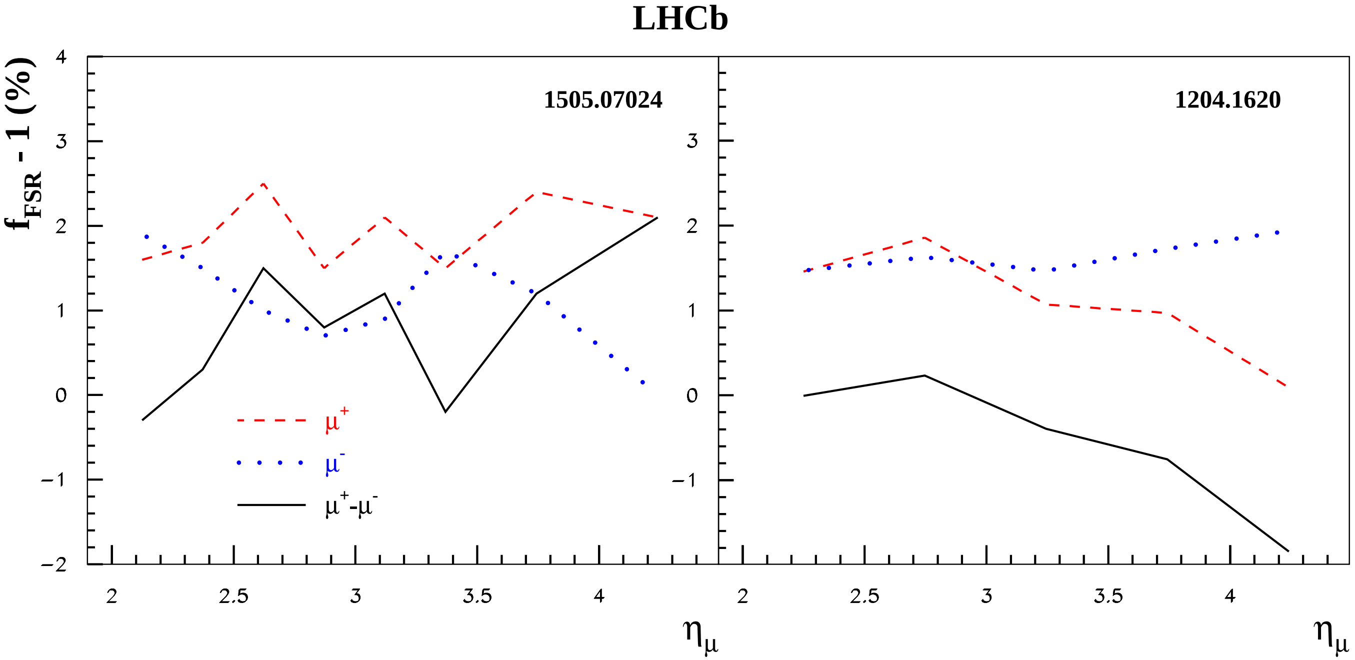
- High precision data from LHC ATLAS, CMS, LHCb and Tevatron D0
  - statistically significant $NDP = 112$
  - differential distributions extend to forward region
  - sensitivity to light quark flavors at $x \simeq 10^{-4}$

Experiment	ATLAS	CMS	D0		LHCb	
$\sqrt{s}$ (TeV)	7	7	1.96		7	8
Final states	$W^+ \to l^+\nu$ $W^- \to l^-\nu$ $Z \to l^+l^-$	$W^+ \to \mu^+\nu$ $W^- \to \mu^-\nu$	$W^+ \to \mu^+\nu$ $W^- \to \mu^-\nu$	$W^+ \to e^+\nu$ $W^- \to e^-\nu$	$W^+ \to \mu^+\nu$ $W^- \to \mu^-\nu$ $Z \to \mu^+\mu^-$	$Z \to e^+e^-$
Reference	1109.5141	1312.6283	1309.2591	1412.2862	1505.07024	1503.00963
Cut on the lepton P_T	$P_T^l > 20$ GeV	$P_T^\mu > 25$ GeV	$P_T^\mu > 25$ GeV	$P_T^e > 25$ GeV	$P_T^\mu > 25$ GeV	$P_T^e > 20$ GeV
Luminosity (1/fb)	0.035	4.7	7.3	9.7	1.	2.
NDP	30	11	10	13	31	17

Theory issues (I)

Final-state-radiation effects

- QED corrections in $W^{\pm}$- and Z-boson decays applied to data of LHCb
  - left: FSR effects from mean of simulations with Herwig++ and Pythia8 with anomalous irregularity at $\eta_\mu = 3.375$
  - right: earlier analysis of LHCb with smooth FSR corrections from PHOTOS Monte Carlo Golonka, Was '05

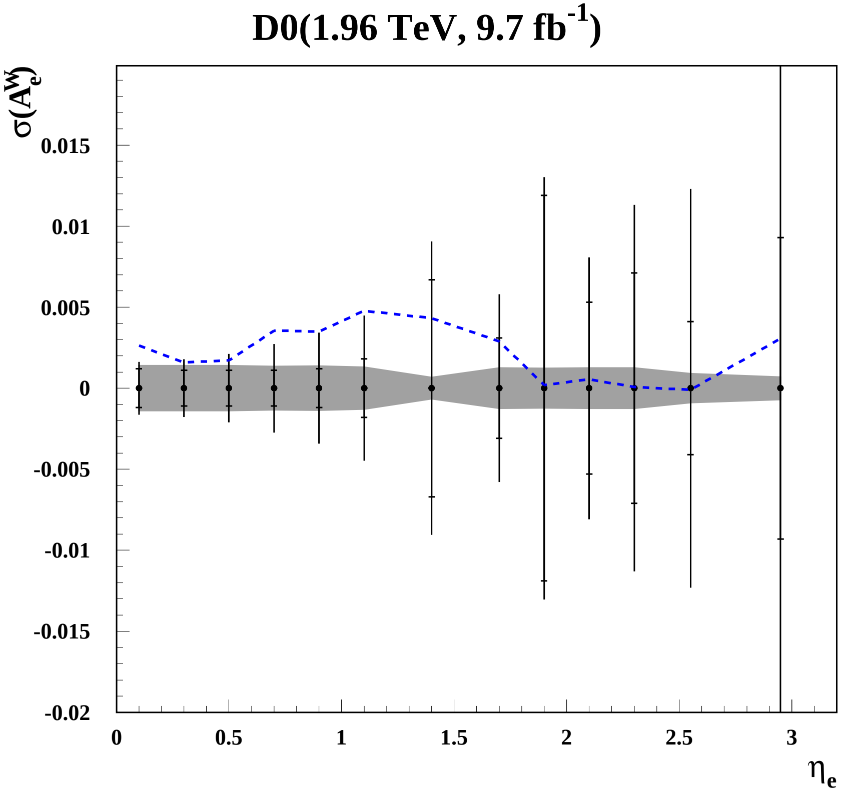


- Dropping problematic data points at $\eta_\mu = 3.375$ reduces χ^2 value by some 10 units

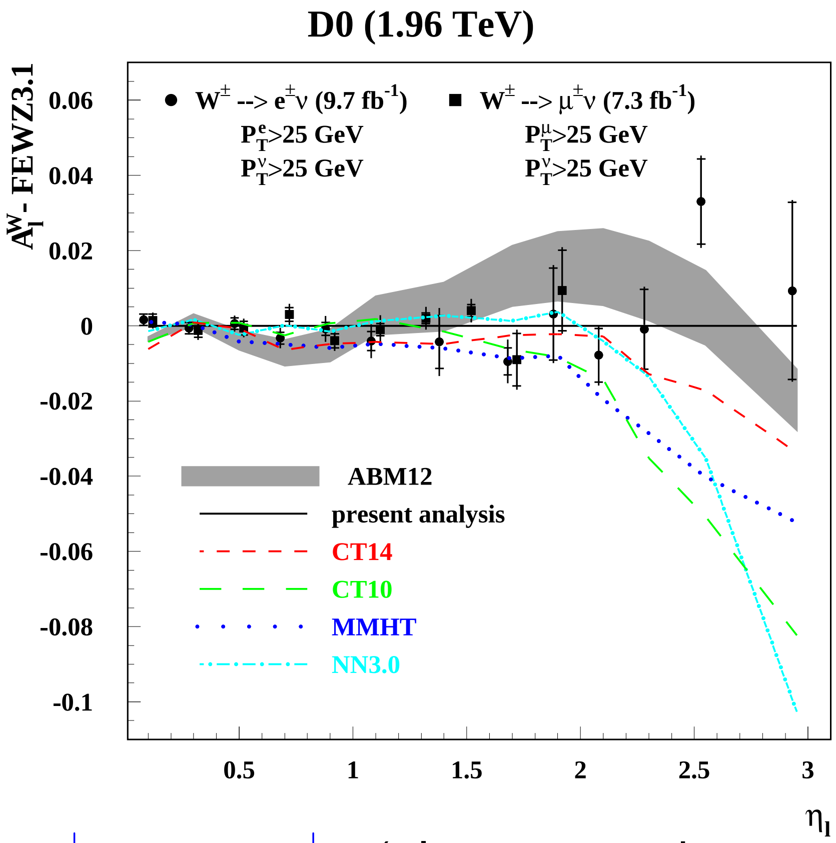
Theory issues (II)

NNLO QCD corrections

- Precision of experimental uncertainties challenges accuracy of numerical integration in QCD NNLO predictions
  - data on electron asymmetry with high precision at central rapidities D0
  - numerical accuracy of NNLO grids (shaded area) obtained with FEWZ
  - NNLO corrections in coefficient functions not uniform in η_e (dashed curve)

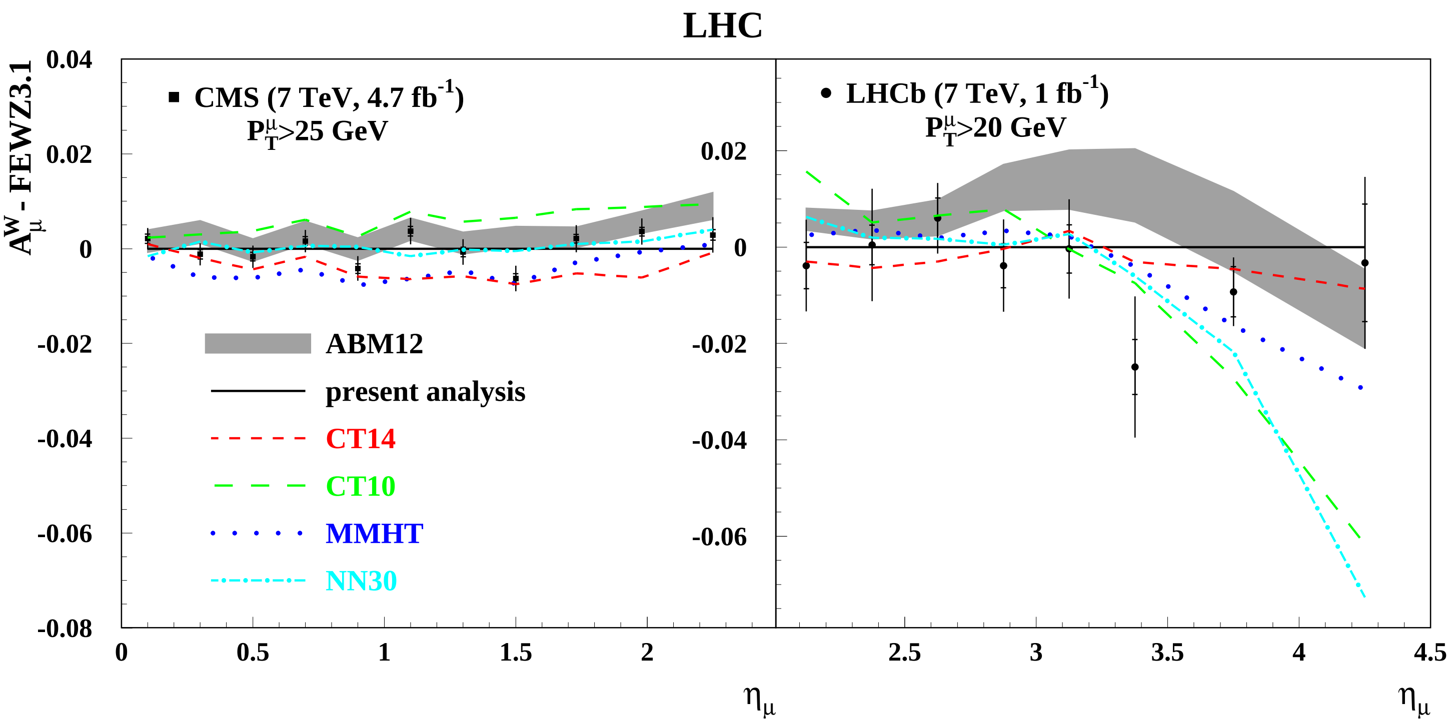


Tevatron charged lepton asymmetry



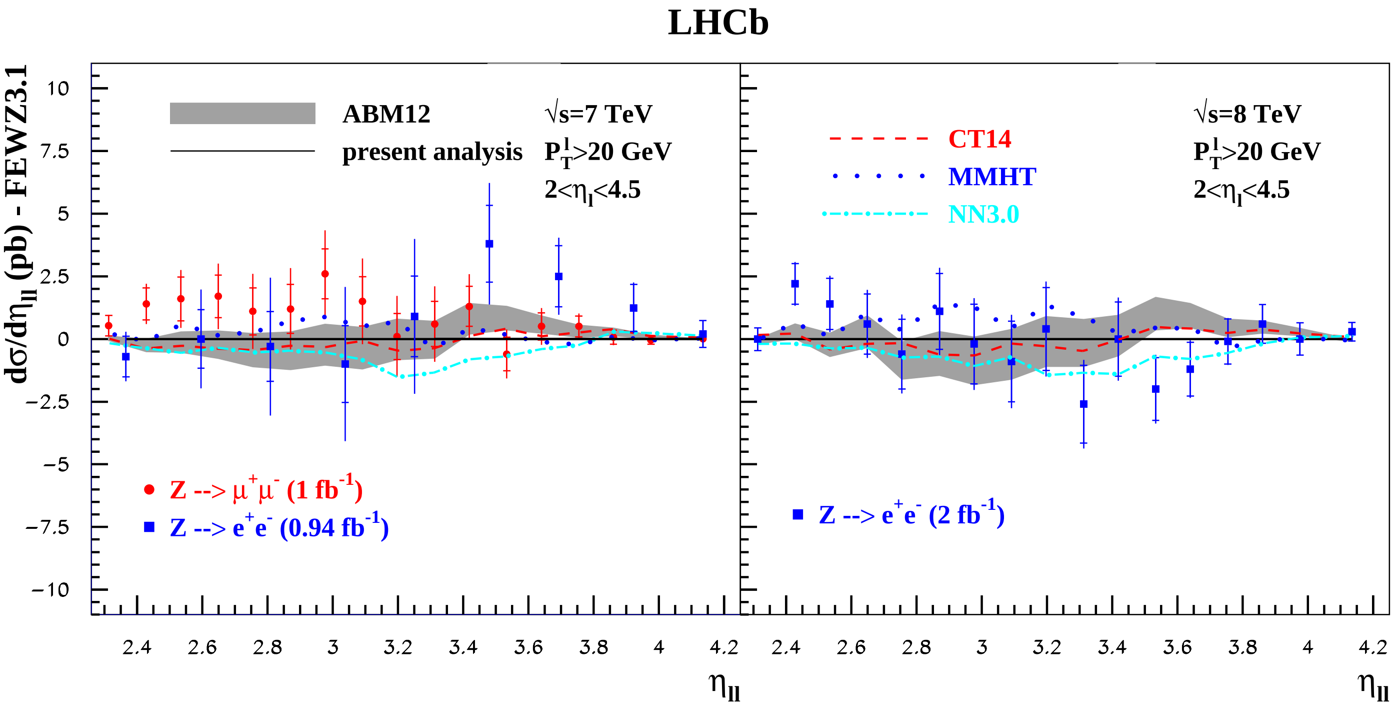
- D0 data for $p\bar{p} \to W^\pm + X \to l^\pm\nu$ (electrons and muons) at $\sqrt{s} = 1.96$ TeV
- Charged lepton asymmetry as function of pseudo-lepton rapidity η_l
- NNLO QCD predictions with FEWZ (version 3.1)
- Comparison with ABM12 (including combined PDF+α_s uncertainty), CT10, CT14, MMHT, and NN3.0

Muon charge asymmetry from LHC



- CMS and LHCb data for $pp \to W^\pm + X \to \mu^\pm\nu$ at $\sqrt{s} = 7$ TeV
- Problematic data points at $\eta_\mu = 3.375$ in LHCb data are omitted in fit

Z-boson production from LHC



- LHCb data for $pp \to Z + X \to l\bar{l}$ (muon and electron) at $\sqrt{s} = 7$ TeV and $\sqrt{s} = 8$ TeV
- Comparison with ABM12 (including combined PDF+α_s uncertainty), CT14, MMHT, and NN3.0

Fit quality

Experiment		ATLAS	CMS	D0		LHCb	
$\sqrt{s}$ (TeV)		7	7	1.96		7	8
Final states		$W^+ \to l^+\nu$ $W^- \to l^-\nu$ $Z \to l^+l^-$	$W^+ \to \mu^+\nu$ $W^- \to \mu^-\nu$	$W^+ \to \mu^+\nu$ $W^- \to \mu^-\nu$	$W^+ \to e^+\nu$ $W^- \to e^-\nu$	$W^+ \to \mu^+\nu$ $W^- \to \mu^-\nu$ $Z \to \mu^+\mu^-$	$Z \to e^+e^-$
Reference		1109.5141	1312.6283	1309.2591	1412.2862	1505.07024	1503.00963
NDP		30	11	10	13	31	17
χ^2	this work	29.8	22.5	16.9	18.0	44.1	18.2
	this work[a]	32.3	19.5(13.5[b])	13.5	9.5	34.7	19.1
	ABM12	34.5	–	–	–	–	–
	CT14	42	–[c]	–	34.7	–	–
	`HERAFitter`	–	–	13	19	–	–
	MMHT14	39	–	21	–	–	–
	NN3.0	35.4	18.9	–	–	–	–

---

[a] Variants with all collider DY and $W^\pm$-boson data excluded except the one given.
[b] Value obtained assuming systematic uncertainties to be uncorrelated.
[c] Statistically less significant data with the cut of $P_T^\mu > 35$ GeV are used.

Fit quality (zoom in)

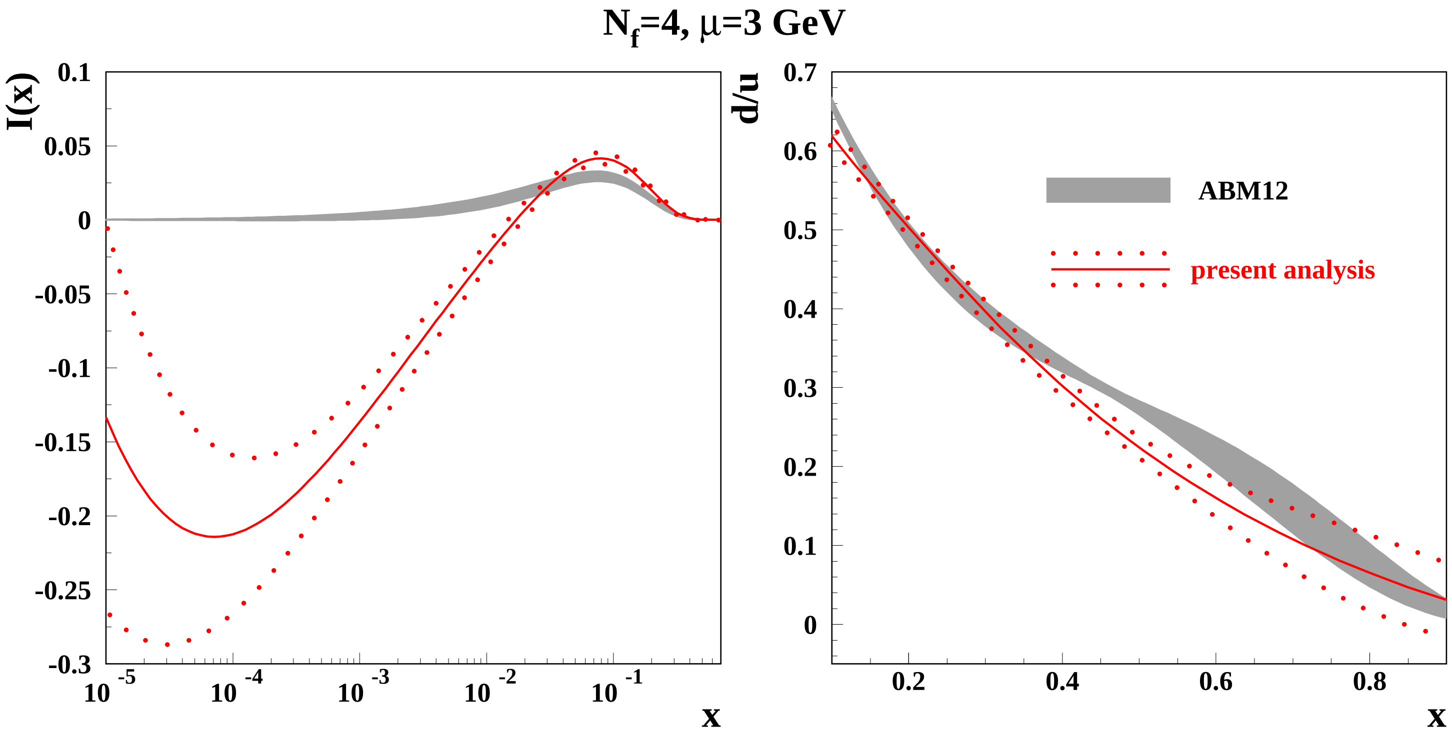
ATLAS data at $\sqrt{s} = 7$ TeV

$NDP = 30$

PDF sets	χ^2	theory accuracy	theory method
this work	32.3	**NNLO**	FEWZ3.1
ABM12	34.5	**NNLO**	FEWZ3.1 and DYNNLO
CT14	42	**(NNLL)**	ResBos
MMHT14	39	**(NNLO)**	APPLGrid, NNLO K-factors (constant)
NN3.0	35.4	**NNLO**	APPLGrid, C-factors (kinematic dependence with FEWZ3.1)

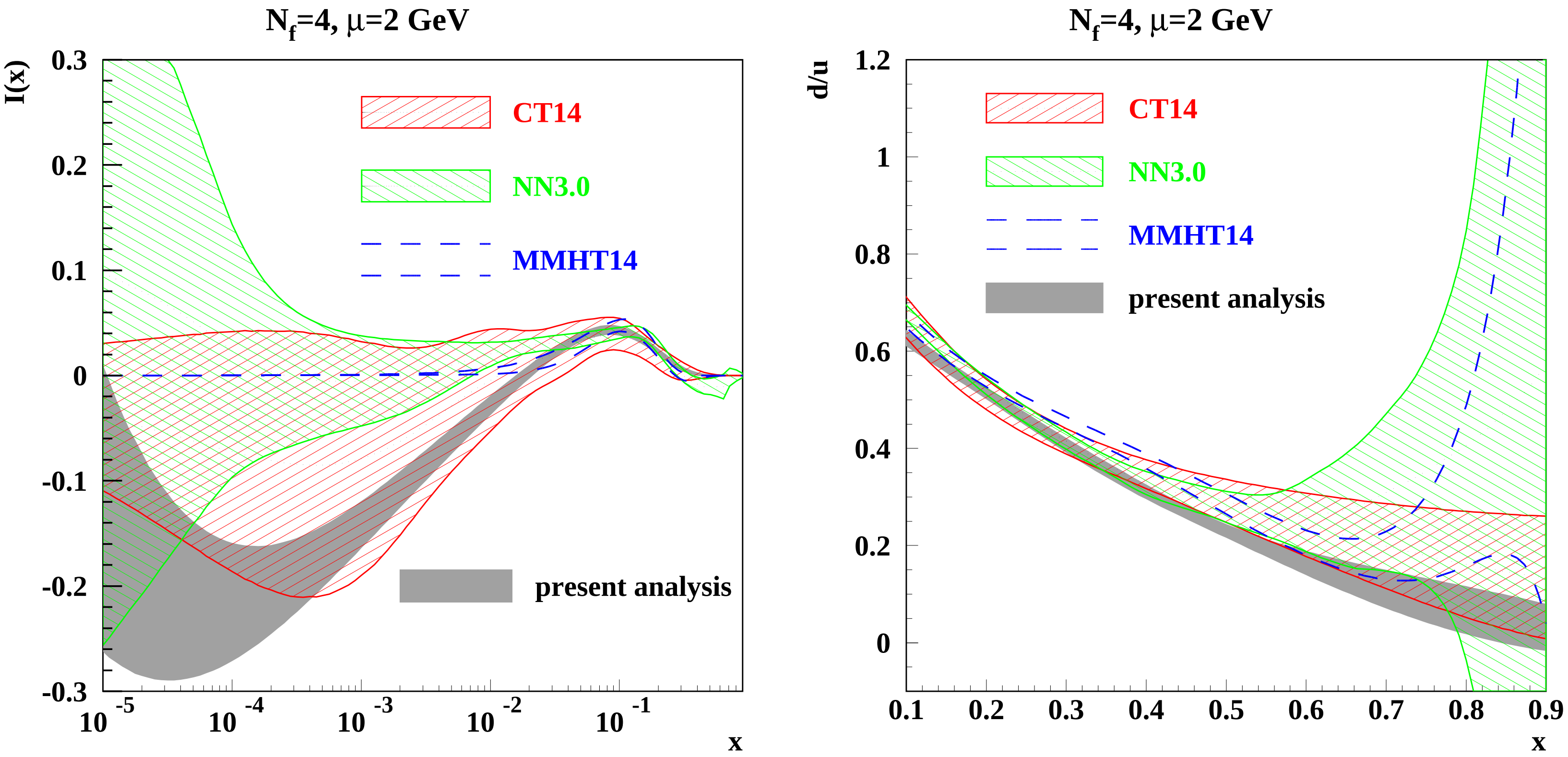
- Use of exact NNLO QCD results with full dependence on kinematics mandatory for goog fit quality
    - significantly deterioration in χ^2 values for CT14 and MMHT

Light flavor PDFs



- Light flavor decomposition not well constrained in DIS data
  - ratio d/u at large x from fixed target Drell-Yan data E-605, E-866 at the price of modelling nuclear corrections
- Iso-spin asymmetry of sea $I(x) = \bar{d} - \bar{u}$
  - Regge theory arguments for small x predict $I(x) \simeq 0$
  - $I(x)$ at small x constrained by new Tevatron and LHC data
- Upshot: non-vanishing $I(x)$ at small $x \simeq 10^{-4}$

Comparision with other PDFs

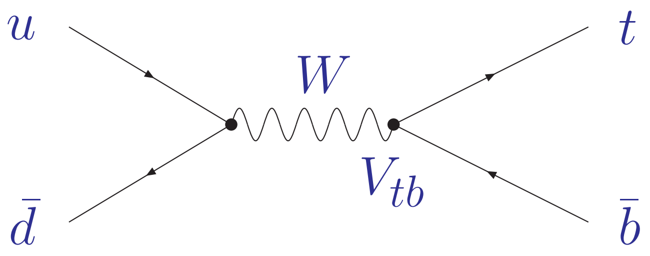


- Iso-spin asymmetry of sea $I(x)$ at small x and ratio d/u at large x with 1σ uncertainty band
- Comparison with CT14, MMHT14, NN3.0
  - CT14 finds non-vanishing $I(x)$ from fit to Tevatron charged lepton asymmetry (D0 data), but with large uncertainties

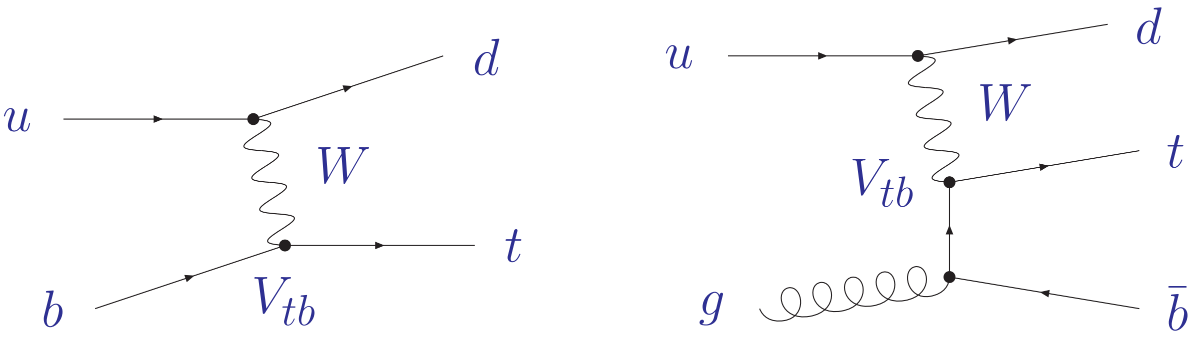
Implications for single top-quark production

Single top-quark production

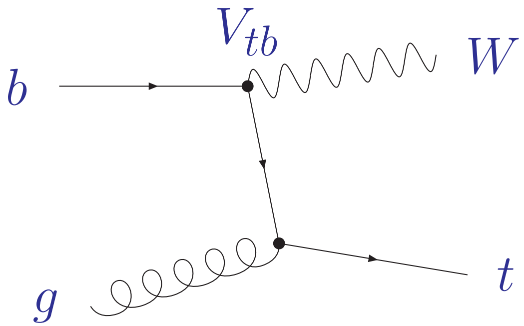
- Study of charged-current weak interaction of top quark
- s-channel production



- t-channel production
  - sensitivity to light flavor PDFs
  - bg-channel at NLO enhanced by gluon luminosity

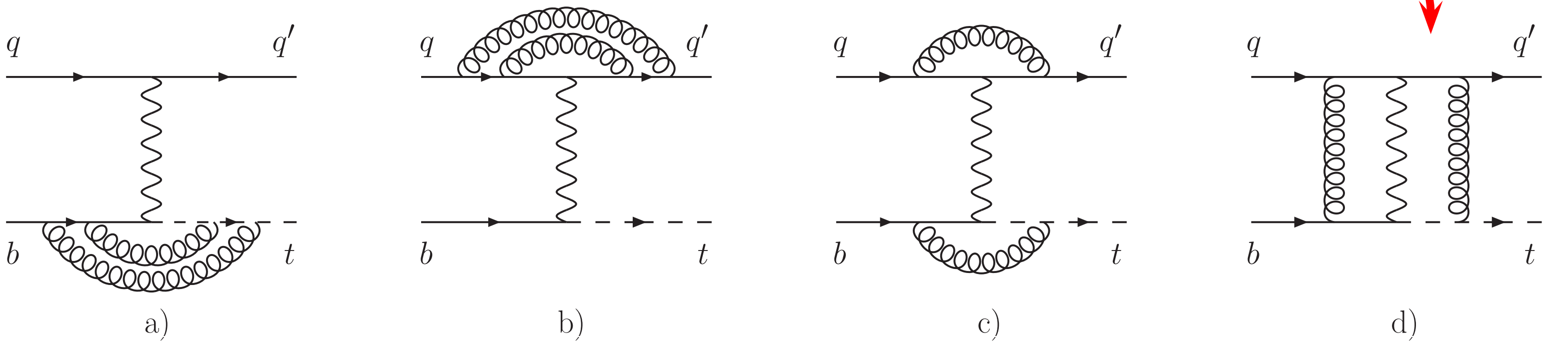


- Wt-production
  - contributes at LHC (small at Tevatron)



QCD corrections at NNLO

- Computation of NNLO QCD corrections Brucherseifer, Caola, Melnikov '14
  - fully differential, with cuts on p_T
- QCD corrections treated in structure function approach
  - non-factorizable contributions neglected (neglected diagrams $\mathcal{O}(1/N_c^2)$ supressed)

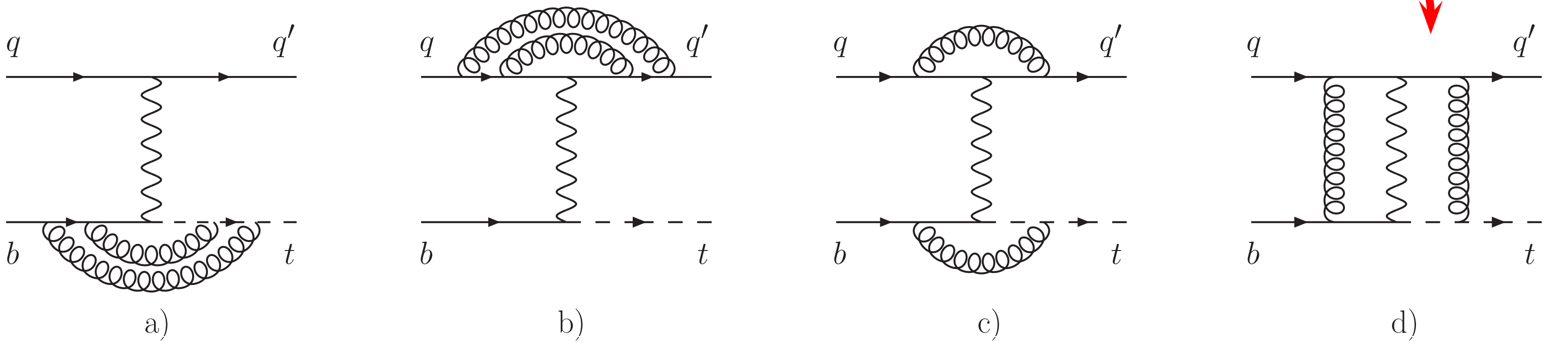


- QCD corrections to t-channel single top quark production at LHC8

$p_\perp$	σ_{LO}, pb	σ_{NLO}, pb	δ_{NLO}	σ_{NNLO}, pb	δ_{NNLO}
0 GeV	$53.8^{+3.0}_{-4.3}$	$55.1^{+1.6}_{-0.9}$	$+2.4\%$	$54.2^{+0.5}_{-0.2}$	-1.6%
20 GeV	$46.6^{+2.5}_{-3.7}$	$48.9^{+1.2}_{-0.5}$	$+4.9\%$	$48.3^{+0.3}_{-0.02}$	-1.2%
40 GeV	$33.4^{+1.7}_{-2.5}$	$36.5^{+0.6}_{-0.03}$	$+9.3\%$	$36.5^{+0.1}_{+0.1}$	-0.1%
60 GeV	$22.0^{+1.0}_{-1.5}$	$25.0^{+0.2}_{+0.3}$	$+13.6\%$	$25.4^{-0.1}_{+0.2}$	$+1.6\%$

QCD corrections at NNLO

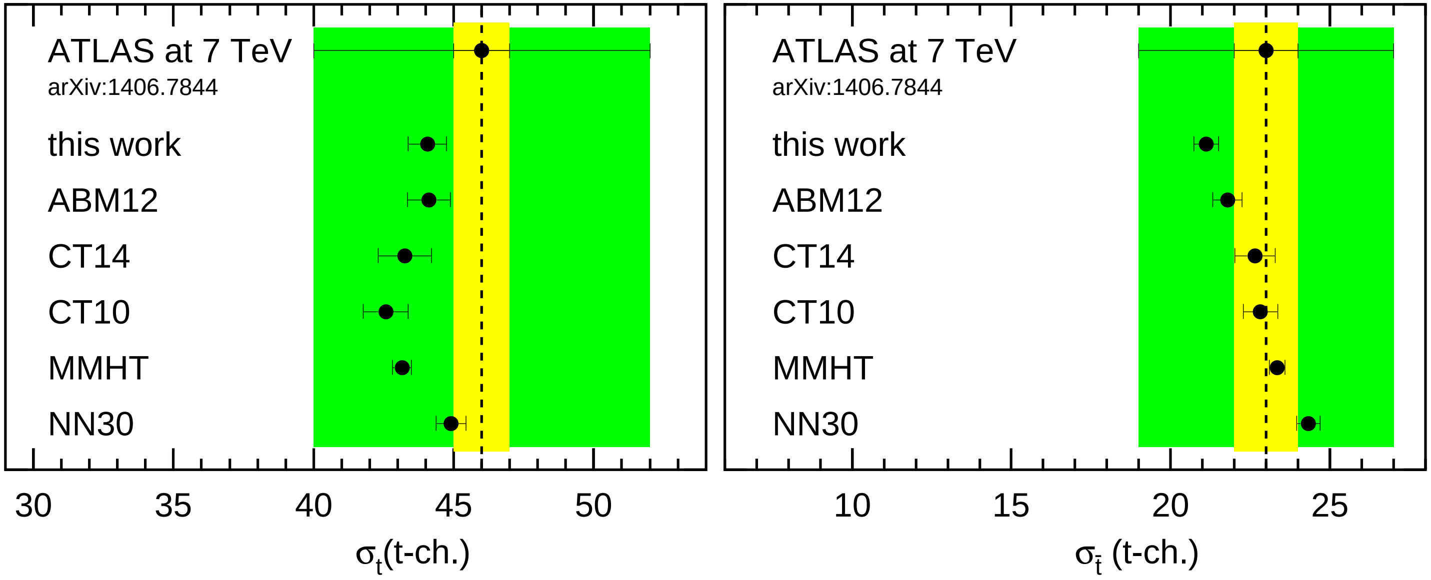
- Computation of NNLO QCD corrections Brucherseifer, Caola, Melnikov ‘14
    - fully differential, with cuts on p_T
- QCD corrections treated in structure function approach
    - non-factorizable contributions neglected
      (neglected diagrams $\mathcal{O}(1/N_c^2)$ supressed)



- QCD corrections to t-channel single anti-top quark production at LHC8

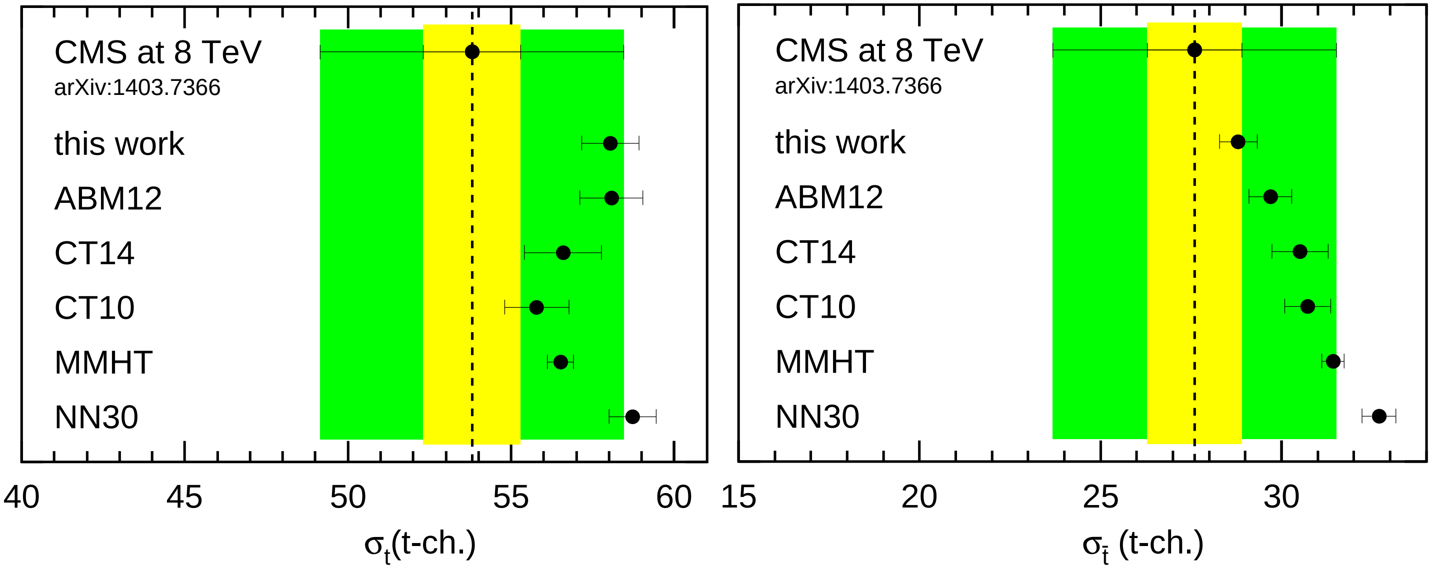
$p_\perp$	$\sigma_{\rm LO}$, pb	$\sigma_{\rm NLO}$, pb	$\delta_{\rm NLO}$	$\sigma_{\rm NNLO}$, pb	$\delta_{\rm NNLO}$
0 GeV	$29.1^{+1.7}_{-2.4}$	$30.1^{+0.9}_{-0.5}$	$+3.4\%$	$29.7^{+0.3}_{-0.1}$	-1.3%
20 GeV	$24.8^{+1.4}_{-2.0}$	$26.3^{+0.7}_{-0.3}$	$+6.0\%$	$26.2^{-0.01}_{-0.1}$	-0.4%
40 GeV	$17.1^{+0.9}_{-1.3}$	$19.1^{+0.3}_{+0.1}$	$+11.7\%$	$19.3^{-0.2}_{+0.1}$	$+1.0\%$
60 GeV	$10.8^{+0.5}_{-0.7}$	$12.7^{+0.03}_{+0.2}$	$+17.6\%$	$12.9^{-0.2}_{+0.2}$	$+1.6\%$

Inclusive cross sections (I)



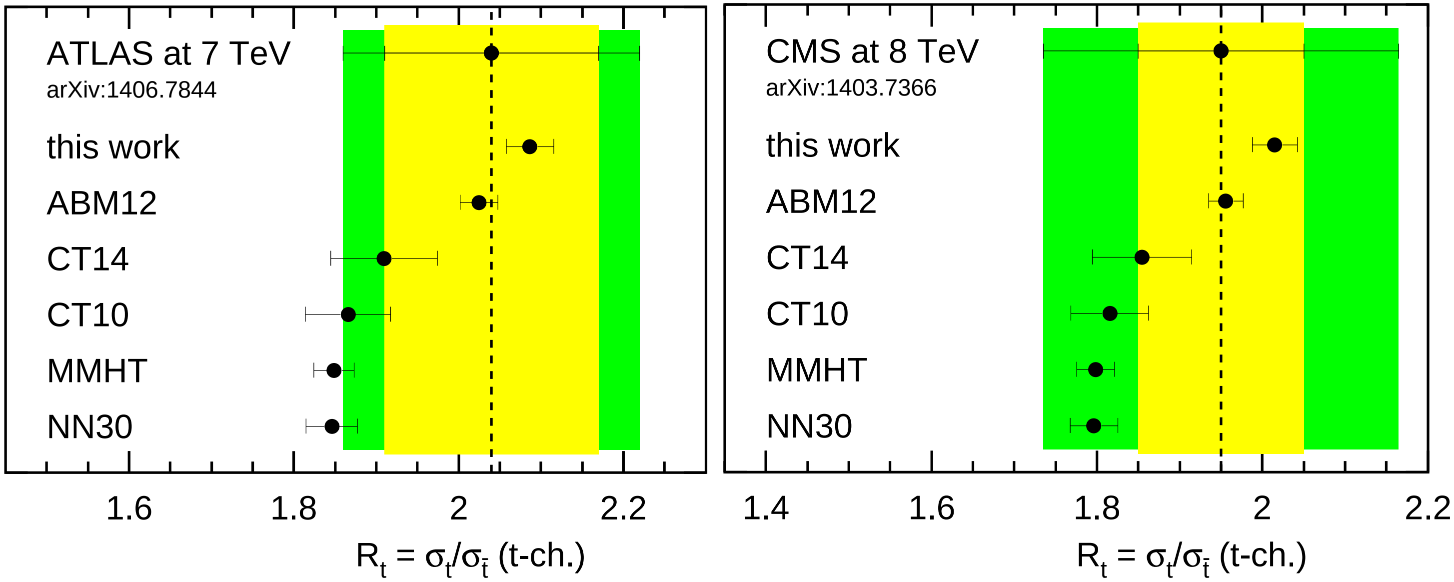
- Cross sections for t-channel production of single (anti)top-quarks at LHC with 1σ PDF uncertainties
  - computation of hard cross section to NLO in QCD with Hathor for $\overline{\mathrm{MS}}$ mass $m_t(m_t) = 163$ GeV at scale $\mu_R = \mu_F = m_t(m_t)$
- Data at $\sqrt{s} = 7$ TeV from ATLAS
  - inner (yellow) band for statistical uncertainty and outer (green) band for combined statistics and systematics uncertainty

Inclusive cross sections (II)



- Cross sections for t-channel production of single (anti)top-quarks at LHC with 1σ PDF uncertainties
  - computation of hard cross section to NLO in QCD with Hathor for $\overline{\mathrm{MS}}$ mass $m_t(m_t) = 163$ GeV at scale $\mu_R = \mu_F = m_t(m_t)$
- Data at $\sqrt{s} = 8$ TeV from CMS
  - inner (yellow) band for statistical uncertainty and outer (green) band for combined statistics and systematics uncertainty

Cross section ratio



- Cross section ratio $R_t = \sigma_t/\sigma_{\bar{t}}$ is very sensitive probe
  - data from ATLAS and CMS dominated by inner (yellow) band for statistical uncertainty, systematics largey cancel
- Theory predictions sensitive to ratio d/u of PDFs
  - 1σ PDF uncertainties in R_t small

Upshot

- Production of single top-quarks at LHC can serve as standard candle for the light quark flavor content of proton

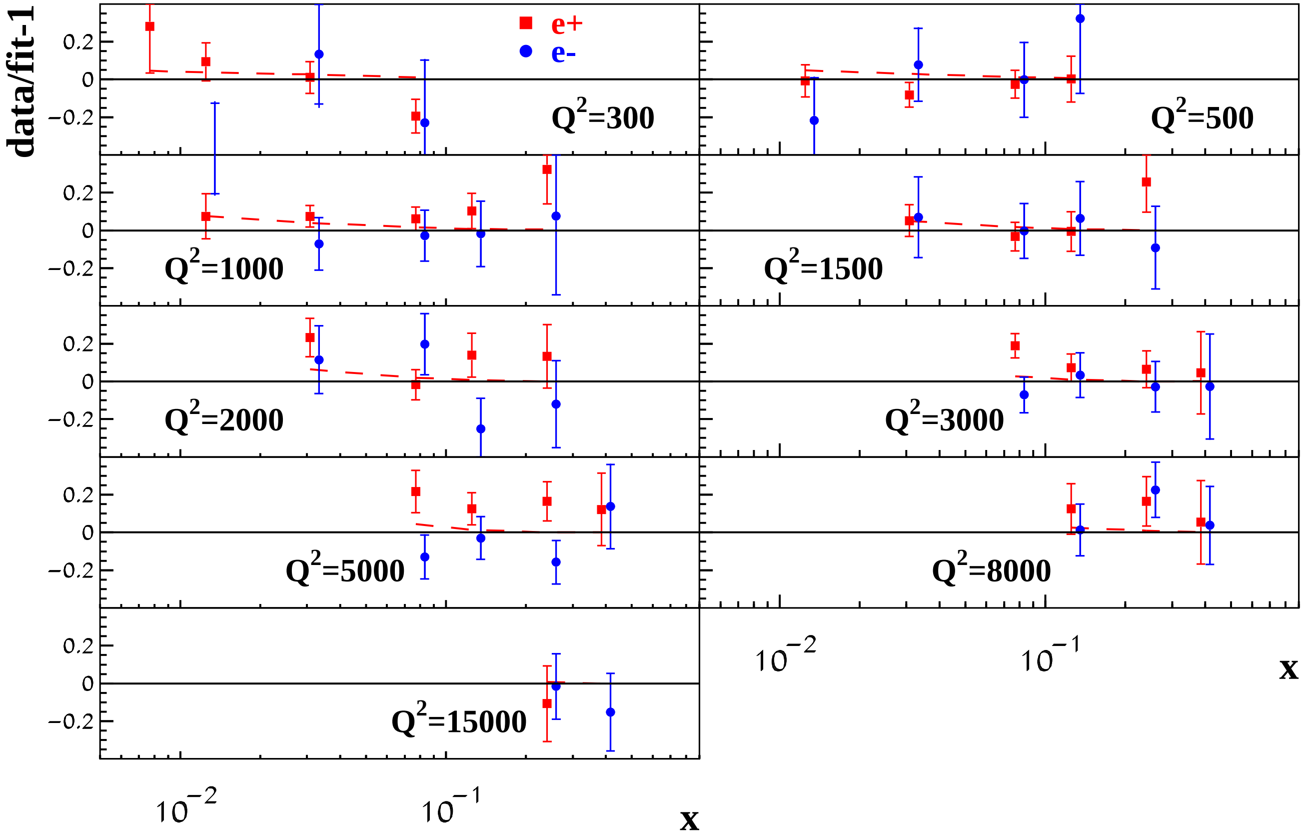
Strange sea in the proton

Strange sea determination

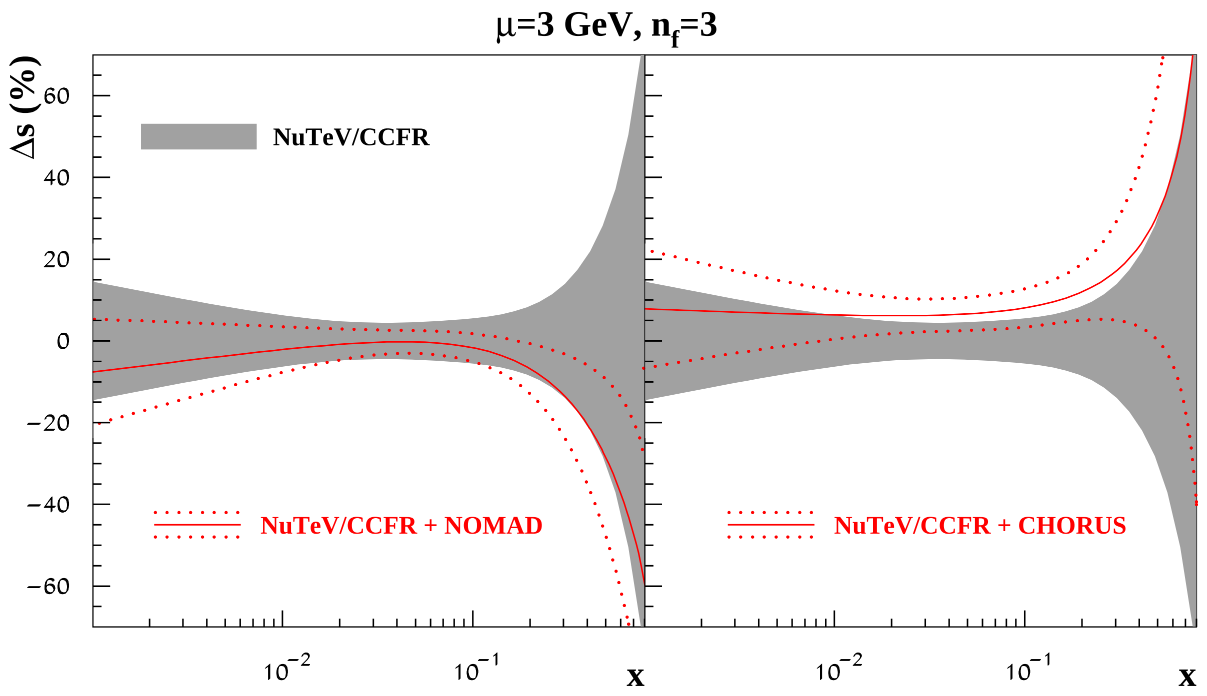
Charged current DIS

Alekhin, Bümlein, Caminada, Lipka, Lohwasser, S.M. Petti, Placakyte '14

- CC DIS inclusive data (HERA), CC DIS di-muon production data (NOMAD) and CC DIS charmed-hadron production data (CHORUS)
- Theory description with exact NLO QCD corrections and asymptotic NNLO terms at large $Q^2 \gg m^2$ Buza van Neerven '97



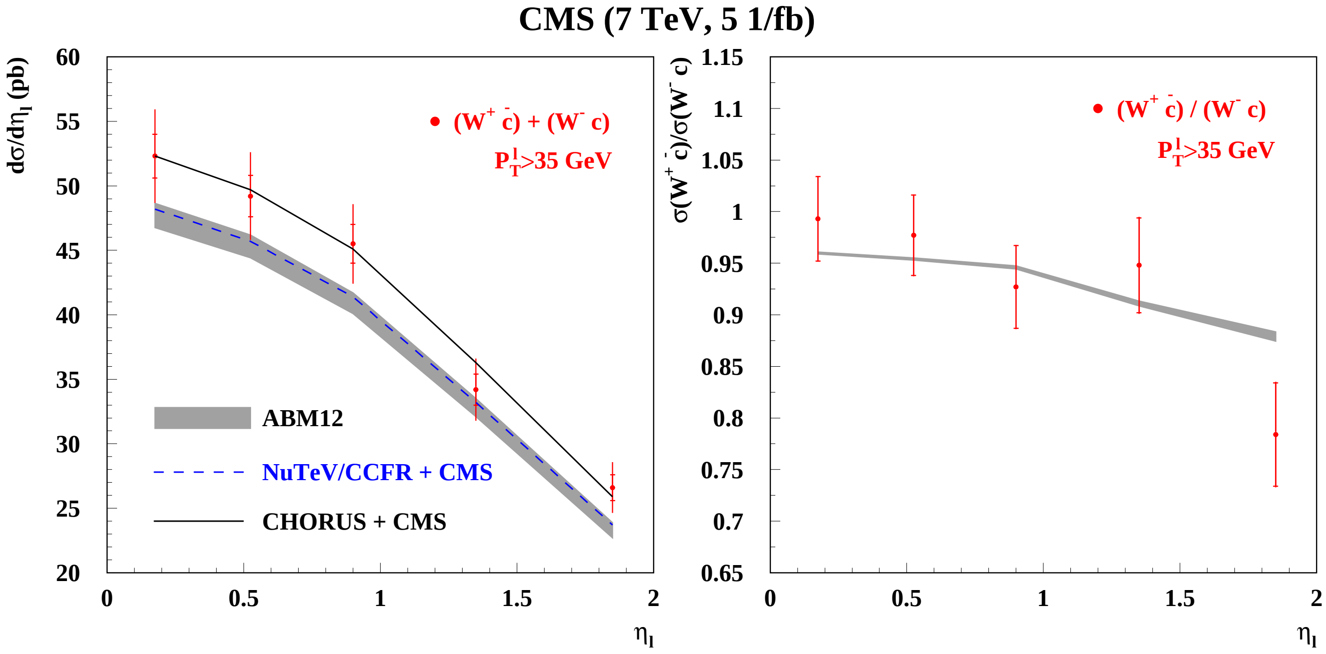
Strange sea from new fixed target data



- Nomad data on ratio of di-muon sample to incl. CC DIS with statistics of 15000 events (much more than CCFR and NuTeV samples)
  - systematics, nuclear corrections, etc. cancel in ratio
  - pull down strange quarks at $x > 0.1$; sizable reduction of uncertainty
  - $m_c(m_c) = 1.23 \pm 0.03(\mathrm{exp.})\mathrm{GeV}$
- Chorus data pull strangeness up
  - statistical significance of the effect is poor

W+charm production at LHC

- Cross check with LHC data for W+charm production

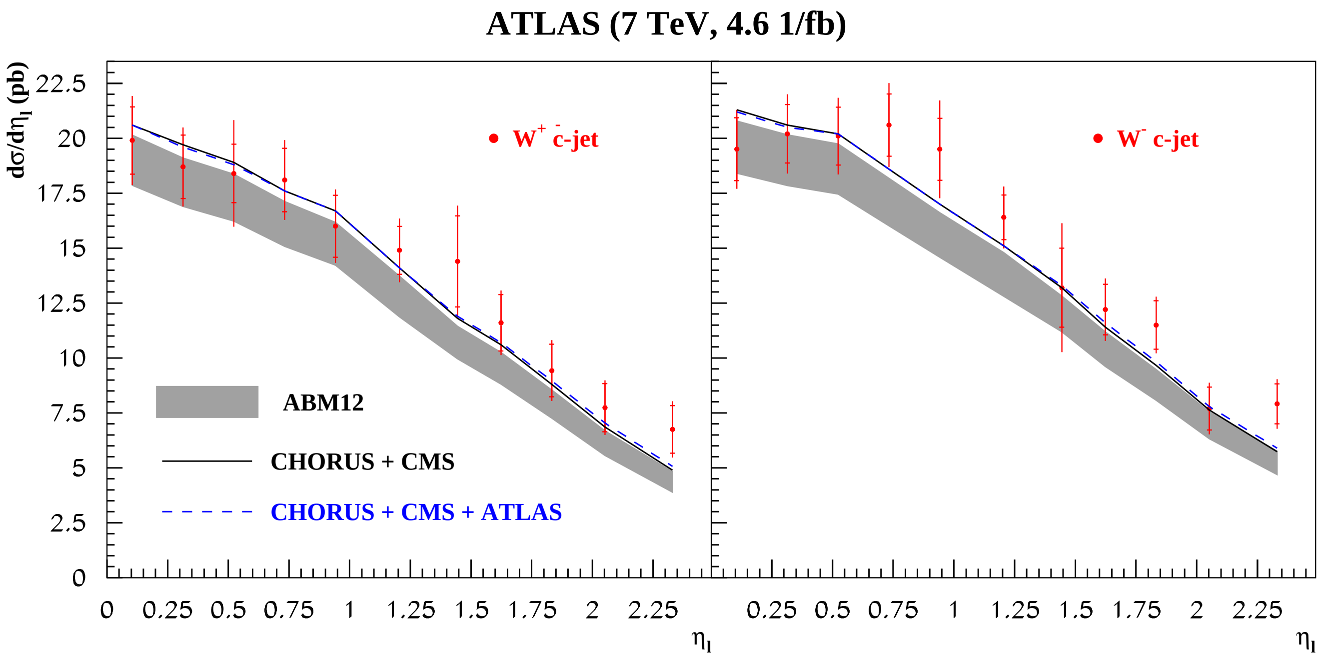


CMS

- CMS data above NuTeV/CCFR by 1σ
- Charge asymmetry in a good agreement with charge-symmetric strange sea

W+charm production at LHC

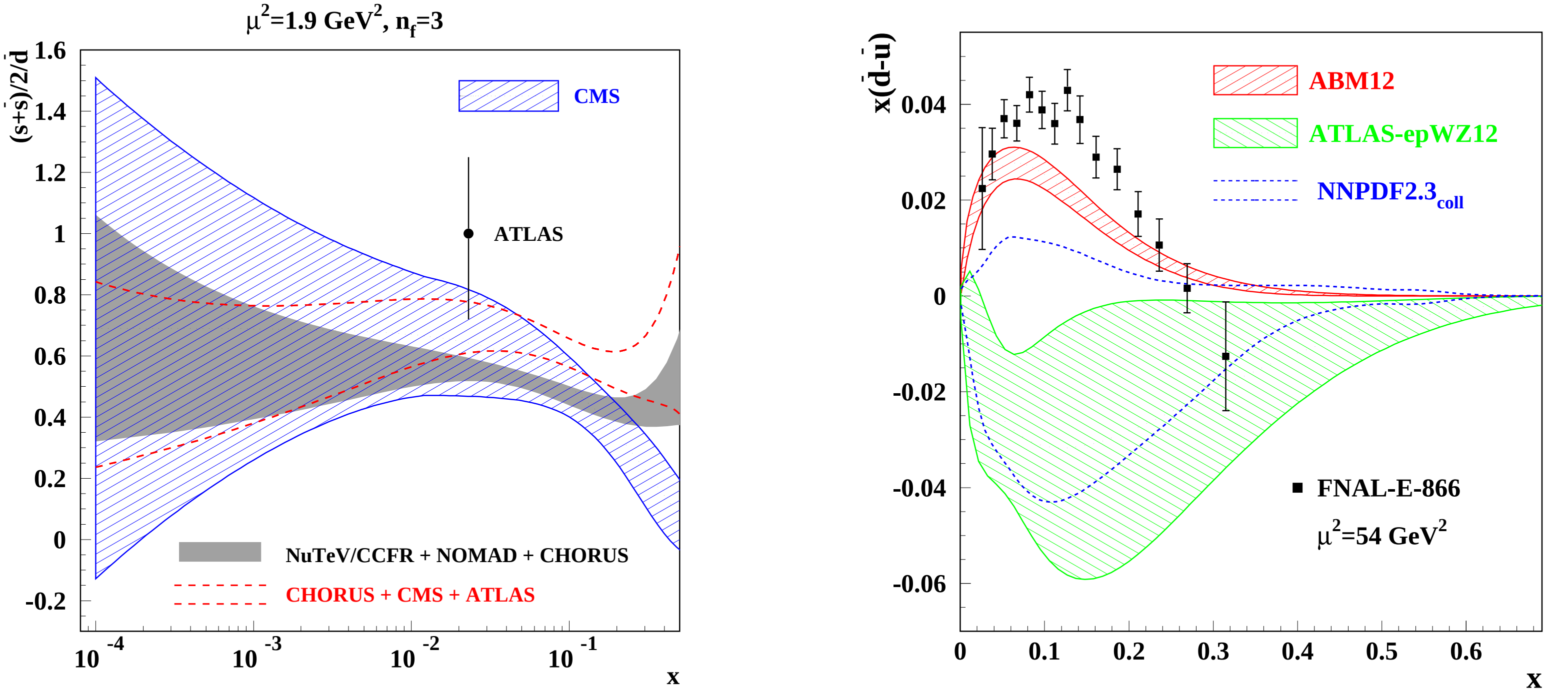
- Cross check with LHC data for W+charm production



ATLAS

- ATLAS data in good agreement with NuTeV/CCFR
- Highest bin in η_l deviates

Comparision with earlier determinations



- ABM update (NuTeV/CCFR+NOMAD+CHORUS) in good agreement with CMS results
- ATLAS strange-sea in enhanced, but correlated with d-quark sea suppression (disagreement with the FNAL-E-866 data)
- Upper margin of ABM analysis (CHORUS+CMS+ATLAS) is lower than ATLAS

Strong coupling constant

Strong coupling constant (1992)

	$\alpha_s(\mathrm{M_Z^2})$
R_τ	$0.117 \begin{smallmatrix} +\ 0.010 \\ -\ 0.016 \end{smallmatrix}$
DIS	0.112 ± 0.007
Υ Decays	0.110 ± 0.010
$R_{e^+e^-}(s < 62$GeV$)$	0.140 ± 0.020
$p\overline{p} \rightarrow W + jets$	0.121 ± 0.024
$\Gamma(Z \rightarrow \mathrm{hadrons})/\Gamma(Z \rightarrow l\bar{l})$	0.132 ± 0.012
Jets at LEP	0.122 ± 0.009
Average	0.118 ± 0.007

G. Altarelli (1992)
in *QCD - 20 Years Later,*
CERN-TH-6623-92

Essential facts

- World average 1992 $\alpha_s(M_Z) = 0.118 \pm 0.007$
- Central value at NLO QCD
  - still right, but for very different reasons
- Error at NLO QCD
  - now down to $\sim 0.0050 - 0.0040$ (theory scale uncertainty)

Strong coupling constant (2015)

Measurements at NNLO

- Values of $\alpha_s(M_Z)$ at NNLO from PDF fits

SY	0.1166 ± 0.013	F_2^{ep}	Santiago, Yndurain ‘01
	0.1153 ± 0.063	$xF_3^{\nu N}$ (heavy nucl.)	
A02	0.1143 ± 0.013	DIS	Alekhin ‘01
MRST03	0.1153 ± 0.0020		Martin, Roberts, Stirling, Thorne ‘03
BBG	$0.1134\ {}^{+\,0.0019}_{-\,0.0021}$	valence analysis, NNLO	Blümlein, Böttcher, Guffanti ‘06
GRS	0.112	valence analysis, NNLO	Glück, Reya, Schuck ‘06
A06	0.1128 ± 0.015		Alekhin ‘06
JR08	0.1128 ± 0.0010	dynamical approach	Jimenez-Delgado, Reya ‘08
	0.1162 ± 0.0006	including NLO jets	
ABKM09	0.1135 ± 0.0014	HQ: FFNS $n_f = 3$	Alekhin, Blümlein, Klein, S.M. ‘09
	0.1129 ± 0.0014	HQ: BSMN	
MSTW	0.1171 ± 0.0014		Martin, Stirling, Thorne, Watt ‘09
Thorne	0.1136	[DIS+DY, HT*] (2013)	Thorne ‘13
ABM11$_J$	$0.1134 \ldots 0.1149\ \pm 0.0012$	Tevatron jets (NLO) incl.	Alekhin, Blümlein, S.M. ‘11
NN21	0.1173 ± 0.0007	(+ heavy nucl.)	NNPDF ‘11
ABM12	0.1133 ± 0.0011		Alekhin, Blümlein, S.M. ‘13
	0.1132 ± 0.0011	(without jets)	
CT10	0.1140	(without jets)	Gao et al. ‘13
CT14	$0.1150\ {}^{+\,0.0060}_{-\,0.0040}$	$\Delta\chi^2 > 1$ (+ heavy nucl.)	Dulat et al. ‘15
MMHT	0.1172 ± 0.0013	(+ heavy nucl.)	Martin, Motylinski, Harland-Lang, Thorne ‘15

Strong coupling constant (2015)

Other measurements of α_s at NNLO

- Values of $\alpha_s(M_Z)$ at NNLO from measurements at colliders

3-jet rate	0.1175 ± 0.0025	Dissertori et al. 2009	arXiv:0910.4283
e^+e^- thrust	$0.1131 \ {}^{+\,0.0028}_{-\,0.0022}$	Gehrmann et al.	arXiv:1210.6945
e^+e^- thrust	0.1140 ± 0.0015	Abbate et al.	arXiv:1204.5746
C-parameter	0.1123 ± 0.0013	Hoang et al.	arXiv:1501.04111
CMS	0.1151 ± 0.0033	$t\bar{t}$	arXiv:1307.1907
NLO Jets ATLAS	$0.111 \ {}^{+\,0.0017}_{-\,0.0007}$		arXiv:1312.5694
NLO Jets CMS	0.1148 ± 0.0055		arXiv:1312.5694

Differences in α_s determinations

Why α_s values from MSTW and NNPDF are large

- Differences result from different physics models and analysis procedures
  - target mass corrections (powers of nucleon mass M_N^2/Q^2)
  - higher twist $F_2^{\mathrm{ht}} = F_2 + ht^{(4)}(x)/Q^2 + ht^{(6)}(x)/Q^4 + \dots$
  - error correlations
- Hadroproduction of jets known NLO only
  - strictly speaking $\alpha_s(M_Z)$ value only NLO (systematically larger)

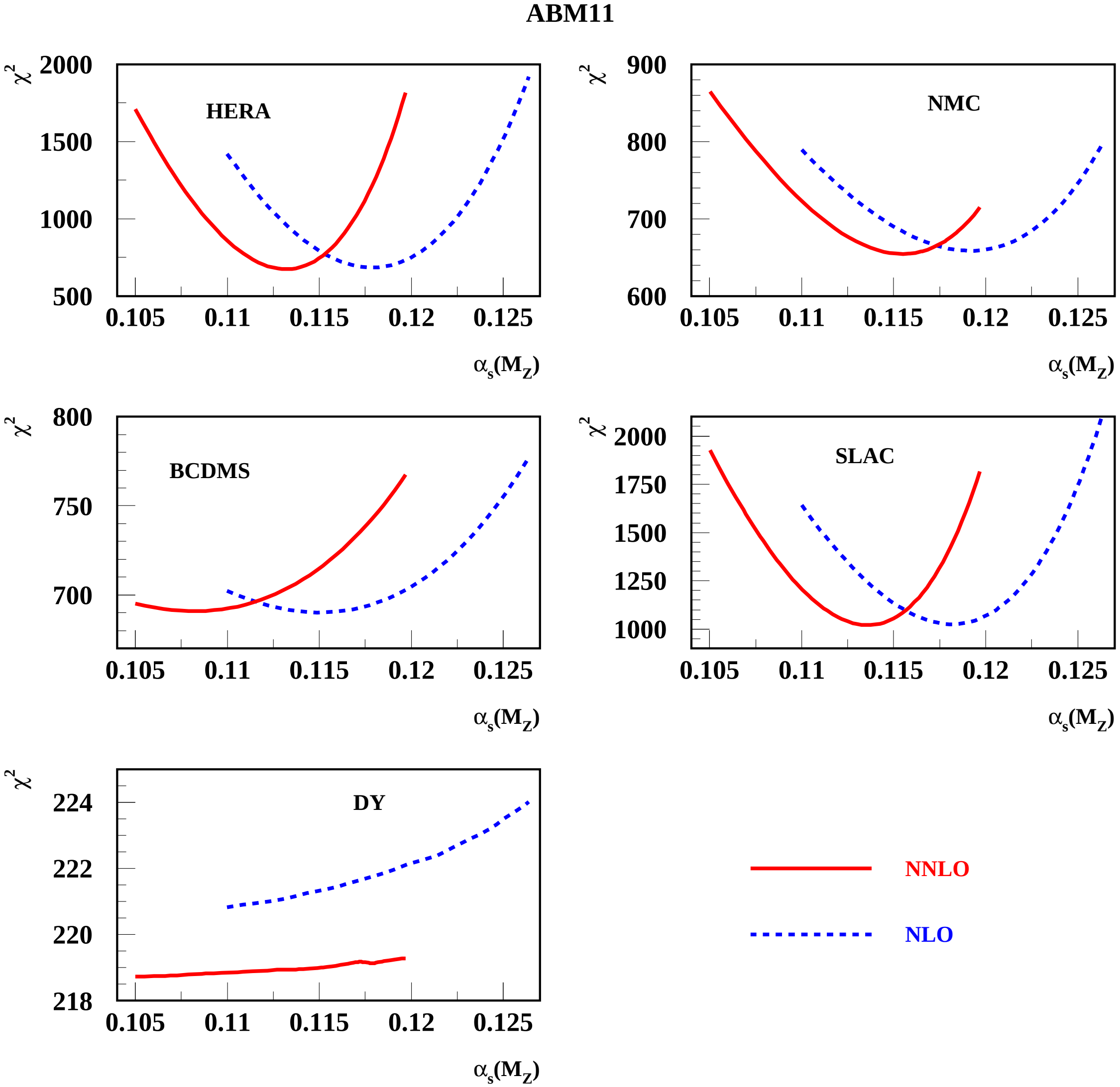
	α_s	NNLO	target mass corr.	higher twist	error correl.
ABM12	0.1132 ± 0.0011	**yes**	**yes**	**yes**	**yes**
NNPDF21	0.1173 ± 0.0007	**(yes)**	**yes**	**no**	**yes**
MSTW	0.1171 ± 0.0014	**(yes)**	**no**	**no**	**no**
MMHT	0.1172 ± 0.0013	**(yes)**	**no**	**no**	–

- Effects for differences are understood
  - variants of ABM with no higher twist etc. reproduce larger α_s values

    Alekhin, Blümlein, S.M. ‘11

Zooming in on ABM

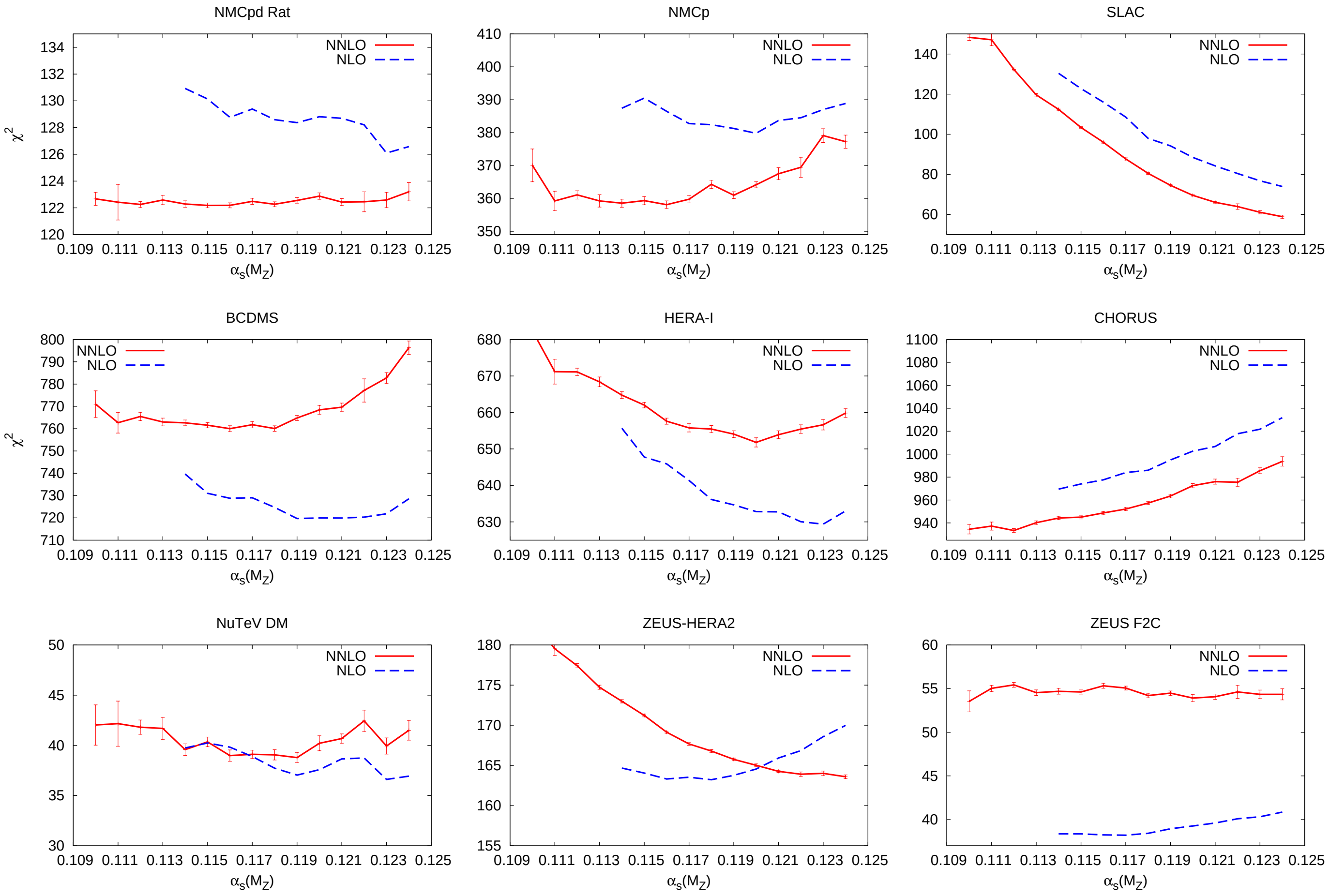
α_s from DIS and PDFs



- Profile of χ^2 for different data sets in ABM11 PDF fit Alekhin, Blümlein, S.M. ‘12

Zooming in on NNPDF

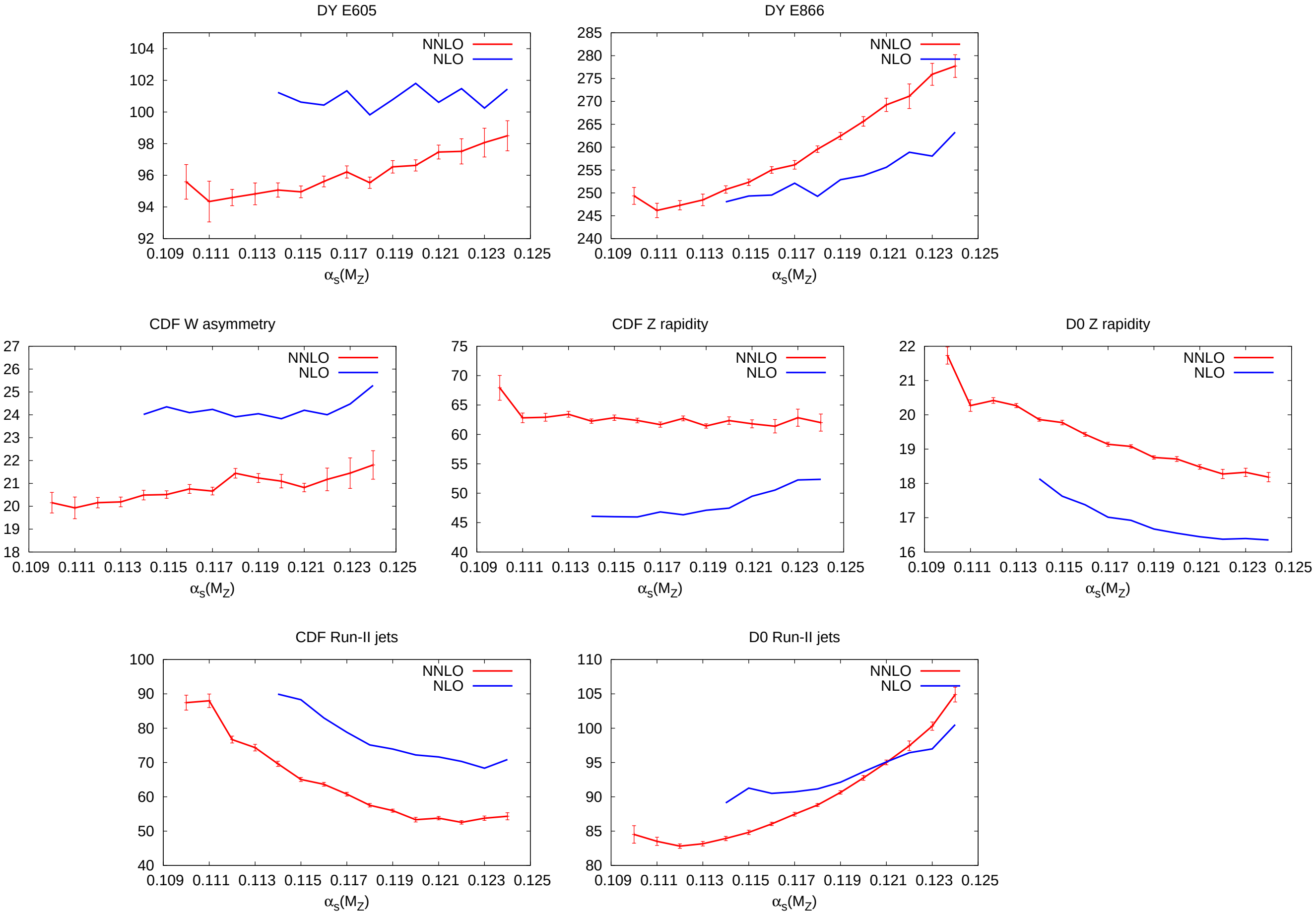
α_s from DIS and PDFs



- Profile of χ^2 for different data sets in NNPDF21 fit Ball et al. ‘11

Zooming in on NNPDF

α_s from DIS and PDFs



- Profile of χ^2 for different data sets in NNPDF21 fit Ball et al. ‘11

Summary

Parton distributions at the LHC

- Precision determinations of non-perturbative parameters is essential
  - parton content of proton (PDFs)
  - strong coupling constant $\alpha_s(M_Z)$
  - quark masses m_c, m_b, m_t
- Experimental precision of $\lesssim 1\%$ puts pressure on accuracy of theoretical predictions
  - NNLO in QCD is *conditio sine qua non*
- Precision measurements require careful definition of observable
  - differences between PDFs due to differences in underlying theory description
  - confronting LHC data requires continuous benchmarking of individual PDF sets
- Lots of challenging tasks in the future . . .