Shape Coexistence far from Stability

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Outline

- Basics
- Shape coexistence and the Islands of Inversion
- IoI at N=20 and N=28 and their merging
- ^{68}Ni: The Portal to the N=40 IoI
- ^{72}Kr and Shape Entanglement
- Conclusions

Introduction

- **The two basic players in the nuclear dynamics are the spherical mean field and the multipole hamiltonian:** $H = \mathcal{H}_m + \mathcal{H}_M$
- **Magic numbers are associated to large energy gaps in the spherical mean field. Therefore, to promote particles above the Fermi level costs a large amount of energy.**
- **The Multipole Hamiltonian is responsible for the very strong nuclear correlations**
- **It is proper to the nucleus that, quite often, certain highly correlated configurations (dubbed ”intruders”) overwhelm their loss of mean field energy with their huge gains in correlation energy.**

The Spherical Mean Field (Monopole Hamiltonian)

$$\mathcal{H}_m = \sum n_i \epsilon_i + \sum \frac{1}{(1+\delta_{ij})} \overline{V}_{ij}\, n_i (n_j - \delta_{ij})$$

the coefficients $\overline{V}$ are angular averages of the two body matrix elements, or centroïds of the two body interaction:

$$\overline{V}_{ij} = \frac{\sum_J V^J_{ijij} [J]}{\sum_J [J]}$$

the sums run over Pauli allowed values.

The Spherical Mean Field (Monopole Hamiltonian)

This can be written as well as:

$$\mathcal{H}_m = \sum_i n_i \left[\epsilon_i + \sum_j \frac{1}{(1+\delta_{ij})} \overline{V}_{ij} \, (n_j - \delta_{ij}) \right]$$

Thus

$$\mathcal{H}_m = \sum_i n_i \;\; \hat{\epsilon}_i(\left[n_j\right])$$

We call these $\hat{\epsilon}_i(\left[n_j\right])$ effective single particle energies (ESPE)

Effective Single Particle Energies

They give the evolution of the underlying (non observable) spherical mean field (aka, shell evolution) as we add particles in the valence space, as well the variations of the spherical mean field in a single nucleus for states which have different configurations.

They are the control parameter for the nuclear dynamics, given the universality of the nuclear correlators.

Monopole anomalies of the realistic NN interactions

They are the more blatant in the neutron-neutron interaction; for instance not producing neither a magic ^{48}Ca, nor the right location of the neutron drip line in the Oxygen isotopes

On the contrary their monopole neutron proton tensor part is correct, and the spin orbit splittings well accounted for.

Nowadays the blame is put in the missing residual three body effects

The Nuclear Correlators (Multipole Hamiltonian)

- The multipole hamiltonian is responsible for the collective nuclear behavior. It is universal and well given by the realistic NN interactions. Its main components are:
- BCS-like isovector and isoscalar pairing. When pairing dominates, as in the case of nuclei with only neutrons (or only protons) on top of a doubly magic nucleus, it produces nuclear superfluids.
- Quadrupole-Quadrupole and Octupole-Octupole terms of very simple nature ($r^\lambda Y_\lambda \cdot r^\lambda Y_\lambda$) which tend to make the nucleus deformed. In this limit, the pairing correlations mainly show up as responsible for the moment of inertia of the nuclear rotors.

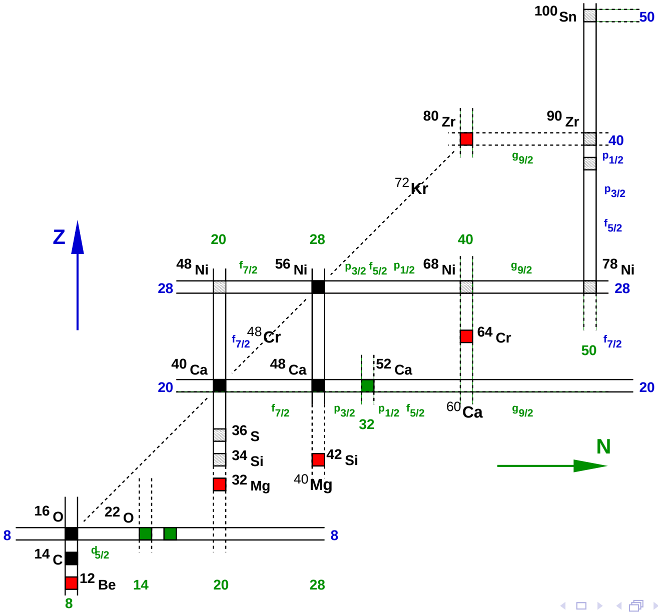
When do the quadrupole correlations thrive in the nucleus?

- The fact that the spherical nuclear mean field is close to the HO has profound consequences, because the dynamical symmetry of the HO, responsible for the accidental degeneracies of its spectrum, is SU(3), among whose generators it is the quadrupole operator.
- When valence protons an neutrons occupy the degenerate orbits a major oscillator shell, and for an attractive Q·Q interaction, the many body problem has an analytical solution in which the ground state of the nucleus is maximally deformed (Elliott's model)

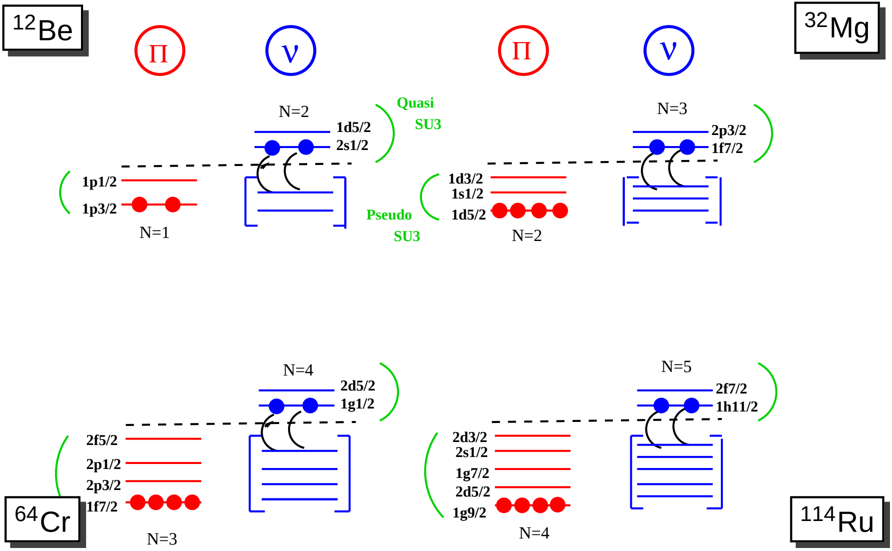
When do the quadrupole correlations thrive in the nucleus?

- **In cases when both valence neutrons and protons occupy quasi-degenerate orbits with Δj= 2 and Δj=2, including j=p+1/2 (Quasi-SU3), or quasi-spin multiplets (Pseudo-SU3)**
- **For example, $0f_{7/2}$ and $1p_{3/2}$, or $0g_{9/2}$ $1d_{5/2}$ and $2s_{1/2}$ form Quasi-SU3 multiplets and $0f_{5/2}$, $1p_{3/2}$ and $1p_{1/2}$ a Pseudo-SU3 triplet**

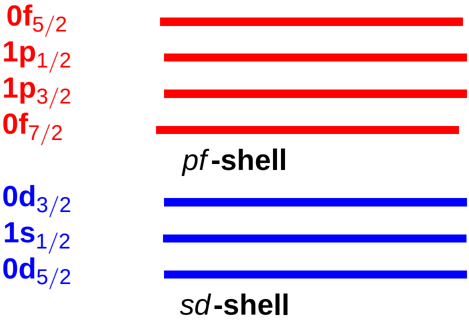
Landscape of medium mass exotica



How deformation sets in at N=8, 20, 40, 70. Universality

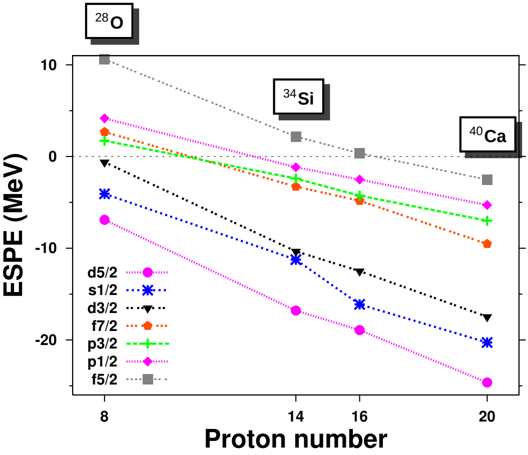


N=20 to N=28. The Valence Space; *sd*-*pf*



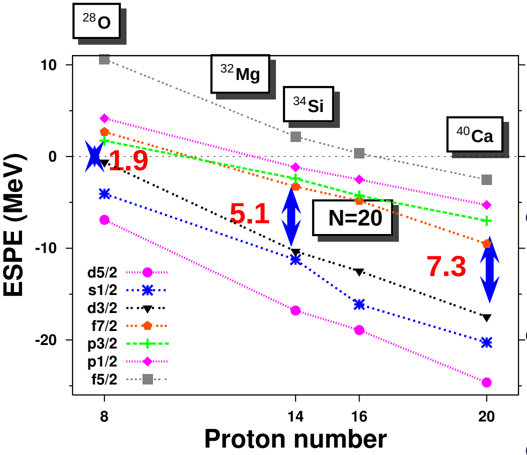
EFFECTIVE INTERACTION **SDPF-U-MIX**

Neutron (Effective) Single Particle Energies: N=20



- At the neutron drip line, the ESPE's of ^{28}O are completely at variance with those of ^{40}Ca at the stability valley. The change from the ESPE's of ^{16}O to the ones in ^{28}O is totally due to the interactions of *sd* shell neutrons among themselves. The N=14 gap seen in ^{22}O must have the same origin, being absent in ^{17}O.
- Notice that the *sd* shell orbits remain barely below the *pf* shell and that the $\nu 0f_{\frac{7}{2}}$ and $\nu 0p_{\frac{3}{2}}$ orbitals DO get inverted
- The monopole part of the neutron-proton interaction restores the N=20 shell gap when the valley of stability is approached.
- The Spin-Tensor decomposition of the effective interaction shows that this is mainly a Central and Tensor effect

Effective Single Particle Energies (ESPE): Trends



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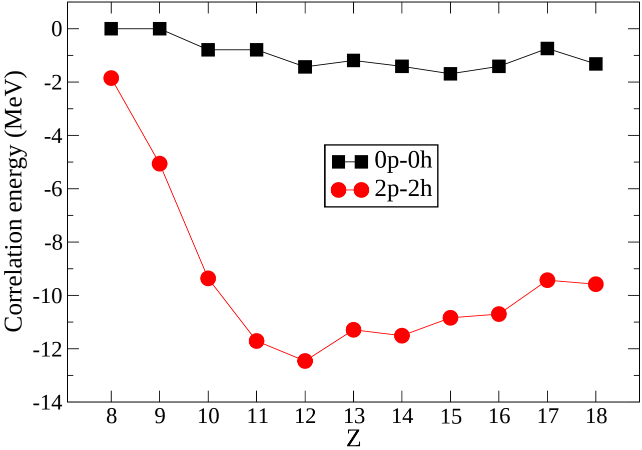
Correlation Energies

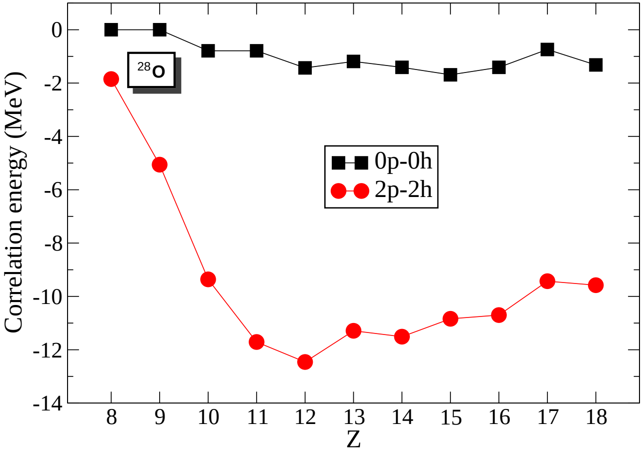
- **Let's consider the configurations with closed N=20 ; $(sp)^{(8,8)}$ $(sd)^{(12,Z-8)}$ (normal filling) and the ones with two neutrons blocked in the *pf*-shell $(sp)^{(8,8)}$ $(sd)^{(10,Z-8)}$ $(pf)^{(2,0)}$ (intruder)**
- **And calculate the energy of the ground states at fixed configuration, with and without correlations**

Correlations energies: normal vs 2p-2h intruder



E. Caurier, F. Nowacki and A. Poves, PRC90 014302 (2014)

Correlations energies: normal vs 2p-2h intruder



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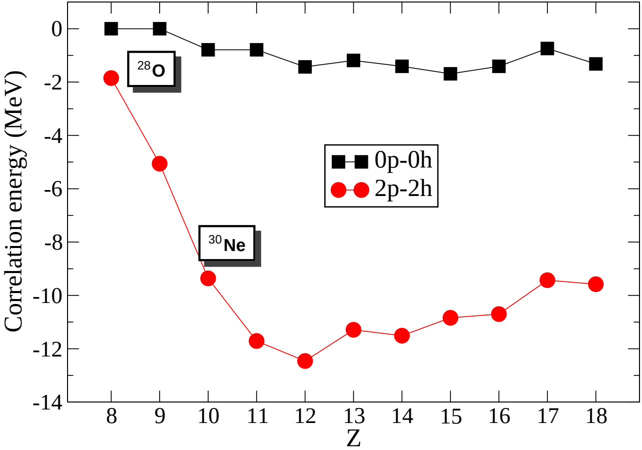

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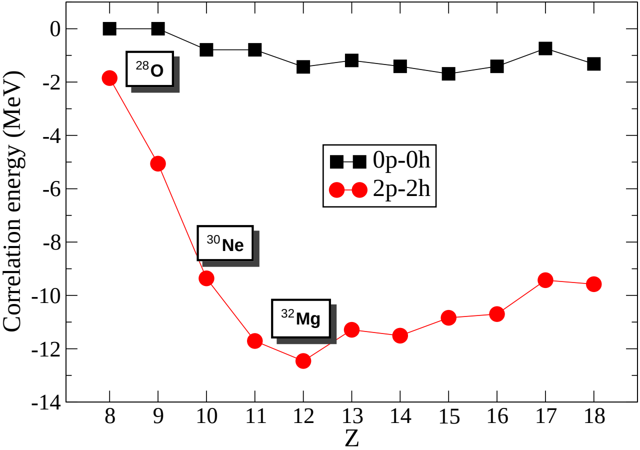

E. Caurier, F. Nowacki and A. Poves, PRC90 014302 (2014)

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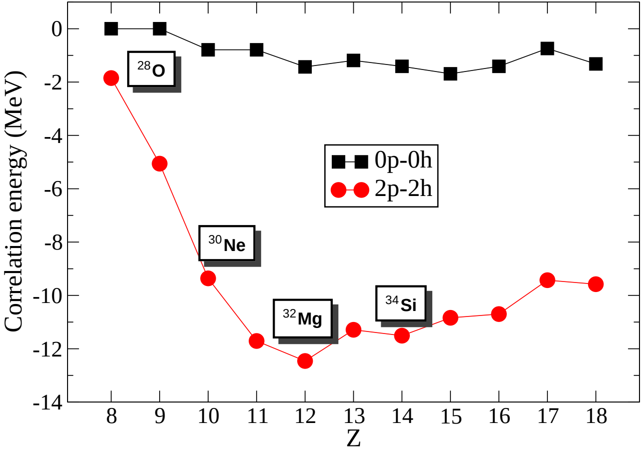
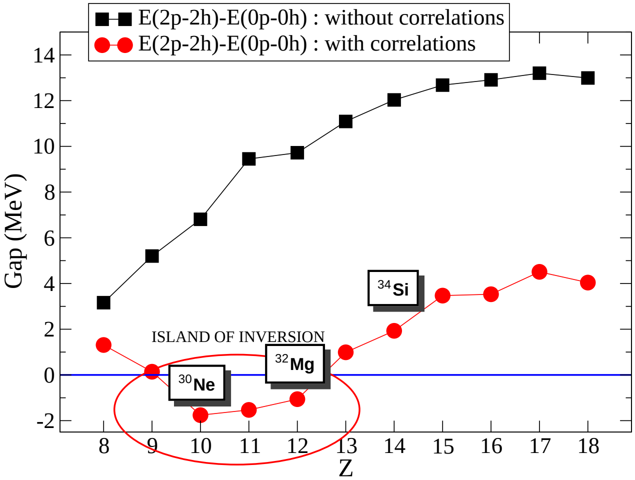


E. Caurier, F. Nowacki and A. Poves, PRC90 014302 (2014)

Gaps: normal vs 2p-2h intruder

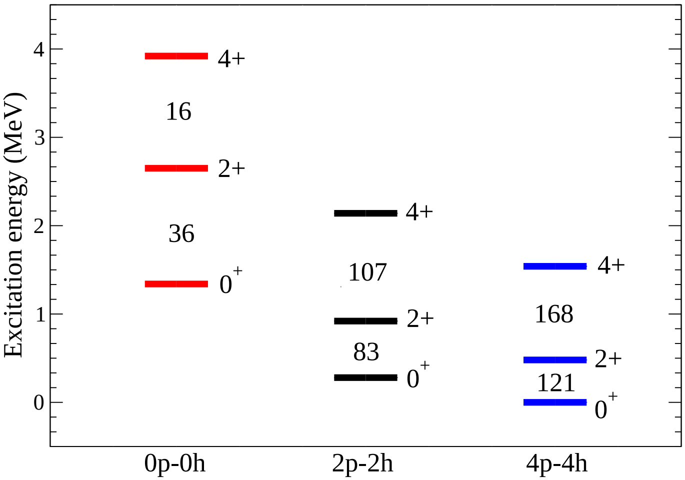


E. Caurier, F. Nowacki and A. Poves, PRC90 014302 (2014)

ESPE's vs. Correlation Energies

- It is evident that the occurrence of the "islands of inversion" depends of subtle cancellations between monopole losses and correlations gains by the intruder states
- Notice that the correlation energies can be huge. For instance, in ^{32}Mg, the correlation energies of the lowest 0^{+} states in the 0p-0h, 2p-2h and 4p-4h configurations are respectively, 1.5 MeV (spherical), 12.5 MeV (deformed) and 21 MeV !! (superdeformed)

Deformed and Superdeformed states in ^{32}Mg



Why Nuclear Shape?

- Because we are still heirs of the semiclassical liquid-drop like models
- The very concept of shape requires to break the rotational (and reflection) invariance, or, equivalently to define an intrinsic reference frame. But even if the symmetry is broken, we need to rely on semiclassical models, liquid-drop like, to define a vocabulary which describes properties akin to the concept of shape.
- The surface of a drop can be expressed in the basis of the spherical harmonics $Y_{\lambda,\mu}(\theta,\phi)$. The coefficients of the development, $\alpha_{\lambda,\mu}$, are the shape parameters. To speak about nuclear shape, we need a protocol to extract the best information about these intrinsic shape parameters from the nuclear wave functions in the laboratory frame.

Shape Parameters; The Case of Quadrupole Deformation

- **From the value of Q_0 one can get the deformation parameter β using different recipes, for instance:**

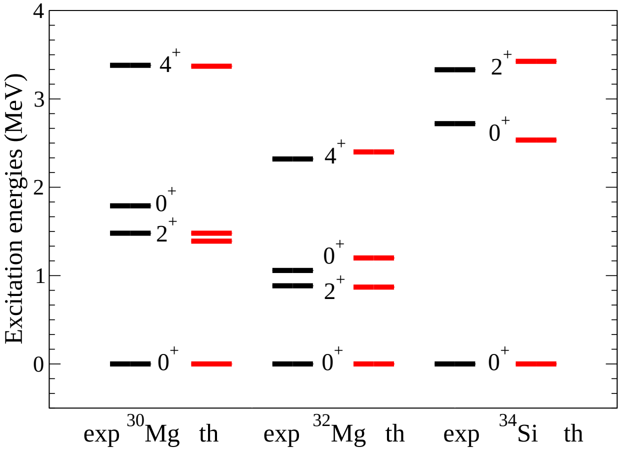
$$Q_0 = \frac{3}{\sqrt{5\pi}} R^2 \, Z \, (1 + 0.16 \, \beta) \, \beta \qquad (1)$$

- **If the nucleus is not axially symmetric, the situation becomes more convoluted, because now we need to recover two shape parameters, β and γ. The former can in most cases be extracted from the B(E2)'s as in the axial case, but for γ we have to resort to other expediencies. Davidov and Filipov use the collective model to extract the values of γ from the B(E2)values of the transitions between the yrast and the γ bands.**

Shape Parameters; The Case of Quadrupole Deformation

- Another possibility is to rely on the use of the expectation values of scalars constructed with the quadrupole operator like $(Q_2 \times Q_2)^0$ or $(Q_2 \times Q_2 \times Q_2)^0$ as proposed by Kumar. These expectation values can be written in terms of the shape parameters. Finally, one could use a basis in the intrinsic frame to perform laboratory frame calculations as in the MCSM (Monte Carlo Shell Model), and keep track of the shape parameter content of the physical solutions.

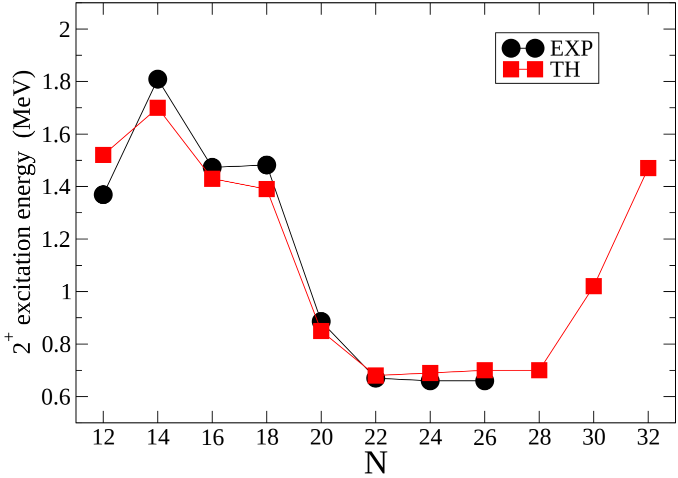
The Portals to the N=20 IoI; Shape Coexistence in ^{30}Mg and ^{34}Si



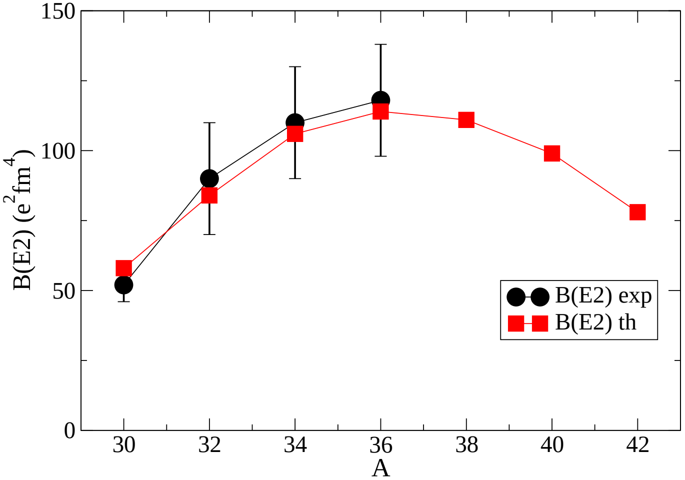
The structure of the two 0^+'s of ^{32}Mg

- **They have a rather weird structure; the ground state is 9% 0p-0h, 54% 2p-2h, 35% 4p-4h and 1% 6p-6h, thus, it is a mixture of deformed and superdeformed prolate shapes and it makes sense to speak of shape mixing.**
- **However, the excited 0^+ has 33% 0p-0h, 12% 2p-2h, 54% 4p-4h and 1% 6p-6h, a surprising hybrid of spherical and superdeformed shapes, whose direct mixing matrix element is strictly zero. Clearly, it is not a case of shape mixing, could it be an example of** *shape entanglement***?**

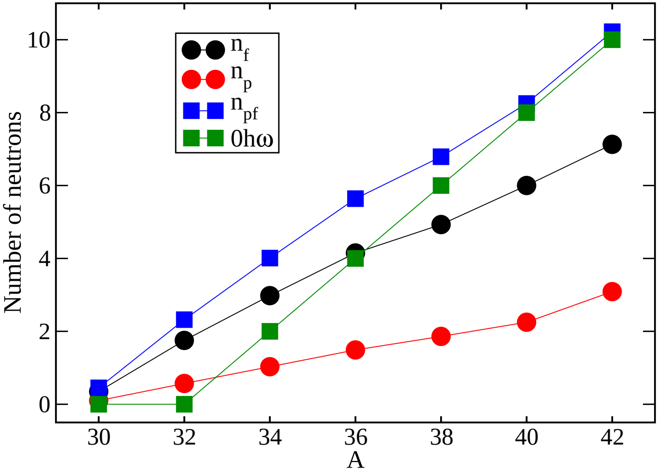
Mg, the N=20 and N=28 islands of inversion merge



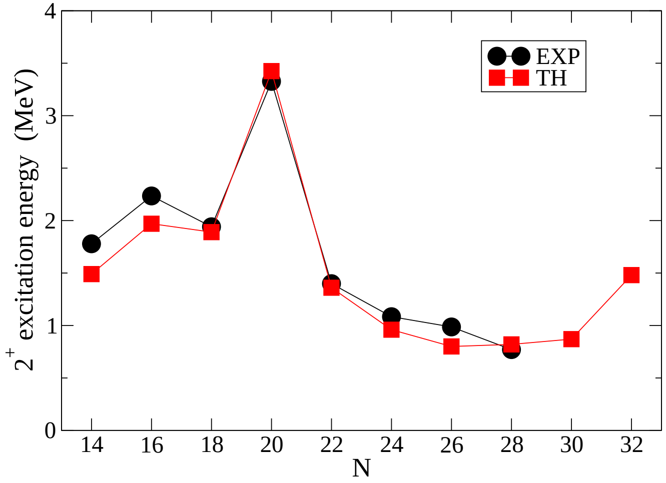
The Magnesium isotopes; B(E2)'s



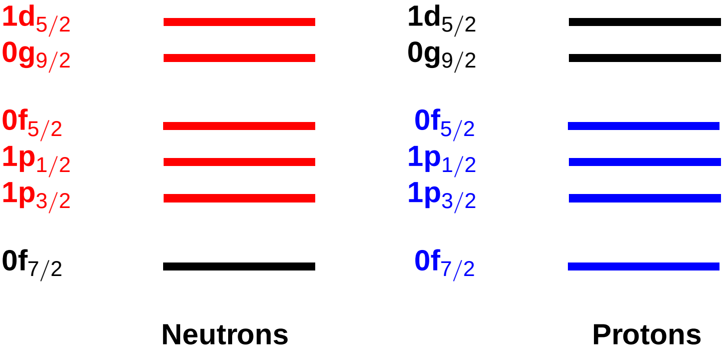
The Mg isotopes; Occupation numbers; Halos?



The Silicon isotopes from the proton to the neutron dripline; SDPF-U-MIX interaction



The valence space for the N=40 isotones



The island of inversion south of ^{68}Ni

Island of inversion around ^{64}Cr

S. Lenzi, F. Nowacki, A. Poves and K. Sieja

Phys. Rev. C 82, 054301, 2010.

The LNPS interaction:

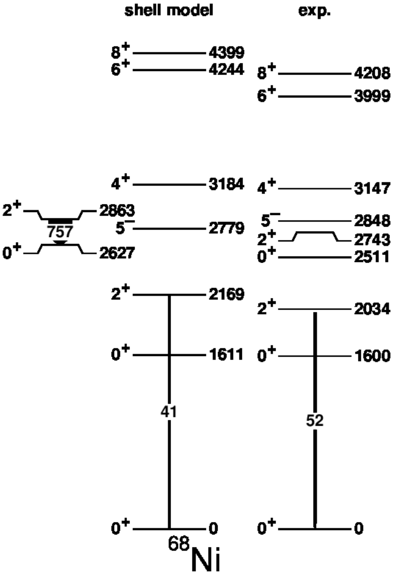
- is based on realistic TBME's plus monopole corrections
- and incorporates a new pf shell interaction (KB3GR, E. Caurier).
- has the $g_{9/2}$-$d_{5/2}$ gap set to 1.5 MeV in ^{68}Ni

The Calculations:

- include up to 14p-14h excitations across Z=28 and N=40 gaps
- reach dimensions up to 10^{10}
- utilize the m-scheme code ANTOINE (non public version)

Triple coexistence in ^{68}Ni

- **In a first approximation, ^{68}Ni has a double shell closure in its *gs***
- **But its low energy spectrum is much more complex**
- **Three coexisting 0^+ states appear between 0 and $\sim$ 2.5 MeV**
- **The 0_2^+ state excitation energy has been remeasured recently by F. Recchia et al. The new value is 1.6 MeV**
- **The 0_3^+ at 2.7 MeV, is predicted by the LSSM calculations to be the head of a superdeformed band of $6p6h$ nature**



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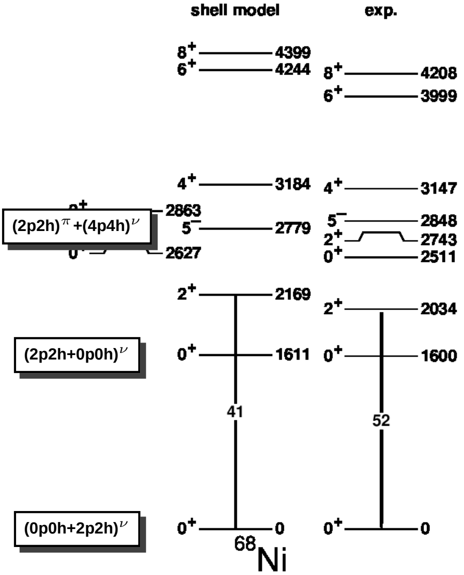

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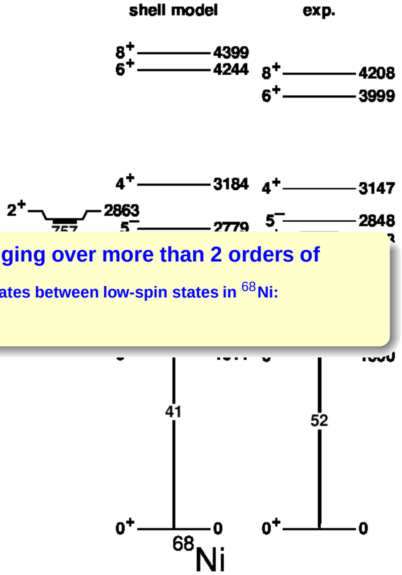
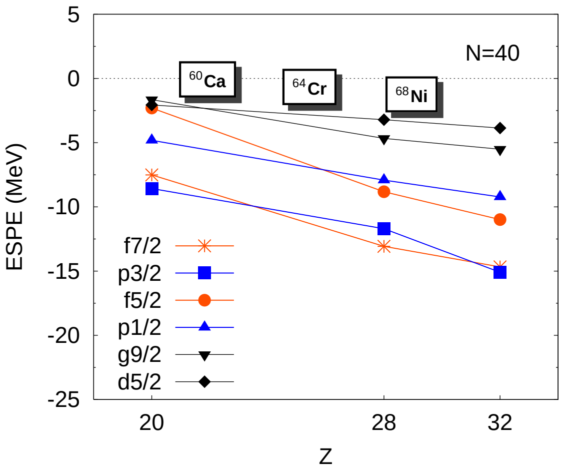


The calculations describe transition rates ranging over more than 2 orders of magnitude Configuration mixing and relative transition rates between low-spin states in ^{68}Ni:

F. Recchia et al., Phys. Rev. C88, 041302(R) (2013)

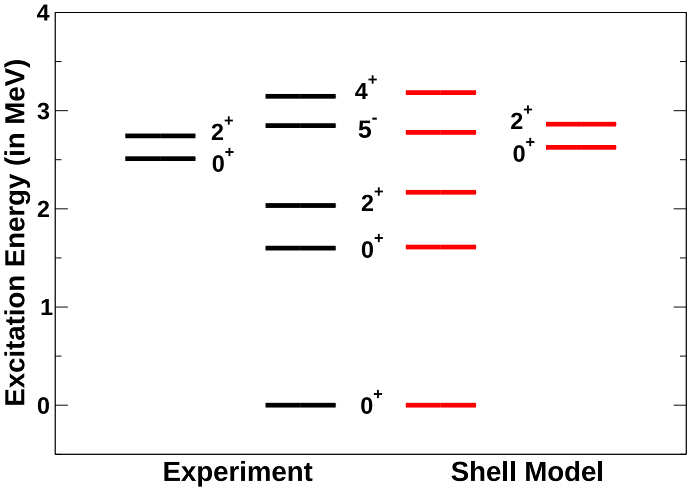
- The 0^+_3 at 2.7 MeV, is predicted by the LSSM calculations to be the head of a superdeformed band of $6p6h$ nature

Neutron ESPE's; main features:



- **Removing $f_{7/2}$ protons the $\nu f_{5/2}$-$g_{9/2}$ gap shrinks**
- **The quasi-SU3 partner orbitals $g_{9/2}$ and $d_{5/2}$ get inverted in ^{60}Ca. This happens in the "ab initio" Coupled Cluster calculations as well.**

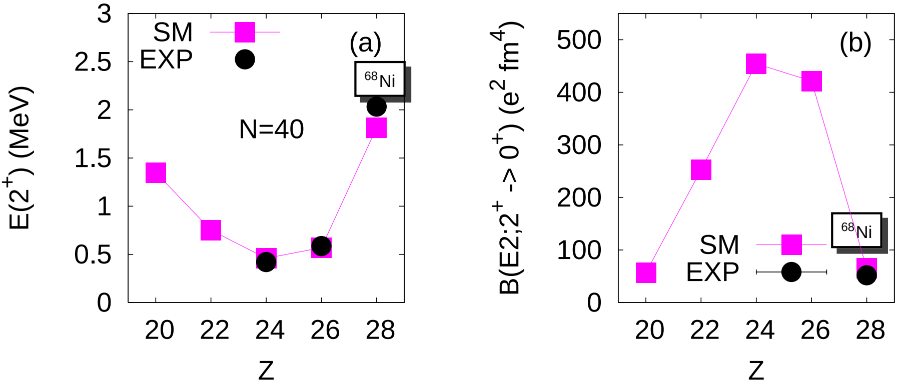
The Portals To the IoIs: ^{68}Ni



The island of inversion south of ^{68}Ni

- **Removing protons from the $0f_{7/2}$ orbit, activates the quadrupole collectivity, which, in turn, favors the np-nh neutron configurations across N=40, that take advantage of the quasi-SU3 coherence of the doublet $0g_{9/2}$- $1d_{5/2}$.**
- **Large scale SM calculations in the valence space of the full *pf*-shell for the protons and the $0f_{5/2}$ $1p_{3/2}$ $1p_{1/2}$ $0g_{9/2}$ and $1d_{5/2}$ orbits for the neutrons, predict a new region of deformation centered at ^{64}Cr.**

Shape transition at N=40



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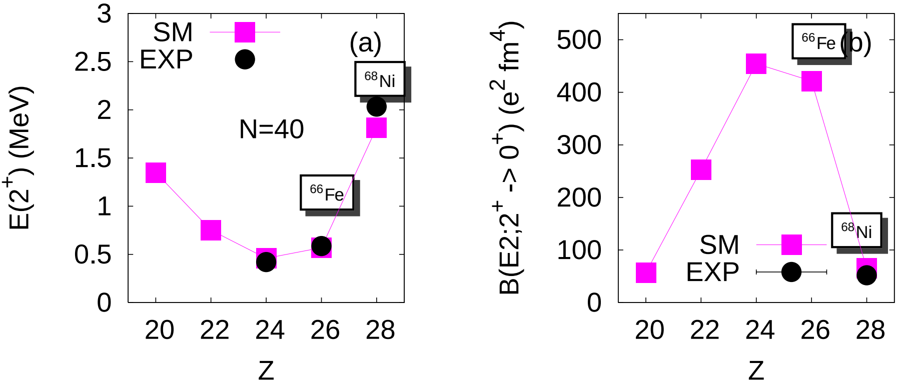

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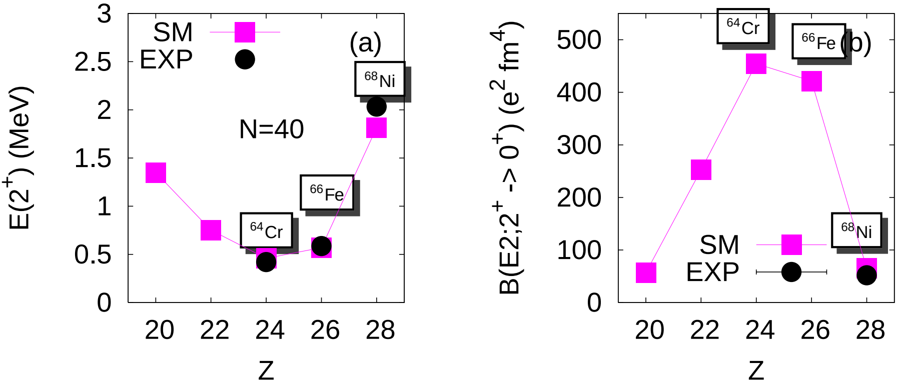
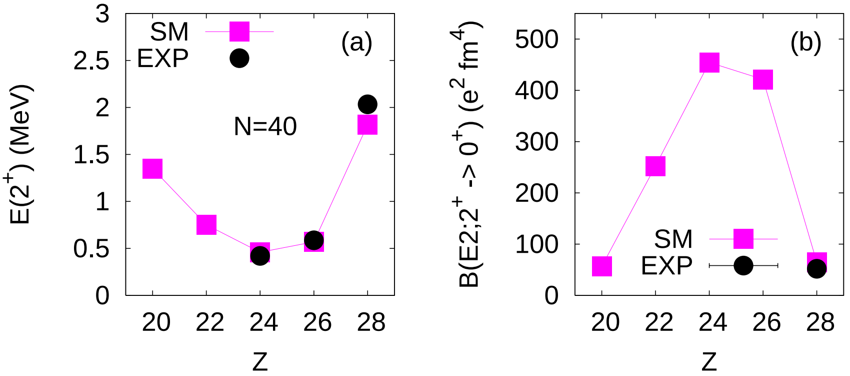
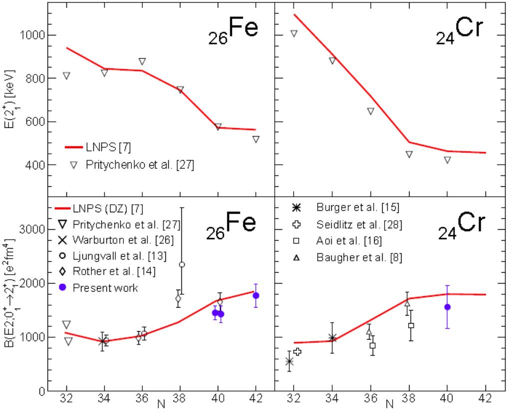


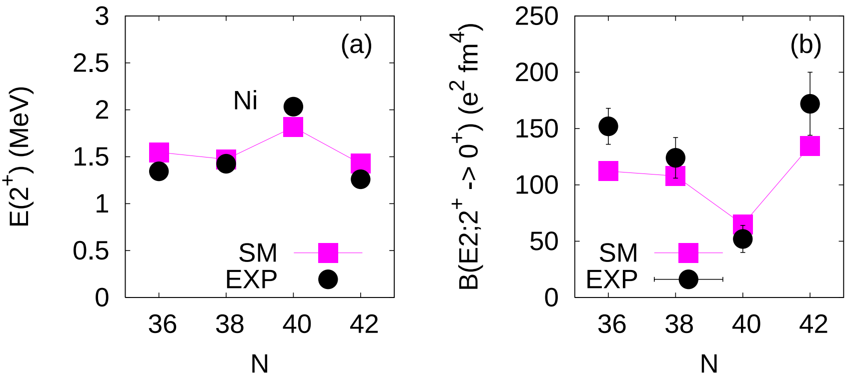
The N=40 isotones



The Iron and Chromium Isotopes



The Nickel Isotopes



^{72}Kr, and shape entanglement

- It is common lore to speak of prolate-oblate or prolate-spherical coexistence when an excited 0^+ state appears at very low energy. This is the case in ^{72}Kr, whose first excited state is a 0^+ at 671 keV followed by a 2^+ at 710 keV. The very large B(E2) of the transition $4^+ \rightarrow 2^+$, 2100 e^2fm^4, strongly suggest that the 2^+ belongs to a prolate band which extends up to J=16^+. But, if so, where is the band head? And why is the $2^+ \rightarrow 0^+$; 810(150) e^2fm^4 smaller that the value expected in the prolate case?
- If we follow down the J(J+1) sequence from the upper part of the band we should expect it 250 keV below the 2^+, which is very close to the experimental excitation energies of the 2^+ in ^{76}Sr and ^{80}Kr. Obviously the distortion must be due to the mixing of the prolate band-head with a near lying oblate state.

^{72}Kr, prolate oblate mixing, a (very) simple model

- **The first element to take into account is that the oblate and prolate 4p-4h states do not mix directly;** *i.e.* $\langle p|H|o\rangle$ **= 0**
- **The mixing should then proceed through 2p-2h or 6p-6h states. Lets take these to be represented by an auxiliary state $|I\rangle$, and further assume that it lies at about ΔE=4 MeV (as our calculations show) and that its coupling to both prolate and oblate states is equal to δ. Taking them degenerated for simplicity, the mixing matrix reads:**
- $$\begin{pmatrix} 0 & 0 & \delta \\ 0 & 0 & \delta \\ \delta & \delta & \Delta E \end{pmatrix}$$

^{72}Kr, prolate oblate mixing, a (very) simple model

- **The mixing can proceed through a cloud of N states, then the matrix is**
- $$\begin{pmatrix} 0 & 0 & \beta & \beta & \beta & \ldots \\ 0 & 0 & \beta & \beta & \beta & \ldots \\ \beta & \beta & \Delta E & 0 & 0 & \ldots \\ \beta & \beta & 0 & \Delta E & 0 & \ldots \\ \beta & \beta & 0 & 0 & \Delta E & \ldots \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \end{pmatrix}$$
- **Which has the same two lowest eigenvalues and eigenvectors if δ=$\sqrt{N}\ \beta$**

^{72}Kr, prolate oblate mixing, a (very) simple model

- For $\delta \sim 1$ MeV, which is a sensible choice, the eigenvalues are: –0.5 MeV, 0.0 MeV and +4.5 MeV. They fit nicely the experimental energies.
  The eigenstates corresponding to the two lower eigenvalues are:
  $|0^+_1\rangle$ = 43% $|p\rangle$ + 43% $|o\rangle$ + 16% $|I\rangle$ and
  $|0^+_2\rangle$ = 50% $|p\rangle$ + 50% $|o\rangle$
- Therefore, the B(E2)($2^+ \rightarrow 0^+_1$) will be approximately one half of the expected value for the prolate band in full accord with the experimental data
- What is the shape of an object which is an even mixture of prolate and oblate? What is the nature of this mixing of shapes? Or should we speak of a shape entangled state?

Conclusions

- The physics around magic or semi-magic closures depends of subtle balances between the spherical mean field and the (very large) correlation energies of the open shell configurations at play
- There is a common mechanism explaining the appearance of ”islands of inversion/deformation” in nuclei with large neutron excess, and shape coexistence usually shows up as a its portal
- The ”islands of inversion” at N=20 and N=28 merge in the Magnesium isotopes.
- ^{68}Ni is a case of triple coexistence, precursor of the N=40 island of inversion
- In some cases, the concepts of shape coexistence and shape mixing don not describe the nuclear behavior which looks more like a kind of shape entanglement