W^+W^- + jet at NLO

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.

based on

.

J. Campbell, D. Miller, TR

[Phys.Rev. D92 (2015) 1, 014033]

and work in progress

IKTP, TU Dresden

QCD, EW and tools at 100 TeV

CERN

7.10.2015

WW + jet: Motivation from experiment

WW [+jet(s)] at the LHC (and beyond)

- Measurement of **WW production cross section** [e.g. ATLAS, JHEP01(2015)049; CMS, 1507.03268]
- $h \rightarrow W\,W$ measurement [e.g. ATLAS, arXiv:1507.04548; CMS, EPJC 75 (2015) 212]
- **spin-/ parity determination** of Higgs [e.g. ATLAS, arXiv:1506.05669; CMS, PRD 92(2015) 012004]
- limits on **anomalous couplings** [e.g. ATLAS, JHEP01(2015)049 ; CMS, arXiv: 1507.03268]
- background for **BSM searches** (e.g. heavy scalars) [e.g. ATLAS, 1509.00389; CMS, arXiv:1504.00936]
- ...
- **K-factors** $\sim 1.2 - 1.8$ [depending on analysis details, cuts, etc...]

[listed are most recent publications]

in more detail...

Process we are interested in

$$p\,p \;\rightarrow\; W^+\,W^-\,\textbf{jet} \;\rightarrow\; (\ell\bar{\nu}_\ell)\;(\bar{\ell}'\,\nu_{\ell'})\;\textbf{jet}$$

at NLO, offshell W's, spin correlations

omitting g g induced contributions

obviously, not the first calculation...

- **previous results:** Campbell, Ellis, Zanderighi [CEZ] (2007); Dittmaier, Kallweit, Uwer [DKU] (2008/ 2010), Sanguinetti/ Karg [BGKKS] (2008)
- **together with shower merging/ matching:** Cascioli ea (2014) in Sherpa/ OpenLoops framework
- also **"ad hoc" available from automatized tools** (personally tested: MG5/aMC@NLO, others probably similar...)

WW+jet @ NLO

Why (yet) another calculation ??

- Main motivation: want to have a **fast and stable code**

⇒ important as **ingredient for NNLO calculations**

⇒ a lot of (recent) progress here, Chachamis ea (2008), Gehrmann ea (2014/ 2015), von Manteuffel, Tancredi (2015), Caola ea (2015),...]

⇒ our approach: use **unitarity-based techniques**, derive **completely analytic expressions**

- tool/ user-interface: ⇒ **implementation in MCFM**

Unitarity methods: a brief recap

Unitary methods: basic idea

$$\mathcal{A}(\{p_i\}) = \sum_j d_j I_4^j + \sum_j c_j I_3^j + \sum_j b_j I_2^j + R\,.$$

$\Rightarrow$ know that all one-loop calculations can be reduced to **integral basis, + rational terms** [Passarino, Veltman, '78]

$\Rightarrow$ idea: **project out coefficients in front of basis integrals by putting momenta in the loop on mass shell**

(Bern, Dixon, Dunbar, Kosower ('94); Britto, Cachazo, Feng ('04))

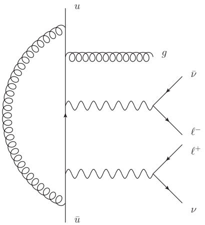
- putting 2/3/4 particles on their mass shell projects out coefficients of a **bubble/ triangle/ box** contribution

WWj @ NLO in more detail

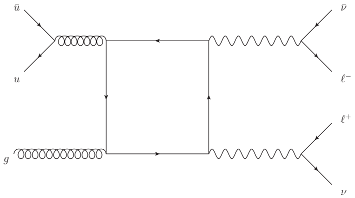
We consider

$$q\,\bar{q} \;\rightarrow\; W^+\,W^-\,g \quad [+\text{ permutations}]$$

diagram classes



(a)



(b)

[+ diagrams for $q\,\bar{q} \;\rightarrow\; (Z/\gamma)\,g \;\rightarrow\; ...$]

Real radiation processes [WW+ dijet]

- ... cf. previous page, with **additional radiation/ gluon splitting**
- ... but **also** [as usual]: **new initial states,** e.g.

$$g\,g;\; u\,d;\; \bar{u}\bar{d}$$

- the latter: exhibit **LO features**, such as scale depenencies, etc...

Implementation: in practise

- calculation: **fully analytic, using generalized unitarity**

⇒ **fully implemented in MCFM framework**, i.e. in combination with Born, real radiation, ...

⇒ **MCFM output** (distributions/ cuts implementation/ interfaces/ etc...)

[comment: also implemented in multi-core version [Campbell, Ellis, Giele, 2015], now standard]

many cross checks: overall agreement:
amplitude/ coefficient level: 10^{-6} or better
cross section level: always within integration errors
✓

Phenomenology

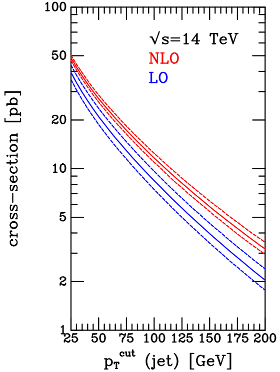
- **total cross section** as a function of $p_{T,\text{jet}}^{\text{cut}}$ for pp collisions **@ 13/ 14/ 100 TeV**
- **differential distributions,** including more specific cuts
- … for **spin- parity determination of Higgs @ 14 TeV**
- … for **searches of extra heavy scalars @ 100 TeV**

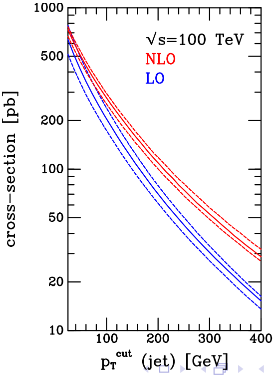
jet definitions: anti-k_T, $p_T^{jet} > 25$ GeV , $|\eta^{jet}| < 4.5$, $R = 0.5$

scales: $\mu_R = \mu_F = \frac{1}{2} \sum_i p_T^i$

Phenomenology: total cross sections, as function of $p_{T,\text{jet}}^{\text{cut}}$

$\sqrt{s}$	σ_{LO} [pb]	σ_{NLO} [pb]
13 TeV	34.9 (-11.0%, +11.4%)	42.9 (-3.7%, +3.7%)
14 TeV	39.5 (-11.0%, +11.7%)	48.6 (-4.0%, +3.8%)
100 TeV	648 (-19.3%, +22.3%)	740 (-9.3%, +4.5%)





More phenomenology: specific studies as background

- e.g. **spin/ parity determination of SM Higgs** (ATLAS, 1503.03643) ⇒ @ 14 TeV
- e.g. **searches for additional scalars at high masses** (CMS, 1504.00936) ⇒ @ 100 TeV

with cuts roughly following above studies...

Results

order	cm energy	no cuts	K	cuts	K
LO	14 TeV	462.0(2)fb		67.12(4)fb	
NLO	14 TeV	568.4(2)fb	1.23	83.91(5)	1.25
LO	100 TeV	6815(1)fb		1237(1)fb	
NLO	100 TeV	7939(5)	1.16	1471(1)fb	1.19

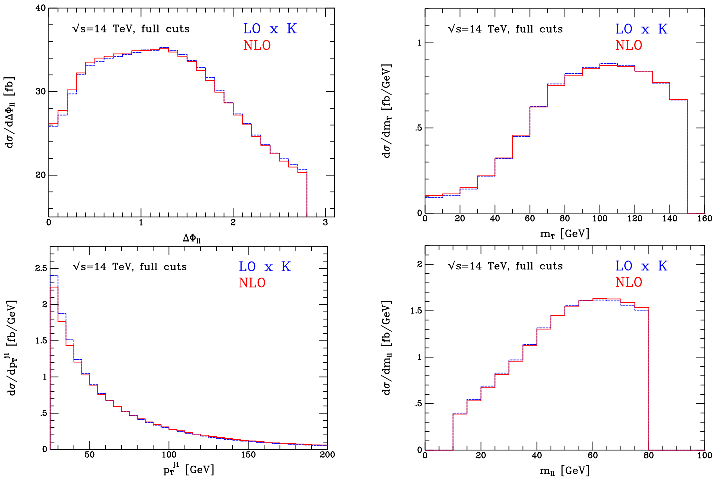
Cuts

variable	14 TeVanalysis	100 TeVanalysis
$p_{\perp,j}$	> 25 GeV	30 GeV
$\lvert\eta_j\rvert$	< 4.5	4.5
η_ℓ	≤ 2.5	2.5
$p_{\perp,\ell_1}$	> 22 GeV	50 GeV
$p_{\perp,\ell_2}$	> 15 GeV	10 GeV
$m_{\ell\ell}$	$\in [10; 80]\,\mathrm{GeV}$	–
$p_\perp^{\text{miss}}$	> 20 GeV	20 GeV
$\Delta\Phi_{\ell\ell}$	< 2.8	
m_T	≤ 150 GeV	$\geq 80\mathrm{GeV}$
$\max[m_T^{\ell_1}, m_T^{\ell_2}]$	> 50 GeV	–

$$\left[m_T^2 \;=\; 2\, p_T^{\ell\ell}\, E_T^{\text{miss}}\, \left(1 - \cos\Delta\Phi(\vec{p}_T^{\,\ell\ell}, \vec{E}_T^{\,\text{miss}})\right)\right]$$

Results for 14 TeV after all cuts

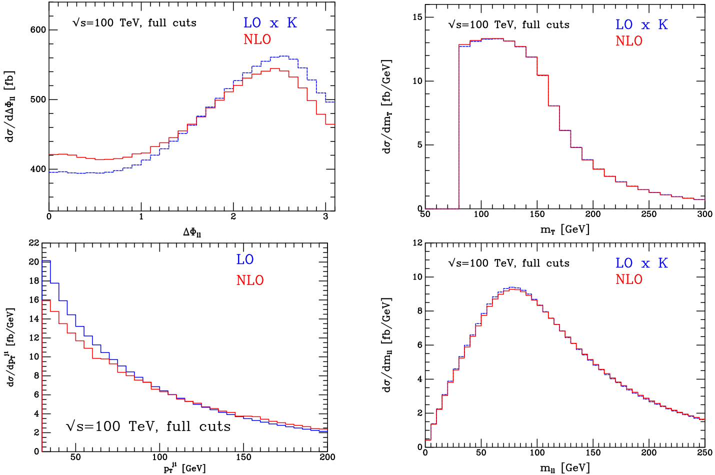
$$\sigma_{\mathrm{LO}} = 67.128(30)\mathrm{fb};\ \sigma_{\mathrm{NLO}} = 83.923(47)\mathrm{fb}$$



[plot: $\sigma_{\mathrm{LO}} \times K$]

Results for 100 TeV after all cuts

$$\sigma_{\text{LO}} = 1237.2(4)\text{fb}; \quad \sigma_{\text{NLO}} = 1472.0(7)\text{fb}$$



[plot: $\sigma_{\text{LO}} \times K$]

Questions one can ask at a 100 TeV hh collider

[loose compilation]

⇒ **Question 1** (for the BSM physicist):
do I have to rethink my cut strategies ??

- tentative answer: in the end, MVA might do it all for you, but maybe worth thinking about it anyways

⇒ **Question 2:** within the SM, is the **behaviour the same at higher energies ??**

[more specific, what about **role of "heavy" objects** for increased c.o.m energies ⇒ probably will be discussed in follow up talks...]

- buzzword for 2: **giant K-factors**

⇒ **Question 3:** ... ??

Setup for 100 TeV studies

jet definitions: anti-k_T, $p_T^{jet} > 25$ GeV , $|\eta^{jet}| < 4.5$, $R = 0.5$
scales: $\mu_R = \mu_F = \frac{1}{2} \sum_i p_T^i$

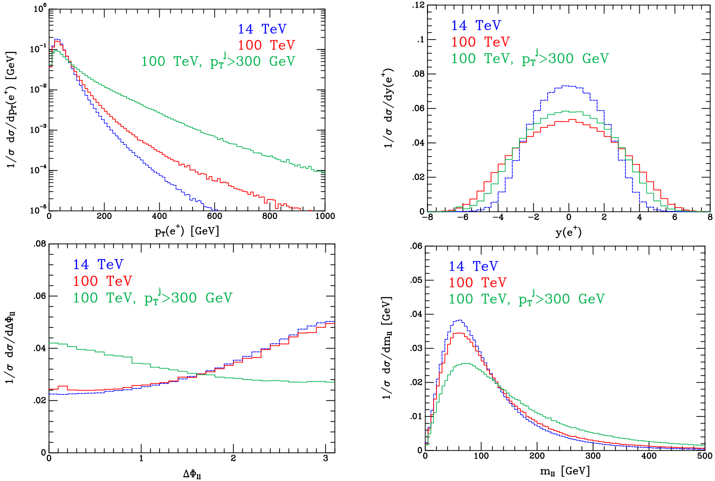
$\sqrt{s}$	$p_{\perp,\text{cut}}^{\text{jet}}$	σ_{LO} [pb]	σ_{NLO} [pb]
14 TeV	25 GeV	$39.5^{+11.7\%}_{-11.0\%}$	$48.6^{+3.8\%}_{-4.0\%}$
100 TeV	25 GeV	$648^{+22.3\%}_{-19.3\%}$	$740^{+4.5\%}_{-9.3\%}$
100 TeV	300 GeV	$30.3^{+11.22\%}_{-10.56\%}$	$53.7^{+8.0\%}_{-7.6\%}$

$\Rightarrow$ note **larger K factor** when $p_{\perp,\text{cut}}$ is increased $\Leftarrow$

[all following results: summarized in a note sent to Giulia and Michelangelo...]

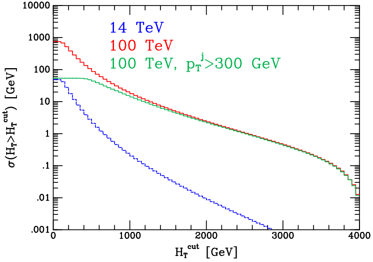
Normalized differential distributions [at 14/ 100 TeV]

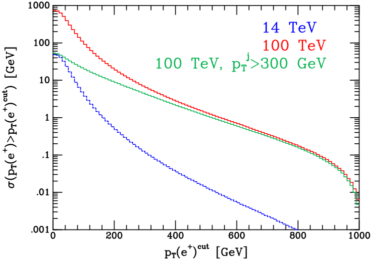
$\sigma_{\rm LO} \sim$ 40/ 650/ 30 pb, K-factors $\sim$ 1.23/1.14/1.77



[plot: $\sigma_{\rm LO} \times K$]

Same in terms of integrated cross sections...





...

$\sigma[\text{pb}]$ as a function of $H_{\perp,\text{cut}}$ $\sigma[\text{pb}]$ as a function of $p_{\perp,\text{lep}}$

answering the **question** **"How many will we still see"**

[future: will extend plot(s) to even higher $H_\perp$ values...]

More on 100 TeV: giant K factors

$\Rightarrow$ **giant K factors: well-known phenomenon**, especially in connotation with "heavy" object

[widely discussed in literature]

- for vector bosons $\Rightarrow$ electroweak Sudakov factors
- what happens ? **NLO/LO larger than assumed/ "OK"** w perturbation theory
- phenomenon **especially interesting when larger $p_\perp$ cuts are applied**
- physical interpretation: understanding: not α_s, but

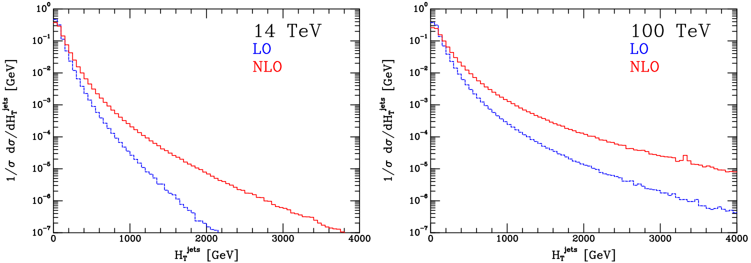
$$\alpha_s \times \log^2 [m_W/p_{\text{cut}}]$$

is the decisive variable

- **new initial states** come into the game

[also changes **scale dependence** (new processes enter at LO)]

Giant K factors: A second example [differential]



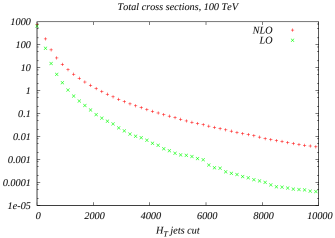
differential distributions of $H_{\perp}$

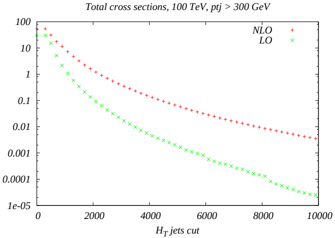
for 14 TeV (left) and 100 TeV (right),

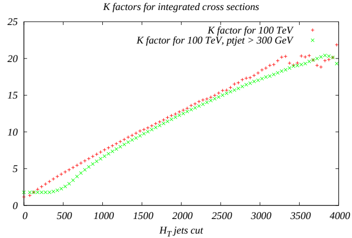
already rescaled

K-factors $\sim$ 1.23 **(14 TeV)/** $\sim$ 1.14 **(100 TeV)**

… and integrated… [preliminary]







K factors **large, but similar** independent of $p_\perp$ cuts

Plans for the future [not finished due to time reasons...]

Disentangle this for our process

(a) initial states also existing at LO (in our calculation):
$q\,\bar{q}$ (ossf), $g\,q$, $g\,\bar{q}$

(b) new initial states at NLO, type I: gg

(c) new initial states at NLO, type II: qq', $q\bar{q}'$, $\bar{q}\bar{q}'$,
(': different flavour)

- in practise: $(b), (c)$ often suppressed using **2nd jet veto** [well-known procedure...]
- **type II also contribute on cases of same sign W's**
  (obviously in different flavour combinations)

  $\Rightarrow$ **test channel contributions wrt different cut setups** $\Leftarrow$

[all above: excludes WBF channels]

Summary and outlook

$$q\,\bar{q} \rightarrow W^+ W^- g$$

available and implemented in MCFM, running, rendering stable results

- virtual contributions: calculated using **unitarity methods** $\Rightarrow$ **available in analytic format**

$\Rightarrow$ **extensively tested** on coefficient, amplitude, and cross section level ✓

$\Longrightarrow$ for 100 TeV $\Longleftarrow$

$\Rightarrow$ depending on background suppression, **"good" BSM selection criteria can turn back**

$\Rightarrow$ for certain distributions [shown here: $\sigma_{\text{tot},p_{\perp,\text{jet}}>p_{\perp,\text{cut}}}$, H_T] **giant K-factors (re)appear**; behaviour understood [should still be investigated in more detail]

any comments/ questions ?? (just started)

Summary and outlook [OLD]

$$q\,\bar{q} \rightarrow W^+\,W^-\,g$$

available and implemented in MCFM, running, rendering stable results

- virtual contributions: calculated using **unitarity methods** ⇒ **available in analytic format**

⇒ **extensively tested** on coefficient, amplitude, and cross section level ✓

⇒ **important ingredient for NNLO calculations**, ready to be used

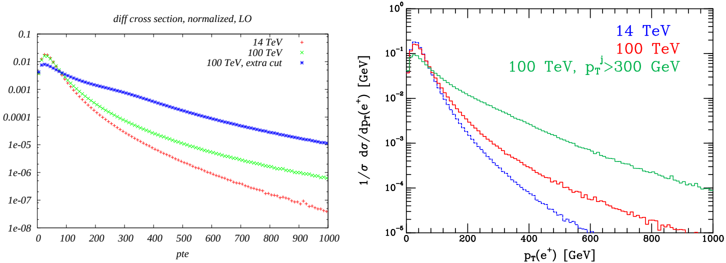
⇒ obviously, **similarly useful for stand alone NLO calculations**

- provided sample applications for typical **Higgs spin/ parity studies @ 14 TeV, heavy scalar searches @ 100 TeVpp colliders**

⟹ **Thanks for listening** ⟸

Appendix

pt for lepton, tree and loop



$q\,\bar{q}'$ states

- W^+ : $u \rightarrow W^+ d$, $\bar{d} \rightarrow W^+ \bar{u}$
- W^- : $\bar{u} \rightarrow W^- \bar{d}$, $d \rightarrow W^- u$
- W^+W^+ initial states:

$$u\,u;\,u\,\bar{d};\,\bar{d}\bar{d}$$

- W^-W^- initial states:

$$d\,d;\,d\,\bar{u};\,\bar{u}\bar{u}$$

- W^-W^+ initial states:

$$u\,\bar{u};\;d\,\bar{d};\,u\,d;\;\bar{d}\,u$$

discussion restricted to first generation for simplicity

Other (related) (reduction) methods and implementations

- purely analytic: **generalized unitarity** [Britto, (Buchbinder), Cachazo, Feng (2005, 2006); Britto, Feng, Mastrolia (2006), Forde (2007), Badger (2009), Mastrolia (2009), ...]
- other widely used approach: **reduction on the amplitude level** [del Aguila, Pitta (2004); Ossola, Papadopoulos, Pittau (2006)]
- **implemented** in many (publicly available) codes: CutTools (OPP, 2007), Samurai (Mastrolia ea, 2010), Gosam (Cullen ea, 2011), MadLoop/aMC@NLO (Hirschi ea, 2011, Frederix ea, 2011), Helac-NLO (van Hameren ea, 2010; Bevilacqua ea, 2013)
- **many other important generic NLO codes and tools,** build around reduction/ recursion principles: Blackhat (Berger ea, 2008), Rocket (Giele, Zanderighi, 2008), Golem (Binoth ea, 2008) NGluon (Badger ea, 2011), OpenLoops (Cascioli ea, 2011), PJFry (Fleischer ea, 2011), Collier (Denner ea, 2014), Ninja (Peraro, 2014),...

Unitarity methods: purely analytic approaches

- as you start with a $d(4)$-dimensional loop integral, **cutting 4 legs is easier than cutting 2**
- **boxes** $\Rightarrow$ straightforward, using **quadruple cuts with complex momenta (BCF)**
- **triangles** $\Rightarrow$ relatively straightforward, using **Fordes method**
- **bubbles** $\Rightarrow$ can get quite complicated, use **spinor integration (BBCF)**
- **rational parts** $\Rightarrow$ long but OK, use **effective mass term (Badger)**

Previous NLO calculations in the SM using analytic expressions from unitarity methods in MCFM

... on the amplitude level ...

- $e\,e \rightarrow 4$ **quarks**: Bern, Dixon, Kosower, Weinzierl (1996); Bern, Dixon, Kosower (1997)
- **Higgs and four partons** (in various configurations): Dixon, Sofianatos (2009); Badger, Glover, Mastrolia, Williams (2009); Badger, Campbell, Ellis, Williams (2009)
- $t\,\bar{t}$ **production**: Badger, Sattler, Yundin (2011)

... generalized unitarity implemented ...

- **Higgs + 2 jets** Campbell, Ellis, Williams (2010)
- **W + 2 b-jets** Badger, Campbell, Ellis (2011)
- $g\,g \rightarrow W\,W$ Campbell, Ellis, Williams (2011, 2014)
- $\gamma\gamma\gamma$ Campbell, Williams (2014)
- $\gamma\gamma\gamma\gamma$ Dennen, Williams (2014)

WWj @ NLO w/ unitarity: complexity

	(a)	(b)
boxes	13	1
triangles	8	4
bubbles	18	2
rational	13	5

Table: **Number of independent** (via singularity structure and/ or symmetries) **coefficients** [neglecting contributions from Z/γ current]

- involving **1,2,3-mass boxes and triangles**,
- **bubbles: 16 different underlying structures**, involving (0/1/2) quadratic poles, e.g. [terms before spinor integration]

$$\frac{[\ell a]^2\,[\ell b]\,[\ell c]}{[\ell d][\ell e]\,\langle\ell|P|\ell]^4},\quad \frac{[\ell a]\,[\ell b]\,[\ell c]}{\langle\ell|P|\ell]^4\,\langle\ell|Q|\ell]},\quad \frac{[\ell a]\,[\ell b]\,[\ell c]\,[\ell d]}{\langle\ell|P|\ell]^4\,\langle\ell|Q|\ell]\,\langle\ell|Q_2|\ell]},\quad \cdots$$

Coefficients: three-massive box

$$d_4\left(s_{56}, s_{34}, 0, s_{17}; s_{127}, s_{234}\right) = \frac{1}{s_{34}-m_W^2}\,\frac{1}{s_{56}-m_W^2}\,\frac{\langle 12\rangle^2\,[2|P|2\rangle}{\langle 27\rangle\,\langle 17\rangle}\times$$
$$\left([42]-\frac{\langle 2|P|4]}{D_1}\right)\left(\langle 3|2+4|6]-\frac{\langle 23\rangle\langle 2|P|6]}{D_1}\right)\left(\frac{[71]\langle 15\rangle}{\langle 2|P|7]}+\frac{\langle 25\rangle}{D_1}\right)$$

$$P \;=\; s_{17}\,p_{34}+s_{234}\,p_{17},\; D_2 \;=\; [2|(3+4)\,(1+7)|2],\; D_1 \;=\; \langle 2|(3+4)\,(1+7)|2\rangle.$$

- in principle: also contributions with second denimonator $D_2 = [2|(3+4)\,(1+7)|2]$ (here: $=0$)
- $D_1\, D_2 \sim$ Gram determinant

Coefficients: easiest bubble and triangle

$$\mathbf{I_2^{LC}(s_{156})} \sim$$

$$\frac{\langle 65\rangle\,[43]}{\langle 27\rangle} \times \left\{ \frac{\langle 73\rangle^2\,\langle 7|P|6]\,[76]}{\langle 7|P|1]\,\langle 7|P|7]} \left[\frac{1}{\langle 7|P|7]} \left(\frac{[7|P|3\rangle}{\langle 37\rangle} + \frac{[76]\,s_{156}}{2\,\langle 7|P|6]} \right) + \frac{[1|P|3\rangle}{\langle 37\rangle\,[1|P|7\rangle} \right.\right.$$

$$\left.\left. - \frac{\langle 15\rangle[56]\,[1|P|3\rangle^2}{s_{156}\,\langle 1|P|1]\,[1|P|7\rangle} \left[\frac{\langle 15\rangle[56]}{2\,\langle 1|P|1]} + \frac{\langle 7|P|6]}{[1|P|7\rangle} \right] \right\}, \quad P = p_{156}$$

$$\mathbf{I_3^{LC}(s_{34}, s_{27}, s_{156})} \quad \sim \quad \frac{1}{2} \sum_{\gamma=\gamma_{1,2}} \frac{s_{27}[4K_2^\flat][72][65]\langle K_1^\flat 2\rangle\langle K_1^\flat 3\rangle\langle 15\rangle^2}{(\gamma - s_{27})\,[7K_2^\flat]\;\langle K_1^\flat 1\rangle\langle K_1^\flat 7\rangle\;\langle 27\rangle}$$

where

$$K_1^\flat = \frac{\gamma\,[\gamma\,p_{27} + s_{27}\,p_{34}]}{\gamma^2 - s_{27}\,s_{34}}, \quad K_2^\flat = -\frac{\gamma\,[\gamma\,p_{34} + s_{34}\,p_{27}]}{\gamma^2 - s_{27}\,s_{34}},$$

$$\gamma_{1,2} = p_{27} \cdot p_{34} \pm \sqrt{(p_{27} \cdot p_{34})^2 - s_{27}\,s_{34}}$$

One slide of self-commercial

- calculation of **bubble-coefficients: process-independent**

$\Rightarrow$ **mathematica-based library,** with (all) librarised poles for $\sim$ 20 different structures

$\Rightarrow$ can **apply these completely straightforward to any other calculation** where the same structures appear in bubble (and can also obviously extend this)

- current interface: **me**
- future plan: **make public** in librarized format

all tested (✓)

but obviously not every possible structure available at the moment

Basis integrals, type (a)

$D^{(1)}$	$I_4(0,0,s_{56},s_{234};s_{17},s_{156})$
$D^{(2)}$	$I_4(s_{34},s_{56},0,s_{27};s_{127},s_{156})$
$D^{(3)}$	$I_4(0,0,s_{34},s_{156};s_{27},s_{234})$
$D^{(4)}$	$I_4(s_{56},s_{34},0,s_{17};s_{127},s_{234})$
$D^{(5)}$	$I_4(0,0,0,s_{127};s_{17},s_{27})$
$C^{(1)}$	$I_3(0,s_{234},s_{156})$
$C^{(2)}$	$I_3(s_{56},s_{17},s_{234})$
$C^{(3)}$	$I_3(s_{34},s_{27},s_{156})$
$C^{(4)}$	$I_3(0,s_{27},s_{127})$
$C^{(5)}$	$I_3(0,s_{17},s_{127})$
$C^{(6)}$	$I_3(0,0,s_{17})$
$C^{(7)}$	$I_3(0,s_{56},s_{156})$
$C^{(8)}$	$I_3(s_{56},s_{34},s_{127})$
$C^{(9)}$	$I_3(0,0,s_{27})$
$C^{(10)}$	$I_3(0,s_{34},s_{234})$
$B^{(1)}$	$I_2(s_{156})$
$B^{(2)}$	$I_2(s_{234})$
$B^{(3)}$	$I_2(s_{56})$
$B^{(4)}$	$I_2(s_{17})$
$B^{(5)}$	$I_2(s_{34})$
$B^{(6)}$	$I_2(s_{127})$

$D^{(6)}$	$I_4(s_{56},s_{34},0,s_{17};s_{127},s_{234})$
$D^{(7)}$	$I_4(0,0,0,s_{127};s_{27},s_{12})$
$D^{(8)}$	$I_4(0,0,0,s_{127};s_{17},s_{12})$
$D^{(9)}$	$I_4(s_{34},0,0,s_{567};s_{234},s_{12})$
$D^{(10)}$	$I_4(s_{34},s_{12},s_{56},0;s_{567},s_{347})$
$D^{(11)}$	$I_4(0,0,s_{56},s_{347};s_{12},s_{156})$
$D^{(12)}$	$I_4(s_{34},s_{12},0,s_{56};s_{567},s_{127})$
$D^{(13)}$	$I_4(0,s_{12},s_{56},s_{34};s_{127},s_{347})$
$C^{(11)}$	$I_3(0,0,s_{27})$
$C^{(12)}$	$I_3(0,s_{34},s_{234})$
$C^{(13)}$	$I_3(0,s_{34},s_{347})$
$C^{(14)}$	$I_3(0,s_{347},s_{156})$
$C^{(15)}$	$I_3(0,s_{127},s_{12})$
$C^{(16)}$	$I_3(0,0,s_{12})$
$C^{(17)}$	$I_3(s_{34},s_{567},s_{12})$
$C^{(18)}$	$I_3(s_{56},s_{347},s_{12})$
$B^{(7)}$	$I_2(s_{567})$
$B^{(8)}$	$I_2(s_{347})$
$B^{(9)}$	$I_2(s_{12})$

Table: scalar integrals of type (a) *left* leading colour and *right* additional subleading color amplitude.

Basis integrals, type (b)

$D^{(10)}$	$I_4(s_{34}, s_{12}, s_{56}, 0; s_{567}, s_{347})$
$D^{(12)}$	$I_4(s_{34}, s_{12}, 0, s_{56}; s_{567}, s_{127})$
$D^{(13)}$	$I_4(0, s_{12}, s_{56}, s_{34}; s_{127}, s_{347})$
$C^{(8)}$	$I_3(0, s_{56}, s_{567})$
$C^{(10)}$	$I_3(s_{56}, s_{34}, s_{127})$
$C^{(13)}$	$I_3(0, s_{34}, s_{347})$
$C^{(15)}$	$I_3(0, s_{127}, s_{12})$
$C^{(17)}$	$I_3(s_{34}, s_{567}, s_{12})$
$C^{(18)}$	$I_3(s_{56}, s_{347}, s_{12})$
$B^{(3)}$	$I_2(s_{56})$
$B^{(5)}$	$I_2(s_{34})$
$B^{(6)}$	$I_2(s_{127})$
$B^{(7)}$	$I_2(s_{567})$
$B^{(8)}$	$I_2(s_{347})$
$B^{(9)}$	$I_2(s_{12})$

Table: Definitions of the scalar integrals that appear in the calculation of the diagrams of type (b), leading colour only.

Appearance of quadratic poles/ square roots

- have a cut leading to a propagator $\sim \frac{1}{s_{\ell_1\, p}}$, where $p^2 \neq 0$ (e.g. $p = p_1 + p_2$)
- for spinor integration $\ell_1 \longrightarrow \frac{P^2}{[\ell|P|\ell\rangle}\,\ell$, where P is momentum over the cut
- i.e., use

$$s_{\ell_1,p} = \langle \ell_1 | \not{p} | \ell_1] + p^2 \rightarrow \frac{P^2 \, \langle \ell | \not{p} | \ell]}{\langle \ell | \not{P} | \ell]} + p^2 = \frac{\langle \ell | \not{Q} | \ell]}{\langle \ell | \not{P} | \ell]},$$

- contributions often appear together with factors $\sim \frac{1}{[\ell|P|\ell\rangle}$

$\Rightarrow$ contains poles $\sim \frac{1}{\langle \ell | PQ \ell \rangle}$

- leads to two possible solutions for $|\ell\rangle$ where $\langle \ell | PQ | \ell \rangle = 0$ (pole)

Electroweak parameters

m_W	80.385 GeV	Γ_W	2.085 GeV
m_Z	91.1876 GeV	Γ_Z	2.4952 GeV
e^2	0.095032	g_W^2	0.42635
$\sin^2\theta_W$	0.22290	G_F	0.116638×10^{-4}

Effect of neglecting diagrams containing Higgs/ diagrams with top loops

- im MG5/aMC@NLO: top and Higgs included
- check: run with $m_t\, m_H \times 10\,(100)$
- results

calculation	parameters	σ^{NLO} [pb]
MCFM	default	14.571 (18)
MG5	default	14.547 (19)
MG5	$m_h \times 10, m_t \times 10$	14.615 (21)
MG5	$m_h \times 100, m_t \times 100$	14.563 (19)
DKU	default	14.678 (10)

Other (related) (reduction) methods [non-exhaustive listing]

- purely analytic: **generalized unitarity** [Britto, (Buchbinder), Cachazo, Feng (2005, 2006); Britto, Feng, Mastrolia (2006), Forde (2007), Badger (2009), Mastrolia (2009), ...]
- other approaches: **recursion/ reduction methods** [Berends, Giele (1987); del Aguila, Pitta (2004); Ossola, Papadopoulos, Pittau (2006)]
- **numerical implementions** in many (publicly available) codes: (in order of appearance) CutTools (OPP, 2007), Samurai (Mastrolia ea, 2010), Gosam (Cullen ea, 2011), MadLoop/aMC@NLO (Hirschi ea, 2011, Frederix ea, 2011), Helac-NLO (van Hameren ea, 2010; Bevilacqua ea, 2013)
- other numerical implementations (in order of appearance): Blackhat (Berger ea, 2008), Rocket (Giele, Zanderighi, 2008), NGluon (Badger ea, 2011), OpenLoops (Cascioli ea, 2011), Ninja (Peraro, 2014)