The quantum/stochastic ϕ in de Sitter

Gerasimos Rigopoulos (Newcastle University)

Work with Bjoern Garbrecht, Yi Zhu, Florian Gautier

Cosmo 2015
Warsaw 9 September 2015

Papers

- B. Garbrecht & G. Rigopoulos 1105.0418
- B. Garbrecht, G. Rigopoulos & Y. Zhu 1310.0367
- B. Garbrecht, F. Gautier, G. Rigopoulos & Y. Zhu 1412.4893
- G. Rigopoulos 1305.0229

Motivation

Scalar field(s) + Gravity the main players of early universe models.

IR physics beyond leading order (tree level) not quite understood.

Simple model:

$$V(\phi) = \frac{1}{2}m^2\phi^2 + \frac{\lambda}{4!}\phi^4 \quad + \quad \text{de Sitter}$$

IR divergences for $m = 0$.
Perturbation expansion meaningful only for

$$m^2 \ll H^2\,, \quad \lambda \ll m^4/H^4$$

Is this all we can do?

Scalar field ϕ in de Sitter

$$\mathcal{L} = \sqrt{-g}\left[\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)\right], \quad ds^2 = -dt^2 + e^{Ht}d\mathbf{x}^2$$

Starobinsky proposed

$$\ddot{\hat{\phi}} + 3H\dot{\hat{\phi}} + \frac{\partial V}{\partial\hat{\phi}} = 0 \qquad\qquad \dot{\phi}(t,\mathbf{x}) + \frac{1}{3H}\frac{\partial V}{\partial\phi} = \xi$$

$$\Rightarrow$$

$$\left[\hat{\phi}(\mathbf{x},t)\,,\,\dot{\hat{\phi}}(\mathbf{x}',t)\right] = i\hbar\frac{\delta(\mathbf{x}-\mathbf{x}')}{e^{3Ht}} \qquad\qquad \langle\xi(t)\xi(t')\rangle = \frac{H^3}{4\pi^2}\delta(t-t')$$

QFT

Begin at t_i with state $|\psi_i\rangle$ and some Hamiltonian $\hat{\mathcal{H}}$. Compute

$$\langle \hat{\mathcal{O}}(\phi_{t'})\hat{\mathcal{O}}(\phi_t)\rangle$$

$$\langle \psi_i \left| \mathcal{U}^\dagger(t', t_i)\mathcal{O}(t_i)\mathcal{U}(t', t_i)\, \mathcal{U}^\dagger(t, t_i)\mathcal{O}(t_i)\mathcal{U}(t, t_i)\right| \psi_i\rangle =$$

$$\langle \psi_i \left| \mathcal{U}^\dagger(t', t_i)\mathcal{O}(t_i)\mathcal{U}^\dagger(t', T)\, \mathcal{U}(T, t)\mathcal{O}(t_i)\mathcal{U}(t, t_i)\right| \psi_i\rangle$$

$$Z = \int D\phi_+ D\phi_-\, e^{iS[\phi_+] - iS[\phi_-]}$$

Schwinger-Keldysh, or CTP, or "in - in" formalism. We call it *Amphichronous* QFT

Stochastic dynamics

$$\dot{\phi} + \frac{\partial_\phi V}{3H} = \xi(t),$$

$$\langle \xi(t)\xi(t')\rangle = \frac{H^3}{4\pi^2}\delta(t-t')\,,$$

$$\mathcal{Z} \equiv \int D[\xi]\mathrm{e}^{-\frac{1}{2}\int dt \xi^2 \frac{4\pi^2}{H^3}} \int D[\phi]\delta(\dot{\phi} + \partial_\phi V/3H - \xi) = 1$$

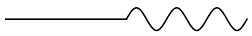
$$\Downarrow$$

$$\mathcal{Z} = \int D[\phi]D[\psi]\mathrm{e}^{-\int dt \left\{ \frac{\mathrm{i}}{H^2}\psi\left(\dot{\phi} + \frac{\partial_\phi V}{3H}\right) + \frac{1}{8\pi^2 H}\psi^2 \right\}}\,.$$

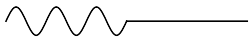
Diagram Elements

QFT Feynman diagrams

$$\partial_t^2 + 3H\partial_t - \frac{\nabla^2}{a^2} + m^2$$

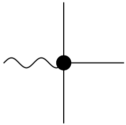


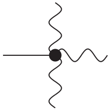
$-\mathrm{i}G^R(x,x')$



$-\mathrm{i}G^A(x,x')$

$F(x,x')$

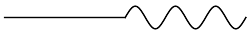




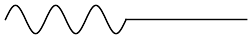
$-\mathrm{i}\frac{\lambda}{2}\int d^4x\ a^4(x)$

Stochastic diagrams

$$\partial_t + \frac{m^2}{3H^2}$$

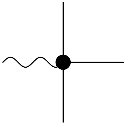


$-\mathrm{i}G^R(t,t')$



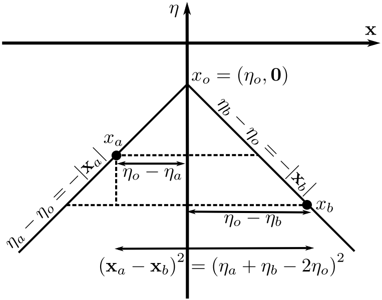
$-\mathrm{i}G^A(t,t')$

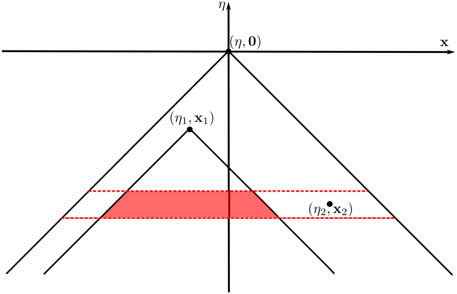
$F(t,t')$



$-\mathrm{i}\frac{\lambda}{3H^3}\int d\tau$

Any QFT diagram coincides with the corresponding stochastic diagram in the IR. Contributions from the lightcone and large spacelike separations are suppressed.





No IR divergences

Starobinsky matches deSitter QFT in the IR order by order.

But Starobinsky can be re-summed non-pertubatively with no IR divergences.

Late times:

$$P(\phi) \propto e^{\frac{-8\pi^2 V(\phi)}{3H^4}}$$

Mass $\neq 0$

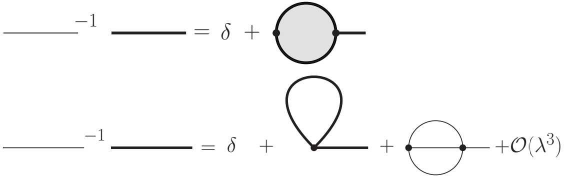
Dynamical mass generation: $m^2_{\rm dyn} \sim \sqrt{\lambda} H^2$

Stochastic (still heuristic)

$$\langle \frac{d}{dt}\phi(t)\phi(0)\rangle + \lambda\langle\phi^3(t)\phi(0)\rangle/(18H) \approx 0\,,$$

$$\frac{d}{dt}\langle\phi(t)\phi(0)\rangle \sim \frac{\lambda}{H}\langle\phi^2(t)\rangle\langle\phi(t)\phi(0)\rangle\,.$$

QFT Schwinger-Dyson



Correlation length $\neq \infty$

Decay of correlations

Stochastic (use de Sitter invariance of QFT)

$$\langle \phi(t,\mathbf{x})\phi(t',\mathbf{x})\rangle \simeq \frac{3H^4}{8\pi^2 m^2} e^{-\frac{m^2}{3H}|t-t'|}$$
$$\Updownarrow$$
$$\langle \phi(t,\mathbf{x})\phi(t,\mathbf{x}')\rangle \simeq \frac{3H^4}{8\pi^2 m^2} \left(\frac{1}{a^2 H^2 |\mathbf{x}-\mathbf{x}'|^2}\right)^{\frac{m^2}{3H^2}}$$

QFT Propagator

$$i\Delta(y) = \frac{H^2}{4\pi^2}\left[-\frac{1}{y} + \frac{3H^2}{2m^2}\left(-\frac{1}{y}\right)^{\frac{m^2}{3H^2}} + \mathcal{O}\left(y^{-2}\frac{m^2}{H^2}\right)\right]$$

Thermal stochastic jitter?

Horizons in GR $\Leftrightarrow$ Thermodynamics

- Black Holes
- Cosmology

de Sitter temperature

$$T_{DS} = \frac{H}{2\pi}$$

Thermodynamic interpretation of T_{DS} on long wavelengths?

Inflationary fluctuations do not have a thermal spectrum.

Fluctuation and Dissipation are dual processes in nature

- Brownian motion

$$m\frac{d\mathbf{v}}{dt} + \gamma\mathbf{v} = \vec{\xi}(t)$$

$$\langle \xi_i(t)\xi_j(t')\rangle = 2\gamma T\delta(t-t')\delta_{ij}\,,$$

- Johnson-Nyquist noise

$$L\frac{d^2Q}{dt^2} + R\frac{dQ}{dt} + \frac{Q}{C} = U_{\rm ext} + \xi_U(t)$$

$$\langle \xi_V(t)\xi_V(t')\rangle = 2RT\delta(t-t')$$

In both cases the magnitude of the friction term in the dynamical equations (dissipation) determines along with the temperature the amplitude of the thermally induced random force (fluctuation). This is a facet of the (classical) fluctuation-dissipation theorem of statistical physics.

Consider full Klein Gordon

$$\ddot{\phi} + 3H\dot{\phi} - e^{-2Ht}\nabla^2\phi + \frac{\partial V}{\partial \phi} = 0\,,$$

Argue by analogy for physical scales $r > 1/H$ ($r = e^{Ht}x$) where "friction" H is important

$$\ddot{\phi} + 3H\dot{\phi} + \frac{\partial V}{\partial \phi} = \xi(t)$$

where the amplitude of the fluctuation term should conform to a fluctuation dissipation relation

$$\langle \xi(t)\xi(t') \rangle = \frac{(2 \times 3H \times T_{DS})}{\frac{4\pi}{3}\left(\frac{1}{H}\right)^3}\,\delta(t - t')\,.$$

- A light minimally coupled scalar in DS executes Brownian motion in equilibrium with temperature $T = \frac{H}{2\pi}$.
- Kinetic energy is significant. For $V = \frac{\lambda}{4}\phi^4$:

$$\frac{1}{2}\langle\dot{\phi}^2\rangle = 2\langle V\rangle = \frac{3H^4}{16\pi^2}\,.$$

- Backreaction? Naive guess

$$H(t) = \frac{H}{\left(\frac{9}{16\pi^2}\frac{H^3}{M_p^2}t + 1\right)^{\frac{1}{3}}}\,,$$

Conclusions

- IR QFT = Stochastic Starobinsky for ϕ in de Sitter.
- No IR divergences through dynamical mass $m^2_{\mathrm{dyn}} \sim \sqrt{\lambda} H^2$
- Gravity? (Path integral + better approach to IR effective theory)
- Relation to Euclidean calculations?
- de Sitter invariant stochastic formulation?
- Is IR ϕ in de Sitter thermal - Fluctuation Dissipation relation?
- Backreaction?

Split long- and short- wavelength fields

$$\Phi = \phi(\mathbf{k} \lesssim \epsilon a H) + \varphi(\mathbf{k} \gtrsim \epsilon a H)$$

using a time dependent window function

$$W_k(t) = \left(1 - \frac{k^3}{(\epsilon a H)^3}\right) \frac{1}{H}\, \Theta\!\left[\ln\left(\frac{\epsilon a H}{k}\right)\right]$$

for which

$$\ddot{W}_k(t) + 3H\dot{W}_k(t) = \delta(t - \frac{1}{H}\ln\left(k/\epsilon H\right))$$

Integrate out short fields to obtain Effective Long Wavelength theory with action A

$$e^{iA_{\mathcal{C}}[\phi]} = e^{iS_{\mathcal{C}}[\phi]} \int D\varphi_{\mathcal{C}}\; e^{i\int_{\mathcal{C}} \mathcal{L}_{\rm int}[\phi,\varphi]}\; e^{i\int_{\mathcal{C}} a^3\left[\frac{1}{2}\varphi\hat{\mathcal{O}}\varphi + \varphi\left(\hat{\mathcal{O}}\phi\right)\right]}$$

(Ignoring $\mathcal{L}_{\rm int}$)

$$A = \int a^3 \left[-\psi \left(\ddot{\phi} + 3H\dot{\phi} - \frac{H^2\epsilon^2}{a^2}\nabla^2_{\tilde{\mathbf{x}}}\phi + \frac{\partial V}{\partial \phi} \right) + 2\sum_{m=1}^{\infty} \frac{V^{(2m+1)}}{(2m+1)!} \left(\frac{\psi}{2}\right)^{2m+1} (\epsilon H) \right.$$

$$+ \frac{i}{2}\frac{9H^5}{4\pi^2} \int d^4\tilde{x} d^4\tilde{x}'\, a^3(t) a^3(t')\, \psi(\tilde{x}) \mathcal{N}(\tilde{x}, \tilde{x}') \psi(\tilde{x}')$$

$\tilde{\mathbf{x}} = \mathbf{x}\epsilon H$, $\phi = \frac{1}{2}(\phi_+ + \phi_-)$, $\psi = \frac{(\phi_+ - \phi_-)}{(\epsilon H)^3}$,
$\mathcal{N}(\tilde{x}, \tilde{x}') = \frac{\sin\left(a|\tilde{\mathbf{x}} - \tilde{\mathbf{x}}'|\right)}{a|\tilde{\mathbf{x}} - \tilde{\mathbf{x}}'|} \delta(t - t')$.

Setting $\mathcal{N}(x, x') \to \delta(t - t') \frac{\delta(\tilde{\mathbf{x}} - \tilde{\mathbf{x}})}{a^3}$

$$\langle \mathcal{Q}(\phi) \rangle = \prod_{\tilde{\mathbf{x}}} \int [D\phi][D\psi]\ \mathcal{Q}(\phi)\ e^{i\int dt a^3 \left[-\psi\left(\ddot{\phi} + 3H\dot{\phi} + \frac{\partial V}{\partial \phi}\right) + \frac{i}{2}\frac{9H^5}{4\pi^2}\psi^2 \right]}.$$

Different spatial points are uncorrelated so focus on the functional for a single point. In reality each "point" corresponds to a region of physical size $R \sim 1/H$.

Effective Theory = (quasi-)Classical Limit + Stochastic dynamics (MSRJD functional)

Perturbation theory $\rightarrow$ loop corrections grow

Full non-perturbative dynamics are equivalent to

$$\ddot{\phi} + 3H\dot{\phi} + \frac{\partial V}{\partial \phi} = \xi$$

where the noise term is gaussian with

$$\langle \xi(t)\xi(t')\rangle = \int [D\xi]\xi(t)\xi(t')e^{-\int dt\, a^3 \left[\frac{2\pi^2}{9H^5}\xi^2\right]} = \frac{9H^5}{4\pi^2}\delta(t-t')\,.$$

For an integrable potential an equilibrium should be reached.

Probability to find the field value ϕ at time t, given the field value ϕ_i at t_i

$$\mathcal{P}\left(\phi, t|\phi_i, t_i\right) = \int_{\phi_i}^{\phi_f} [D\phi][Dy][Dp][D\rho]\ e^{-\int_{t_i}^{t} dt\left[q\dot{\phi}+p\dot{y}-H(p,q,y,\phi)\right]}$$

"Hamiltonian"

$$H(p,q,y,\phi) = \frac{9H^5}{8\pi^2}p^2 - p\left(3Hy + \frac{\partial V}{\partial \phi}\right) + qy$$

With $p = -\partial_y$ and $q = -\partial_\phi$ and normal ordering, "Shrödinger" $=$ Fokker-Planck

$$\partial_t \mathcal{P} = \left(\frac{9H^5}{8\pi^2}\frac{\partial^2}{\partial y^2} + 3H\frac{\partial}{\partial y}y + \frac{\partial V}{\partial \phi}\frac{\partial}{\partial y} - y\frac{\partial}{\partial \phi}\right)\mathcal{P}$$

Equilibrium distn

$$\mathcal{P}(\phi, \dot{\phi}) = \frac{e^{-\frac{8\pi^2}{3H^4}\frac{\dot{\phi}^2}{2}}e^{-\frac{8\pi^2}{3H^4}V}}{N}$$

This is the Maxwell-Boltzmann distribution!