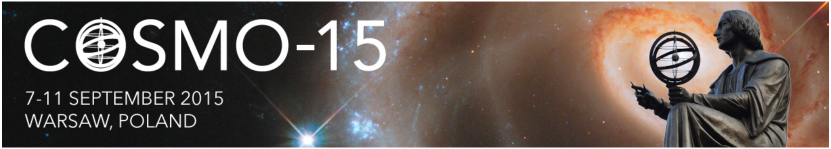
Baryogenesis with Electroweak Symmetry Realised Non-linearly

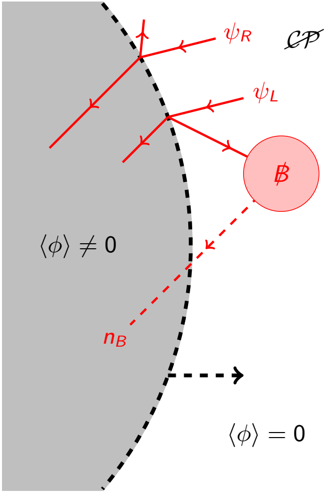
Jason Yue

September 8, 2015



based on: work with A. Kobakhidze and L. Wu

Electroweak Baryogenesis



- Many papers to explain[a]

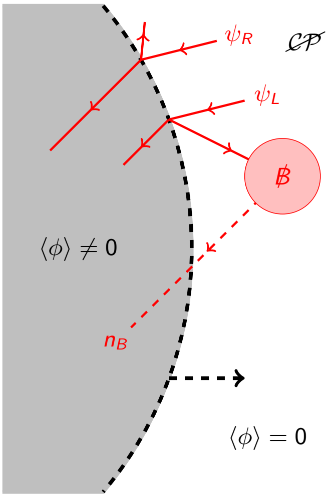
$$\frac{n_B}{s} = (8.59 \pm 0.11) \times 10^{-11} (Planck)$$

- Baryogenesis at the electroweak phase transition[b]

[a] P. A. R. Ade *et al.* [Planck Collaboration], Astron. Astrophys. **571** (2014) A16

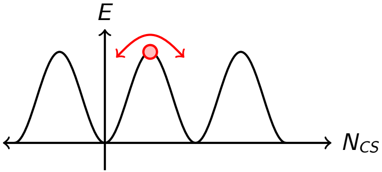
[b] V. A. Kuzmin *et al.*, Phys. Lett. B **155** (1985) 36;
G. R. Farrar and M. E. Shaposhnikov, Phys. Rev. Lett. **70** (1993) 2833;
P. B. Arnold and O. Espinosa, Phys. Rev. D **47** (1993) 3546;
G. W. Anderson and L. J. Hall, Phys. Rev. D **45** (1992) 2685.

Electroweak Baryogenesis



- $\not{B}$ must occur via non-perturbative effects

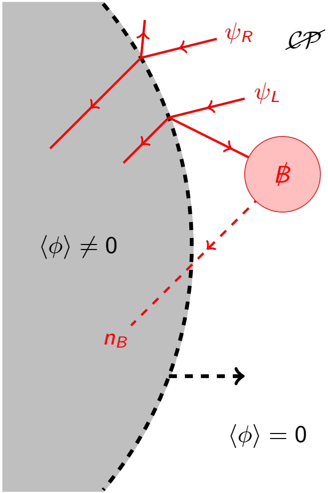
$$\partial_\mu j^\mu_B = \partial_\mu j^\mu_L = n_f \left(\frac{g_2^2}{32\pi^2} \mathrm{Tr}\, \mathbf{W}_{\mu\nu} \tilde{\mathbf{W}}^{\mu\nu} \right)$$



$$N_{CS}(t) := \frac{g_2}{32\pi^2} \int d^3x\, \epsilon^{ijk} \mathrm{Tr} \left(\mathbf{W}_i \partial_j \mathbf{W}_k + \frac{2i}{3} \mathbf{W}_i \mathbf{W}_j \mathbf{W}_k \right)$$

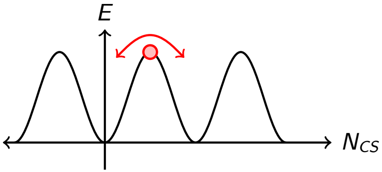
- Instanton suppression lifted by thermal effects

$$E_{sph}(T=0) \sim (5\ \mathrm{TeV}) B \left(\frac{\lambda}{g_2^2} \right)$$



- $\not{B}$ must occur via non-perturbative effects

$$\Delta_B = \Delta_L = n_f(N_{CS}(t_f) - (N_{CS}(t_i))$$

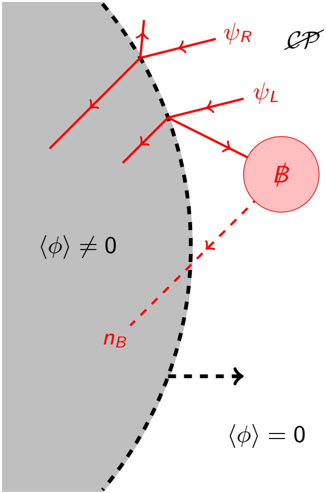


$$N_{CS}(t) := \frac{g_2}{32\pi^2}\int d^3x\ \epsilon^{ijk}\mathrm{Tr}\left(\mathbf{W}_i\partial_j\mathbf{W}_k + \frac{2i}{3}\mathbf{W}_i\mathbf{W}_j\mathbf{W}_k\right)$$

- Instanton suppression lifted by thermal effects

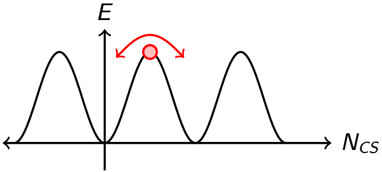
$$E_{sph}(T=0) \sim (5\ \mathrm{TeV})B\left(\frac{\lambda}{g_2^2}\right)$$

Electroweak Baryogenesis



- $\not{B}$ must occur via non-perturbative effects

$$\Delta_B = \Delta_L = n_f(N_{CS}(t_f) - (N_{CS}(t_i))$$



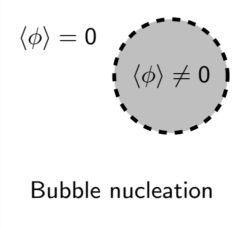
$$N_{CS}(t) := \frac{g_2}{32\pi^2}\int d^3x\ \epsilon^{ijk}\mathrm{Tr}\left(\mathbf{W}_i\partial_j\mathbf{W}_k + \frac{2i}{3}\mathbf{W}_i\mathbf{W}_j\mathbf{W}_k\right)$$

- Sphaleron decoupling in Higgs phase

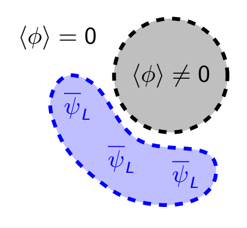
$$\Gamma(T_c) = e^{-\frac{E_{sph}(T_c)}{T_c}} < 1.66\frac{\sqrt{g_*(T)}}{M_P}T_c^2 = H(T_c)$$

Electroweak Baryogenesis

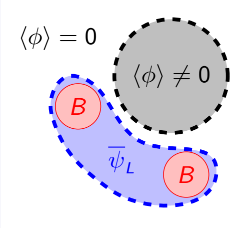
First order phase transition



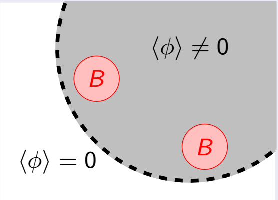
$\mathcal{CP}$ and $\mathcal{C}$ violation



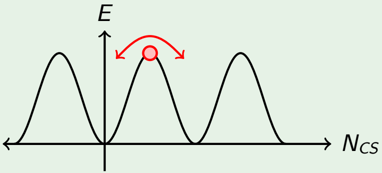
B asymmetry



Bubble expansion



$B + L$ violation with EW Sphaleron

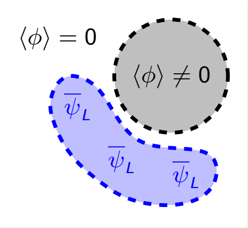


Electroweak Baryogenesis

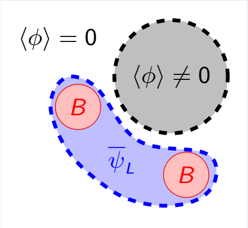
First order phase transition

$m_h < 72$ GeV

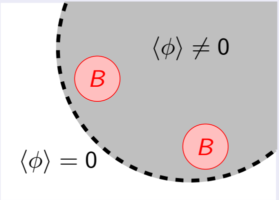
$\mathcal{CP}$ and $\mathcal{C}$ violation



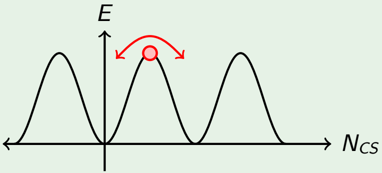
B asymmetry



Bubble expansion



$B + L$ violation with EW Sphaleron



Electroweak Baryogenesis

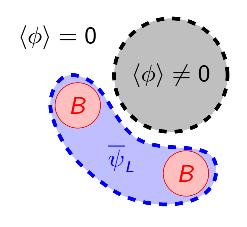
First order phase transition

$m_h < 72$ GeV

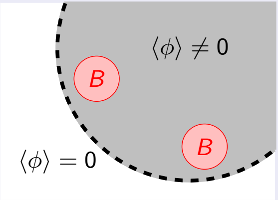
$\mathcal{CP}$ and $\mathcal{C}$ violation

Insufficient CKM

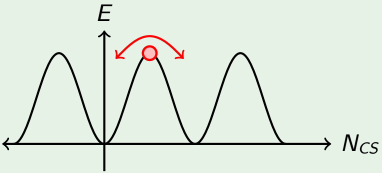
B asymmetry



Bubble expansion



$B + L$ violation with EW Sphaleron



Electroweak Baryogenesis

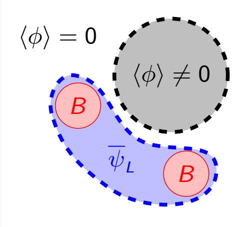
First order phase transition

$m_h < 72$ GeV

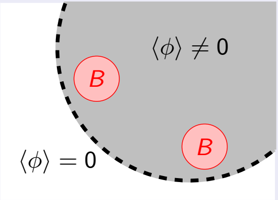
$\mathcal{CP}$ and $\mathcal{C}$ violation

Insufficient CKM

B asymmetry



Bubble expansion



$B + L$ violation with EW Sphaleron

$$\frac{v(T_c)}{T_c} \gtrsim 1.1$$

$$m_h \lesssim 35 \text{ GeV}$$

Remedy (Keeping the SM Particle Content)

- Enhance first order character[a,b]

$$\mathcal{O}_6 = \frac{1}{\Lambda^2}(H^\dagger H)^3$$

- $\mathcal{CP}$-violation[b,c]

$$\mathcal{O}_{t\bar{t}h} = \frac{1}{\Lambda^2}(H^\dagger H)\overline{Q}_L H t_R$$

[a] C. Grojean *et al.*, Phys. Rev. D **71** (2005) 036001; S. W. Ham and S. K. Oh, Phys. Rev. D **70** (2004) 093007; C. Delaunay, C. Grojean and J. D. Wells, JHEP **0804** (2008) 029

[b] D. Bodeker *et al.*, JHEP **0502** (2005) 026; L. Fromme and S. J. Huber, JHEP **0703** (2007) 049; T. Konstandin, Phys. Usp. **56** (2013) 747,S. J. Huber, M. Pospelov and A. Ritz, Phys. Rev. D **75** (2007) 036006

[c] J. Shu and Y. Zhang, Phys. Rev. Lett. **111** (2013) 9, 091801

Non-linear Realisation

- Does the physical Higgs h reside in the $SU(2)_L$ doublet[a]?

$$H = \frac{\rho}{\sqrt{2}}\Sigma, \quad \Sigma := \exp\left[-\frac{i}{2}\left(\sigma^a\pi^a - \mathbb{1}\pi^3\right)\right]\begin{pmatrix}0\\1\end{pmatrix}$$

- SSB

$$\rho(x) = v + h(x)$$

Realisation[a] of $SU(2)_L \otimes U(1)_Y$

- *linearly realised*
  - ▹ operators are constructed in terms of the H field
- *non-linearly realised*
  - ▹ operators are constructed in terms of the h field

---

[a] R. Foot and A. Kobakhidze, Mod. Phys. Lett. A **26** (2011) 461; M. Gonzalez-Alonso, *et al.*, Eur. Phys. J. C **75** (2015) 3, 128

Higgs Potential

- Extra cubic term!

$$V(\rho) = -\frac{\mu^2}{2}\rho^2 + \frac{\kappa}{3}\rho^3 + \frac{\lambda}{4}\rho^4$$

- Leaves the gauge sector unmodified

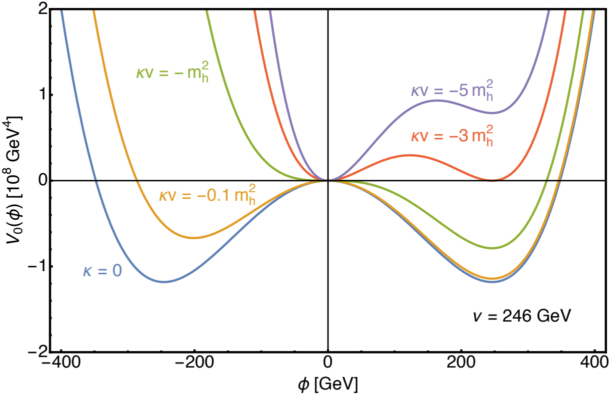
$$\mathcal{L} \supset \frac{\rho^2}{2}(D_i\Sigma)^\dagger(D^i\Sigma)$$

$$m_Z^2(\rho) = \frac{g_2^2 + g_1^2}{4}\rho^2, \quad m_W^2(\rho) = \frac{g_2^2}{4}\rho^2$$

Higgs Potential

- Extra cubic term!

$$V(\rho) = -\frac{\mu^2}{2}\rho^2 + \frac{\kappa}{3}\rho^3 + \frac{\lambda}{4}\rho^4$$



$\mathcal{CP}$-violating Yukawa Couplings

- Lagrangian

$$\mathcal{L}_Y \supset -\left(m_{ij}^{(u)\prime} + \frac{y_{ij}^{(u)}}{\sqrt{2}}\rho\right)\overline{Q}_L^i \Sigma u_R^j - \left(m_{ij}^{(d)\prime} + \frac{y_{ij}^{(d)}}{\sqrt{2}}\rho\right)\overline{Q}_L^i \tilde{\Sigma} d_R^j$$
$$- \left(m_{ij}^{(\ell)\prime} + \frac{y_{ij}^{(\ell)}}{\sqrt{2}}\rho\right)\overline{L}_L^i \Sigma \ell_R^j + h.c.$$

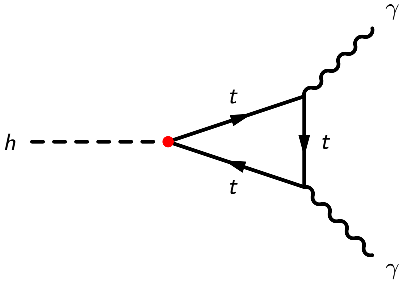
- Simplified model:

$$m_{ij}^{(d,\ell)\prime} = 0, \quad m_{ij}^{(u)\prime} = \mathrm{diag}(0, 0, m_t')$$
$$y_{ij}^{(d,\ell)} = y_{SM}^{(d,\ell)}{}_{ij}, \quad y_{ij}^{(u)} = \mathrm{diag}\left(\frac{\sqrt{2}m_u}{v}, \frac{\sqrt{2}m_c}{v}, y_t e^{i\xi}\right)$$

- $\mathcal{CP}$-violating top-Higgs sector:

$$\mathcal{L}_{\text{pheno}} \supset -\frac{y_t}{\sqrt{2}}\bar{t}(\cos\xi + i\gamma^5 \sin\xi)th + h.c$$

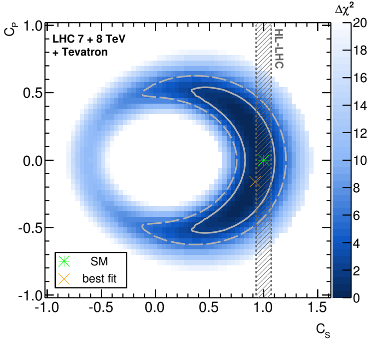
Constraining y_t and ξ with $h \to \gamma\gamma$



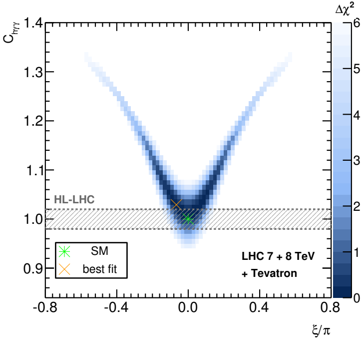
- Modified fermion couplings $(C_S, C_P) := (y_t \cos\xi, y_t \sin\xi)$ will influence the $h\gamma\gamma$, hgg and $hZ\gamma$ loop functions

$$\Gamma_{h\to\gamma\gamma} \approx \frac{m_h^3\alpha^2}{256\pi^3 v^2}\left[\left(-8.32 + \frac{1.83 y_t \cos\xi}{y_t^{SM}}\right)^2 + \left(\frac{2.79 y_t \sin\xi}{y_t^{SM}}\right)^2\right]$$

Constraints on ξ



LHC 7+8 TeV



LHC 7+8 TeV

Allowed 95% C.L region[a]

$C_S \in [-0.1, 1.2]$ and $C_P \in [-0.6, 0.6]$

$y_t/y_t^{SM} \in [0.7, 1.2]$ at $(\xi = 0)$

to

$y_t/y_t^{SM} \in [0.4, 0.6]$ at $(\xi = \pi/2)$

95% C.L — dashed
68% C.L. — solid

[a] A. Kobakhidze *et al.*, JHEP **1410** (2014) 100

Remedy (Nonlinear realisation)

- Enhance first order character[a,b]

$$\mathcal{O}_6 = \frac{1}{\Lambda^2}(H^\dagger H)^3 \quad \longrightarrow \quad V(\rho) \supset \frac{\kappa}{3}\rho^3$$

- $\mathcal{CP}$-violation[b,c]

$$\mathcal{O}_{t\bar{t}h} = \frac{1}{\Lambda^2}(H^\dagger H)\overline{Q}_L H t_R \quad \longrightarrow \quad \mathcal{L}_Y \supset -\left(m_{ij}^{(u)\prime} + \frac{y_{ij}^{(u)}}{\sqrt{2}}\rho\right)\overline{Q}_L^i \Sigma u_R^j$$

[a] C. Grojean *et al.*, Phys. Rev. D **71** (2005) 036001; S. W. Ham and S. K. Oh, Phys. Rev. D **70** (2004) 093007; C. Delaunay, C. Grojean and J. D. Wells, JHEP **0804** (2008) 029

[b] D. Bodeker *et al.*, JHEP **0502** (2005) 026; L. Fromme and S. J. Huber, JHEP **0703** (2007) 049; T. Konstandin, Phys. Usp. **56** (2013) 747,S. J. Huber, M. Pospelov and A. Ritz, Phys. Rev. D **75** (2007) 036006

[c] J. Shu and Y. Zhang, Phys. Rev. Lett. **111** (2013) 9, 091801

Quantum corrections to the potential

- $\overline{\text{DR}}$ scheme

$$V^{(1)}(\rho) = \sum_i \frac{n_i m_i^4(\rho)}{64\pi^2}\left(\ln\left(\frac{m_i^2(\rho)}{\mu_R^2}\right) - \frac{3}{2}\right)$$

- Thermal correction

$$V_T^{(1)}(\rho) = \frac{T^4}{2\pi^2}\sum_{i=W,Z,t,h}\int_0^\infty dx\; x^2 \ln\left[1-(-1)^{2s} e^{-\sqrt{x^2+\beta^2 m_i^2(\phi)}}\right]$$

- High temperature expansion $(m_i^2(\rho) \ll T^2)$

$$J_i(x) = \begin{cases} -\frac{\pi^4}{45} + \frac{\pi^2}{12}x - \frac{\pi}{6}x^{3/2} - \frac{x^2}{32}\ln\frac{x}{c_B} + \mathcal{O}\left(x^6\right), & \text{for bosons} \\ \frac{7\pi^4}{360} - \frac{\pi^2}{24}x - \frac{x^2}{32}\ln\frac{x}{c_F} + \mathcal{O}\left(x^6\right), & \text{for fermions} \end{cases}$$

$$c_B := 16\pi^2\exp(3/2 - 2\gamma_E) \quad \text{and} \quad c_F := \pi^2\exp(3/2 - 2\gamma_E)$$

Quantum corrections to the potential

- $\overline{\text{DR}}$ scheme

$$V^{(1)}(\rho) = \sum_i \frac{n_i m_i^4(\rho)}{64\pi^2}\left(\ln\left(\frac{m_i^2(\rho)}{\mu_R^2}\right) - \frac{3}{2}\right)$$

- Thermal correction

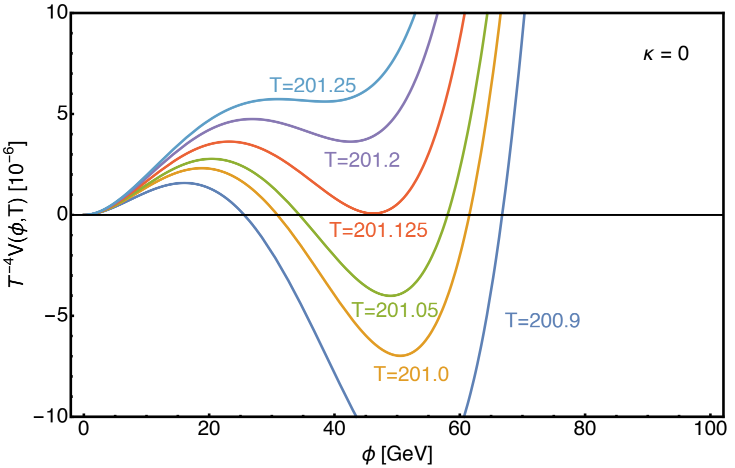
$$V_T^{(1)}(\rho) = \frac{T^4}{2\pi^2}\sum_{i=W,Z,t,h} n_i J_i\left[\frac{m_i^2(\phi)}{T^2}\right]$$

- High temperature expansion $(m_i^2(\rho) \ll T^2)$

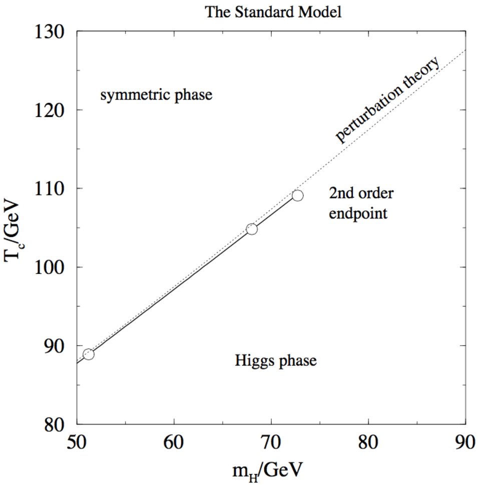
$$J_i(x) = \begin{cases} -\frac{\pi^4}{45} + \frac{\pi^2}{12}x - \frac{\pi}{6}x^{3/2} - \frac{x^2}{32}\ln\frac{x}{c_B} + \mathcal{O}\left(x^6\right), & \text{for bosons} \\ \frac{7\pi^4}{360} - \frac{\pi^2}{24}x - \frac{x^2}{32}\ln\frac{x}{c_F} + \mathcal{O}\left(x^6\right), & \text{for fermions} \end{cases}$$

$$c_B := 16\pi^2\exp(3/2 - 2\gamma_E) \quad \text{and} \quad c_F := \pi^2\exp(3/2 - 2\gamma_E)$$

Thermal potential — SM

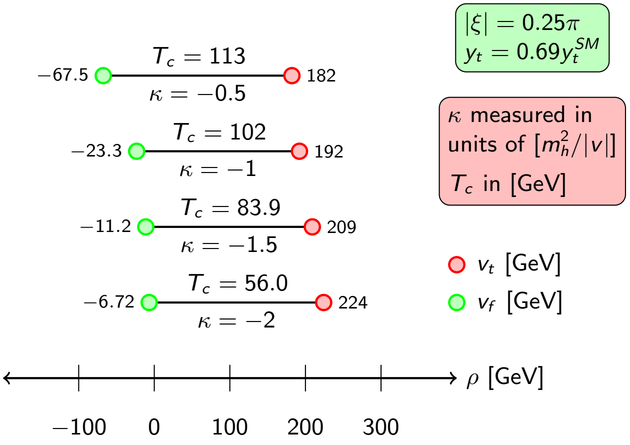


Thermal potential — SM



[Nucl. Phys. Proc. Suppl. 73 (1999) 180]

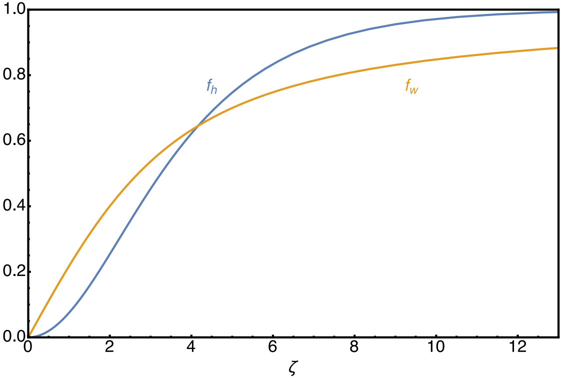
Electroweak Baryogenesis



Sphaleron

- Energy of sphaleron[a] with $O(3)$ *Ansatz*:

$$E_{sph} = \frac{4\pi|v|}{g_2}\int_0^\infty d\zeta \left[4\left(\frac{df_W}{d\zeta}\right)^2 + \frac{8}{\zeta^2}[f_W(1-f_W)]^2 + \frac{1}{2}\zeta^2\left(\frac{df_h}{d\zeta}\right)^2 + [f_h(1-f_W)]^2 \right.$$
$$\left. + \frac{\zeta^2}{g_2^2 v^4}\left(\frac{\lambda}{4}(vf_h)^4 + \frac{\kappa}{3}(vf_h)^3 - \frac{\mu^2}{2}(vf_h)^2 + c\right)\right], \quad (\zeta := |v|g_2 r)$$



[a] X. Zhang *et al.*, Phys. Rev. D **51** (1995) 5327; F. R. Klinkhamer and N. S. Manton, Phys. Rev. D **30** (1984) 2212.

Sphaleron

- Weak sphaleron rate[a]

$$\Gamma_{ws} \sim \frac{4k\omega_-}{g_2 \langle \phi(T) \rangle T^3} \left(\frac{g_2^2 T}{16\pi^2} \right)^4 \mathcal{N}_{tr} \mathcal{N}_{rot} \left(\frac{4\pi \langle \phi(T) \rangle}{g_2 T} \right)^7 \exp\left(-\frac{E_{sph}(T)}{T} \right)$$

- Approximate scaling law[b]

$$E_{sph}(T) = E_{sph}(T=0) \frac{\langle \phi(T) \rangle}{\langle \phi(0) \rangle}$$

$\kappa\ \left[m_h^2/\lvert v \rvert\right]$	-0.5	-1.0	-1.5	-2	-2.5
$E_{sph}\ [4\pi \lvert v \rvert / g_2]$	1.95	1.93	1.91	1.88	1.85

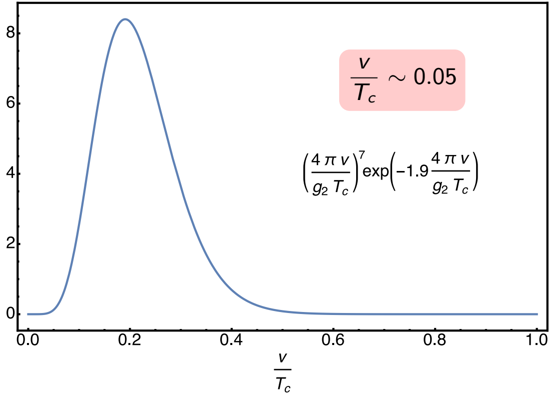
[a] Y. Burnier *et al.*, JCAP **0602** (2006) 007;H. H. Patel and M. J. Ramsey-Musolf, JHEP **1107** (2011) 029 ; P. B. Arnold and L. D. McLerran, Phys. Rev. D **36** (1987) 581;L. Carson *et al.*, Phys. Rev. D **42** (1990) 2127; P. B. Arnold and L. D. McLerran, Phys. Rev. D **37** (1988) 1020.

[b] S. Braibant *et al.*, Int.J.Mod.Phys. A8 (1993) 5563; Y. Brihaye and J. Kunz, Phys. Rev. D **48** (1993) 3884

Sphaleron

- Weak sphaleron rate[a]

$$\Gamma_{ws} \sim \frac{4k\omega_-}{g_2\langle\phi(T)\rangle T^3}\left(\frac{g_2^2 T}{16\pi^2}\right)^4 \mathcal{N}_{tr}\mathcal{N}_{rot}\left(\frac{4\pi\langle\phi(T)\rangle}{g_2 T}\right)^7 \exp\left(-\frac{E_{sph}(T)}{T}\right)$$



[a] Y. Burnier *et al.*, JCAP **0602** (2006) 007;H. H. Patel and M. J. Ramsey-Musolf, JHEP **1107** (2011) 029 ; P. B. Arnold and L. D. McLerran, Phys. Rev. D **36** (1987) 581;L. Carson *et al.*, Phys. Rev. D **42** (1990) 2127; P. B. Arnold and L. D. McLerran, Phys. Rev. D **37** (1988) 1020.

Chemical potential

- $5n_f+1$ massless non-interacting gas of particles

$$n_i-\overline{n_i}=\frac{k_iT^3}{6}\left(\mu_i\beta+\mathcal{O}((\beta\mu_i)^3)\right)$$

- Top transport[a]

$$D_QQ''-v_wQ'-\Gamma_y\left(\frac{Q}{k_Q}-\frac{H}{k_H}-\frac{T}{k_T}\right)-\Gamma_m\left(\frac{Q}{k_Q}-\frac{T}{k_T}\right)$$
$$-6\Gamma_{ss}\left(\frac{2Q}{k_Q}-\frac{T}{k_T}+\frac{9(Q+T)}{k_B}\right)+S_t^{CPV}=0$$
$$D_TT''-v_wT'-\Gamma_y\left(-\frac{Q}{k_Q}+\frac{H}{k_H}+\frac{T}{k_T}\right)-\Gamma_m\left(-\frac{Q}{k_Q}+\frac{T}{k_T}\right)$$
$$-3\Gamma_{ss}\left(-\frac{2Q}{k_Q}+\frac{T}{k_T}-\frac{9(Q+T)}{k_B}\right)-S_t^{CPV}=0$$
$$D_HH''-v_wH'-\Gamma_y\left(-\frac{Q}{k_Q}+\frac{H}{k_H}+\frac{T}{k_T}\right)-\Gamma_h\left(\frac{H}{k_H}\right)=0$$

[a] P. Huet and A. E. Nelson, Phys. Rev. D **53** (1996) 4578; D. Bodeker *et al.*, JHEP **0502** (2005) 026; L. Fromme and S. J. Huber, JHEP **0703** (2007) 049; T. Konstandin, Phys. Usp. **56** (2013) 747

$\mathcal{CP}$-violating source term

- $\mathcal{CP}$ source term[a]

$$S_t^{CPV}(z) = \frac{3}{2\pi^2} T \gamma_w v_w m_t^2(z) \theta_t'(z) + \mathcal{O}\left(v_w^2, \left(\frac{m_t}{T}\right)^4, \left(\frac{T}{L}\right)^2\right)$$

$$m_t(z) := \sqrt{\left(m_t' + \frac{y_t}{\sqrt{2}} h(z) \cos\xi\right)^2 + \left(\frac{y_t}{\sqrt{2}} h(z) \sin\xi\right)^2}$$

$$\tan\theta_t(z) := \frac{y_t h(z) \sin\xi}{\sqrt{2} m_t' + y_t h(z) \cos\xi}$$

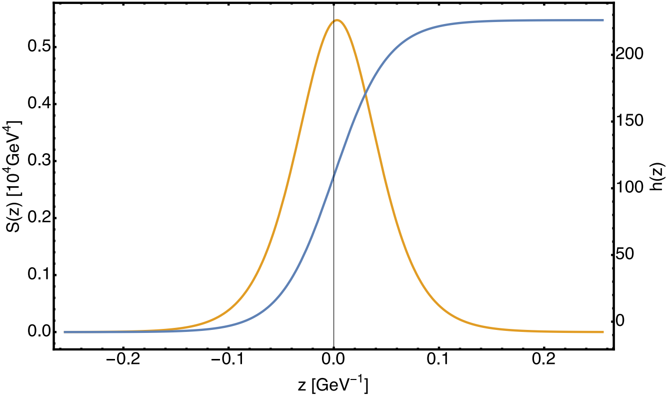
- Higgs profile:

$$h(z) = v_f + \frac{v_t - v_f}{2}\left[1 + \tanh\left(\frac{z}{L_w}\right)\right]$$

$\mathcal{CP}$-violating source term

- $\mathcal{CP}$ source term[a]

$$S_t^{CPV}(z) = \frac{3}{2\pi^2} T\gamma_w v_w m_t^2(z)\theta_t'(z) + \mathcal{O}\left(v_w^2, \left(\frac{m_t}{T}\right)^4, \left(\frac{\tau}{L}\right)^2\right)$$



[a] J. Shu and Y. Zhang, Phys. Rev. Lett. **111** (2013) 9, 091801;P. Huet and A. E. Nelson, Phys. Lett. B **355** (1995) 229; P. Huet and E. Sather, Phys. Rev. D **51** (1995) 379

Baryon asymmetry

- Lepton number[a]

$$n_L(z) = -\frac{3}{28}\left(\frac{D_q H''(z) - v_w H'(z)}{\Gamma_{ss}}\right) \qquad (\Gamma_y \gg \Gamma_{ss})$$

- Diffusion equation

$$D_q n_B''(z) - v_w n_B'(z) - 3\Gamma_{ws}(z) n_L(z) = 0$$

- Baryon asymmetry

$$n_B(z>0) = -\frac{3\Gamma_{ws}}{v_w}\int_{-\infty}^{0} dz_0\ n_L(z_0)$$

$k=1$	$\kappa\ [m_h^2/\lvert v\rvert]$	$\xi = 0.25\pi$		
		$y_t = 0.62 y_t^{SM}$	$y_t = 0.69 y_t^{SM}$	$y_t = 0.76 y_t^{SM}$
n_B/s	-0.5	9.28×10^{-10}	3.41×10^{-9}	2.41×10^{-8}
	-2.0	2.02×10^{-5}	1.92×10^{-5}	1.38×10^{-5}

[a] P. Huet and A. E. Nelson, Phys. Rev. D **53** (1996) 4578

Summary

- It is important to pin down the $\mathcal{CP}$ properties of the Higgs with LHC data

- Non-linear realisation may lead to anomalous Higgs couplings that is not yet ruled out

- Electroweak baryogenesis may be possible with non-linear realisation of $SU(2)_L \times U(1)_Y$
  - Cubic Higgs self-coupling may contribute to first order phase transition
  - Extra $\mathcal{CP}$-violation provided in the top Yukawa sector

Thank You!

SM fields

- dimension of action $S = \int d^4x \, \mathcal{L}$ zero in natural units $\Longrightarrow$ dim $[\mathcal{L}] = 4$
- dim $[\phi] = 1$, dim $[\psi] = 3/2$, dim $[D_\mu] = 1$, dim $[X_\mu] = 1$
- SM fields $(SU(3)_c \times SU(2)_L \times U(1)_Y)$:

$$\begin{pmatrix} u \\ d \end{pmatrix}_L = \quad Q_L \sim (\mathbf{3}, \mathbf{2}, \tfrac{1}{3}) \qquad u_L^c \sim (\bar{\mathbf{3}}, \mathbf{1}, -\frac{4}{3}) \qquad d_L^c \sim (\bar{\mathbf{3}}, \mathbf{1}, \frac{2}{3})$$

$$\begin{pmatrix} \nu_\ell \\ \ell \end{pmatrix}_L = \quad \ell_L \sim (\mathbf{1}, \mathbf{2}, -1) \qquad \ell_L^c \sim (\bar{\mathbf{1}}, \mathbf{1}, 2)$$

$$H \sim (\mathbf{1}, \mathbf{2}, 1) \qquad \tilde{H} = i\sigma_2 \phi \sim (\mathbf{1}, \mathbf{2}, 1)$$

$$G_\mu^a \sim (\mathbf{8}, \mathbf{1}, 0) \qquad W_\mu^a \sim (\mathbf{1}, \mathbf{3}, 0) \qquad B_\mu^a \sim (\mathbf{1}, \mathbf{1}, 0)$$

Which vev?

- Minimum

$$V'(\rho) = -\mu^2\rho + \kappa\rho^2 + \lambda\rho^3 = 0$$

- Higgs mass

$$m_h^2(\rho) := V''(\rho) = -\mu^2 + 2\kappa\rho + 3\lambda\rho^2$$

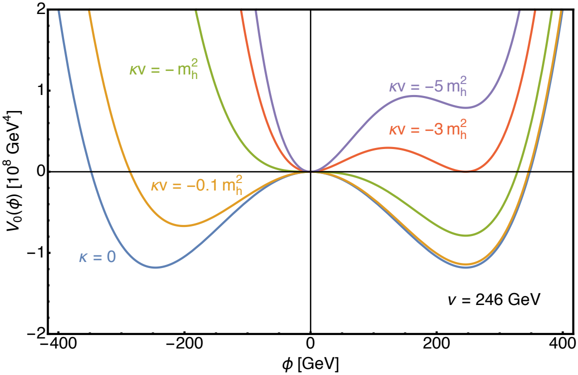
- Impose relations

$$\mu^2 = \frac{1}{2}\left(m_h^2 + v\kappa\right),$$
$$\lambda = \frac{1}{2v^2}\left(m_h^2 - v\kappa\right) > 0 \quad \text{(stability)}$$

Exercise — Classical Potential Analysis

$$\mu^2 = \frac{1}{2}\left(m_h^2 + v\kappa\right), \quad \lambda = \frac{1}{2v^2}\left(m_h^2 - v\kappa\right) > 0$$

- $\mu^2 > 0$ ($\rho = 0$ is a maximum)
  - $|v\kappa| < m_h^2$
  - $v\kappa < 0$ (compare the minima)
- $\mu^2 < 0$ ($\rho = 0$ is a minimum)
  - $v\kappa < 0$ and so $|v\kappa| > m_h^2$
  - $|\kappa v| < 3m_h^2$ (imposing $V(v) < V(0)$)



$\mathcal{CP}$-violating Yukawa Couplings

- beyond SM process contribute via a tower of non-renormalisable higher-dimensional effective operators

$$\mathcal{L} = \mathcal{L}_{SM}^{(\leq 4)} + \frac{1}{\Lambda} \sum_i c_i^{(5)} \mathcal{O}_i^{(5)} + \frac{1}{\Lambda^2} \sum_i c_i^{(6)} \mathcal{O}_i^{(6)} + \dots$$

- dim-6 operators[a,b]

$$\mathcal{O}^{\text{dim } 6} \supset (H^\dagger H)\left(\overline{Q_L} t_R \tilde{H} \pm h.c.\right), \quad (H^\dagger D_\mu H + (D_\mu H)^\dagger H)\left(\overline{t_{L,R}} \gamma^\mu t_{L,R}\right)$$

- Yukawa couplings (no Higgs derivative):

$$\mathcal{L}_{\text{eff}} \supset -\left(\alpha + \beta \frac{H^\dagger H}{\Lambda^2}\right) \overline{Q_L} t_R \tilde{H} + h.c.$$

---

[a] W. Buchmuller and D. Wyler, *Nucl. Phys. B* 268 (1986) 621; C. N. Leung, S. T. Love, and S. Rao, *Z. Phys. C* 31 (1986) 433

[b] J.A. Aguilar-Saavedra, *Nucl. Phys. B* 821 (2009) 215; T. Han and R. Ruiz hep-ph/1312.3324

$\mathcal{CP}$-violating Yukawa Couplings

- dim 4 operators considered by Appelquist[a] *et al.* and Bagan[b] *et al.*

- only one dim 5 operator for SM fields[c].

$$\mathcal{O}^{(5)} = \left(\overline{\ell^c_{Li}}\tilde{\phi}^*\right)\left(\tilde{\phi}^\dagger \ell^c_{Li}\right) \pm h.c. \tag{1}$$

  and it violates lepton number[d].

- SSB with $H = (0, v + h/\sqrt{2})^T$

$$\mathcal{L}_{\text{pheno}} \supset -\frac{y_t}{\sqrt{2}}\bar{t}_L(\cos\xi + i\sin\xi)t_R h + \text{h.c}$$

- with $y_t^{SM} \sim 10^{-2}$ TeV and new physics at TeV scale ($\Lambda \sim 10v$)
  ξ admits a value over full range $(-\pi, \pi]$ if $|\beta| \sim 1$

---

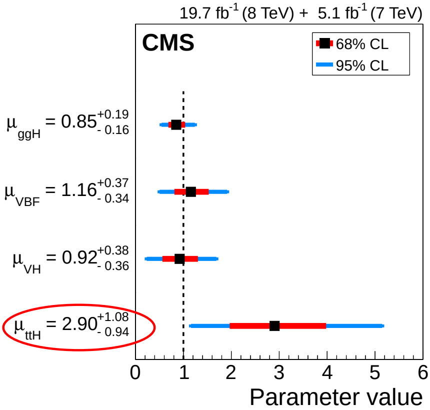
[a] T. Appelquist, M. Bowick, E. Cohler and A.Hauser, *Phys. Rev. D* 31 (1985) 1676

[b] E. Bagan, D. Espriu and J. Manzano, *Phys. Rev. D.* 60 (1999) 114035

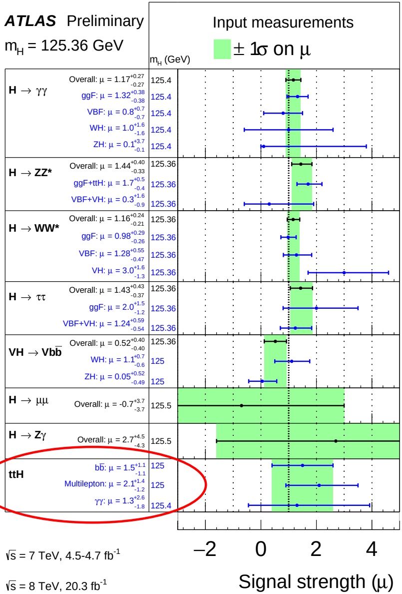
[c] S. Weinberg, *Phys. Rev. Lett.* 43 (1979) 1566

[d] V. Cirgliano, Lectures

Higgs — Couplings

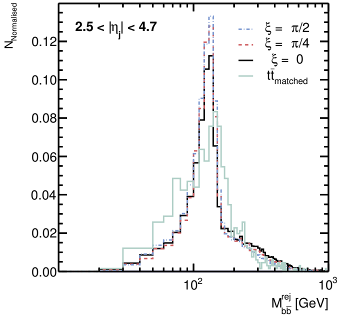


[EPJC 75 (2015) 212]



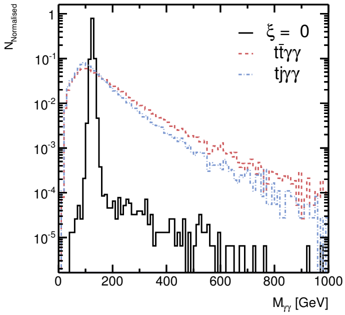
[ATLAS-CONF-2015-007]

Cutflow ($h \to b\bar{b}$)



Cuts		σ [fb]			
		thj			$t\bar{t}_{\mathrm{matched}}$
		$\xi = 0$	$\xi = \pi/4$	$\xi = \pi/2$	
(C1)	$\Delta R_{ij} > 0.4$, $i, j = b, j$ or ℓ $p_T^b > 25$ GeV, $\lvert\eta_b\rvert < 2.5$ $p_T^\ell > 25$ GeV, $\lvert\eta_\ell\rvert < 2.5$ $p_T^j > 25$ GeV, $\lvert\eta_j\rvert < 4.7$	0.3169	0.6700	2.1860	712.4
(C2)	$M_{b\ell} < 200$ GeV	0.3152	0.6582	2.1446	708.7
(C3)	$\lvert\eta_j\rvert > 2.5$	0.1492	0.3314	1.1002	80.33
(C4)	$\lvert M_{b_1\bar{b}_2} - m_h\rvert < 15$ GeV	0.0443	0.1102	0.3762	15.82
$S/\sqrt{B}$ with 3000 fb^{-1}		0.610	1.517	5.180	
S/B		0.28%	0.70%	2.38%	

Cutflow ($h \to \gamma\gamma$)



Cuts		σ [10^{-3} fb]						
		$t(\to \ell\nu_\ell b)h(\to \gamma\gamma)j$			$t\bar{t}\gamma\gamma$			$tj\gamma\gamma$
		$\xi = 0$	$\xi = 0.25\pi$	$\xi = 0.5\pi$	$\xi = 0$	$\xi = 0.25\pi$	$\xi = 0.5\pi$	
(C1)	$\Delta R_{ij} > 0.4$, $i,j = b, j, \ell, \gamma$ $p_T^b > 25$ GeV, $\lvert\eta_b\rvert < 2.5$ $p_T^\ell > 25$ GeV, $\lvert\eta_\ell\rvert < 2.5$ $p_T^j > 25$ GeV, $\lvert\eta_j\rvert < 4.7$ $p_T^\gamma > 20$ GeV, $\lvert\eta_\gamma\rvert < 2.5$	4.545	10.32	42.79	145.0	145.8	144.4	299.4
(C2)	$p_T^{\gamma 1} > 50$ GeV, $p_T^{\gamma 2} > 25$ GeV	4.194	9.599	39.69	88.11	88.24	87.59	155.2
(C3)	$M_{b\ell} < 200$ GeV	4.059	9.104	37.44	64.05	64.10	63.68	151.3
(C4)	$\lvert M_{\gamma\gamma} - m_h\rvert < 5$ GeV	3.219	6.866	28.47	3.295	3.493	3.393	9.031
$S/\sqrt{B}$ with 3000 fb^{-1}		1.59	3.36	13.99				
S/B		0.261	0.548	2.29				

Reconstruction

- For each *thj* event, the configuration of ℓ and ν_ℓ with minimal χ^2:

$$\chi^2 := \left(\frac{m_t - m_{\ell\nu_\ell j_b}}{\Delta m_t}\right)^2$$

  and the jet is taken as that with the next highest p_T

- neutrino kinematics
  - transverse: MET
  - longitudinal W on shell condition

$$p_\nu := \left(0, \not{E}^x, \not{E}^y, \frac{1}{2p_{\ell T}^2}\left(Ap_{\ell L} \pm E_\ell\sqrt{A^2 - 4p_{\ell T}^2 \not{E}_T^2}\right)\right)$$

where $A := m_W^2 + 2\mathbf{p}_{eT} \cdot \not{\mathbf{E}}_T$

$\mathcal{CP}$-violating Yukawa Couplings

- Top loop induced cubic coupling

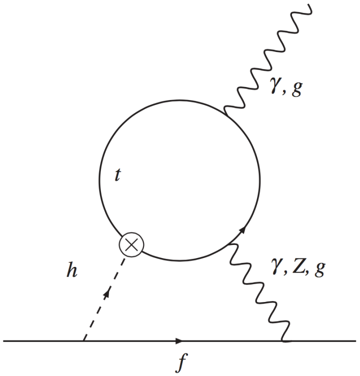
$$\kappa_{t-\text{loop}} \approx \frac{-3\sqrt{2} m_t' y_t^3 \cos\xi}{16\pi^2} \left[2\cos^2\xi + 3\ln\left(\frac{m_t'^2}{v^2}\right)\right]$$

- m_t' as a function of y_t, ξ and m_t

$$m_t'^{(\pm)} = \frac{1}{2}\left(\pm\sqrt{4m_t^2(v) - 2y_t^2 v^2 \sin^2\xi} - \sqrt{2} y_t v \cos\xi \right)$$

EDM

- Barr Zee type diagrams[a]



[a] C. Y. Chen *et al.*, JHEP **1506** (2015) 056; J. Brod *et al.*, JHEP **1311** (2013) 180; S. J. Huber *et al.*, Phys. Rev. D **75** (2007) 036006; K. Cheung *et al.*, JHEP **1406** (2014) 149; C. Lee, J. Phys. Conf. Ser. **69** (2007) 012036; S. Khatibi and M. M. Najafabadi, Phys. Rev. D **90** (2014) 7, 074014

$\mathcal{CP}$-violating Yukawa Couplings

		$\kappa_{\text{loop-induced}}$ $[m_h^2/\lvert v \rvert]$, m_t' [GeV]
ξ	y_t/y_t^{SM}	$v = 246$ GeV
0	0.77	0.0675, 39.8 0.193, −301.21
	0.99	0.0194, 1.73 0.558, −344.
	1.2	−0.243, 34.6 1.26, −381.
0.25π	0.62	0.0322, 79.6 0.0102, −231.
	0.69	0.0440, 66.6 0.0168, −235.
	0.76	0.0562, 52.9 0.0254, −239.
0.5π	0.46	0, 154. 0, −154.
	0.52	0, 148. 0, −148.
	0.57	0, 142. 0, −142.

Quantum corrections to the potential

- 1-loop correction

$$V_i^{(1)}(\phi) = \frac{(-1)^{2s_i}}{2} \int \frac{d^4 p}{(2\pi)^4} \ln \det K_i$$

- *Bosons*

$$K_i = (p^0)^2 + |\mathbf{p}|^2 + m_i^2(\rho)$$

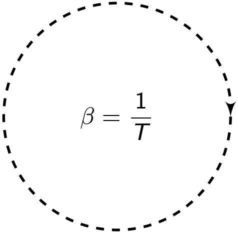
- *Fermions*

$$K_i = \gamma_0 p_0 + \gamma_i p_i - (y_S(\rho) + m' + i\gamma^5 y_P(\rho)).$$

- Recover boson case with effective mass

$$m_t^2(\rho) = \left(m' + \frac{y_t}{\sqrt{2}}\rho\cos\xi\right)^2 + \left(\frac{y_t}{\sqrt{2}}\rho\sin\xi\right)^2$$

Finite temperature effects



$$p_E^0 = \omega_n = \begin{cases} 2n\pi T, & \text{bosons} \\ (2n+1)\pi T, & \text{fermions} \end{cases} \qquad (n \in \mathbb{Z})$$

- Compactified time dimension

$$\int \frac{d^4 p}{(2\pi)^4} \to T \sum_{n=-\infty}^{\infty} \int \frac{d^3 p}{(2\pi)^3}$$

- Thermal function

$$\frac{T^4}{2\pi^2} J_i(\beta^2 m_i^2(\phi)) := \frac{T^4}{2\pi^2} \int_0^\infty dx\ x^2 \ln\left[1 - (-1)^{2s} e^{-\sqrt{x^2+\beta^2 m_i^2(\phi)}}\right]$$

Phase transition — $\xi = 0$

$\kappa \left[\frac{m_h^2}{\lvert v \rvert}\right]$		$\lvert\xi\rvert = 0$		
		$y_t = 0.77 y_t^{SM}$	$y_t = 0.99 y_t^{SM}$	$y_t = 1.2 y_t^{SM}$
-0.1	$\langle\phi(T_c)\rangle$	$-189., 214$	–	–
	T_c	77.6	–	–
-0.5	$\langle\phi(T_c)\rangle$	$-61.5, 175.$	–	–
	T_c	110.4	–	–
-1.0	$\langle\phi(T_c)\rangle$	$-19.9, 187.$	$20.8, 144.$	–
	T_c	98.4	104.	–
-1.5	$\langle\phi(T_c)\rangle$	$-9.44, 206.$	$2.90, 190.$	$41.7, 151.$
	T_c	80.5	83.2	92.1
-2.0	$\langle\phi(T_c)\rangle$	$-5.46, 220.$	$-1.51, 211.$	$6.82, 200.$
	T_c	53.0	55.9	63.4
-2.5	$\langle\phi(T_c)\rangle$	–	–	–
	T_c	–	–	–

Phase transition — $|\xi| = 0.25\pi$

$\kappa \left[\frac{m_h^2}{\lvert v \rvert}\right]$		$\lvert\xi\rvert = 0.25\pi$		
		$y_t = 0.62 y_t^{SM}$	$y_t = 0.69 y_t^{SM}$	$y_t = 0.76 y_t^{SM}$
-0.1	$\langle\phi(T_c)\rangle$	$-197.$, 224	$-194.$, $219.$	$-187.$, $213.$
	T_c	72.9	75.7	81.2
-0.5	$\langle\phi(T_c)\rangle$	-73.4, $188.$	-67.5, $182.$	-58.8, $174.$
	T_c	114.	113.	113.
-1.0	$\langle\phi(T_c)\rangle$	-26.6, $196.$	-23.3, $192.$	-18.7, $187.$
	T_c	104.	102.	101.
-1.5	$\langle\phi(T_c)\rangle$	-13.0, $212.$	-11.2, $209.$	-9.03, $206.$
	T_c	86.3	83.9	81.9
-2.0	$\langle\phi(T_c)\rangle$	-7.79, $226.$	-6.72, $224.$	-5.59 , $221.$
	T_c	58.7	56.0	54.2
-2.5	$\langle\phi(T_c)\rangle$	–	–	–
	T_c	–	–	–

Phase transition — $|\xi| = 0.5\pi$

$\kappa \left[\frac{m_h^2}{\|v\|}\right]$		$\|\xi\| = 0.5\pi$		
		$y_t = 0.46 y_t^{SM}$	$y_t = 0.52 y_t^{SM}$	$y_t = 0.57 y_t^{SM}$
0	$\langle\phi(T_c)\rangle$	$\mathbb{Z}_2$		
	T_c			
-1	$\langle\phi(T_c)\rangle$	–	74.68, 99.68	60., 113.94
	T_c	–	159.447	152.532
-1.5	$\langle\phi(T_c)\rangle$	36.1, 186.	31.3, 188.	16.3, 190.
	T_c	142.	135.	128.
-2	$\langle\phi(T_c)\rangle$	6.31, 218.	4.80, 219.	3.66, 219.
	T_c	115.	107.	101.
-2.5	$\langle\phi(T_c)\rangle$	-0.666, 238.	-1.33, 238.	-1.83, 238.
	T_c	77.0	67.7	58.6
-3	$\langle\phi(T_c)\rangle$	–	–	–
	T_c	–	–	–

Sphaleron

$$E_{sph} = \int d^3x \left(-\frac{1}{4} W^a_{ij} W^{a,ij} - \frac{1}{4} F_{ij} F^{ij} + \frac{\rho^2}{2} (D_i \Sigma)^\dagger (D^i \Sigma) - V(\rho) \right),$$

$$\begin{aligned} W^a_{\mu\nu} &:= \partial_\mu W^a_\nu - \partial_\nu W^a_\mu - i g_2 \epsilon^{abc} W^b_\mu W^c_\nu, \\ A_{\mu\nu} &:= \partial_\mu A_\nu - \partial_\nu A_\mu \\ D_\mu &:= \partial_\mu - \frac{i}{2} g_2 \sigma^a W^a_\mu \Phi - \frac{i}{2} g_1 A^\mu \Phi \end{aligned}$$

Sphaleron

$$\frac{i}{2} g_2 \sigma^a W_i^a dx^i = f_W(\zeta) dU^\infty (U^\infty)^{-1}$$
$$\phi = \frac{v}{\sqrt{2}} f_h(\zeta) U^\infty \begin{pmatrix} 0 \\ 1 \end{pmatrix},$$

$$\zeta := g_2 |v| r$$

$$U^\infty := \frac{1}{r} \begin{pmatrix} z & x + iy \\ -x + iy & z \end{pmatrix}.$$

$$\zeta^2 \frac{d^2 f_W}{d\zeta^2} = 2 f_W (1 - f_W)(1 - 2 f_W) - \frac{\zeta^2}{4} f_h^2 (1 - f_W)$$
$$\frac{d}{d\zeta}\left(\zeta^2 \frac{d f_h}{d\zeta}\right) = 2 f_h (1 - f_W)^2 + \frac{\zeta^2}{g_2^2}\left(\lambda f_h^3 + \frac{\kappa}{v} f_h^2 - \frac{\mu^2}{v^2} f_h\right),$$

Sphaleron

- Weak sphaleron rate[a]

$$\Gamma_{ws} \sim \frac{4k\omega_-}{g_2\langle\phi(T)\rangle T^3}\left(\frac{g_2^2 T}{16\pi^2}\right)^4 \mathcal{N}_{tr}\mathcal{N}_{rot}\left(\frac{4\pi\langle\phi(T)\rangle}{g_2 T}\right)^7 \exp\left(-\frac{E_{sph}(T)}{T}\right)$$

- For $\lambda = g_2^2$

$$\mathcal{N}_{tr} = 26, \quad \mathcal{N}_{rot} \sim 5.3 \times 10^3, \quad k \sim \mathcal{O}(1), \quad \omega_- \sim m_W$$

[a] Y. Burnier *et al.*, JCAP **0602** (2006) 007;H. H. Patel and M. J. Ramsey-Musolf, JHEP **1107** (2011) 029 ; P. B. Arnold and L. D. McLerran, Phys. Rev. D **36** (1987) 581;L. Carson *et al.*, Phys. Rev. D **42** (1990) 2127; P. B. Arnold and L. D. McLerran, Phys. Rev. D **37** (1988) 1020.

Diffusion equation

$$-v_w H' + \overline{D} H'' - \overline{\Gamma} H + \overline{S}_t^{CPV} = 0$$

$$\overline{D} = \frac{D_Q(9k_T k_Q + 4k_T k_B + k_B k_Q) + D_H k_H(k_B + 9k_Q + 9k_T)}{(9k_T k_Q + 4k_T k_B + k_B k_Q) + k_H(k_B + 9k_Q + 9k_T)} = \frac{9D_Q + 7D_H}{16}$$

$$\overline{\Gamma} = \frac{(\Gamma_m + \Gamma_h)(9k_T + 9k_Q + k_B)}{(9k_T k_Q + 4k_T k_B + k_B k_Q) + k_H(k_B + 9k_Q + 9k_T)} = \frac{7(\Gamma_m + \Gamma_h)}{32}$$

$$\overline{S}_t^{CPV} = \frac{S_t^{CPV} k_H(9k_T + 9k_Q + k_B)}{(9k_T k_Q + 4k_T k_B + k_B k_Q) + k_H(k_B + 9k_Q + 9k_T)} = \frac{7S_t^{CPV}}{16}$$

$$k_Q = 2k_T = 2k_B = 3k_H = 6$$

Bubble expansion and diffusion parameters

- Diffusion and interaction rates

$$D_H = \frac{20}{T} \qquad D_Q = D_T = \frac{6}{T}$$

$$\Gamma_m = \frac{m_t^2(z,T)}{63T} \qquad \Gamma_h = \frac{m_W^2(z,T)}{50T}$$

- n_L to n_B conversion:

$$\begin{aligned} n_L(z) &= Q(z) + Q_1(z) + Q_2(z) \\ &= \frac{9k_Q k_T - 5k_Q k_B - 8k_T k_B}{k_H(k_B + 9k_Q + 9k_T)} H(z) + \mathcal{O}\left(\frac{1}{\Gamma_{ss}}, \frac{1}{\Gamma_y}\right) \\ &= -\frac{3}{28}\left(\frac{D_q H''(z) - v_w H'(z)}{\Gamma_{ss}}\right) \qquad (\Gamma_y \gg \Gamma_{ss}) \end{aligned}$$

- Bubble expansion must occur subsonically otherwise diffusion will be inefficient.

- Baryon asymmetry peaks at around $v_w = 0.01$ [1506.04741]

- The wall thickness has been determined to be $3 \lesssim L_w T_c \lesssim 16$ [JEHP 02 (2005) 026]

Baryon asymmetry

	$\kappa\left[\frac{m_h^2}{\lvert v\rvert}\right]$	$\xi = 0.25\pi$		
		$y_t = 0.62 y_t^{SM}$	$y_t = 0.69 y_t^{SM}$	$y_t = 0.76 y_t^{SM}$
n_B/s	-0.5	9.28×10^{-10}	3.41×10^{-9}	2.41×10^{-8}
	-2.0	2.02×10^{-5}	1.92×10^{-5}	1.38×10^{-5}
	0.5	8.31×10^{-10}	3.98×10^{-9}	3.35×10^{-8}
	2.0	6.18×10^{-6}	1.00×10^{-5}	1.01×10^{-5}

	$\kappa\left[\frac{m_h^2}{\lvert v\rvert}\right]$	$\xi = 0.5\pi$		
		$y_t = 0.46 y_t^{SM}$	$y_t = 0.52 y_t^{SM}$	$y_t = 0.57 y_t^{SM}$
n_B/s	-1.5	1.14×10^{-6}	1.72×10^{-6}	1.59×10^{-6}
	-2.0	8.02×10^{-8}	3.48×10^{-8}	1.24×10^{-8}
	-2.5	2.10×10^{-12}	5.71×10^{-10}	1.21×10^{-8}