The squeezed limit of the bispectrum for multi-field inflation

Zac Kenton

Based on arxiv:1507.08629 with David Mulryne

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Take-home message

- Standard δN calculates bispectrum in the equilateral limit
- We find the bispectrum in the squeezed limit

Outline

Primordial Curvature Perturbation ζ

Squeezed Limit of the Bispectrum

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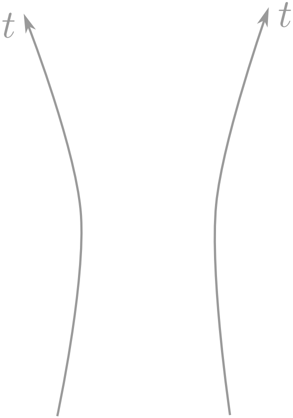
Primordial Curvature Perturbation ζ

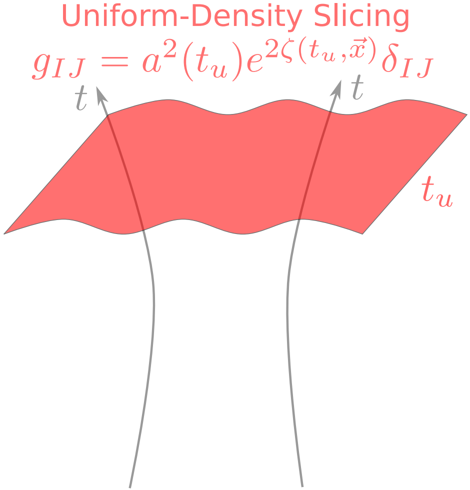
- Definition of ζ
- The δN Formula
- Correlation Functions of ζ

Squeezed Limit of the Bispectrum

- Squeezed Limit
- Single Field
- Multiple Fields
- Observational Prospects



t
t


Uniform-Density Slicing
$g_{IJ} = a^2(t_u)e^{2\zeta(t_u,\vec{x})}\delta_{IJ}$
t
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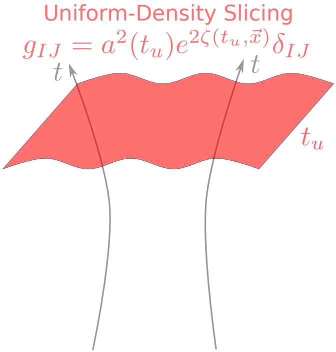
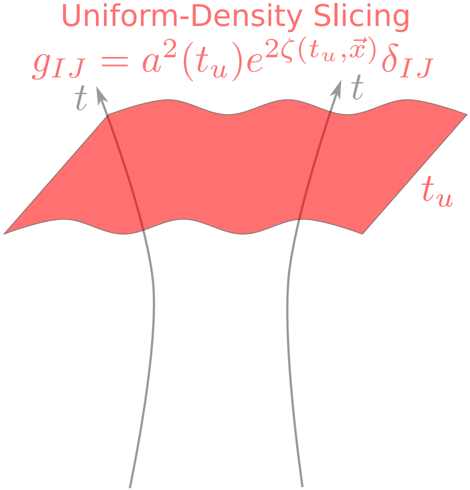
Primordial Curvature Perturbation, ζ



- This defines the ‘Primordial Curvature Perturbation’, ζ
- ζ important for observations of CMB and LSS
- ζ originates from inflation

Outline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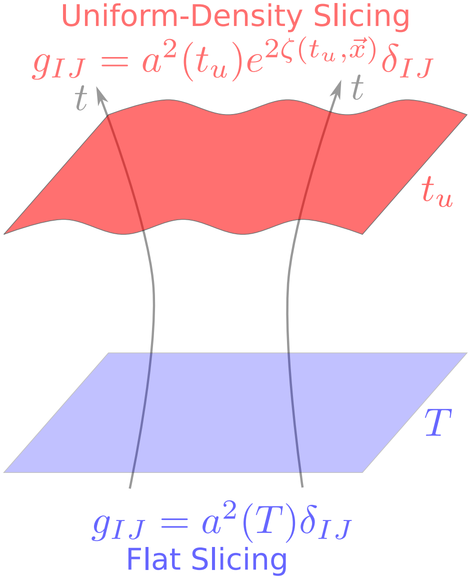

Uniform-Density Slicing
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Flat Slicing

δN For Inflation

- Inflation with multiple scalar fields $\phi_i(T, \vec{x}) = \phi_i^{(T)} + \delta\phi_i^{(T)}(\vec{x})$.

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  $\zeta(t_u,\vec{x}) = N_i^{(T)}\delta\phi_i^{(T)}(\vec{x}) + \frac{1}{2}N_{ij}^{(T)}\delta\phi_i^{(T)}(\vec{x})\delta\phi_j^{(T)}(\vec{x}) + \ldots$

  where $N_i^{(T)} \equiv \frac{\partial N_0(t_u,T)}{\partial\phi_i^{(T)}}$ with $N_0(t_u,T) \equiv \int_T^{t_u} H(t)dt$

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  where $N_i^{(T)} \equiv \frac{\partial N_0(t_u,T)}{\partial\phi_i^{(T)}}$ with $N_0(t_u,T) \equiv \int_T^{t_u} H(t)dt$
- So ζ at t_u can be expressed in perturbations of inflationary fields at early time T, multiplied by background quantities

[Sasaki, Stewart '96; Lyth, Rodriguez '05; ...]

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Bispectrum of ζ from δN

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$k_1 \leq k_2 \leq k_3$. Want bispectrum $\langle\zeta_{\vec{k_1}}\zeta_{\vec{k_2}}\zeta_{\vec{k_3}}\rangle$ in terms of

$$\langle\delta\phi_{i,\vec{k_1}}^{(T)}\delta\phi_{j,\vec{k_2}}^{(T)}\rangle \sim \Sigma_{ij}^{(T)}(k_1)$$

$$\langle\delta\phi_{i,\vec{k_1}}^{(T)}\delta\phi_{j,\vec{k_2}}^{(T)}\delta\phi_{k,\vec{k_3}}^{(T)}\rangle \sim \alpha_{ijk}^{(T)}(k_1,k_2,k_3)$$

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$$\langle \zeta_{\vec{k}_1} \zeta_{\vec{k}_2} \zeta_{\vec{k}_3} \rangle \sim N_i^{(T)} N_j^{(T)} N_k^{(T)} \alpha_{ijk}^{(T)}(k_1, k_2, k_3) + N_i^{(T)} N_{jk}^{(T)} N_l^{(T)} \left[\Sigma_{ij}^{(T)}(k_1) \Sigma_{kl}^{(T)}(k_2) + 2 \text{ perms} \right]$$

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T taken to be time k_3 exits the horizon.

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- Equilateral limit $k_1 \approx k_2 \approx k_3 \approx k_*$
  - all modes exit at about the same time, in which case $\Sigma_{ij}^{(3)}(k_A) \approx \Sigma_{ij}^{(*)}(k_*)\Big|_{k_* \mapsto k_A}$ $\alpha_{ijk}^{(3)}(k_1,k_2,k_3) \approx \alpha_{ijk}^{(*)}(k_1,k_2,k_3)\Big|_{k_1\approx k_2\approx k_3}$ have been calculated [Stewart, Lyth '93; Nakamura, Stewart '96; Seery, Lidsey '05;...]

Bispectrum of ζ from δN

$$\langle \zeta_{\vec{k}_1} \zeta_{\vec{k}_2} \zeta_{\vec{k}_3} \rangle \sim N_i^{(T)} N_j^{(T)} N_k^{(T)} \alpha_{ijk}^{(T)}(k_1, k_2, k_3) \\ + N_i^{(T)} N_{jk}^{(T)} N_l^{(T)} \left[\Sigma_{ij}^{(T)}(k_1) \Sigma_{kl}^{(T)}(k_2) + 2 \text{ perms} \right]$$

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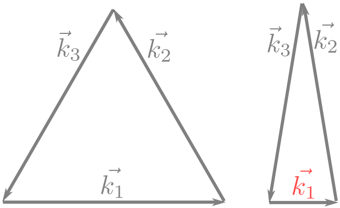
- Squeezed limit $k_1 \ll k_2, k_3$
  - We need $\Sigma_{ij}^{(3)}(k_1)$ and $\alpha_{ijk}^{(3)}(k_1, k_2, k_3)$. This is our work.

Outline

Squeezed Limit

- Soft limit: $k_1 \ll$ other k_a
- Bispectrum: Momentum conservation $\Rightarrow$ closed triangle



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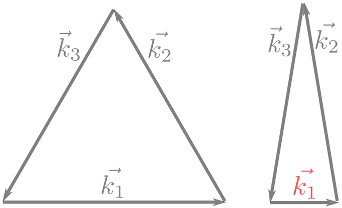


- Soft limit is a squeezed triangle $\equiv$ ‘squeezed limit’
- Observationally good: CMB, LSS, halo bias, μ-distortion
- Theoretically simple
- Sensitive to number of light fields during inflation!

Outline

Single Field Case

$$\lim_{k_1 \ll k_3} \langle \zeta_{\vec{k}_1} \zeta_{\vec{k}_2} \zeta_{\vec{k}_3} \rangle = -(n_s - 1) \langle \zeta_{\vec{k}_1} \zeta_{-\vec{k}_1} \rangle \langle \zeta_{\vec{k}_2} \zeta_{\vec{k}_3} \rangle$$

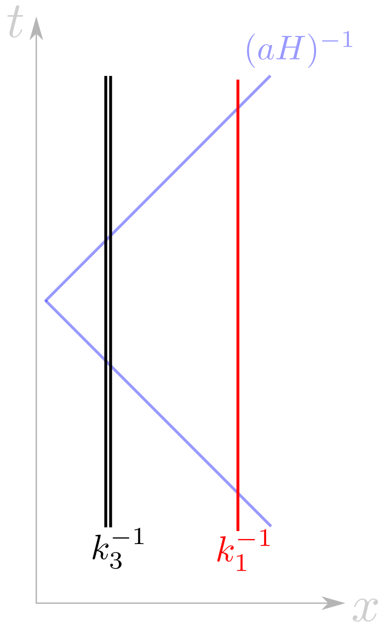
$$n_s - 1 = -0.032 \pm 0.006$$

- In squeezed limit f_{NL} is small
- Result independent of slow roll!

[Maldacena ’03; Creminelli, Zaldarriaga ’04; Cheung et al. ’08;...]

Outline

Multifield Case



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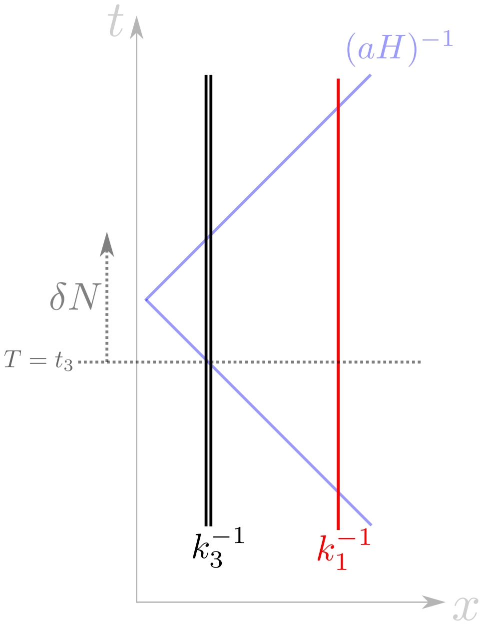

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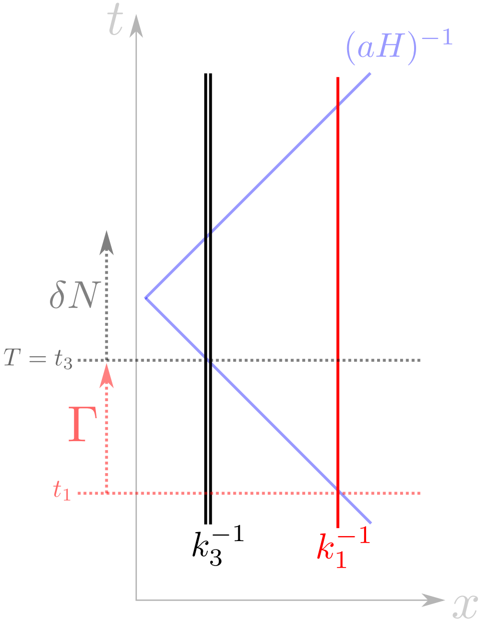

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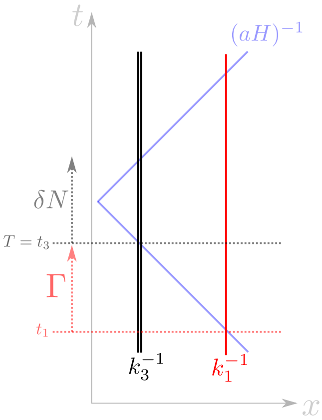

- Multifield: perturbations evolve
- Relate later perturbation at t_3 to earlier one at t_1

$$\delta\phi_i^{(3)}(\mathbf{x}) = \Gamma_{ij}\delta\phi_j^{(1)}(\mathbf{x}) + \dots$$

$$\Gamma_{ij} \equiv \frac{\partial\phi_i^{(3)}}{\partial\phi_j^{(1)}}$$

[ZK & Mulryne '15; Yokoyama et al. '07;..]

- Previous work has approximations to this

$$\Gamma_{ij} \approx \delta_{ij} - (\text{small})_{,ij} \log\left(\frac{k_3}{k_1}\right) + \dots$$

but not valid for large $\log(k_3/k_1)$

[Byrnes et al. '10, Dias, Ribeiro, Seery '13;...]

Multifield Case

- Squeezed limit:

$$\lim_{k_1 \ll k_3, k_2} \langle \zeta_{\vec{k_1}} \zeta_{\vec{k_2}} \zeta_{\vec{k_3}} \rangle \sim N_i^{(3)} N_j^{(3)} N_k^{(3)} \alpha_{ijk}^{(3)}(k_1, k_2, k_3) + 2 N_i^{(3)} N_{jk}^{(3)} N_l^{(3)} \Sigma_{ij}^{(3)}(k_1) \Sigma_{kl}^{(3)}(k_3)$$

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- We need $\Sigma_{ij}^{(3)}(k_1)$ and $\alpha_{ijk}^{(3)}(k_1, k_2, k_3)$
- First find $\Sigma_{ij}^{(3)}(k_1)$:
- Γ-evolution gives

$$\Sigma_{ij}^{(3)}(k_1) = \Gamma_{il} \Gamma_{jm} \Sigma_{lm}^{(1)}(k_1) = \Gamma_{il} \Gamma_{jl} \frac{{H^{(1)}}^2}{2k_1^3}$$

Multifield Case: Find α

- Next find $\alpha^{(3)}_{ijk}(k_1, k_2, k_3)$
- Equilateral case known [Seery, Lidsey '05]
- Squeezed case [ZK & Mulryne '15]

$$\lim_{k_1 \ll k_3, k_2} \alpha^{(3)}_{ijk}(k_1, k_2, k_3) \approx \Sigma^{(3)}_{im}(k_1) \frac{\partial}{\partial \phi^{(3)}_m} \Sigma^{(3)}_{jk}(k_3).$$

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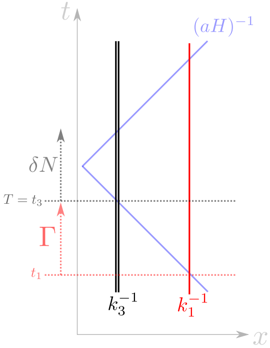
- Similar results in [Allen et al. '05; Li & Wang '08]

Multifield Case



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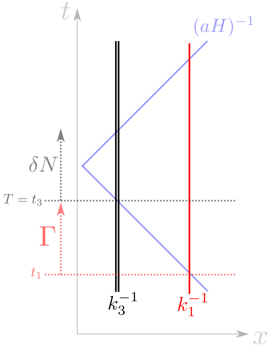

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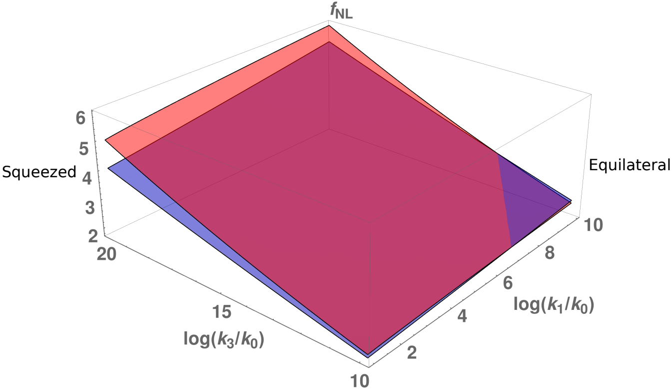
Put these into:

$$\lim_{k_1 \ll k_3, k_2} \langle \zeta_{\vec{k_1}}\zeta_{\vec{k_2}}\zeta_{\vec{k_3}}\rangle \sim N^{(3)}_i N^{(3)}_j N^{(3)}_k \alpha^{(3)}_{ijk}(k_1, k_2, k_3)$$
$$+ 2N^{(3)}_i N^{(3)}_{jk} N^{(3)}_l \Sigma^{(3)}_{ij}(k_1)\Sigma^{(3)}_{kl}(k_3)$$

Outline

Example: Interacting Curvaton

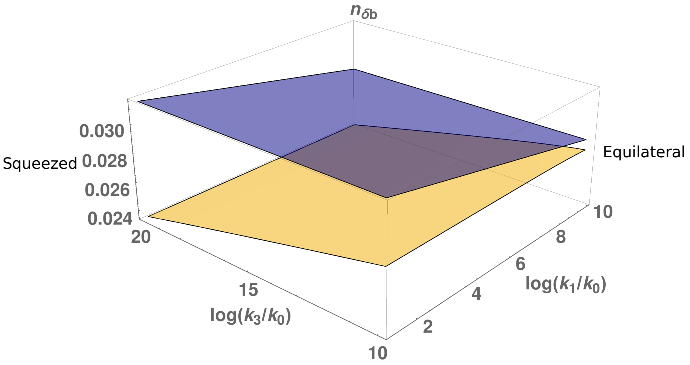
- Blue is us
- Other colours are previous work
- Reduced bispectrum
  $f_{NL}(k_1, k_2, k_3) \sim B_\zeta(k_1, k_2, k_3)/P_\zeta(k_1)P_\zeta(k_3)$



Example: Interacting Curvaton

- Blue is us
- Other colours are previous work
- Spectral index of the halo bias

$$n_{\delta b} \sim \frac{d \log f_{\rm NL}(k_1, k_2, k_3)}{d \log k_1}$$



Summary

- Done:
  - Calculated squeezed limit of bispectrum for multifield inflation
  - Our approach with Γ-evolution allows a larger squeezing than previous work
  - Observations: DES, Euclid, μ-distortions will probe large squeezing
  - Theoretical correction of about 20% for large squeezing

Summary

- Done:
  - Calculated squeezed limit of bispectrum for multifield inflation
  - Our approach with Γ-evolution allows a larger squeezing than previous work
  - Observations: DES, Euclid, μ-distortions will probe large squeezing
  - Theoretical correction of about 20% for large squeezing
- To do:
  - Higher point correlators, e.g. trispectrum
  - Multiple soft limits
  - Soft internal momenta
  - Understand from symmetries? [Assassi, Baumann, Green '12]
  - Can α term ever be large? Can't be in equilateral case. [Lyth, Zaballa '05]

Thanks!

Special thanks to David Mulryne and Steve Thomas

Take-home message

- Standard δN calculates bispectrum in the equilateral limit
- We find the bispectrum in the squeezed limit

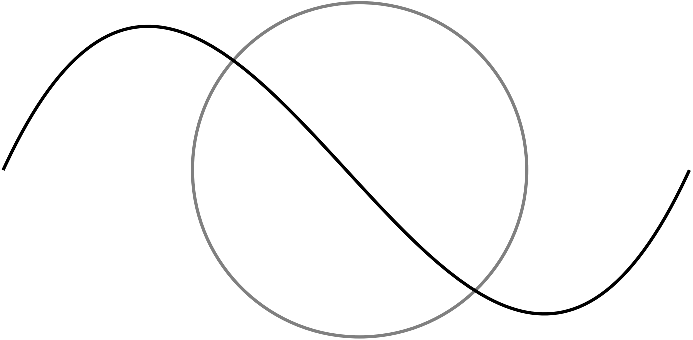
Multifield Case: Heuristic Proof of α result

More detailed proof in the paper, in position space.
Let $q \sim k_1 \ll k_2, k_3$

$$\begin{aligned}
\lim_{k_1 \ll k_{2,3}} \alpha^{(3)}_{ijk}(k_1, k_2, k_3) &\sim \lim_{k_1 \ll k_{2,3}} \langle \delta\phi^{(3)}_{i,\mathbf{k_1}} \phi^{(3)}_{j,\mathbf{k_2}} \phi^{(3)}_{k,\mathbf{k_3}} \rangle \\
&\sim \langle \delta\phi^{(3)}_{i,\mathbf{k_1}} \langle \phi^{(3)}_{j,\mathbf{k_2}} \phi^{(3)}_{k,\mathbf{k_3}} \rangle_{\delta\phi^{(3)}_{\mathbf{q}}} \rangle \\
&\sim \langle \delta\phi^{(3)}_{i,\mathbf{k_1}} (\langle \phi^{(3)}_{j,\mathbf{k_2}} \phi^{(3)}_{k,\mathbf{k_3}} \rangle + \delta\phi^{(3)}_{m,\mathbf{q}} \frac{\partial}{\partial \phi^{(3)}_m} \langle \phi^{(3)}_{j,\mathbf{k_2}} \phi^{(3)}_{k,\mathbf{k_3}} \rangle + ..) \rangle \\
&\sim 0 + \langle \delta\phi^{(3)}_{i,\mathbf{k_1}} \delta\phi^{(3)}_{m,\mathbf{q}} \rangle \frac{\partial}{\partial \phi^{(3)}_m} \langle \phi^{(3)}_{j,\mathbf{k_2}} \phi^{(3)}_{k,\mathbf{k_3}} \rangle \\
&\sim \Sigma^{(3)}_{im}(k_1) \frac{\partial}{\partial \phi^{(3)}_m} \Sigma^{(3)}_{jk}(k_3)
\end{aligned}$$

Other work this year

- ZK & Steve Thomas:
  - D-Brane Potentials in the Warped Resolved Conifold & Natural Inflation
  - JHEP 1502 (2015) 127. arxiv:1409.1221
- ZK, David J. Mulryne & Steve Thomas:
  - Generating Asymmetry with g_{NL}
  - PRD 92 (2015) 023505. arxiv:1504.05736



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