Shaft Inflation

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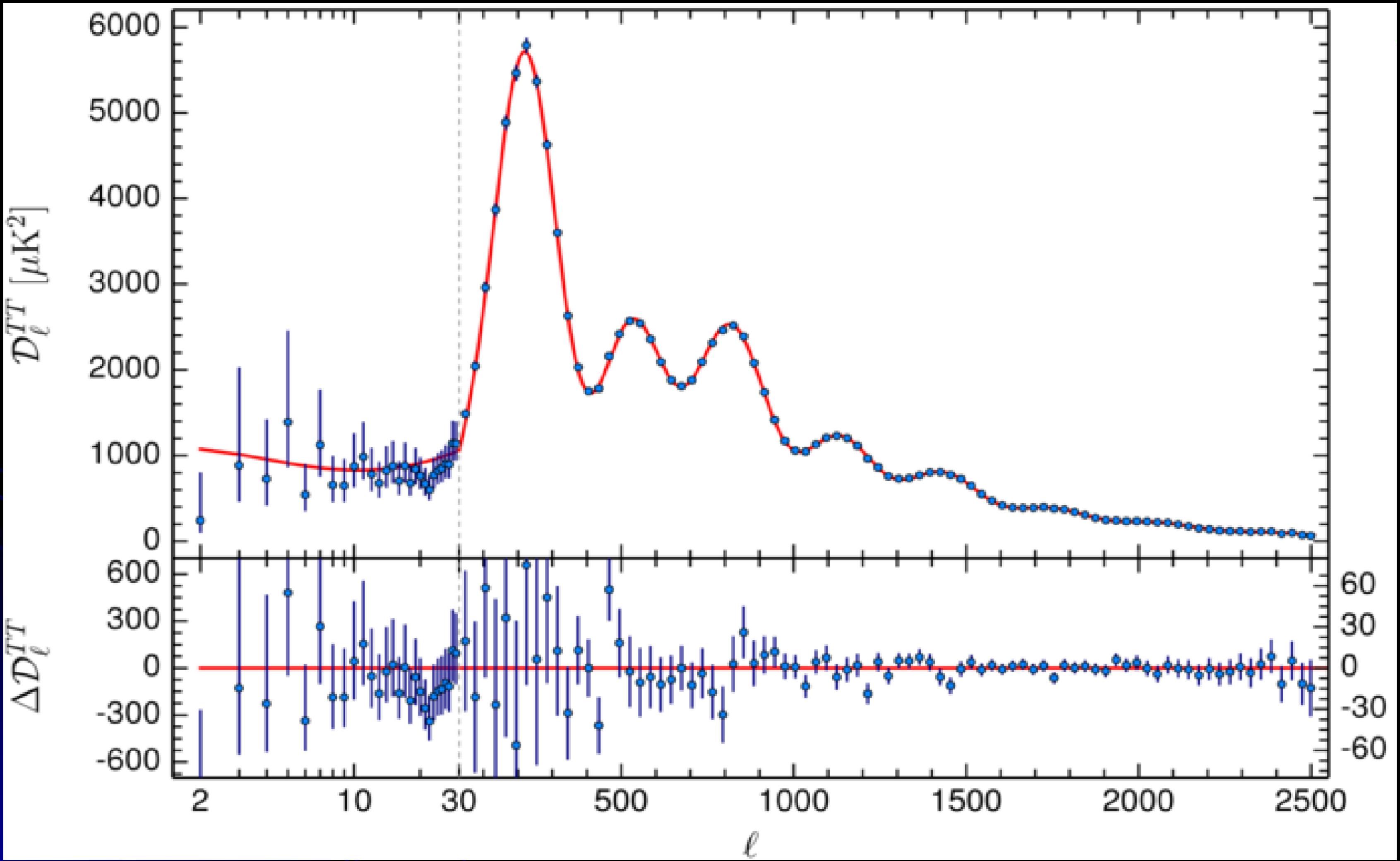
Inflation and Planck

- 

Inflation and Planck

- **Planck satellite CMB observations confirm the vanilla predictions of cosmic inflation for the primordial curvature perturbation:**
  - Gaussian
  - Adiabatic
  - Statistically isotropic & homogeneous
  - (almost) Scale-invariant (with red tilt)

$$n_s = 0.968 \pm 0.006$$

- **Planck data favour single field inflation** Planck2015, 1502.01589
- **In conjunction with other data Planck favours inflationary plateau**
- **Prominent examples:** R^2 Inflation, Higgs Inflation and T-model Inflation

  A.A.Starobinski, PLB91(1980)99; F.L.Bezrukov & M.Shaposhnikov, PLB659(2008)703; A.Linde, arXiv:1402.0526 [hep-th]
- Power-law approach to inflationary plateau: **Shaft Inflation**

KD, PLB735(2014)75

Bottom-up vs Top-down

- **Top-down scenario: Models based on “realistic” constructions**
  - String-inspired, SUSY/SUGRA etc.
  - Look for specific signatures in data (e.g. non-Gaussianity)
  - Planck observations favour “easy” constructions
- **Bottom-up scenario: Model constructions “suggested” by data**
  - Data-inspired, “guess-stimates”
  - Uses Early Universe as Lab to investigate fundamental physics
- **Shaft Inflation proposes power-law approach to inflationary plateau in context of global SUSY**

KD, PLB735(2014)75

The Inflationary Potential

- Toy model superpotential: $$W = M^2 \frac{\Phi^{nq+1}}{(\Phi^n + m^n)^q}$$

▸ When $|\Phi| \gg m$: $W \simeq M^2\Phi$ O' Raifearteagh → De Sitter Inflation

▸ When $|\Phi| \ll m$: $W \propto \Phi^{nq+1}$ Chaotic monomial Inflation

- Further take $q = -1/n$: $W = M^2(\Phi^n + m^n)^{1/n}$ **Shaft Inflation**

- Scalar Potential: $\Rightarrow V = M^4|\Phi|^{2(n-1)}|\Phi^n + m^n|^{2(\frac{1}{n}-1)}$

$\Phi \equiv \phi\, e^{i\theta}$

$\phi, \theta \in \mathbb{R}$ & $\phi > 0$ $$\Rightarrow V = \frac{M^4\phi^{2(n-1)}}{[\phi^{2n} + m^{2n} + 2\cos(n\theta)m^n\phi^n]^{\frac{n-1}{n}}}$$

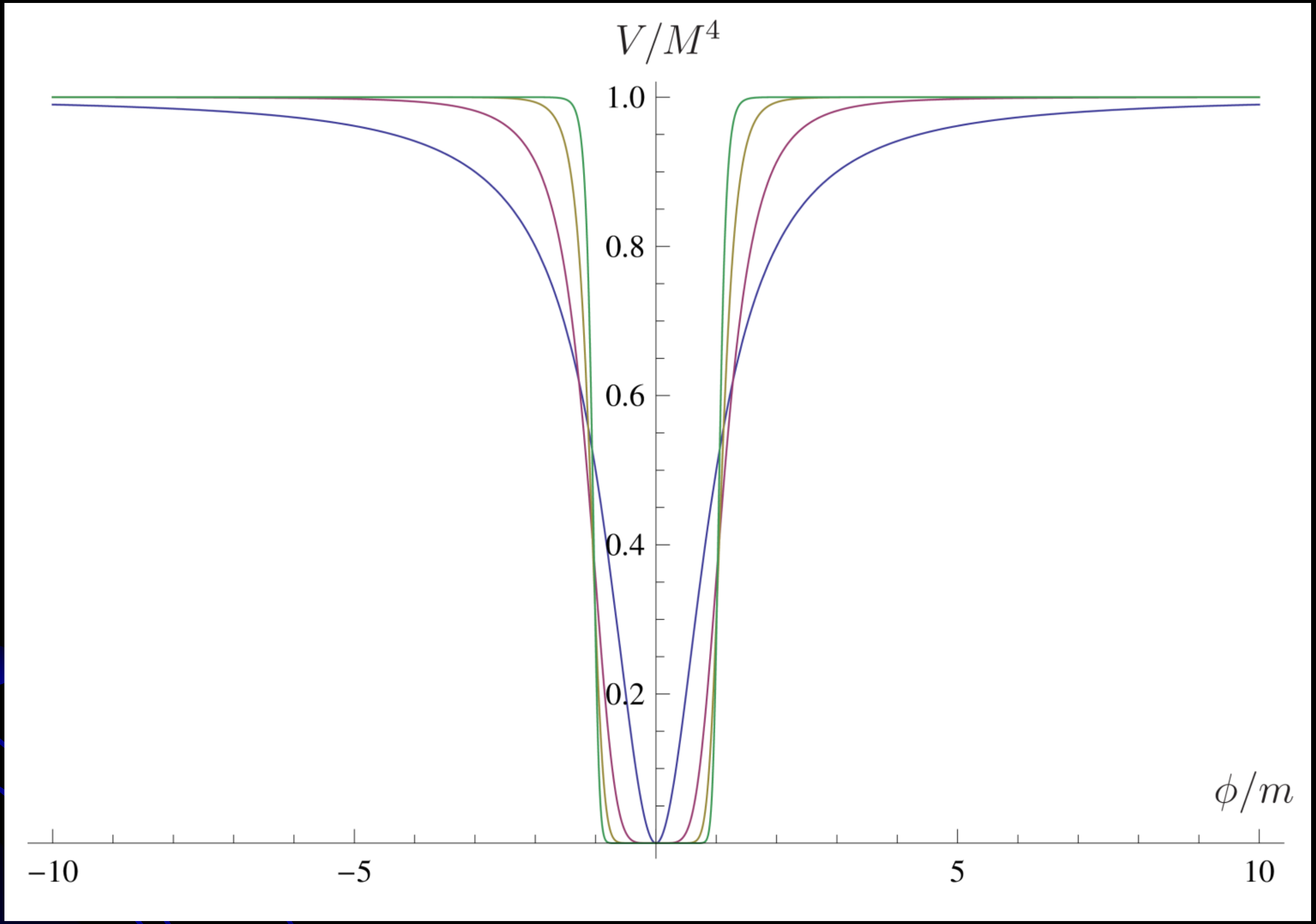
Minimised: $n\theta = 2\ell\pi$ with $\ell \in \mathbb{N}$

Further $-\phi = \phi e^{i\pi}$ so $V(\phi) = V(-\phi)$ with $n = 2\ell$ **even**

▸ When $\phi \gg m$: $V \approx M^4$ $\Rightarrow$ $V(\phi) = M^4\phi^{2n-2}(\phi^n + m^n)^{\frac{2}{n}-2}$

▸ When $\phi \ll m$: $V \propto \phi^{2(n-1)}$ $\Rightarrow n \geq 2$ $\forall\, \phi \in \mathbb{R}$

The Inflationary Potential



- Scalar Potential for Shaft Inflation for $n = 2, 4, 8, 16$
- The shaft becomes more pronounced as n grows.
- Far from the origin $V \approx M^4$, whereas near the origin $V \propto \phi^{2(n-1)}$

Slow-Roll Parameters

$$\epsilon \equiv \frac{1}{2} m_P^2 \left(\frac{V'}{V}\right)^2 = 2(n-1)^2 \left(\frac{m_P}{\phi}\right)^2 \left(\frac{m^n}{\phi^n + m^n}\right)^2$$

$$\eta \equiv m_P^2 \frac{V''}{V} = 2(n-1) \left(\frac{m_P}{\phi}\right)^2 \left(\frac{m^n}{\phi^n + m^n}\right) \frac{(2n-3)m^n - (n+1)\phi^n}{\phi^n + m^n}$$

- Spectral Index:

$$n_s = 1 + 2\eta - 6\epsilon = 1 - 4(n-1) \left(\frac{m_P}{\phi}\right)^2 \frac{m^n[(n+1)\phi^n + n m^n]}{(\phi^n + m^n)^2}$$

Slow-Roll Parameters

- Inflation ends when $|\eta| \simeq 1 \Rightarrow \phi_{\text{end}} \simeq m_P \left[2(n^2-1)\alpha^n\right]^{1/(n+2)}$

where $\alpha \equiv \dfrac{m}{m_P}$ and $\phi > m$

$$N = \frac{1}{m_P^2}\int_{\phi_{\text{end}}}^{\phi} \frac{V}{V'}\mathrm{d}\phi \simeq \frac{1}{2(n-1)(n+2)\alpha^n}\left[\left(\frac{\phi}{m_P}\right)^{n+2} - \left(\frac{\phi_{\text{end}}}{m_P}\right)^{n+2}\right]$$

$$\Rightarrow \quad \phi(N) \simeq m_P\left[2(n-1)(n+2)\alpha^n\left(N + \frac{n+1}{n+2}\right)\right]^{1/(n+2)}$$

n_s and r

$$n_s = 1 - 2\frac{n+1}{n+2}\left(N + \frac{n+1}{n+2}\right)^{-1}$$

B

$$r = 16\epsilon = 32(n-1)^2\alpha^{\frac{2n}{n+2}}\left[2(n-1)(n+2)\left(N+\frac{n+1}{n+2}\right)\right]^{-2\left(\frac{n+1}{n+2}\right)}$$

- **Dependence on** $\alpha \equiv \frac{m}{m_P}$ **only for tensor/scalar ratio**

Examples

► : $n = 2$ $$V(\phi) = M^4 \frac{\phi^2}{\phi^2 + m^2}$$

- Such potential is also featured in S-dual superstring inflation (with $\alpha = \frac{1}{4}$) and also in radion assisted gauge inflation (with $\alpha \sim 10^{-3/2}$)

A.De la Macorra & S.Lola, PLB373(1996)299;
M.Fairbairn, L.Lopez Honorez & M.H.G.Tygat, PRD67(2003)101302

$$n_s = 1 - \frac{3}{2}\left(N + \frac{3}{4}\right)^{-1} \quad \& \quad r = \frac{32\alpha}{\left[8\left(N + \frac{3}{4}\right)\right]^{3/2}}$$

► : $n \gg 1$

$$n_s = 1 - \frac{2}{N+1} \quad \& \quad r = \frac{8\alpha^2}{n^2(N+1)^2} \to 0$$

- Same as R^2 – Inflation, Higgs Inflation and T-model Inflation

Agreement with Planck

- Consider $\alpha = 1$

▶ : $n = 2$

$$N = 60 \Rightarrow n_s = 0.975 \quad r = 0.0030$$
$$N = 50 \Rightarrow n_s = 0.970 \quad r = 0.0039$$

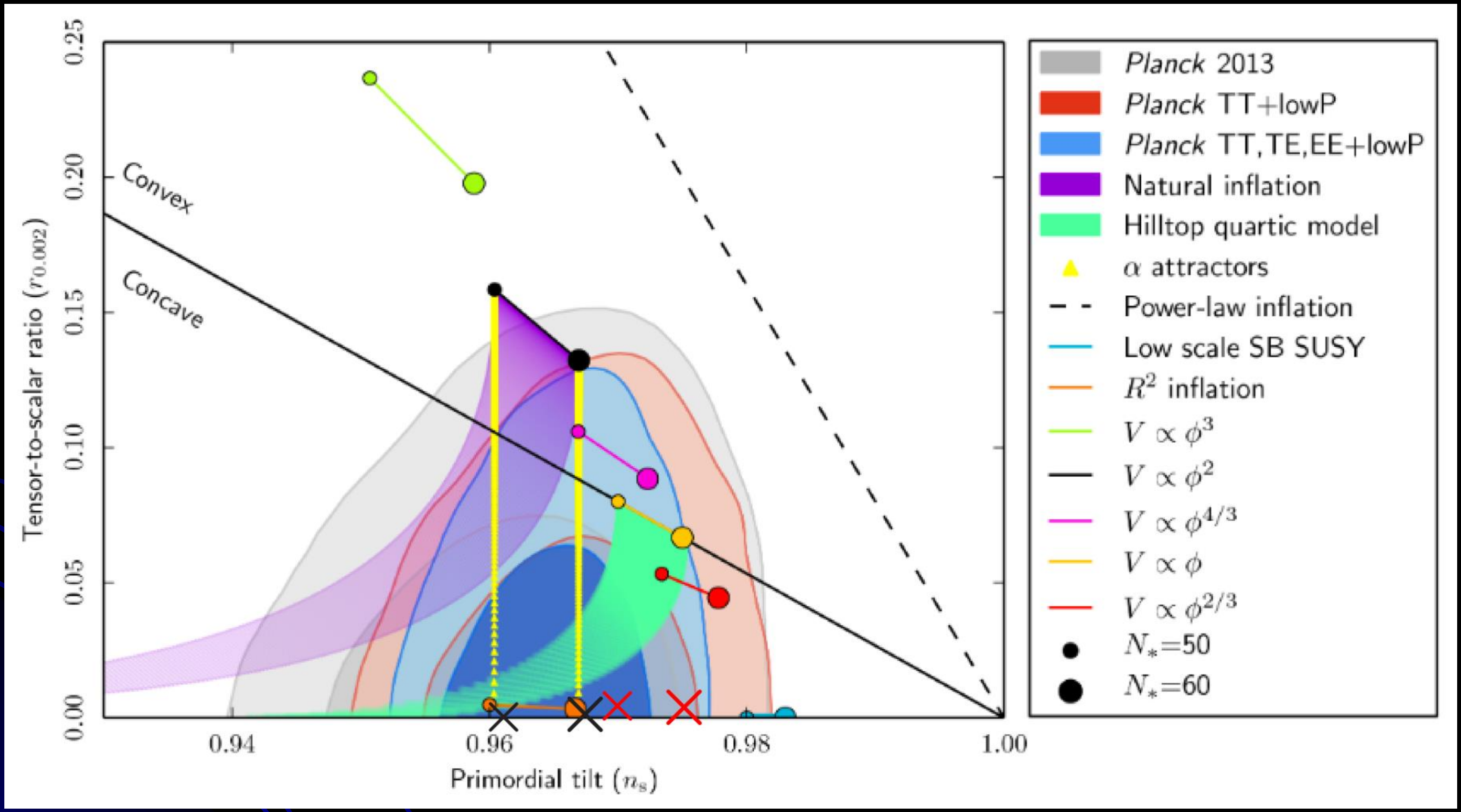
▶ : $n \gg 1$

$$n_s = 0.967 \quad n_s = 0.961$$

for $N = 60$ & $N = 50$

Agreement with Planck

- Consider $\alpha = 1$



- Big (small) red cross corresponds to $n = 2$ with $N = 60$ (50)
- Big (small) black cross corresponds to $n \gg 1$ with $N = 60$ (50)

Gravitational Waves

- Planck observations (with BICEP2 and Keck Array) suggest: $r < 0.1$
- In Shaft Inflation: $r \propto \alpha^{2n/(n+2)}$ $\Rightarrow$
- **Sizable tensors can be attained by widening the shaft**

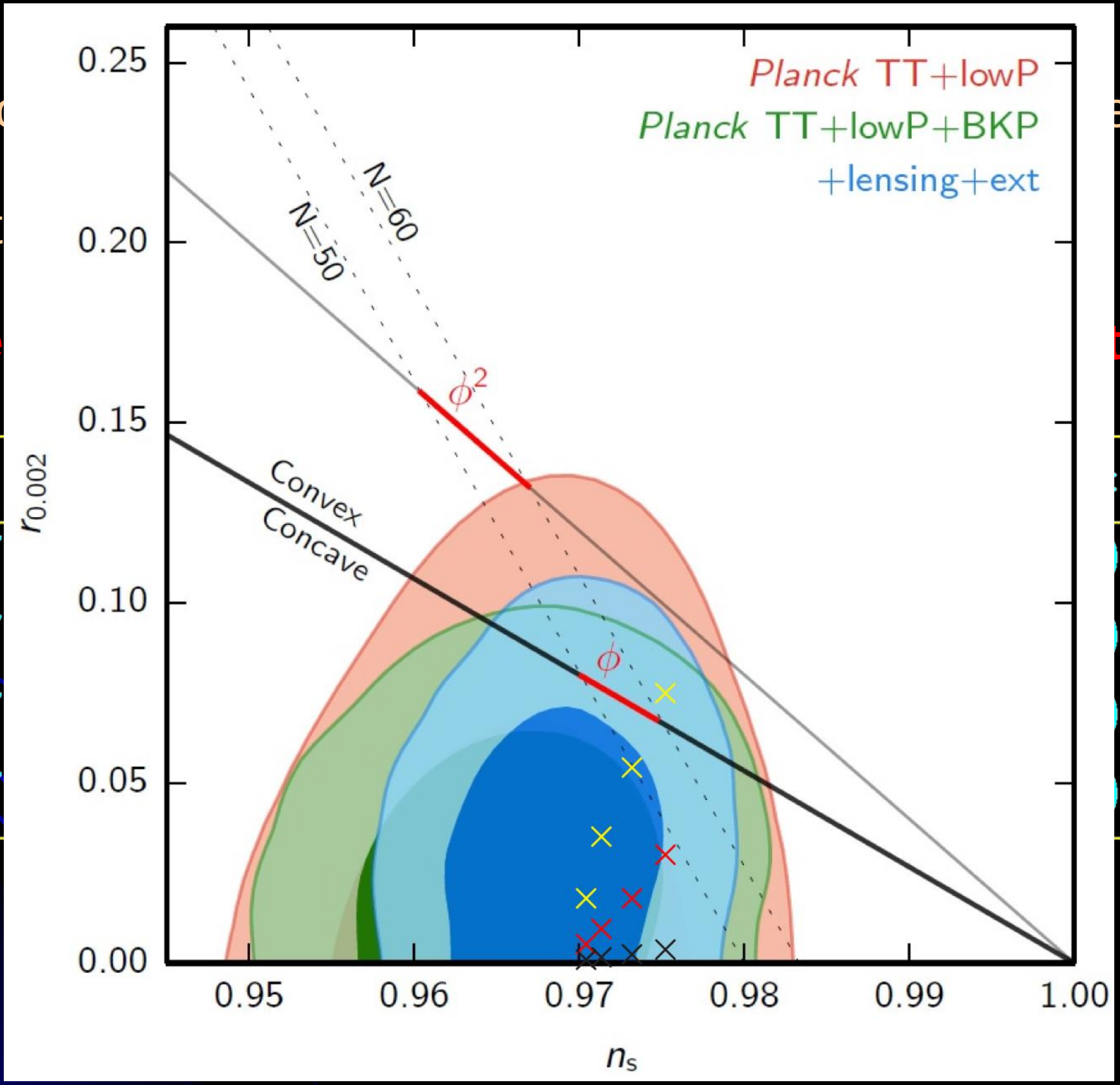
n	n_s	$r\ (\alpha = 1)$	$r\ (\alpha = 2\sqrt{8\pi})$	$r\ (\alpha = 5\sqrt{8\pi})$
2	0.975	0.0030	0.0299	0.0747
4	0.973	0.0008	0.0168	0.0570
6	0.971	0.0003	0.0089	0.0352
8	0.970	0.0001	0.0052	0.0227

$$m = \alpha m_P = \frac{\alpha}{\sqrt{8\pi}} M_P$$

Gravitational Waves

- Planck c … est: $r < 0.1$
- In Shaft …
- **Sizable** … t

n	n_s	… $5\sqrt{8\pi})$
2	0.97 …	…747
4	0.97 …	…570
6	0.97 …	…352
8	0.97 …	…227



More on Shaft Inflation

- Running of spectral index:

$$\frac{\mathrm{d}n_s}{\mathrm{d}\ln k} = -\frac{2\left(\frac{n+1}{n+2}\right)}{\left(N+\frac{n+1}{n+2}\right)^2} \sim -\frac{2}{N^2} \sim -10^{-3}$$

$\Rightarrow$ In agreement with Planck:

$$\frac{\mathrm{d}n_s}{\mathrm{d}\ln k} = -0.003 \pm 0.007$$

- Inflationary energy scale:

$$\sqrt{\mathcal{P}_\zeta} = \frac{1}{2\sqrt{3}\pi}\frac{V^{3/2}}{m_P^3|V'|}$$

$$\left(\frac{M}{m_P}\right)^2 = 4\sqrt{3}(n-1)\alpha^{-\frac{n}{n+2}}\pi\sqrt{\mathcal{P}_\zeta}\left[2(n-1)(n+2)\left(N+\frac{n+1}{n+2}\right)\right]^{-\frac{n+1}{n+2}}$$

$$\sqrt{\mathcal{P}_\zeta} = 4.706\times10^{-5}$$

$n=2$ & $\alpha=1$ & $N=60$

$\Rightarrow$ $M = 7.7\times10^{15}\,\mathrm{GeV}$

~ **GUT** scale

Conclusions

- Planck data suggest single field inflation, characterized by a scalar potential which approaches an inflationary plateau
- In contrast to many other successful models, Shaft Inflation approaches this plateau in a power-law manner
- Shaft Inflation is based on a simple superpotential: $W = M^2(\Phi^n + m^n)^{1/n}$
- Without any fine tunning ($M \sim$ **GUT** and $m \sim m_P$) Shaft Inflation produces a scalar spectral index very close to the Planck sweet spot with very small (negative) running, in agreement with Planck
- Rendering m mildly super-Planckian one can easily obtain potentially observable tensors without affecting the spectral index
- **The challenge in now to obtain realistic setups which can realise the simple Shaft Inflation superpotential**

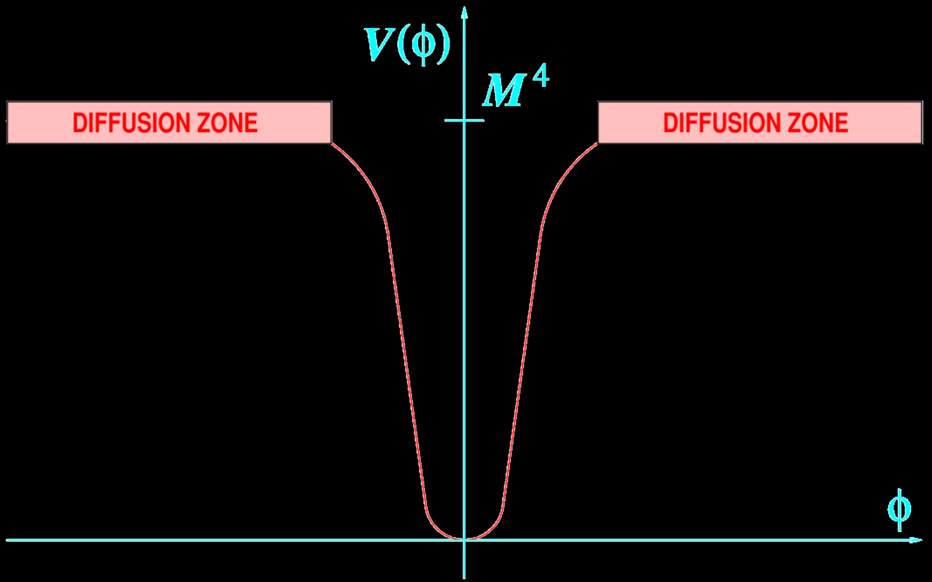
Eternal Inflation

- For a large inflaton, slow-roll can last for a huge number of e-folds
- The vacuum density in now constant and remains sub-Planckian:

$$V(\phi \gg m) \simeq M^4 \quad \text{with} \quad M \ll m_P$$

- Far from the shaft, the inflaton is in the quantum diffusion zone where its variation is dominated by quantum fluctuations

$\Rightarrow$ **eternal inflation**



Eternal Inflation

- For quantum diffusion to dominate over the classical field variation:

$$|\dot{\phi}| \gtrless \frac{\delta\phi}{\delta t} \Leftrightarrow |V'| \gtrless \frac{3}{2\pi}H^3$$

where $\delta\phi = H/2\pi \quad \delta t = H^{-1}$

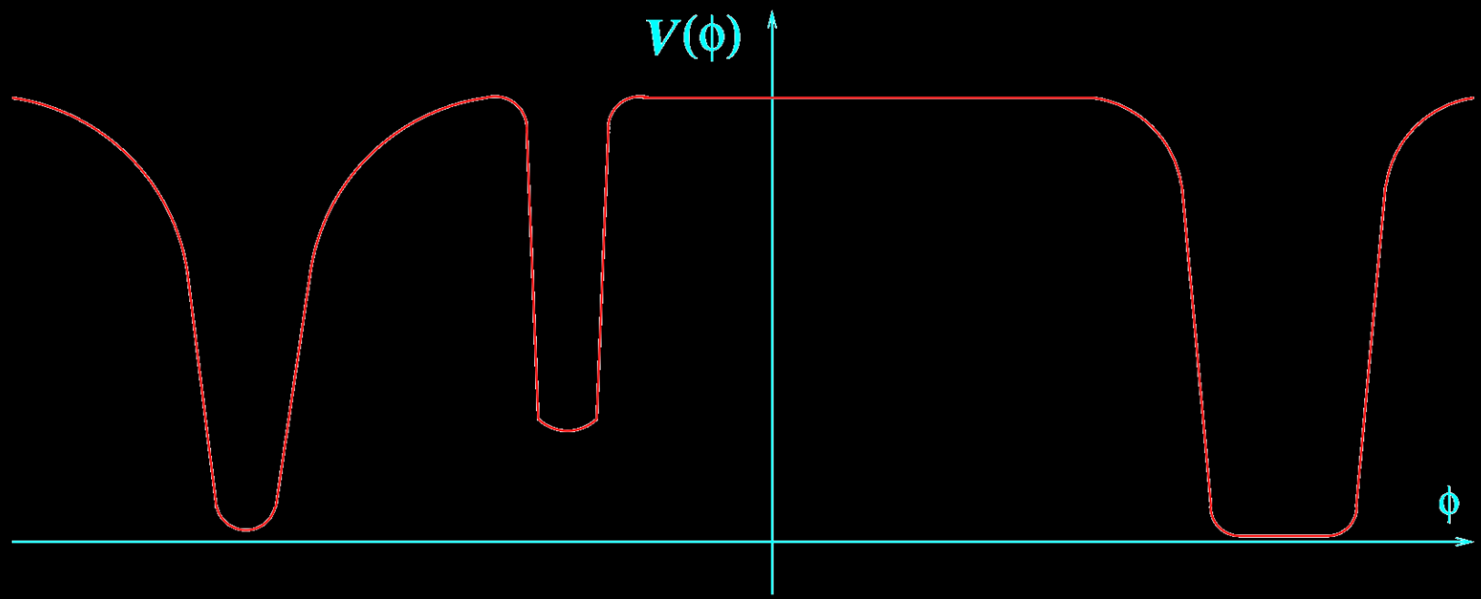
$3H\dot{\phi} \simeq -V' \quad V(\phi) \simeq 3(Hm_P)^2$

$$\Rightarrow \quad \frac{N}{N_*} \simeq \frac{N + \frac{n+1}{n+2}}{N_* + \frac{n+1}{n+2}} \lesssim \left(\sqrt{\mathcal{P}_\zeta}\right)^{-\frac{n+2}{n+1}} \sim 10^{4-6}$$

with $N_* \simeq 60$ and also noting that $1 < \frac{n+2}{n+1} < \frac{3}{2}$

- After exiting the diffusion zone, slow-roll inflation takes over for a few million e-folds before the cosmological scales exit the horizon (and ~ 60 e-folds afterwards). Eventually, inflation ends and the inflaton oscillates at the bottom of the shaft, leading to (p)reheating and the hot big bang.

Shaft Inflation and the Multiverse



- There may be many shafts (with possibly different values of n) puncturing the inflationary plateau (leading to different vacua).
- Some regions of the multiverse are close to the shafts so that eternal inflation is superseded by slow-roll, which attracts the inflaton the shaft in question. Our observable Universe is such a case.
- Meanwhile, elsewhere in the Multiverse, eternal inflation continues…