On Soft Limits of Large-Scale Structure Correlation Functions

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Based on 1411.3225 and 1508.06306

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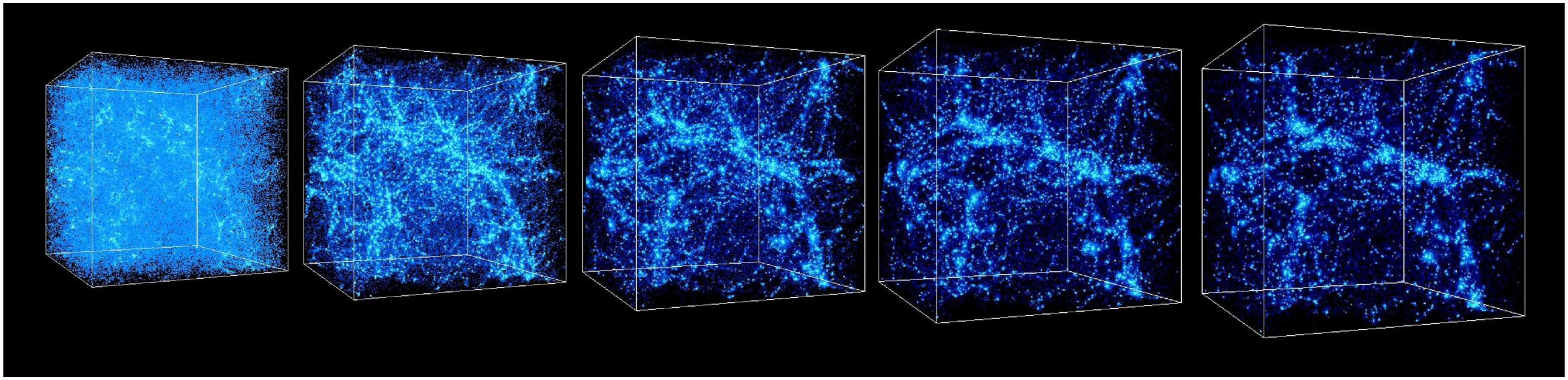
COSMO-15, Warsaw

September 08, 2015

Introduction

Initial matter density perturbations → Present large-scale structure



[cosmicweb.uchicago.edu/filaments.html]

→ Fluid equations for LSS

- Solution: cosmological perturbation theory
- Non-perturbative relations?

Outline

1. Fluid equations
2. Non-perturbative soft limits
   - Bispectrum
   - Power spectrum

   $\Longrightarrow$ Implications for perturbation theory
3. Conclusions

Fluid equations

Fluid equations

- Central quantities of cosmological perturbation theory

$$\delta \quad \text{and} \quad \theta \equiv \boldsymbol{\nabla} \cdot \boldsymbol{v}$$

as doublet

$$\psi_a\,(\boldsymbol{k}, \eta) \equiv \begin{pmatrix} \delta \\ -\theta/\mathcal{H} \end{pmatrix}$$

$\rightarrow$ Conformal expansion rate $\mathcal{H} = a\,H$

$\rightarrow$ Time variable $\eta \equiv \ln a$

Fluid equations

- Fluid equations (continuity, Euler and Poisson equation)

  in a compact form:

$$\partial_\eta \psi_a = -\Omega_{ab}\, \psi_b + \gamma_{abc}\, \psi_b \psi_c$$

[Bernardeau et al., '02]
[Scoccimarro, '06]

→ Ω_{ab} depends on cosmological model

→ Vertex functions γ_{abc}

Fluid equations

- Fluid equations:

$$\partial_\eta \psi_a = -\Omega_{ab}\, \psi_b + \gamma_{abc}\, \psi_b \psi_c$$

- Iteration of correlation functions:

$$\begin{aligned}\partial_\eta \langle \psi_a \psi_b \rangle &= -\Omega_{ac} \langle \psi_c \psi_b \rangle - \Omega_{bc} \langle \psi_a \psi_c \rangle \\ &\quad + \gamma_{acd} \langle \psi_c \psi_d \psi_b \rangle + \gamma_{bcd} \langle \psi_a \psi_c \psi_d \rangle, \\ \partial_\eta \langle \psi_a \psi_b \psi_c \rangle &= \ldots\end{aligned}$$

$\rightarrow$ Infinite hierarchy of evolution equations

Time-flow approach

[Matarrese, Pietroni, '06]

- Correlation functions:

$$\langle \psi_a \psi_b \rangle \sim P_{ab},$$

$$\langle \psi_a \psi_b \psi_c \rangle \sim B_{abc},$$

$$\langle \psi_a \psi_b \psi_c \psi_d \rangle \sim P_{ab}\, P_{cd} + P_{ac}\, P_{bd} + P_{ad}\, P_{bc} + Q_{abcd}$$

- Closure approximation:

$$Q_{abcd} = 0$$

$\rightarrow$ Solution of fluid equations

Soft limit

of the

bispectrum

Soft limit of the bispectrum

Derivation:

- Gaussian initial conditions: $B_{abc}(\eta = 0) = 0$
- Perturbative expansion in the soft momentum q
- Soft limit: $q \to 0$
- Angular average: $(\ldots)^{\text{av}} \equiv \int d\Omega_{kq} / (4\pi)$

$\longrightarrow$ Bispectrum consistency relation:

$$B_{abc}{}^{\text{av}}_{q\to 0}(k, q, \eta)$$

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Generally valid for all cosmologies

Soft limit of the bispectrum

For a flat universe (EdS, ΛCDM):

- At linear order:

$$P_{ab}(k) \simeq P^L(k) \quad \forall\, a, b$$

$$B^{L}_{abc}\,{}^{\mathrm{av}}_{q\to 0} \simeq \left(\frac{1}{21} \begin{pmatrix} 47 & 39 \\ 39 & 31 \end{pmatrix}_{ac} - \frac{1}{3} k\, \partial_k \right) P^L(k)\, P^L(q)$$

→ For $B^{L}_{111}\,{}^{\mathrm{av}}_{q\to 0} \sim \langle \delta\delta\delta \rangle^{\mathrm{av}}_{q\to 0}$:
Coincides with standard perturbation theory (SPT)

[Sherwin, Zaldarriaga, '12]

Physical picture

Physical picture

Goal:

Understand influence of a long-wavelength (soft) mode

on the local dynamics of the universe

[Baldauf et al., '11]

Ansatz:

- Isotropic long-wavelength (soft) perturbation:

$$\Phi_L \simeq \frac{1}{4} H^2 a^2 \delta_L \boldsymbol{x}^2 \ll 1$$

- Flat FRW cosmology in Newtonian gauge:

$$ds^2 = -\left[1 + 2\Phi_L\right] dt^2 + a^2 \left[1 - 2\Phi_L\right] d\boldsymbol{x}^2$$

Physical picture

- Coordinate transformation:

$$t = t_K + f(t_K, \boldsymbol{x}_K)\,, \quad \boldsymbol{x} = \boldsymbol{x}_K(1 + g(t_K, \boldsymbol{x}_K))$$

$\longrightarrow$ Locally curved universe:

$$ds^2 = -dt_K^2 + a_K^2 \frac{d\boldsymbol{x}_K^2}{\left(1+\frac{1}{4}K\boldsymbol{x}_K^2\right)^2}\,,$$

$$K \simeq \frac{5}{3}H^2a^2\delta_L\,, \quad a_K \simeq a\left(1 - \frac{1}{3}\delta_L\right)\,, \quad \delta_K = \delta\left(1 - \delta_L\right)$$

 + flat FRW $\rightarrow$ locally curved universe

Locally curved universe

- Correlation functions in Fourier space:

$$\langle\delta\delta\rangle \sim P\,, \quad \langle\delta\delta\delta\rangle \sim B$$

- Non-perturbative relation for the bispectrum in the soft limit:

$$B_{111\;q\to 0} = P^L(q)\left[\left(1-\frac{1}{3}k\;\partial_k - \frac{1}{3}\partial_\eta\right)P(k) + \frac{5}{3}\;\frac{\partial}{\partial\kappa}P_K(k)\bigg|_{K=0}\right]$$

With $\eta \equiv \ln a\,,\ \kappa = K/(a^2H^2)$

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Hypothetical curved universe

Locally curved universe

$$B_{111\;q\to 0} = P^L(q)\left[\left(1 - \frac{1}{3}k\;\partial_k - \frac{1}{3}\partial_\eta\right)P(k) + \frac{5}{3}\;\frac{\partial}{\partial\kappa}P_K(k)\bigg|_{K=0}\right]$$

- Proposal by Valageas, and by Kehagias, Perrier, Riotto (VKPR):

$$\frac{\partial}{\partial\kappa}P_K(k)\bigg|_{K=0} = \frac{4}{7}\,\partial_\eta P(k)$$

$$\rightarrow \quad B_{111\;q\to 0} = \left(1 - \frac{1}{3}k\,\partial_k + \frac{13}{21}\partial_\eta\right)P(k)\,P^L(q)$$

[Valageas '13]

[Kehagias, Perrier, Riotto '13]

Locally curved universe

- VKPR relation:

  $\rightarrow$ At linear order: exact

  $\rightarrow$ Beyond linear order: very good approximation

  $\Longrightarrow$ Tested with "separate universe" simulations

[Li et al., '14]

[Wagner et al., '14]

[Chiang et al '14]

- Generalization for density and velocity fields:

$$\frac{\partial}{\partial\kappa}\psi^{K}_{ab}\bigg|_{K=0} \simeq \frac{4}{7}\begin{pmatrix} \partial_\eta & 0 \\ 0 & \partial_\eta + 1 \end{pmatrix}_{ab} \psi_b$$

Soft limit

of the power

spectrum

Soft limit of the power spectrum

- Physical picture:

  Directional soft mode $\longrightarrow$ curved anisotropic universe

- Derivation:

  $\longrightarrow$ Insert soft-limit bispectrum in fluid equations

  $\longrightarrow$ Use generalized VKPR relation to approximate curvature dependence

  $\Longrightarrow$ Non-perturbative equation for the power spectrum

Soft limit of the power spectrum

- Non-perturbative equation for the power spectrum:

$$\partial_\eta P_{ab}(q) = -\Omega_{ac} P_{cb}(q) - \Omega_{bc} P_{ac}(q) + \frac{q^2}{2} P^L(q) \left\{ \begin{pmatrix} 0 & 1 \\ 1 & 2 \end{pmatrix} C_{22}(\eta) + \begin{pmatrix} 2 & 1 \\ 1 & 0 \end{pmatrix} C_{12}(\eta) \right\}$$

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negligible

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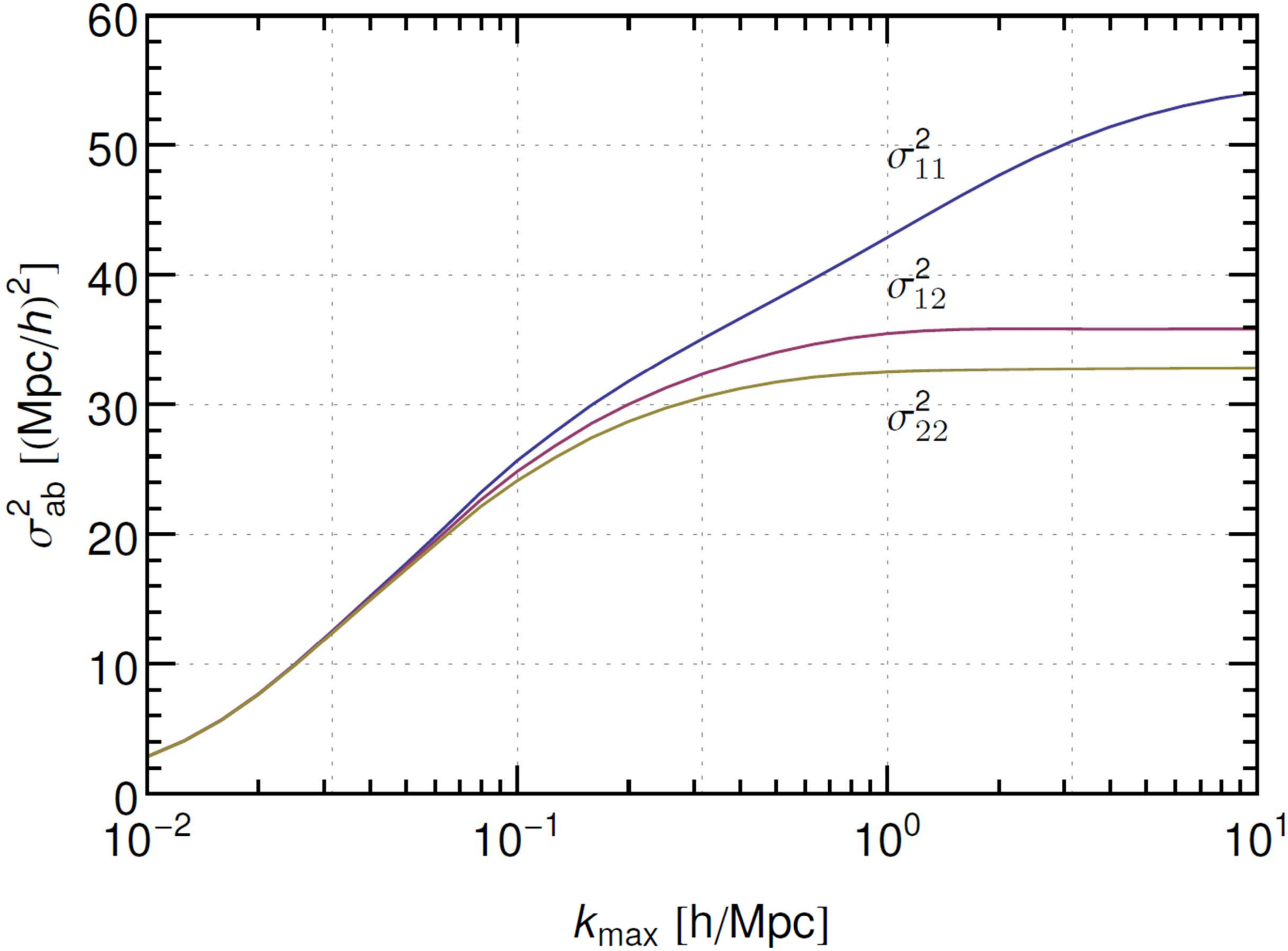
negligible

$$\sim \sigma_{22}^2(\eta) \equiv \frac{4\pi}{3} \int dk \; P_{22}(k,\eta)$$

Soft limit of the power spectrum



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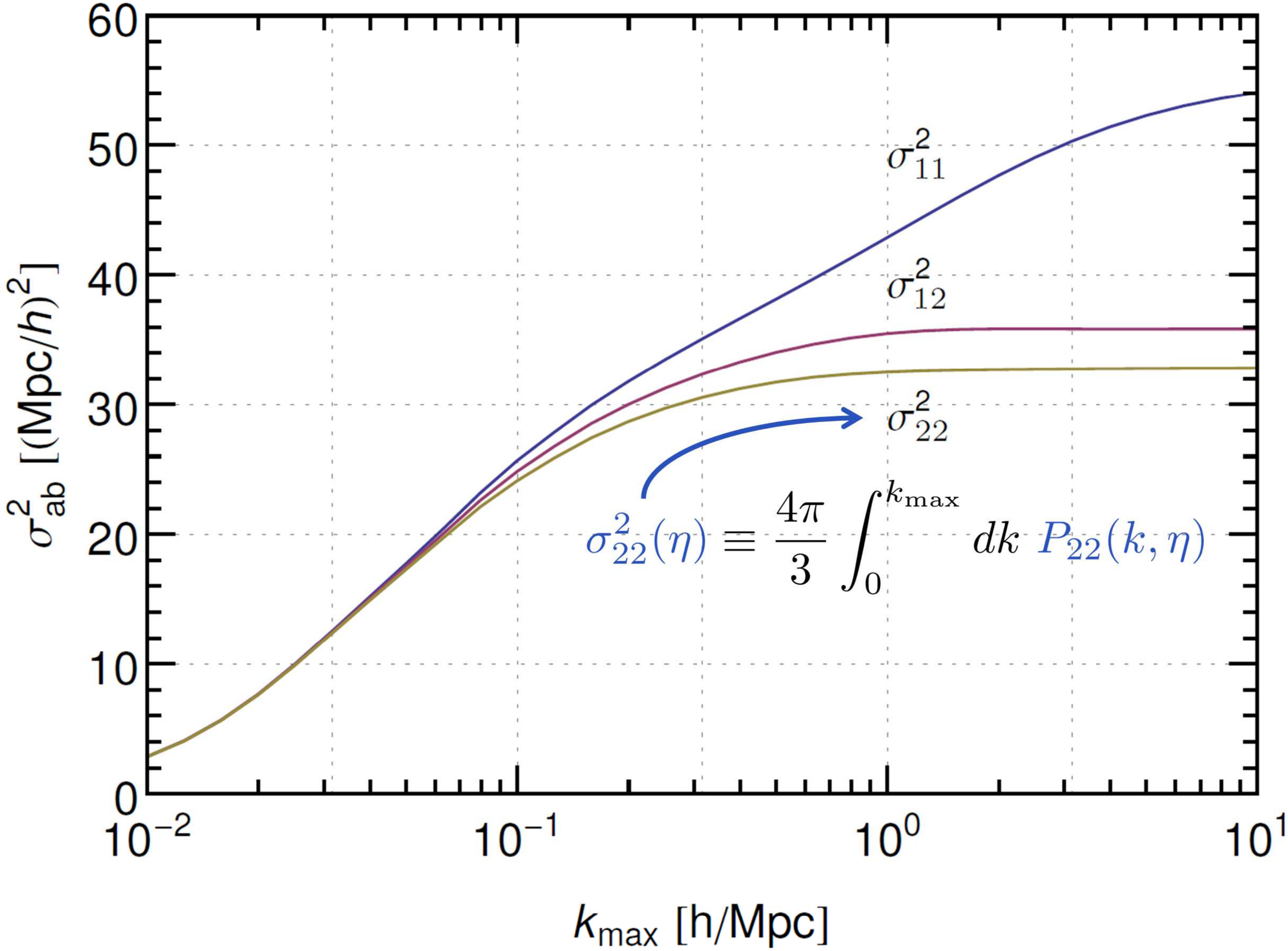

Soft limit of the power spectrum

σ_{22}^2 and hence $P(q)$ depend only weakly on UV modes

- UV sensitivity in SPT: artifact

  $\rightarrow$ perturbative techniques inapplicable beyond non-linear scale

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  $\longrightarrow$ perturbative techniques inapplicable beyond non-linear scale

- In effective field theory (EFT) of LSS:

  $\longrightarrow$ counter-terms for UV dependence

  $\longrightarrow$ leading-order renormalized coefficients:

  mainly modes up to non-linear scale contribute

Conclusions

Conclusions

Non-perturbative soft limits of LSS correlation functions:

- Bispectrum:

  $\longrightarrow$ Simple consistency relation

  as probe for upcoming LSS surveys (e.g. non-Gaussianity)

- Power spectrum:

  $\longrightarrow$ UV dependence in perturbation theory (SPT)

  $\longrightarrow$ Implications for EFT

Conclusions

Future work:

- Non-perturbative power spectrum

  $+$ "anisotropic separate universe" simulations

  $\Longrightarrow$ precisely infer leading-order EFT coefficients

Thank you for your attention!

Backup

Fluid equations

Continuity equation

= conservation of mass:

$$\partial_\tau \delta + \boldsymbol{\nabla} \cdot \left[(1+\delta)\, \boldsymbol{v} \right] = 0$$

Euler equation

= conservation of momentum:

$$\partial_\tau v + \mathcal{H}\, \boldsymbol{v} + (\boldsymbol{v} \cdot \boldsymbol{\nabla})\, \boldsymbol{v} = -\boldsymbol{\nabla} \Phi$$

Poisson equation:

$$\triangle \Phi = \frac{3}{2} \Omega_m \mathcal{H}^2\, \delta$$

→ highly non-linear differential equations

Physical quantities:

- Density contrast $\delta = \dfrac{\rho - \bar{\rho}}{\bar{\rho}}$
- Conformal time τ: $d\tau = \frac{dt}{a}$
- Peculiar velocity $\boldsymbol{v}$
- Expansion rate $\mathcal{H} = a\, H$
- Gravitational potential Φ