
THE OBSERVATIONAL POSITION OF SIMPLE NON-MINIMALLY COUPLED INFLATIONARY SCENARIOS

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Edwards and Liddle; June 2014; arXiv: [1406.5768](#)[astro-ph.CO]

Kallosch, Linde and Roest; Oct. 2013; arXiv: [1310.3950](#)[hep-th]

Kehagias, Dizgah and Riotto; Dec. 2013; arXiv: [1312.1155](#) [hep-th]

Kallosch and Linde; Sep. 2013; arXiv: [1309.2015](#)[astro-ph.CO]

THE UNIVERSAL ATTRACTORS

Kallosh, Linde and Roest; Oct. 2013; arXiv:1310.3950[hep-th]

The Jordan Frame:

$$\mathcal{L}_J = \sqrt{-g} \left[\frac{1}{2} \Omega(\phi) R - \frac{1}{2} (\partial\phi)^2 - V_J(\phi) \right]$$

$$\Omega(\phi) = 1 + \xi f(\phi) \quad ; \quad V_J(\phi) = \lambda^2 f^2(\phi)$$

Transformation to the Einstein Frame:

$$g_{\mu\nu} \rightarrow \Omega^{-1}(\phi) g_{\mu\nu}$$

$$\mathcal{L}_E = \sqrt{-g} \left[\frac{1}{2} R - \frac{1}{2\Omega^2(\phi)} \left(\Omega(\phi) + \frac{3}{2} \left[\frac{d\Omega(\phi)}{d\phi} \right]^2 \right) (\partial\phi)^2 - \frac{V_J(\phi)}{\Omega^2(\phi)} \right]$$

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A journey to observable quantities:

$$\left[\frac{\partial\varphi}{\partial\phi}\right]^2 = \frac{1}{\Omega^2(\phi)} \left(\Omega(\phi) + \frac{3}{2} \left[\frac{d\Omega(\phi)}{d\phi}\right]^2 \right)$$

$$\epsilon \equiv \frac{\Omega^4(\phi)}{V_J^2(\phi)} \left(\frac{d}{d\phi} \left[\frac{V_J(\phi)}{\Omega^2(\phi)} \right] \right)^2 \left(\frac{\partial\phi}{\partial\varphi} \right)^2$$

$$\eta = 2\epsilon + \frac{\partial\epsilon}{\partial\varphi} \frac{1}{\sqrt{2\epsilon}}$$

These take on a form independent of the potential in the
"strong coupling" limit

THE UNIVERSAL ATTRACTORS

Kalosh, Linde and Roest; Oct. 2013; arXiv:1310.3950[hep-th] Edwards and Liddle; June 2014; arXiv:1406.5768[astro-ph.CO]

Evaluate number of e-foldings:

$$N = \int \frac{1}{\sqrt{2\epsilon}} d\varphi = \frac{1}{4} \int \frac{f(\phi)}{f'(\phi)} \frac{2 + 2\xi f(\phi) + 3[\xi f'(\phi)]^2}{1 + \xi f(\phi)} d\phi$$

$$\xi f(\phi) \gg 1$$

$$N \simeq \frac{3}{4} \xi f(\phi_N)$$

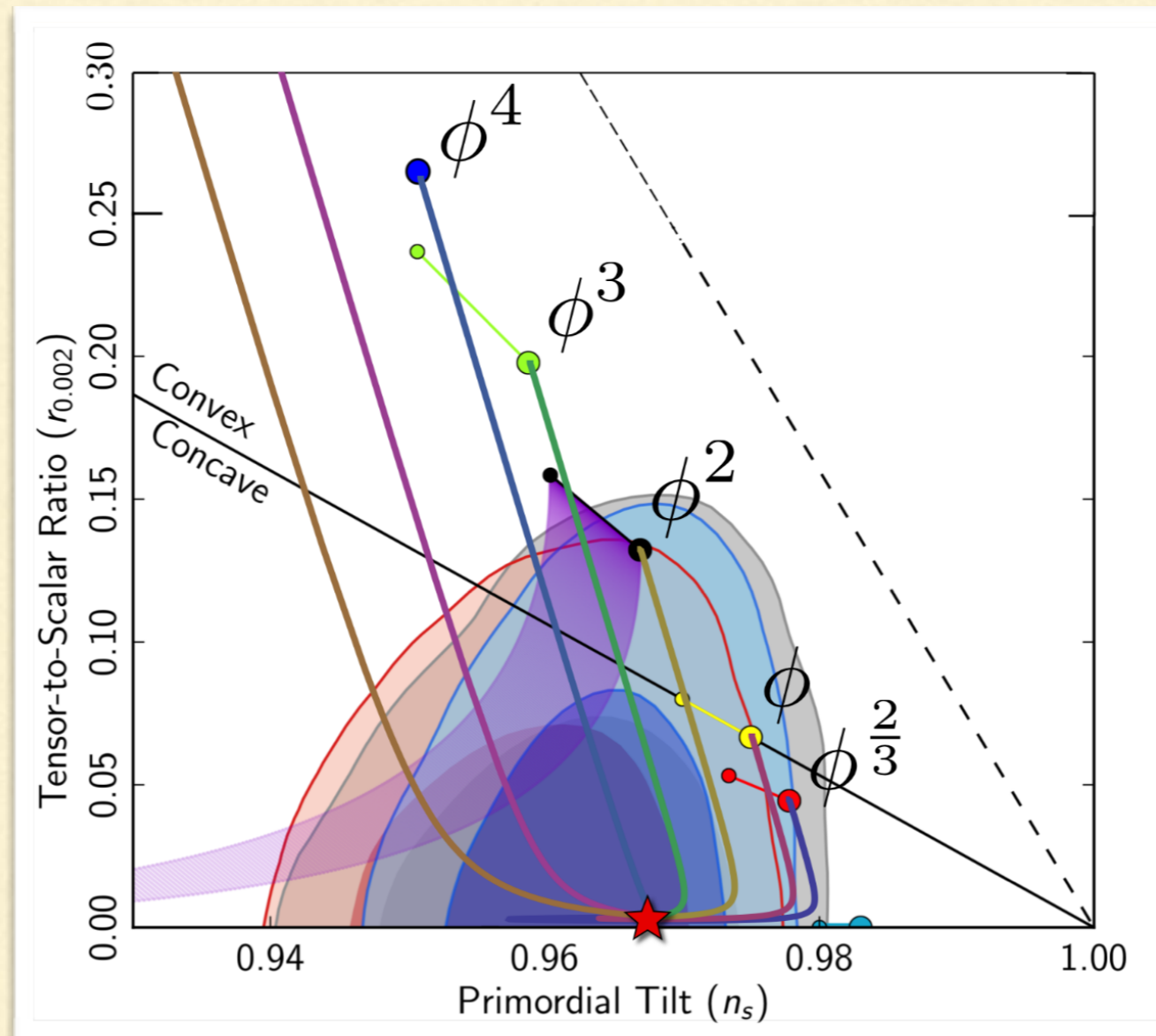
Apply to the expressions for the slow-roll parameters:

$$r = \frac{12}{N^2}$$

$$n_s = 1 - \frac{2}{N}$$

THE UNIVERSAL ATTRACTORS

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THE UNIVERSAL ATTRACTORS?

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Re-evaluate number of e-foldings:

$$\xi f(\phi) \gg 1$$

$$N \simeq \frac{3}{4} \xi f(\phi_N)$$

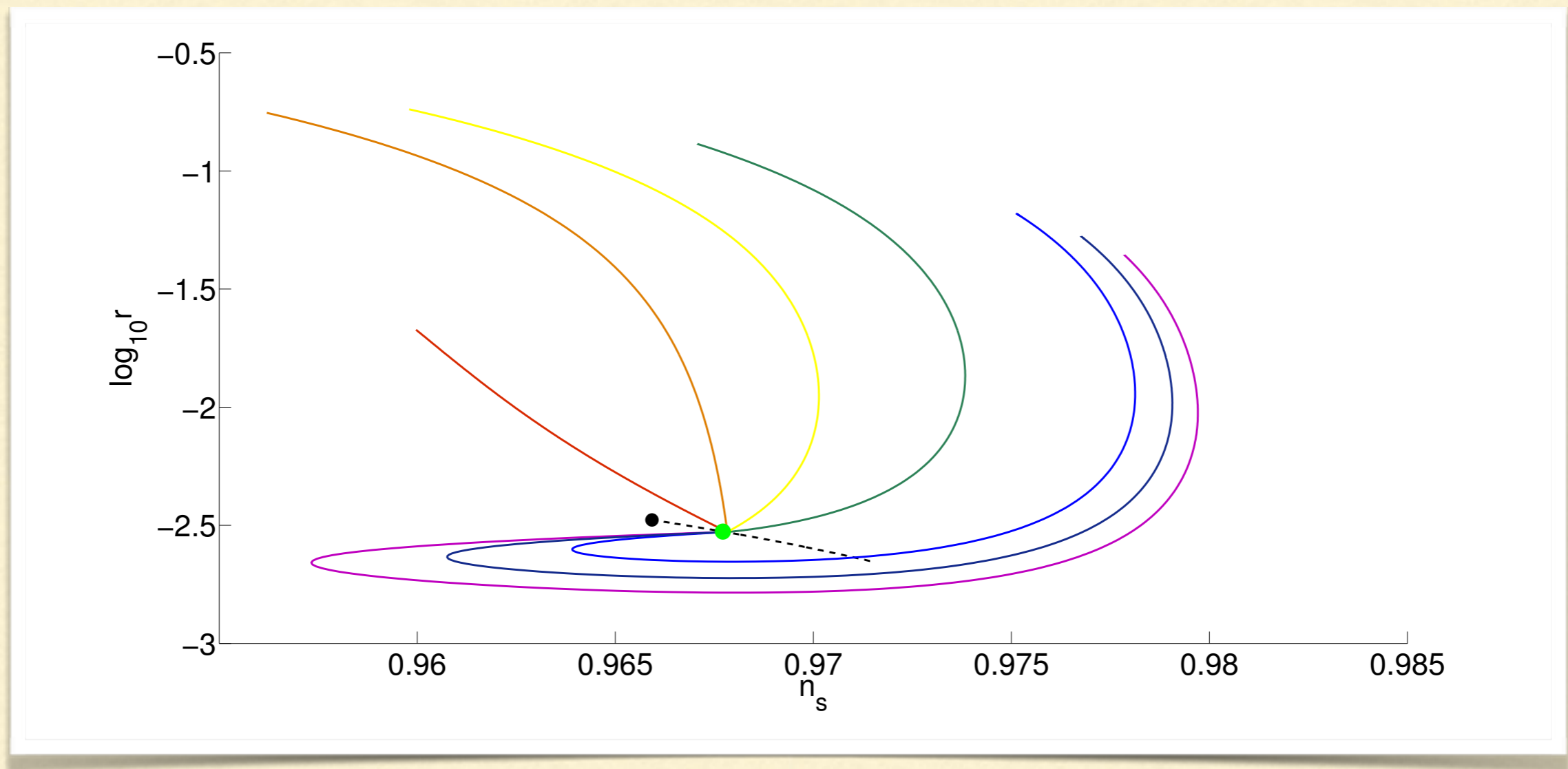
$$N = \frac{3}{4} \xi f(\phi_N) - \frac{3}{4} \log \frac{f(\phi_N)}{f(\phi_{\text{end}})}$$

Apply to the expressions for the slow-roll parameters:

$$\delta r = -\frac{24\delta N}{N^3} = -\frac{12}{N} \delta n_s$$

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APPROACHING THE ATTRACTOR

Edwards and Liddle; June 2014; arXiv:1406.5768[astro-ph.CO]

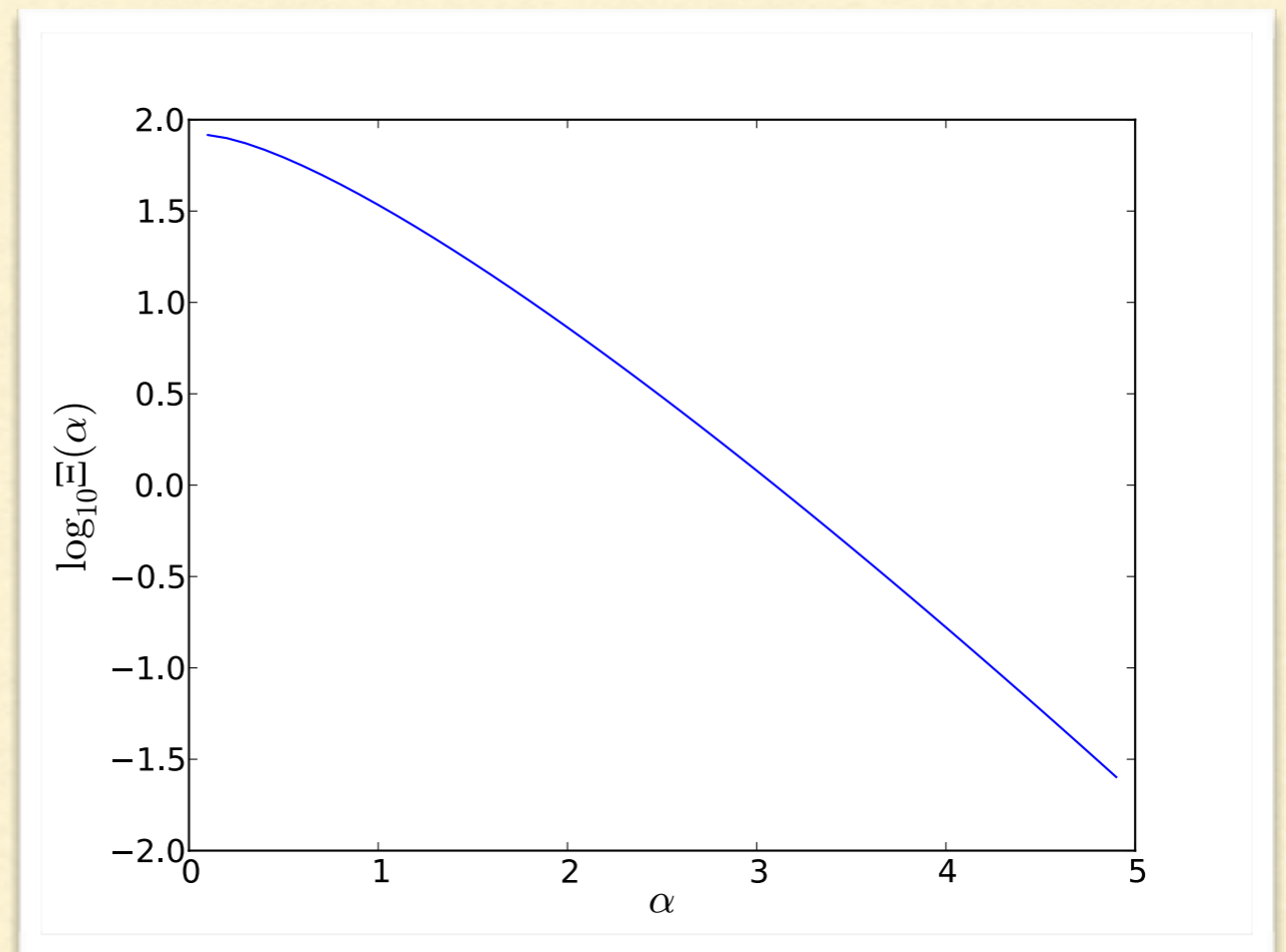
Once a form for the potential is specified, it is possible to quantify where Universal Attractor regime is approached:

$$3 [\xi f'(\phi)]^2 > 2\xi f(\phi)$$

Two specific cases considered:

$$f(\phi) = \phi^{\alpha/2}$$

$$\Xi(\alpha) \equiv \left(\frac{8}{3\alpha^2}\right)^{\alpha/4} \left(\frac{4N}{3}\right)^{1-\alpha/4}$$



APPROACHING THE ATTRACTOR

Edwards and Liddle; June 2014; arXiv:1406.5768[astro-ph.CO]

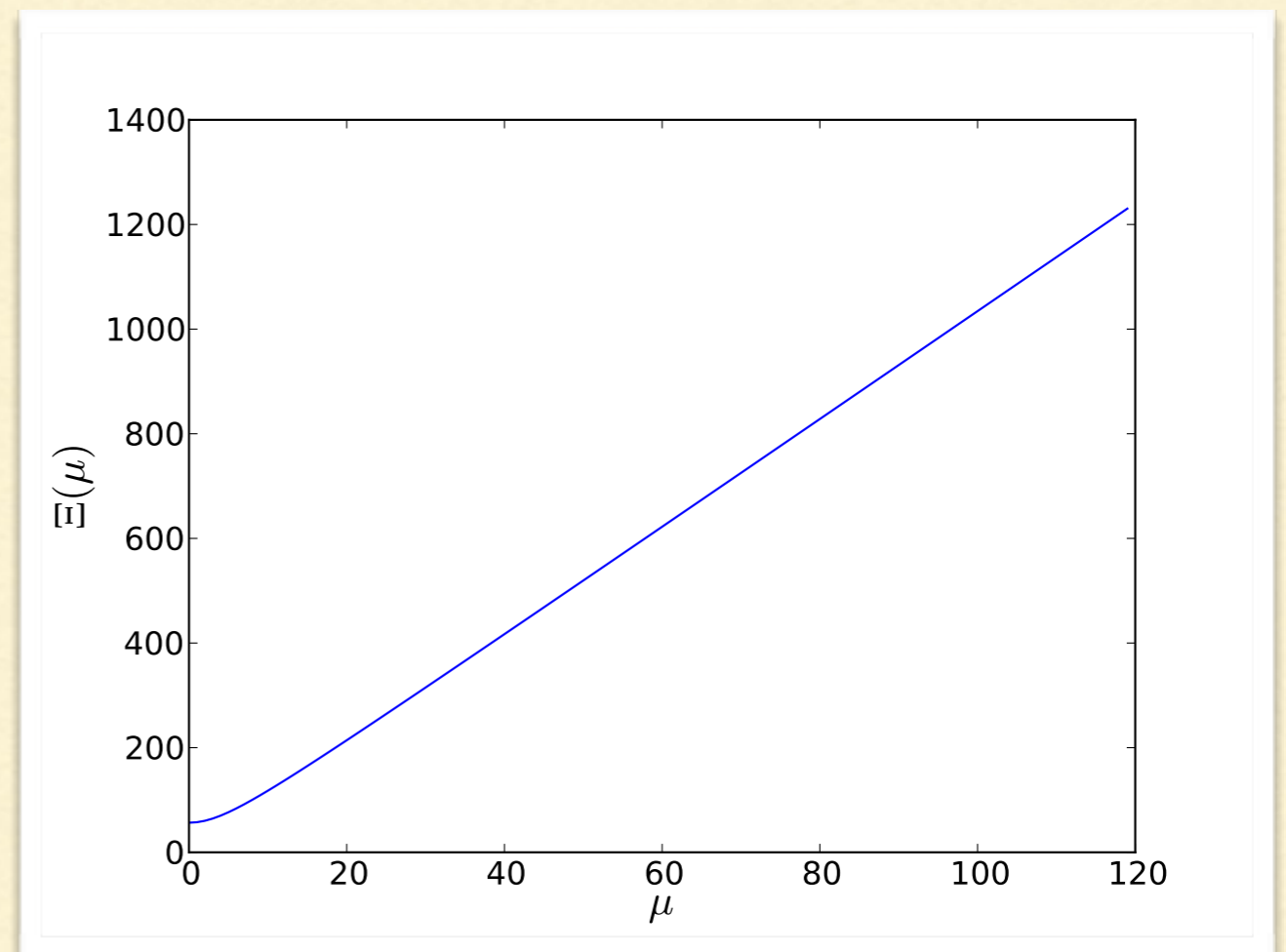
Once a form for the potential is specified, it is possible to quantify where Universal Attractor regime is approached:

$$3 [\xi f'(\phi)]^2 > 2\xi f(\phi)$$

Two specific cases considered:

$$f(\phi) = \sqrt{1 + \cos(\phi/\mu)}$$

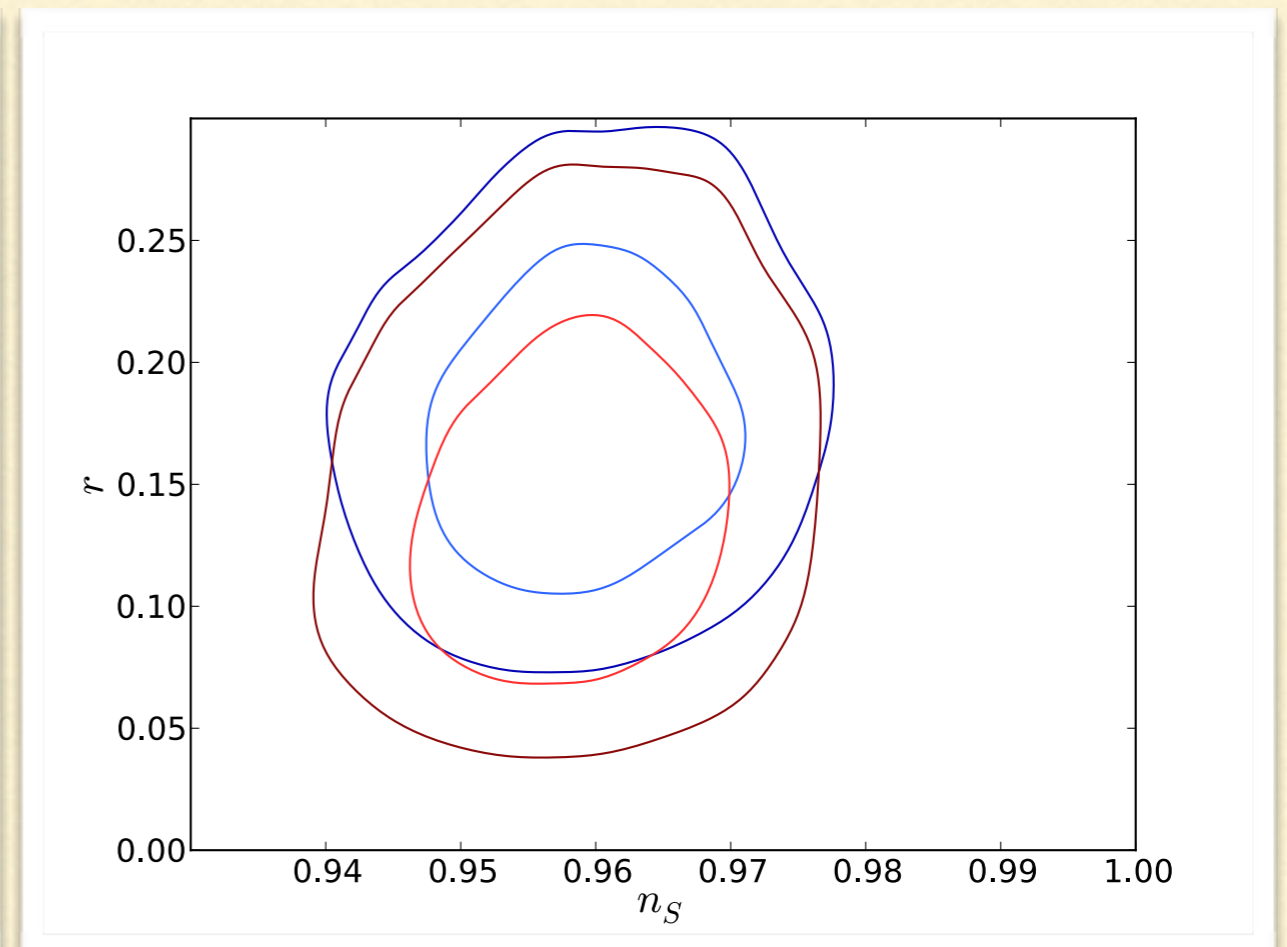
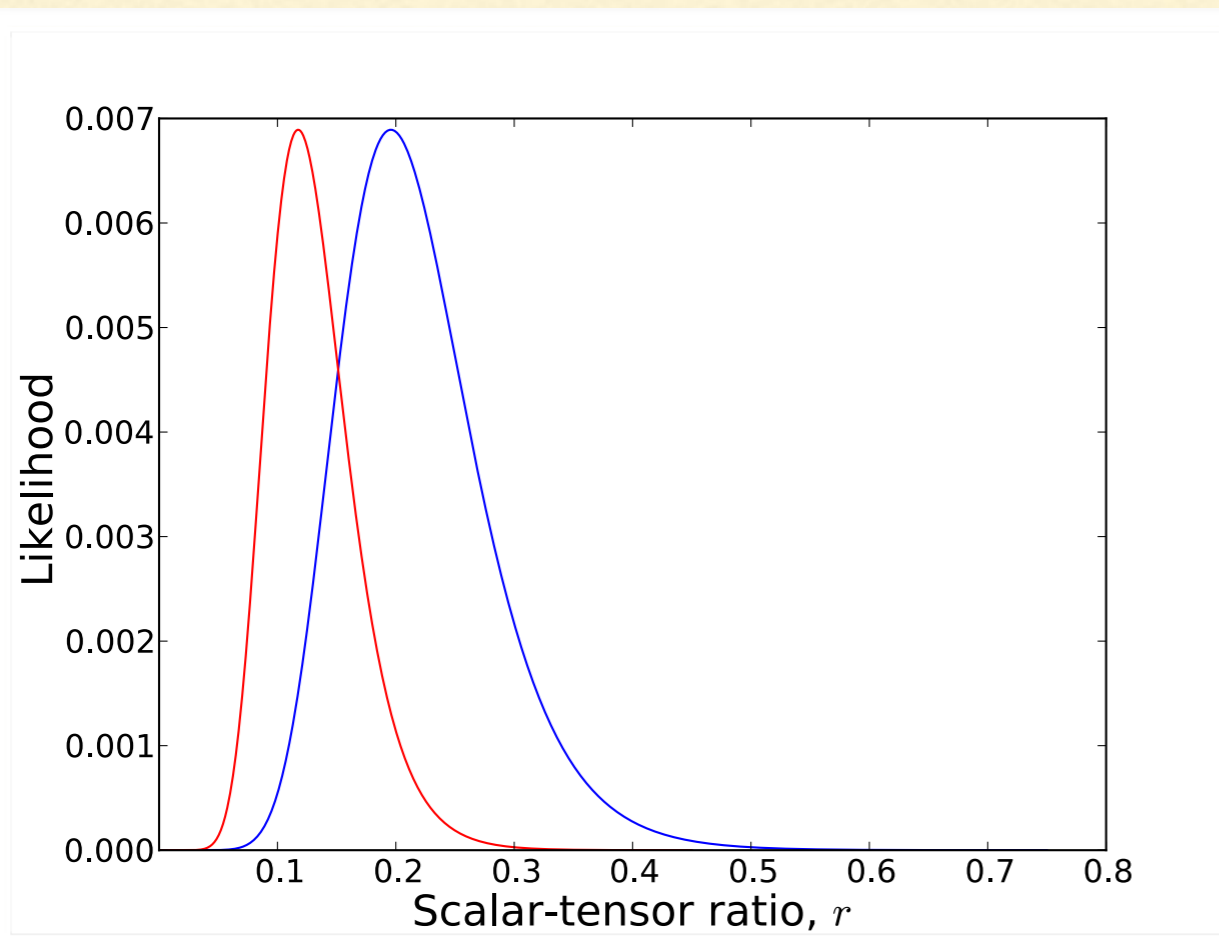
$$\Xi(\mu) \equiv \frac{4}{3} \sqrt{\frac{N^2}{2} + N\mu^2}$$



OBSERVATIONAL CONSTRAINTS

Edwards and Liddle; June 2014; arXiv:1406.5768[astro-ph.CO]

The Universal Attractors give $r \sim 0.005$, but what if we manage to put a lower bound on r greater than this value?



2013 Planck Collaboration Data Release and BICEP "inspired"

OBSERVATIONAL CONSTRAINTS

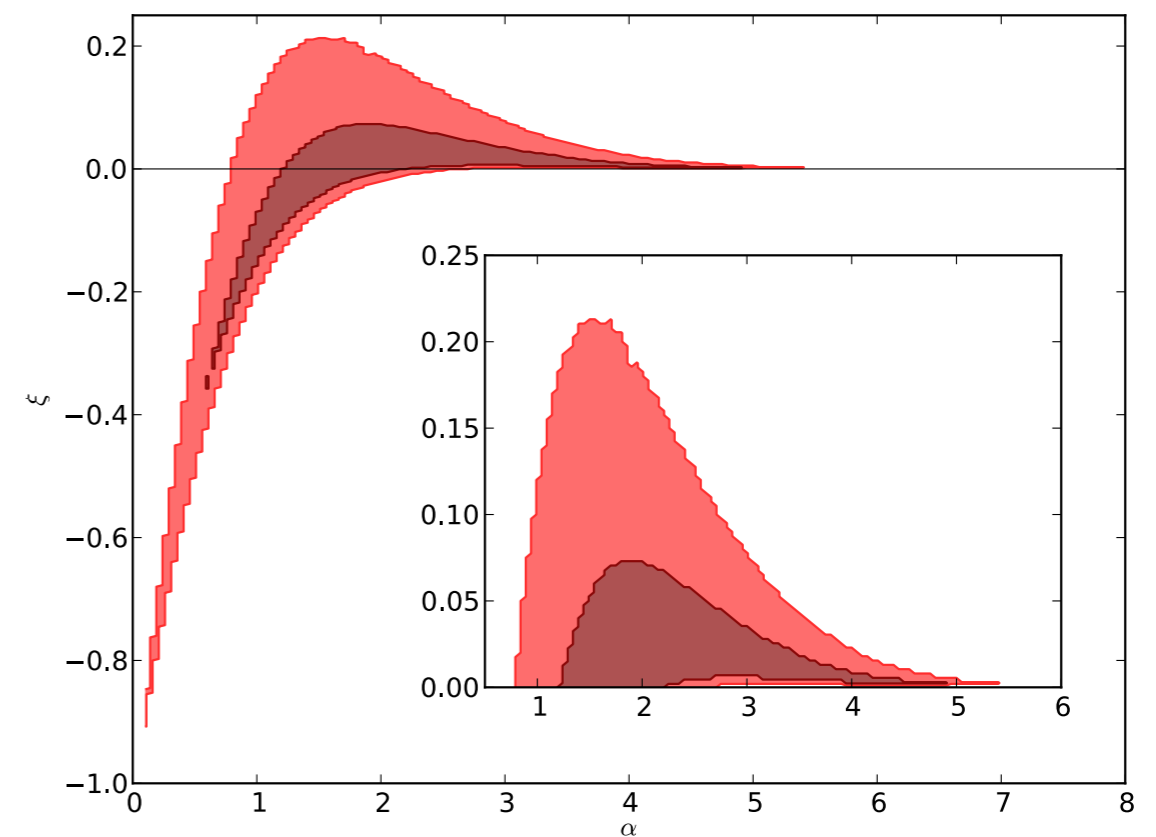
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The more optimistic case, $r > 0.05$

Form of potential and coupling:

$$f(\phi) = \phi^{\alpha/2}$$

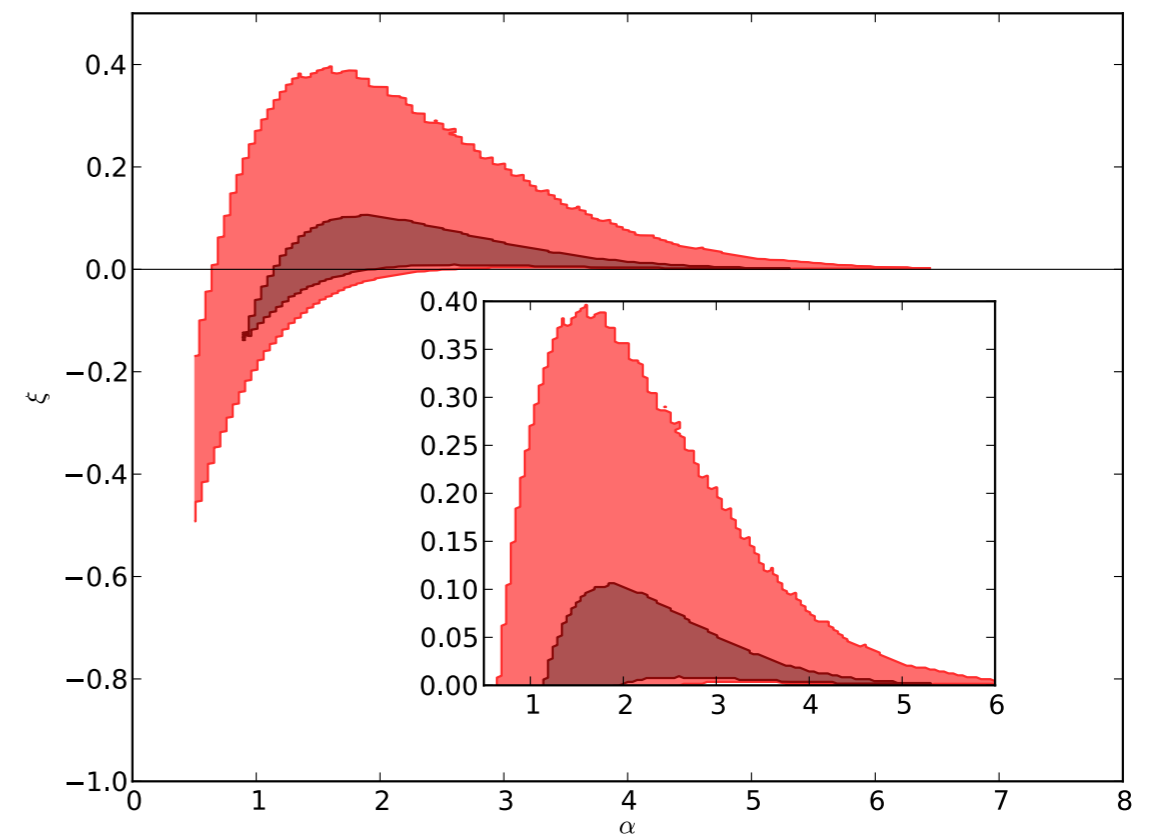
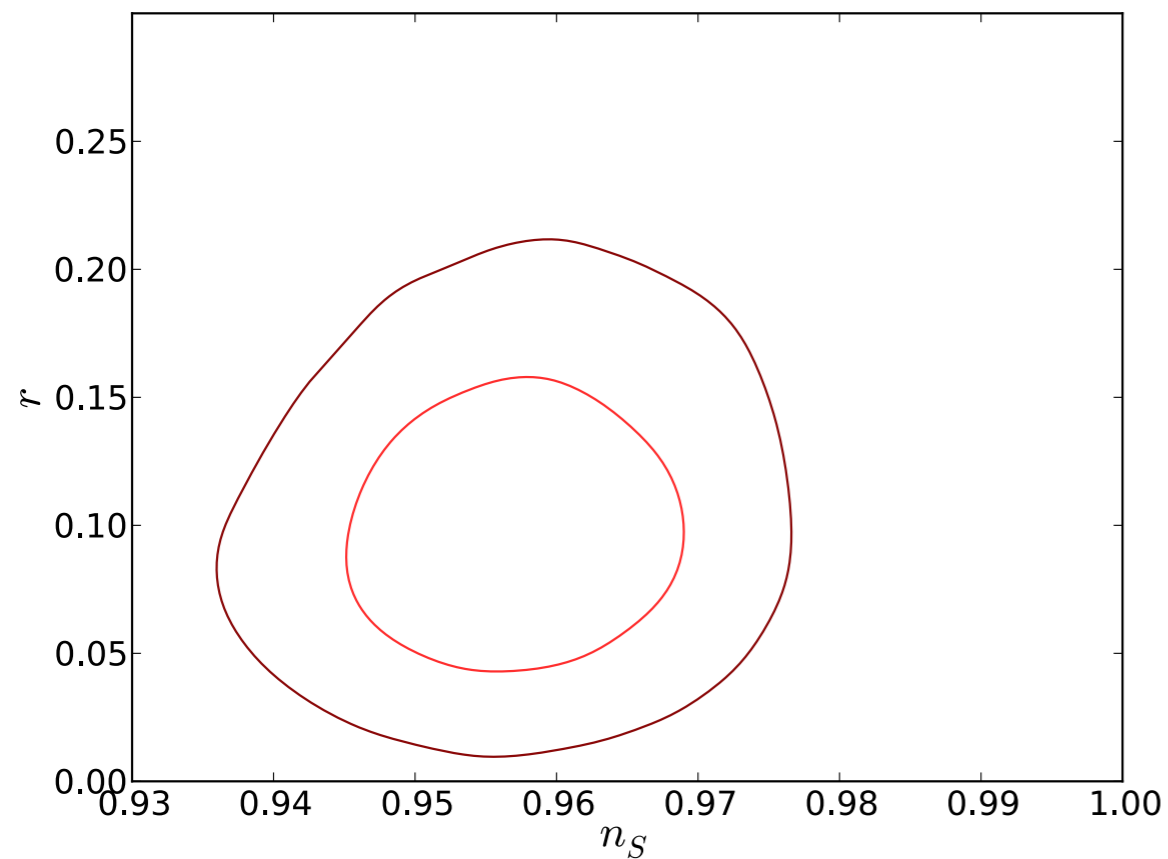
$$\Omega(\phi) = 1 + \xi f(\phi)$$



OBSERVATIONAL CONSTRAINTS

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The less optimistic case, $r > 0.01$



THE NEXT STEP - MORE FIELDS

Kalosh and Linde; September 2013; arXiv:1309.2015[astro-ph.CO]

Starting point:

$$\mathcal{L} = \sqrt{-g} \left[\frac{1}{2} \left((\partial_\mu \chi)^2 - \sum_i (\partial_\mu \phi_i)^2 \right) + \frac{\chi^2 - \sum_i \phi_i^2}{12} R - \frac{F\left(\frac{\phi_i}{\chi}\right)}{36} \left(\chi^2 - \sum_i \phi_i^2 \right)^2 \right]$$

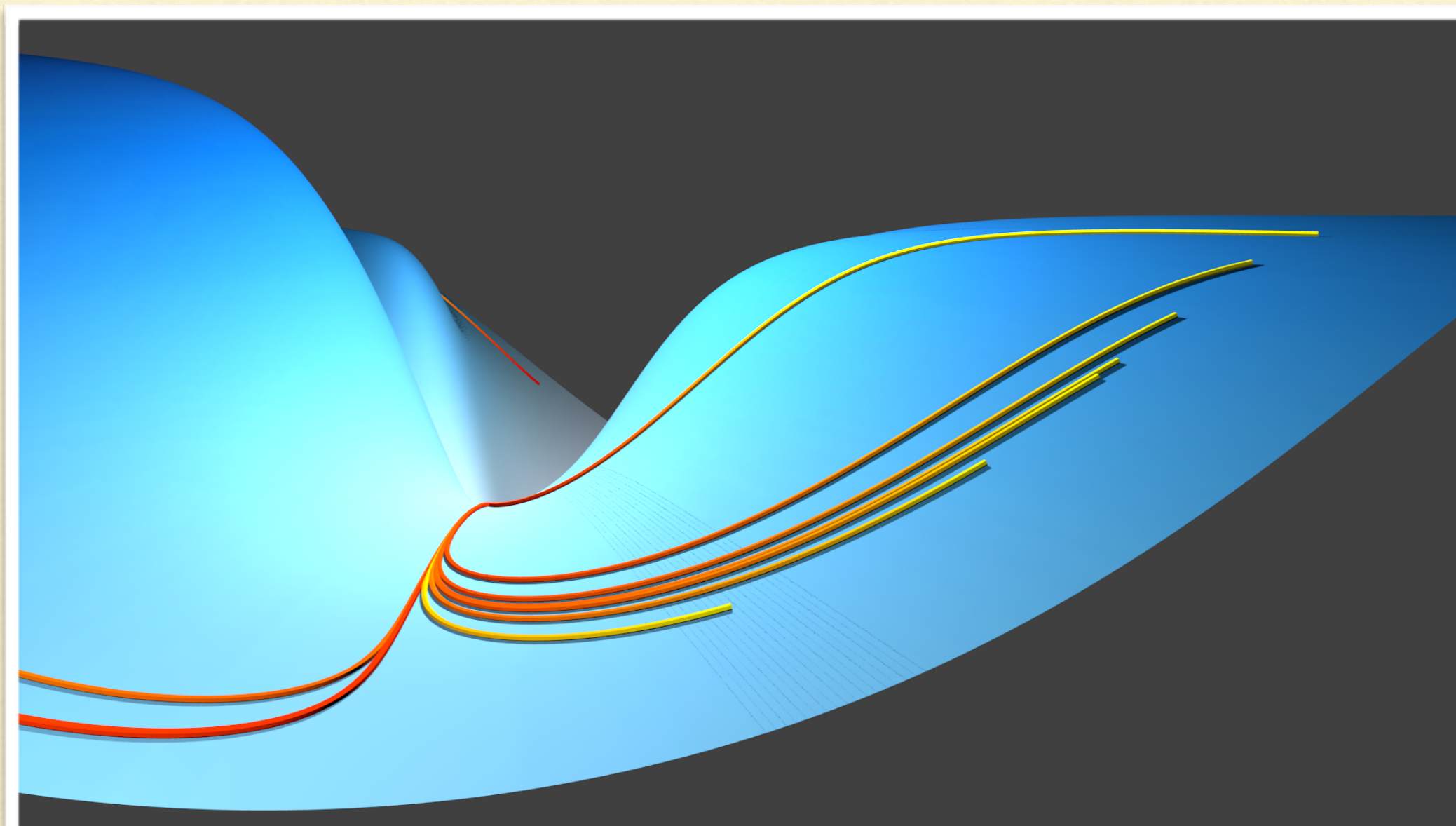
Break the conformal symmetry:

$$\mathcal{L} = \sqrt{-g} \left[- \sum_i (\partial_\mu \phi_i)^2 + \frac{1}{2} R - \frac{\sum_i \phi_i^2}{12} R - \frac{F\left(\frac{\phi_i}{\sqrt{6}}\right)}{36} \left(6 - \sum_i \phi_i^2 \right)^2 \right]$$

Now analyse this using transport code of Dias, Frazer and Seery;
Feb 2015; arXiv:1502.03125[astro-ph.CO]

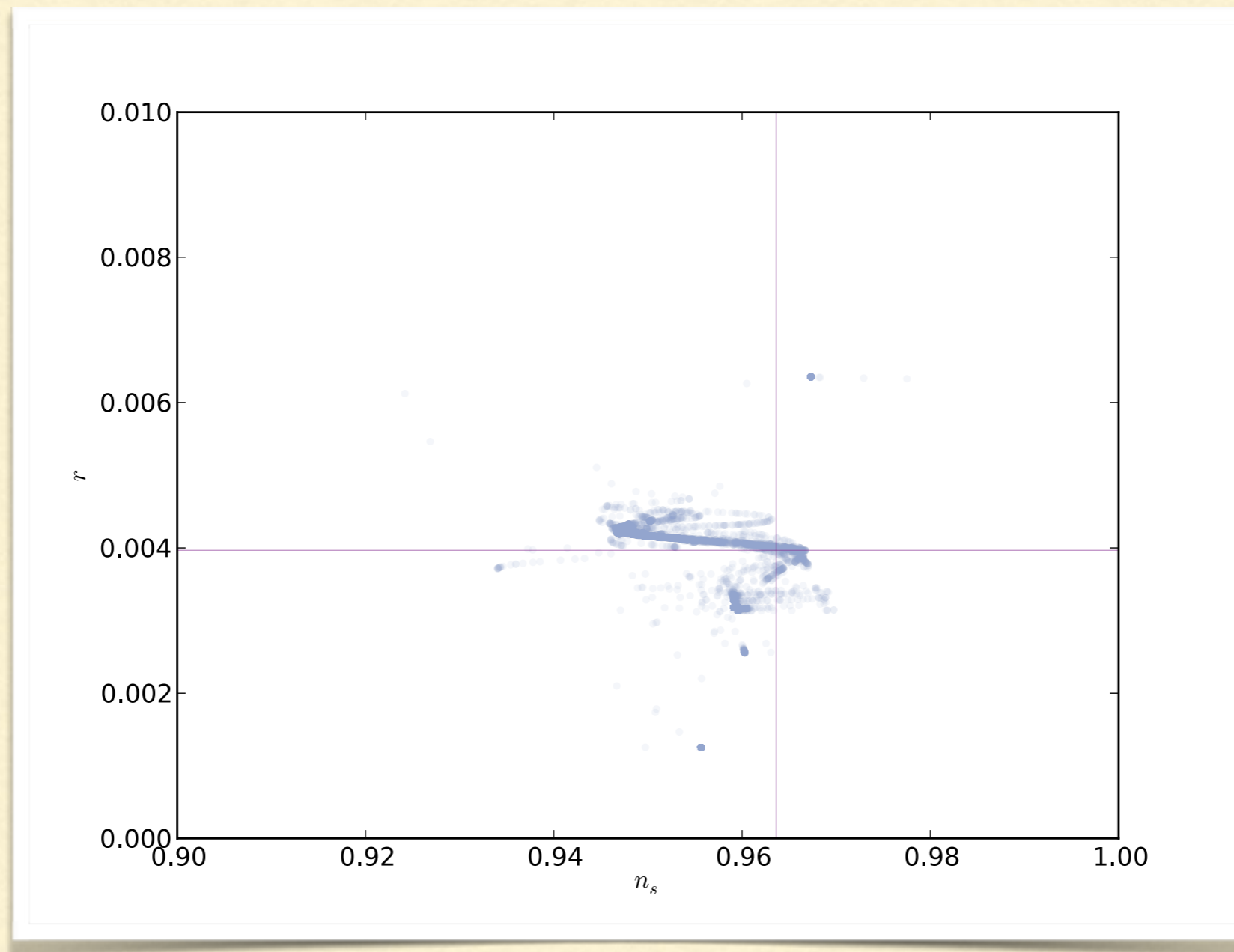
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CONCLUSION

- The Universal Attractors do have a universal prediction.
 - But it is **not** the same as predicted by the Starobinsky model (close though!)
 - A measurement of non-zero r would allow fairly tight constraints on the coupling parameter.
 - Wide range of phenomenon to understand in the 2-D case.
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