Area Law and Second Law for Cosmology

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Cosmology and Quantum Gravity

- There is a growing web of theorems and conjectures that relate quantum information to the area of surfaces.
- Think of these as “coincidences,” embedded in known physics (QFT, GR), but unexplained.
- Analogy: equivalence principle
  - not a theory
  - but testable
  - needs an explanation
  - this guides/constrains the search

Cosmology and Quantum Gravity

- The web of coincidences grew out of Hawking's area theorem, and Bekenstein's proposal that area is entropy.
- Their work applies to event horizons.
- Not every spacetime has an event horizon. What about cosmology?
- I will report on a recently proven, new Area Theorem and an associated new Generalized Second Law, both of which apply in cosmology.

Area Theorem for Event Horizons

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$$dA \geq 0 \; .$$

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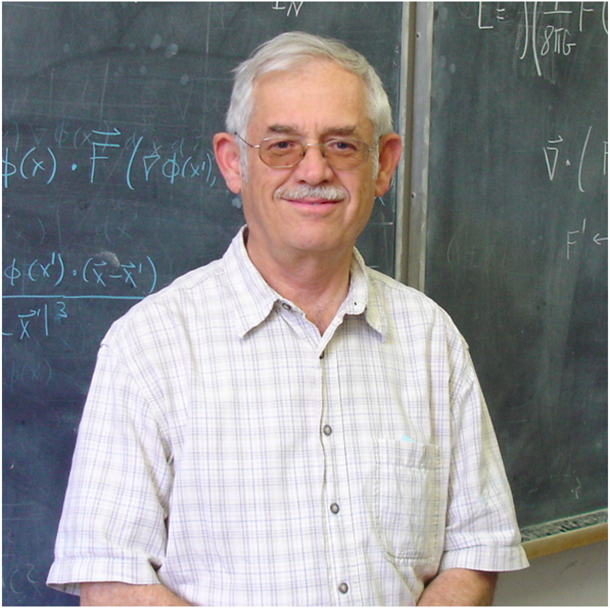
✓Proven, assuming the Null Energy Condition (NEC):

$$T_{kk} \equiv T_{\mu\nu} k^{\mu} k^{\nu} \geq 0 \; .$$

True for classical matter.

But in QFT the NEC can be violated, and so the area theorem can be evaded.

Jacob Bekenstein, 1947-2015



1972 Paper

LETTERE AL NUOVO CIMENTO VOL. 4, N. 15 12 Agosto 1972

Black Holes and the Second Law (*).

J. D. BEKENSTEIN (**)

Joseph Henry Laboratories, Princeton University - Princeton, N. J.

(ricevuto il 22 Maggio 1972)

Black Holes Destroy Matter Entropy

Black-hole physics seems to provide at least two ways in which the second law of thermodynamics may be transcended or violated:

a) Let an observer drop or lower a package of entropy into a black hole; the entropy of the exterior world decreases. Furthermore, from an exterior observer's point of view a black hole in equilibrium has only three degrees of freedom: mass, charge and angular momentum ([1]). Thus, once the black hole has settled down to equilibrium, there is no way for the observer to determine its interior entropy. Therefore, he cannot exclude the possibility that the total entropy of the universe may have decreased in the process. It is in this sense that the second law appears to be transcended ([2]).

Generalized Second Law for Event Horizons

Define the generalized entropy,

$$S_{\text{gen}} \equiv \frac{A}{4G\hbar} + S_{\text{out}} \ .$$

Conjecture the GSL:

$$dS_{\text{gen}} \geq 0 \ .$$

Bekenstein 1972, 1973, 1974 (✓) Proof (free fields, semiclassical limit) A. Wall (2011).

Hawking Radiation

- Had to be there since $T^{-1} = dS/dE$
- Found by a hard calculation. Hawking (1974)
- Black holes are thermodynamic objects.

Do All Areas Bound Some Entropy?

GSL for event horizons $\Longrightarrow$
Black hole must have more entropy than the matter it swallowed:

$$A_{\mathrm{EH}}/4G\hbar \geq S_{\mathrm{matter}}$$

Bekenstein 1981

Could there be a more general relation between entropy and area?

't Hooft 1993, Susskind 1995

E.g., $S(\text{volume}) \leq A(\text{boundary})/4G\hbar$?
Counterexamples!

Fischler & Susskind 1998

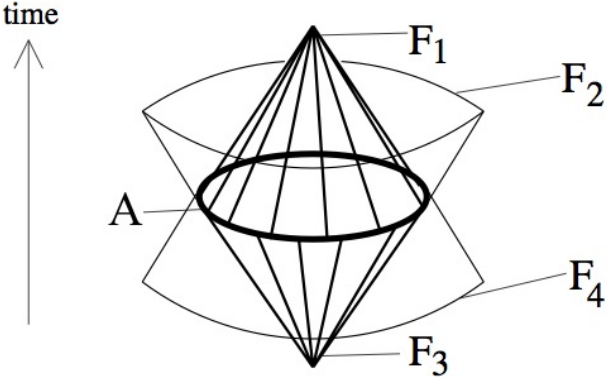
Covariant Entropy Bound

Conjecture:

$$S_{\text{matter}}[\text{light-sheet}] \leq \frac{A}{4G\hbar}$$

RB 1999

A light-sheet is a nonexpanding null hypersurface with initial area A.



The Role of Null Slices

Null hypersurfaces seem to play a distinguished role in quantum gravity.

They are also more closely related to observation than spacelike slices.

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They are also more closely related to observation than spacelike slices.

✓Proof:

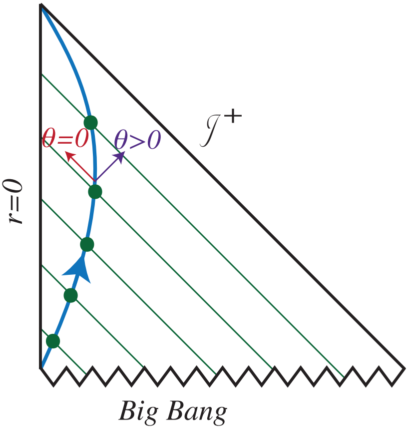
- We have to take fully into account that all observations are made on our past lightcone which is itself perturbed.
  We see density fluctuations which are further away from us, further in the past.
  We cannot observe 3 spatial dimensions but 2 spatial and 1 lightlike, more precisely we measure 2 angles and a redshift.

R. Dürrer, talk at COSMO 15

The World as a Hologram

- Choose a null slicing
- Find maximum area on each slice *r*
- It bounds the entropy on that slice
- The max area 2-surfaces $\sigma(r)$ form a 2+1D Holographic Screen

RB, 1999



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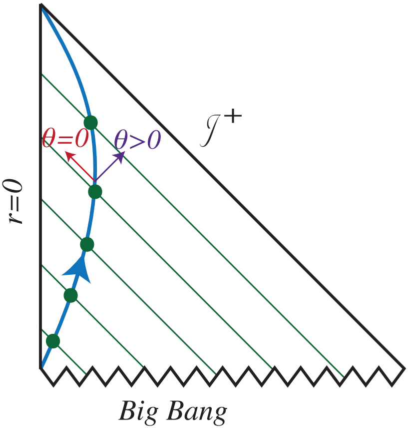


RB, 1999

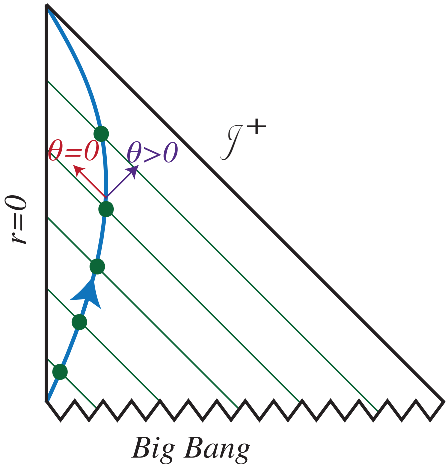
Next: The area of $\sigma(r)$ satisfies an area theorem, $dA/dr > 0$.

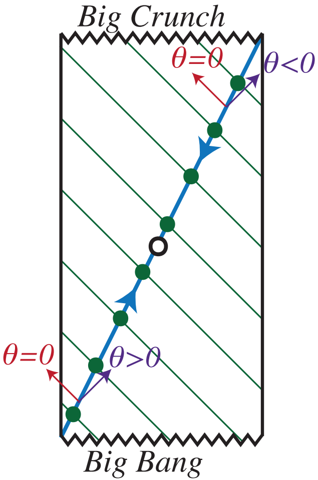
Past and Future Holographic Screens

- Hawking: the area of a future event horizon cannot decrease. $\Longrightarrow$ (by time reversal)
- The area of a past event horizon cannot increase. E.g., white hole.
- Before we can hope to prove a theorem, we need a distinction between past and future holographic screens

Past Holographic Screens

- The maximum area surface σ, by definition, has vanishing expansion in one future null direction.
- Consider the sign of the other expansion.
  $+$: Past Holographic Screen



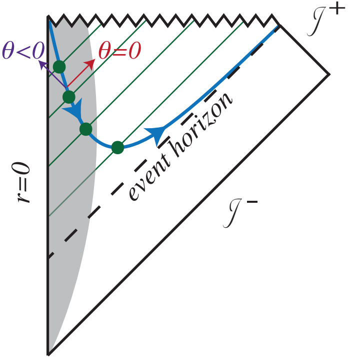


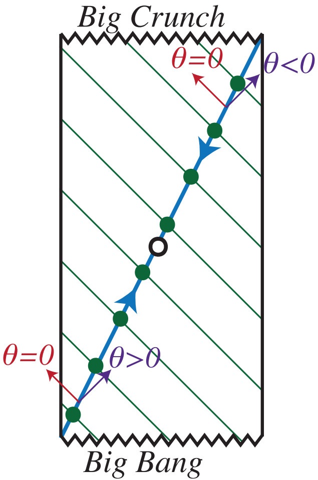
Future Holographic Screens

Negative expansion

$\leftrightarrow$ σ is marginally trapped

$\leftrightarrow$ near big crunch,
inside a black hole...



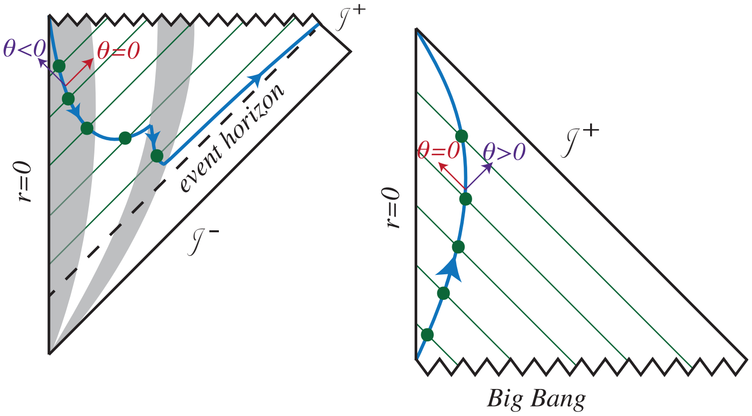


Future Holographic Screens

(Aside: future holographic screens are like the quasilocal black hole boundaries defined by Hayward, and Ashtekar *et al.*, but without the restriction that they be spacelike.)

Definition

A future holographic screen is a 2+1D hypersurface foliated by maginally trapped 2-surfaces $\sigma(r)$.



A past holographic screen is a 2+1D hypersurface foliated by maginally antitrapped 2-surfaces $\sigma(r)$.

Area Theorem for Holographic Screens

Assume Einstein's equations and the NEC. Then

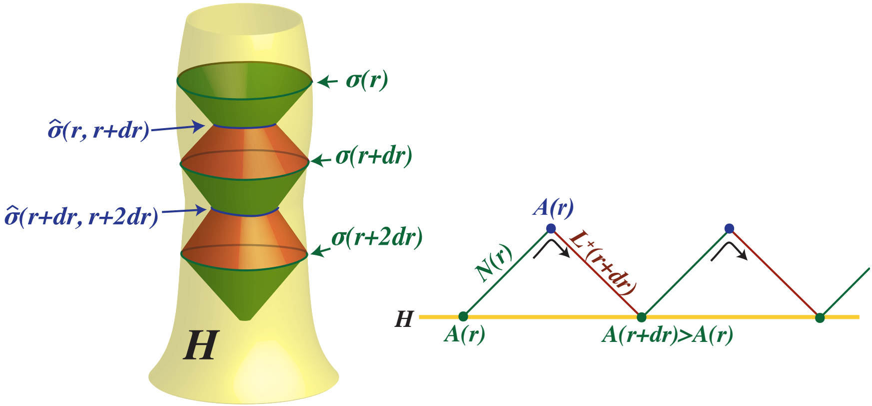
$$\frac{dA}{dr} > 0 \ .$$

RB & Engelhardt, 2015

- The area of a past holographic screen increases monotonically towards the exterior or future.
- The area of a future holographic screen increases monotonically towards the exterior or past.

Proof of the Cosmological Area Theorem

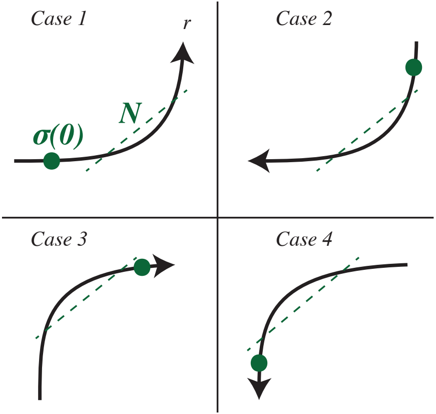
Future Holographic Screen, timelike and spacelike portions:



Theorem holds by definition on each portion.

Proof of the Cosmological Area Theorem

These cases would violate the theorem:



But they would contradict the classical focussing theorem ($\theta' \leq 0$), which follows from the NEC.

Beyond the Classical Limit

The NEC can be violated.

Like Hawking's area theorem for event horizons, the new area theorem for holographic screens fails for an evaporating black hole.

But we can again follow Bekenstein and replace the (proven) area theorem with a (conjectured) Generalized Second Law.

Substitute $A \to 4G\hbar S_{\text{gen}}$.

GSL for Holographic Screens

For each surface $\sigma(r)$ of a holographic screen, consider the generalized entropy

$$S_{\rm gen} \equiv \frac{A}{4G\hbar} + S_{\rm out} \ .$$

Conjecture the GSL:

$$dS_{\rm gen} \geq 0 \ .$$

Quantum Holographic Screens

For the new GSL to work, we also need to change the definition of the holographic screen:

Instead of maximizing the area on each null slice, $\sigma(r)$ is chosen to maximize S_{gen}.

Proof of the new GSL

The new GSL can be proven…

RB & Engelhardt, to appear

Proof of the new GSL

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from another conjecture, the Quantum Focussing Conjecture.

RB, Fisher, Leichenauer & Wall, 2015

This is a quantum generalization of the classical statement that matter focusses (and never repels) light.

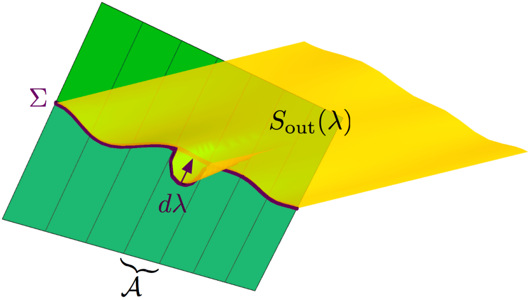
Quantum Focussing Conjecture

The QFC appears to be quite powerful. It implies:

- classical focussing theorem
- Bekenstein's GSL
- our GSL
- covariant entropy bound
- QNEC

Quantum Null Energy Condition

$$\langle T_{kk} \rangle \geq \frac{\hbar}{2\pi} \lim_{\mathcal{A}\to 0} \frac{S''_{\text{out}}}{\mathcal{A}} \; .$$



We recently proved that free bosonic fields satisfy the QNEC.

RB, Fisher, Koeller, Leichenauer & Wall, 2015

Discussion

- Deep connections between quantum information and geometry
- Conjectures have nontrivial implications as $\hbar \to 0$ or $G \to 0$, in GR or QFT.
- Some of these novel results have been rigorously proven,
- in GR: area theorem
- in QFT: QNEC, Bekenstein bound [Casini 2008], "quantum Bousso bound" [RB, Casini, Fisher, Maldacena, 2014])
- But what is the interpretation of the new GSL in cosmology?