Consistent Metric Combinations in Cosmology of Massive Bigravity

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Motivation

- *Can we construct a consistent (ghost free) theory of interacting spin 2 fields?*
- *Can we construct a consistent theory for massive graviton?*
- *Can we explain the late time accelration of the univerese trough this modification of GR?*

Massive generalizaton of GR

Fierz & Pauli (1939):

$$h_{\mu\nu} = g_{\mu\nu} - \eta_{\mu\nu}$$

(linearized metric fluctuations around flat space-time)

The only ghost-free mass term:

$$\frac{m^2}{4}\left(h_{\mu\nu}h^{\mu\nu} - a\, h^{\mu}_{\mu}h^{\nu}_{\nu}\right)\,, \qquad \text{with} \quad a = 1$$

There is a vDVZ discontinuity and there is a need of the Vainshtein mechanism.

Non linear theory of massive gravity

Non-linear theories of massive gravity were conjectured to suffer from ghosts.

2010-11: breakthrough

de Rham & Gabadadze	[arXiv:1007.0443]
de Rham, Gabadadze & Tolley	[arXiv:1011.1232]
Hassan & Rosen	[arXiv:1103.6055]
	[arXiv:1106.3344]
	[arXiv:1109.3515]
	[arXiv:1111.2070]
Hassan, Rosen & Schmidt-May	[arXiv:1109.3230]

- The only ghost-free, massive gravity
- The only ghost-free, bimetric gravity

Massive Gravity

In general it is impossible to construct non trivial interaction terms with only one metric, one has to introduce a second metric

$$g^{\mu\alpha}g_{\alpha\nu} = \delta^{\mu}_{\nu}$$

- *The only ghost-free action for a single massive graviton*

$$S = -M_p^2 \int d^4x\sqrt{-g}\,R(g) + 2M_p^2 m^2 \int d^4x\sqrt{-g}\sum_{n=0}^{3}\beta_n\, e_n(\sqrt{g^{-1}f})$$

- Where f is called reference metric and is not dynamical.

Here $e_n(\mathbb{X})$ *are the elementary symmetric polynomials of the matrix* $\mathbb{X} \equiv \sqrt{g^{-1}f}$

$$e_0(\mathbb{X}) \equiv 1,$$
$$e_1(\mathbb{X}) \equiv [\mathbb{X}],$$
$$e_2(\mathbb{X}) \equiv \frac{1}{2}\left([\mathbb{X}]^2 - [\mathbb{X}^2]\right),$$
$$e_3(\mathbb{X}) \equiv \frac{1}{6}\left([\mathbb{X}]^3 - 3[\mathbb{X}][\mathbb{X}^2] + 2[\mathbb{X}^3]\right),$$
$$e_4(\mathbb{X}) \equiv \det(\mathbb{X}),$$

Massive Bigravity

- *The action of the theory:*

$$\mathcal{L} = -\frac{M_{\rm Pl}^2}{2}\sqrt{-g}R(g) - \frac{M_f^2}{2}\sqrt{-f}R(f) + m^2 M_{\rm Pl}^2\sqrt{-g}\,V(\sqrt{g^{-1}f}) + \sqrt{-g}\mathcal{L}_m\,(g,\Phi_i)$$

Gives dynamics to the reference metric

Admits FLRW backgrounds

- **V**: *interaction potential built out of the matrix*
  m: *interaction scale/graviton mass*
  M_{pl}, M_f: *Planck masses for $g_{\mu\nu}$ and $f_{\mu\nu}$*
  $\beta_{0\ldots4}$: *free parameters of the theory*

Viable models on background level

Model	B_0	B_1	B_2	B_3	B_4	Ω_m	χ^2_{min}	p-value	log-evidence
ΛCDM	free	0	0	0	0	free	546.54	0.8709	-278.50
(B_1, Ω_m^0)	0	free	0	0	0	free	551.60	0.8355	-281.73
~~(B_2, Ω_m^0)~~	~~0~~	~~0~~	~~free~~	~~0~~	~~0~~	~~free~~	~~894.00~~	~~< 0.0001~~	~~-450.25~~
~~(B_3, Ω_m^0)~~	~~0~~	~~0~~	~~0~~	~~free~~	~~0~~	~~free~~	~~1700.50~~	~~< 0.0001~~	~~-850.26~~
(B_1, B_2, Ω_m^0)	0	free	free	0	0	free	546.52	0.8646	-279.77
~~(B_1, B_3, Ω_m^0)~~	~~0~~	~~free~~	~~0~~	~~free~~	~~0~~	~~free~~	~~542.82~~	~~0.8878~~	~~-280.10~~
~~(B_2, B_3, Ω_m^0)~~	~~0~~	~~0~~	~~free~~	~~free~~	~~0~~	~~free~~	~~548.04~~	~~0.8543~~	~~-280.91~~
(B_1, B_4, Ω_m^0)	0	free	0	0	free	free	548.86	0.8485	-281.42
~~(B_2, B_4, Ω_m^0)~~	~~0~~	~~0~~	~~free~~	~~0~~	~~free~~	~~free~~	~~806.82~~	~~< 0.0001~~	~~-420.87~~
~~(B_3, B_4, Ω_m^0)~~	~~0~~	~~0~~	~~0~~	~~free~~	~~free~~	~~free~~	~~685.30~~	~~0.0023~~	~~-351.14~~
$(B_1, B_2, B_3, \Omega_m^0)$	0	free	free	free	0	free	546.50	0.8582	-279.61
~~$(B_1, B_2, B_4, \Omega_m^0)$~~	~~0~~	~~free~~	~~free~~	~~0~~	~~free~~	~~free~~	~~546.52~~	~~0.8581~~	~~-279.56~~
~~$(B_1, B_3, B_4, \Omega_m^0)$~~	~~0~~	~~free~~	~~0~~	~~free~~	~~free~~	~~free~~	~~546.78~~	~~0.8563~~	~~-280.00~~
~~$(B_2, B_3, B_4, \Omega_m^0)$~~	~~0~~	~~0~~	~~free~~	~~free~~	~~free~~	~~free~~	~~549.68~~	~~0.8353~~	~~-282.89~~
$(B_1, B_2, B_3, B_4, \Omega_m^0)$	0	free	free	free	free	free	546.50	0.8515	-279.60
full bigravity model	free	free	free	free	free	free	546.50	0.8445	-279.82

- Where $B_i \equiv \frac{m^2}{H_0^2} \beta_i$

Y. Akrami, T. Koivisto, M. Sandstad [arXiv:1209.0457]

F. Könnig, A. Patil, L. Amendola [arXiv:1312.3208]

Features and Achievements

- *Gives a dynamical dark energy model.*
- *Phantom behaviour is common (w < -1).*
- *Possible non-GR signatures at background level and in structure formation.*
- *Technically natural dark energy candidate .*

Problems

- *Single Coupling:*
  Only one metric describes space-time (directly couples to matter)

  - *Finite Branch: Scalar perturbations are plugged by early-time gradient instabilities.*

  - *Infinite Branch: Vacuum decays immediately because of ghosts.*

Ways Out

- *Cure gradient instabilities nonlinearly (Vainshtein mechanism)*

  E.Mörtsell, J. Enander arXiv:1506.04977

  K. Aoki, K. Maeda, R. Namba arXiv:1506.04543

- *Non-FRLW backgrounds*

  HN, Y. Akrami, L. Amendola arXiv:1502.03988

- *Doubly coupled bigravity, but there are problems*

  Y. Akrami, T. Koivisto, D. Mota, M. Sandstad arXiv:1306.0004

  Y. Akrami, T. Koivisto, A. Solomon arXiv:1404.0006

- *Plank mass hierarchy for two metrics*

  Y. Akrami, S.F. Hassan, F. Könnig, A. Schmidt-May, A. Solomon arXiv:1503.07521

- *Other modifications of the theory*
  (Quasidilaton, Varying mass, Lorentz violation, F(R)-bigravity, trigravity ...)

Possible metric combinations

FRLW-FRLW: *(homogenous-homogenous)*

$$g_{\mu\nu}dx^{\mu}dx^{\nu} = -dt^2 + a^2(t)\,d\vec{x_g}^2,$$
$$f_{\mu\nu}dx^{\mu}dx^{\nu} = -X^2(t)\,dt^2 + b^2(t)\,d\vec{x_f}^2,$$

- Where

$$d\vec{x_g}^2 = \frac{dr^2}{1-k_g r^2} + r^2\left(d\theta^2 + sin^2(\theta)\,d\phi^2\right),$$
$$d\vec{x_f}^2 = \frac{dr^2}{1-k_f r^2} + r^2\left(d\theta^2 + sin^2(\theta)\,d\phi^2\right),$$

FRLW- Lemaître: *(homogenous-inhomogenous)*

$$g_{\mu\nu}dx^{\mu}dx^{\nu} = -dt^2 + a^2(t)\,d\vec{x_g}^2,$$
$$f_{\mu\nu}dx^{\mu}dx^{\nu} = -X^2(t,r)\,dt^2 + Y^2(t,r)\,dr^2 + Z^2(t,r)\,r^2 d\Omega^2,$$

LTB-LTB: *(inhomogenous-inhomogenous)*

$$g_{\mu\nu}dx^{\mu}dx^{\nu} = -dt^2 + A^2(t,r)dr^2 + B^2(t,r)d\Omega^2,$$
$$f_{\mu\nu}dx^{\mu}dx^{\nu} = -X^2(t)dt^2 + Y^2(t,r)dr^2 + Z^2(t,r)d\Omega^2.$$

Bianchi I-FRLW: *(anisotropic-isotropic)*

$$g_{\mu\nu}dx^{\mu}dx^{\nu} = -dt^2 + a_1^2(t)\,dx^2 + a_2^2(t)\,dy^2 + a_3^2(t)\,dz^2,$$
$$f_{\mu\nu}dx^{\mu}dx^{\nu} = -X^2(t)\,dt^2 + b^2(t)\,d\vec{x}^2,$$

Possible combinations

HN, Y. Akrami, L. Amendola arXiv:1502.03988

Metric combinations				
$g_{\mu\nu}$ (physical metric)	$f_{\mu\nu}$ (reference metric)	$T^g_{\mu\nu}$	Possibility	Reference
FLRW (k)	FLRW (k)	PF	✓	Standard
~~FLRW (k_g)~~	~~FLRW (k_f)~~	~~PF~~	~~×~~	~~Present work~~
~~FLRW~~	~~LEMAÎTRE~~	~~PF~~	~~×~~	~~Present work~~
FLRW	LEMAÎTRE	Inhom.	✓	Present work & Ref. [26]
~~LEMAÎTRE~~	~~FLRW~~	~~Any~~	~~×~~	~~Present work~~
~~FLRW~~	~~LTB~~	~~Any~~	~~×~~	~~Present work~~
~~LTB~~	~~FLRW~~	~~Any~~	~~×~~	~~Present work~~
LTB	LTB	PF	✓ [a]	Present work
~~Bianchi I~~	~~FLRW~~	~~Any~~	~~×~~	~~Present work~~
~~FLRW~~	~~Bianchi I~~	~~PF~~	~~×~~	~~Present work~~
FLRW	Bianchi I	Aniso.	✓	Present work
Bianchi Class A	Bianchi Class A	PF	✓	Refs. [30, 74]
Perturbed FLRW	Perturbed FLRW	Perturbed PF	✓	Standard
~~Perturbed FLRW (scalars)~~	~~Unperturbed FLRW (scalars)~~	~~Perturbed PF~~	~~× [b]~~	~~Present work~~
~~Unperturbed FLRW (scalars)~~	~~Perturbed FLRW (scalars)~~	~~Perturbed PF~~	~~× [b]~~	~~Present work~~
~~Perturbed FLRW (tensors)~~	~~Unperturbed FLRW (tensors)~~	~~Perturbed PF~~	~~× [c]~~	~~Present work~~
~~Unperturbed FLRW (tensors)~~	~~Perturbed FLRW (tensors)~~	~~Perturbed PF~~	~~× [c]~~	~~Present work~~

Cosmological implementation of a viable combinations

FRLW-FRLW´:

$$g_{\mu\nu}dx^{\mu}dx^{\nu} = -dt^2 + a^2(t)\left[\frac{dr^2}{1-k_g r^2} + r^2\left(d\theta^2 + sin^2(\theta)\, d\phi^2\right)\right],$$

$$f_{\mu\nu}dx^{\mu}dx^{\nu} = -\tilde{X}^2(\tilde{t})\, d\tilde{t}^2 + \tilde{b}^2(\tilde{t})\left[\frac{d\tilde{r}^2}{1-k_f\tilde{r}^2} + \tilde{r}^2\left(d\tilde{\theta}^2 + sin^2\left(\tilde{\theta}\right) d\tilde{\phi}^2\right)\right]$$

$$\tilde{t} = f(t,r)$$

$$\tilde{r} = g(t,r)$$

$$d\tilde{s}_g^2 = -dt^2 + a^2 dr^2 + a^2 r^2 d\Omega^2$$

$$d\tilde{s}_f^2 = -\left(X^2\dot{f}^2 - b^2\dot{g}^2\right)dt^2 + 2\left(b^2 g'\dot{g} - X^2 f'\dot{f}\right)dt dr + \left(b^2 g'^2 - X^2 f'^2\right)dr^2 + b^2 g^2 d\Omega^2.$$

Attractor solution:

- *At the late times of cosmological evolution, the transformation functions will be*

$$f = \sqrt{\frac{\Lambda_g}{\Lambda_f}}\, t \qquad g = Q_{1,2}\, r.$$

⬇

FRLW-FRLW

⬇

Attractor solution

In preparation

Summary

- *No viable and linear stable model found*
- *Option 1: cure gradient instability nonlinearly?*
- *Option 2: other backgrounds*
- *Option 3: can couple both metrics to matter*
- *Option 4: take small f-metric Planck mass*
- *Option 5: other theories (modifications/generalizations)*

Thank You !

The possibility of different metric combinations

Equations of Motion:

$$G_{\mu\nu}(g) + m^2 V^g_{\mu\nu} = \frac{1}{M^2_{\rm Pl}} T_{\mu\nu},$$
$$\alpha^2 G_{\mu\nu}(f) + m^2 V^f_{\mu\nu} = 0,$$

- Where $\alpha \equiv \frac{M_f}{M_{\rm Pl}}$

Ricci identities + energy-momentum conservation:

$$\nabla^\mu V^g_{\mu\nu} = 0 \iff \nabla^\mu V^f_{\mu\nu} = 0$$

↓

Consistent branches of the theory