# **Consistent Metric Combinations in Cosmology of Massive Bigravity**

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# Cosmo -15

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## Motivation

Can we construct a consistent (ghost free) theory of interacting spin 2 fields?

Can we construct a consistent theory for massive graviton?

Can we explain the late time accelration of the univerese trough this modification of GR?

## Massive generalizaton of GR

### Fierz & Pauli (1939):

 $h_{\mu\nu} = g_{\mu\nu} - \eta_{\mu\nu}$ 

(linearized metric fluctuations around flat space-time)

# The only ghost-free mass term:

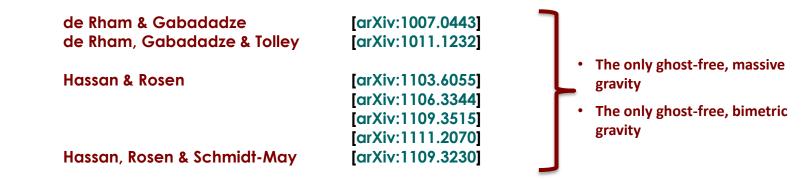
 $\frac{m^2}{4} \left( h_{\mu\nu} h^{\mu\nu} - a \, h^{\mu}_{\mu} h^{\nu}_{\nu} \right) \,, \qquad \text{with} \quad a = 1$ 

There is a vDVZ discontinuity and there is a need of the Vainshtein mechanism.

Non linear theory of massive gravity

Non-linear theories of massive gravity were conjectured to suffer from ghosts.

### 2010-11: breakthrough



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### **Massive Gravity**

In general it is impossible to construct non trivial interaction terms with only one metric, one has to introduce a second metric

 $g^{\mu\alpha}g_{\alpha\nu} = \delta^{\mu}_{\nu}$ 

The only ghost-free action for a single massive graviton

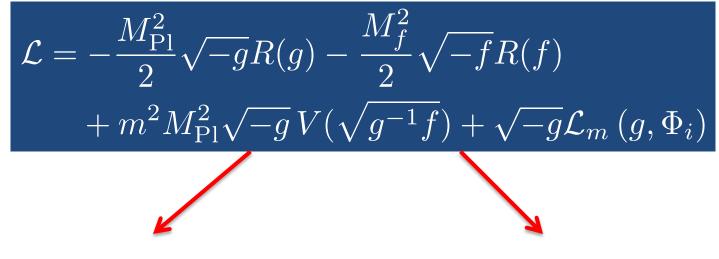
$$S = -M_p^2 \int d^4x \sqrt{-g} R(g) + 2M_p^2 m^2 \int d^4x \sqrt{-g} \sum_{n=0}^3 \beta_n e_n(\sqrt{g^{-1}f})$$

• Where f is called reference metric and is not dynamical. Here  $e_n(X)$  are the elementary symmetric polynomials of the matrix  $X \equiv \sqrt{g^{-1}f}$ 

$$\begin{split} e_0 \left( \mathbb{X} \right) &\equiv 1, \\ e_1 \left( \mathbb{X} \right) &\equiv \left[ \mathbb{X} \right], \\ e_2 \left( \mathbb{X} \right) &\equiv \frac{1}{2} \left( \left[ \mathbb{X} \right]^2 - \left[ \mathbb{X}^2 \right] \right), \\ e_3 \left( \mathbb{X} \right) &\equiv \frac{1}{6} \left( \left[ \mathbb{X} \right]^3 - 3 \left[ \mathbb{X} \right] \left[ \mathbb{X}^2 \right] + 2 \left[ \mathbb{X}^3 \right] \right), \\ e_4 \left( \mathbb{X} \right) &\equiv \det \left( \mathbb{X} \right), \end{split}$$

### **Massive Bigravity**

#### The action of the theory:



*Gives dynamics to the reference metric* 

Admits FLRW backgrounds

V: interaction potential built out of the matrix m: interaction scale/graviton mass  $M_{pl}$ ,  $M_f$ : Planck masses for  $g_{\mu\nu}$  and  $f_{\mu\nu}$  $\beta_{0...4}$ : free parameters of the theory

## Viable models on background level

Model	B <sub>0</sub>	B <sub>1</sub>	B <sub>2</sub>	B <sub>3</sub>	B <sub>4</sub>	$\Omega_{\mathbf{m}}$	$\chi^{2}_{\mathbf{min}}$	p-value	log-evidence
$\Lambda \mathrm{CDM}$	free	0	0	0	0	free	546.54	0.8709	-278.50
$(\mathbf{B_1}, \mathbf{\Omega^0_m})$	0	free	0	0	0	free	551.60	0.8355	-281.73
$(\mathbf{B_2}, \mathbf{\Omega_m^0})$	0	0	free	0	0	free	894.00	< 0.0001	-450.25
$(\mathbf{B_3}, \mathbf{\Omega_m^0})$	0	0	0	free	0	free	1700.50	< 0.0001	-850.26
$(B_1, B_2, \Omega^0_m)$	0	free	free	0	0	free	546.52	0.8646	-279.77
$(\mathbf{B_1},\mathbf{B_3},\mathbf{\Omega_m^0})$	0	free	0	free	0	free	542.82	0.8878	-280.10
$(\mathbf{B_2},\mathbf{B_3},\mathbf{\Omega_m^0})$	0	0	free	free	-0	free	<u>548.04</u>	0.8543	280.91
$(B_1, B_4, \Omega^0_m)$	0	free	0	0	free	free	548.86	0.8485	-281.42
$(\mathbf{B_2},\mathbf{B_4},\mathbf{\Omega_m^0})$	0	0	free	0	free	free	806.82	< 0.0001	-420.87
$(\mathbf{B_3},\mathbf{B_4},\mathbf{\Omega_m^0})$	-0	-0	-0	free	free	free	685.30	0.0023	-351.14
$(B_1,B_2,B_3,\Omega_m^0)$	0	free	free	free	0	free	546.50	0.8582	-279.61
$(B_1, B_2, B_4, \Omega_m^0)$	0	free	free	0	free	free	546.52	0.8581	-279.56
$(\mathbf{B_1},\mathbf{B_3},\mathbf{B_4},\mathbf{\Omega_m^0})$	0	free	0	free	free	free	540.78	0.8563	-280.00
$(\mathbf{B_2},\mathbf{B_3},\mathbf{B_4},\mathbf{\Omega_m^0})$	0	0	free	free	free	free	549.68	0.8353	-282.89
$(B_1,B_2,B_3,B_4,\Omega_m^0)$	0	free	free	free	free	free	546.50	0.8515	-279.60
full bigravity model	free	free	free	free	free	free	546.50	0.8445	-279.82

$$\bullet~$$
 Where  $~B_i\equiv \frac{m^2}{H_0^2}\beta_i$ 

Y. Akrami, T. Koivisto, M. Sandstad [arXiv:1209.0457]

F. Könnig, A. Patil, L. Amendola [arXiv:1312.3208]

### **Features and Achievements**

Gives a dynamical dark energy model.

Phantom behaviour is common (w < -1).

Possible non-GR signatures at background level and in structure formation.

Technically natural dark energy candidate .

### **Problems**

*Single Coupling: Only one metric describes space-time (directly couples to matter)* 

Finite Branch: Scalar perturbations are plugged by early-time gradient instabilities.

Infinite Branch: Vacuum decays immediately because of ghosts.

## Ways Out

#### Cure gradient instabilities nonlinearly (Vainshtein mechanism)

E.Mörtsell, J. Enander arXiv:1506.04977

K. Aoki, K. Maeda, R. Namba arXiv:1506.04543

#### Non-FRLW backgrounds

HN, Y. Akrami, L. Amendola arXiv:1502.03988

### Doubly coupled bigravity, but there are problems

Y. Akrami, T. Koivisto, D. Mota, M. Sandstad arXiv:1306.0004

Y. Akrami, T. Koivisto, A. Solomon arXiv:1404.0006

#### Plank mass hierarchy for two metrics

Y. Akrami, S.F. Hassan, F. Könnig, A. Schmidt-May, A. Solomon arXiv:1503.07521

Other modifications of the theory (Quasidilaton, Varying mass, Lorentz violation, F(R)-bigravity, trigravity ...)

## Possible metric combinations

#### **FRLW-FRLW:**

#### (homogenous-homogenous)

$$g_{\mu\nu}dx^{\mu}dx^{\nu} = -dt^{2} + a^{2}(t) d\vec{x_{g}}^{2},$$
  
$$f_{\mu\nu}dx^{\mu}dx^{\nu} = -X^{2}(t) dt^{2} + b^{2}(t) d\vec{x_{f}}^{2},$$

Where 
$$d\vec{x_g}^2 = \frac{dr^2}{1 - k_g r^2} + r^2 \left( d\theta^2 + \sin^2(\theta) \, d\phi^2 \right),$$
  
 $d\vec{x_f}^2 = \frac{dr^2}{1 - k_f r^2} + r^2 \left( d\theta^2 + \sin^2(\theta) \, d\phi^2 \right),$ 

### FRLW- Lemaître:

#### (homogenous-inhomogenous)

$$g_{\mu\nu}dx^{\mu}dx^{\nu} = -dt^{2} + a^{2}(t) d\vec{x_{g}}^{2},$$
  
$$f_{\mu\nu}dx^{\mu}dx^{\nu} = -X^{2}(t,r) dt^{2} + Y^{2}(t,r) dr^{2} + Z^{2}(t,r) r^{2}d\Omega^{2},$$

#### LTB-LTB:

#### (inhomogenous-inhomogenous)

$$g_{\mu\nu}dx^{\mu}dx^{\nu} = -dt^{2} + A^{2}(t,r)dr^{2} + B^{2}(t,r)d\Omega^{2},$$
  
$$f_{\mu\nu}dx^{\mu}dx^{\nu} = -X^{2}(t)dt^{2} + Y^{2}(t,r)dr^{2} + Z^{2}(t,r)d\Omega^{2},$$

### **Bianchi I-FRLW:**

#### (anisotropic-isotropic)

$$g_{\mu\nu}dx^{\mu}dx^{\nu} = -dt^{2} + a_{1}^{2}(t) dx^{2} + a_{2}^{2}(t) dy^{2} + a_{3}^{2}(t) dz^{2},$$
  
$$f_{\mu\nu}dx^{\mu}dx^{\nu} = -X^{2}(t) dt^{2} + b^{2}(t) d\vec{x}^{2},$$

# **Possible combinations**

#### HN, Y. Akrami, L. Amendola arXiv:1502.03988

Metric combinations									
$g_{\mu\nu}$ (physical metric)	$f_{\mu\nu}$ (reference metric)	$T^g_{\mu\nu}$	Possibility	Reference					
FLRW $(k)$	FLRW $(k)$	PF	$\checkmark$	Standard					
$FLRW(k_g)$	FLRW $(k_f)$	PF	×	Present work					
FLRW	LEMAÎTRE	PF		Present work					
FLRW	LEMAÎTRE	Inhom.	$\checkmark$	Present work & Ref. [26]					
LEMAÎTRE	FLRW	Any	×	Present work					
FLRW	LTB	Any	×	Present work					
LTB	FLRW	Any	×	Present work					
LTB	LTB	PF	√ <sup>a</sup>	Present work					
Bianchi I	FLRW	Any	×	Present work					
FLRW	Bianchi I	PF	×	Present work					
FLRW	Bianchi I	Aniso.	$\checkmark$	Present work					
Bianchi Class A	Bianchi Class A	PF	$\checkmark$	Refs. [30, 74]					
Perturbed FLRW	Perturbed FLRW	Perturbed PF	$\checkmark$	Standard					
Perturbed FLRW (scalars)	Unperturbed FLRW (scalars)	Perturbed PF	× <sup>b</sup>	Present work					
Unperturbed FLRW (scalars)	Perturbed FLRW (scalars)	Perturbed PF	$\times^{\mathrm{b}}$	Present work					
Perturbed FLRW (tensors)	Unperturbed FLRW (tensors)	Perturbed PF	×	Present work					
Unperturbed FLRW (tensors)	Perturbed FLRW (tensors)	Perturbed PF	×°	Present work					

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### Cosmological implementation of a viable combinations

#### FRLW-FRLW':

$$g_{\mu\nu}dx^{\mu}dx^{\nu} = -dt^{2} + a^{2}\left(t\right)\left[\frac{dr^{2}}{1 - k_{g}r^{2}} + r^{2}\left(d\theta^{2} + \sin^{2}\left(\theta\right)d\phi^{2}\right)\right],$$

$$f_{\mu\nu}dx^{\mu}dx^{\nu} = -\tilde{X}^{2}\left(\tilde{t}\right)d\tilde{t}^{2} + \tilde{b}^{2}\left(\tilde{t}\right)\left[\frac{d\tilde{r}^{2}}{1 - k_{f}\tilde{r}^{2}} + \tilde{r}^{2}\left(d\tilde{\theta}^{2} + \sin^{2}\left(\tilde{\theta}\right)d\tilde{\phi}^{2}\right)\right]$$

$$\tilde{t} = f\left(t, r\right)$$

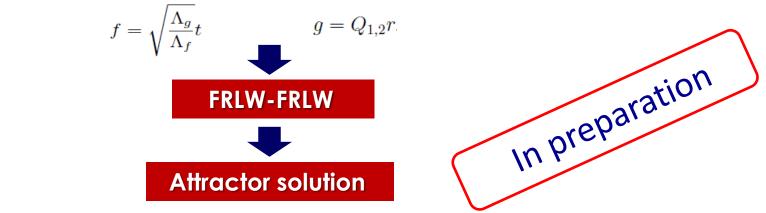
$$\tilde{r} = g\left(t, r\right)$$

$$d\tilde{s}_{q}^{2} = -dt^{2} + a^{2}dr^{2} + a^{2}r^{2}d\Omega^{2}$$

$$d\tilde{s}_{f}^{2} = -\left(X^{2}\dot{f}^{2} - b^{2}\dot{g}^{2}\right)dt^{2} + 2\left(b^{2}g'\dot{g} - X^{2}f'\dot{f}\right)dtdr + \left(b^{2}g'^{2} - X^{2}f'^{2}\right)dr^{2} + b^{2}g^{2}d\Omega^{2}$$

#### Attractor solution:

At the late times of cosmological evolution, the transformation functions will be



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### Summary



- Option 1: cure gradient instability nonlinearly?
- Option 2: other backgrounds
- *Option 3: can couple both metrics to matter*
- *Option 4: take small f-metric Planck mass*



*Option 5: other theories (modifications/generalizations)* 



### The possibility of different metric combinations

#### **Equations of Motion:**

$$G_{\mu\nu}(g) + m^2 V_{\mu\nu}^g = \frac{1}{M_{\rm Pl}^2} T_{\mu\nu},$$
  
$$\alpha^2 G_{\mu\nu}(f) + m^2 V_{\mu\nu}^f = 0,$$

• Where 
$$\alpha \equiv \frac{M_f}{M_{\rm Pl}}$$

#### Ricci identities + energy-momentum conservation:

