THE STRUCTURE OF THE REAL LINE AND COSMOLOGY

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The plan of the presentation

1. Foundations of mathematics
   - Zermelo-Fraenkel set theory with the axiom of choice
   - Model theory
   - Forcing and the structure of the real line
2. Forcing in quantum mechanics
   - Historical remarks
   - Micro to macroscale shift
3. Forcing in cosmology
   - The general assumptions
   - The cosmological constant problem
4. Cosmic inflation driven by exotic smoothness
5. Discussion and perspectives

Foundations of mathematics

Zermelo-Fraenkel set theory with the axiom of choice

ZF(C) = Zermelo-Fraenkel set theory (with the axiom of choice)

Foundation of (almost) all of mathematics

It is widely accepted paradigm that most of the mathematics required by theories of physics is based on the formal theory of sets, e.g. ZFC

- The real line $\mathbb{R}$ lies in the framework of set theory

Axiomatization as the final step of formalization of a theory:
No place for ambiguity?

Model theory

Kurt Gödel (1906 - 1978)

Model of a theory

A mathematical universe (a class) within which the theorems of this theory are fulfilled.

The ZFC set theory has infinitely many non-isomorphic models - there is no unique universe for mathematics

What differs non-isomorphic models of ZFC?

Real numbers

Different view on properties "as seen" from inside and outside of a model: the idea of absolutness of a formula

The set of real numbers is not absolute

Some formal object can be a real number in one model, while in another model it is not.

$\mathcal{M}_1, \mathcal{M}_2$ - non-isomorphic models of ZFC

$\mathbb{R}_{\mathcal{M}_1}, \mathbb{R}_{\mathcal{M}_2} \subset \mathbb{R}$ $\qquad$ $\mathbb{R}_{\mathcal{M}_1} \neq \mathbb{R}_{\mathcal{M}_2}$

Different models of ZFC $\Leftrightarrow$ **Different sets of reals**

Preliminary assumptions

PERHAPS, DURING THE EVOLUTION OF THE UNIVERSE, THERE WERE SOME CHANGES OF THE MODEL OF ZFC (AND SO THE REAL LINE AS WELL)

$\Rightarrow$ WE PROPOSE THE MODEL WHERE IT ACTUALLY HAPENNED (DURING THE INFLATION ERA)

- We need appriopriate mathematical tools to formally describe the change of the model of ZFC.
- Also some connection to QM formalism is required.

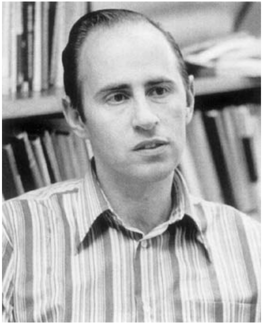
Forcing

FORCING - a formal method of extending given model of ZFC (or ZF)

The 1964 Fields Medal

Using his original method, Paul Cohen showed that CH is independent of the axioms of ZFC.

Continuum Hypothesis (CH): $2^{\aleph_0} = \aleph_1$



Paul Cohen (1934 - 2007)

Forcing adds some "missing" reals to a model

$\mathcal{M}[G]$ - generic extension by special *measure* forcing of a model $\mathcal{M}$

Rarefaction of the real line

$\mathbb{R}_{\mathcal{M}}$ is merely a meager subset of $\mathbb{R}_{\mathcal{M}[G]}$ (it is in precise sense of negligible size)

$\mathcal{M} \xrightarrow{\textit{forcing}} \mathcal{M}[G]$ $\qquad$ $\mathbb{R}_{\mathcal{M}} \subset \mathbb{R}_{\mathcal{M}[G]}$

A Meager Set = countable sum of nowhere-dense sets

($\star$) It has inner (Lebesgue) measure 0 and full outer measure

Lemma

Every Lebesgue measureable subset of a set that fulfills ($\star$) has measure 0.

Forcing in quantum mechanics

Historical remarks

Paul A. Benioff (1976)

Neither any model of ZFC nor its generic extension by forcing are sufficient for grasping the mathematics of QM along with its statistical predictions

⇒ Particular position of QM formalism in mathematics

William Boos (1996) ⋄ Robert A. van Wesep (2006)

If there are "semiclassical states" realizing LHV program, then they are generic ultrafilters (formal objects specific to forcing constructions).

Jerzy Król (2004)

"Dynamical network of models"
(adding result of a measurement by forcing)

Micro to macroscale shift

- ⇒ QM is defined w.r.t. the reals $\mathbb{R}_{\mathcal{M}}$ of some model $\mathcal{M}$ of ZFC.
- ⇒ Then, the model varies and new reals are added (by forcing).

In macroscale (GR) we use the full real line $\mathbb{R}$ for a description of space and time

($\star$) - Suppose that the real numbers, which parametrize space, come from the quantum realm via continous measurement ($\sim$ position observable).

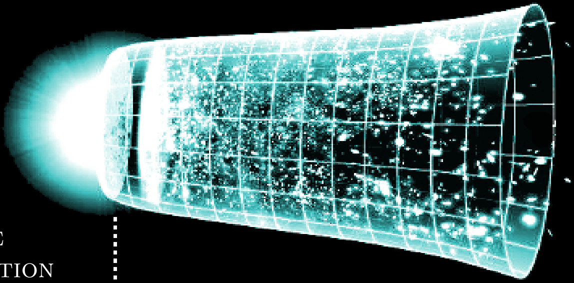
Jerzy Król ⋄ P. K. (2015)

Under ($\star$) there is always a nontrivial forcing on the measure algebra on $\mathbb{R}^3$, when passing from the quantum regime to the classical world described by GR

- ⇒ This is the reason why forcing should be considered when building the cosmological models.

Forcing in cosmology

The general assumptions



SPACETIME PARAMETRIZATION VIA CHANGING $\mathbb{R}_{\mathcal{M}}$'S

$\star$

THE FULL REAL LINE $\mathbb{R}$ PARAMETRIZATION

$\star$ The growth of inherent density of reals $\leftrightarrow$ spacetime inflation

$\Rightarrow$ Observer is connected with the large scale (GR).

The cosmological constant problem

The zero-point energy of quantum field corresponding to a particle of mass m:

$$\frac{E}{V} = \int \frac{d^3k}{(2\pi)^3} \frac{\sqrt{\mathbf{k}^2 + m^2}}{2}$$

In particle physics such contributions to Ω_Λ appear to be too large by many orders of magnitude.

$\Longrightarrow$ Let us evaluate this integral under the assumption that the zero-modes of quantum fields lie in the model $\mathcal{M}$ (the internal space coordinates are parameterized by the real numbers $\mathbb{R}_{\mathcal{M}}$).

The cosmological constant problem

Zero-modes are described as if the space is spanned on the internal $\mathbb{R}^3_{\mathcal{M}}$. However, the degrees of freedom of GR require the full real line $\mathbb{R}$

In this case the integral is evaluated over the non-measureable subset $\mathbb{R}^3_{\mathcal{M}} \subset \mathbb{R}^3$.

$\Rightarrow$ In general such Lebesgue integral does not exist.

$\Rightarrow$ We look for contributions over all measureable subsets of $\mathbb{R}^3_{\mathcal{M}}$.

Since $\mathbb{R}^3_{\mathcal{M}}$ is meager, all such sets have measure 0 and their contributions to Ω_Λ vanish.

Cosmic inflation driven by exotic smoothness

Obtaining non-zero CC

So why the value of density $\Omega_\Lambda \sim 10^{-27} \frac{kg}{m^3}$ is non-zero (and so small)?

⋆ (Torsten Asselmeyer-Maluga, Jerzy Król; 2014)
One can make use of the exotic smooth geometry on $\mathbb{R}^4$

Exotic $\mathbb{R}^4$ $\Rightarrow$ Hiperbolic geometry $\Rightarrow$

$\Rightarrow$ (Codimension-1) foliation of $\mathbb{R}^4$ $\Rightarrow$ Evolution w.r.t. the foliation

The transition:

WILDELY EMBEDDED 3-SPHERE $\longrightarrow$ HOMOLOGY 3-SPHERE θ

generates the realistic inflation in the simple model $S^3 \times_\theta \mathbb{R} \subset \mathbb{R}^4$.

The Morse function of the handle-decomposition of $S^3 \times_\theta \mathbb{R}$ generates the shape of the potential $V(\phi)$. It is realistic if we choose the exotic smooth $\mathbb{R}^4$ (there are infinitely many *-continuum-* different exotic $\mathbb{R}^4$'s and $S^3 \times_\theta \mathbb{R}$'s).

Discussion and perspectives

Summary

- There is no single model of ZFC that can be used for the correct representation of mathematics of QM and its statistical predictions. One needs to refer to some dynamics between models.

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- The structure of the real line (e.g. set-theoretic forcing) encodes the formal shift from quantum to classical position observable.

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- Obtaining full real line from QM via continous measurements must involve nontrivial forcing.

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- Localization of quantum realm within a ZFC model leads to vanishing of zero-modes of quantum fields.

Summary

- If we choose an exotic smooth $\mathbb{R}^4$ structure for a spacetime, then one can obtain the shape of inflation potential $V(\phi)$ and a realistic cosmological constant.

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Plans for future work:

- ⋆ Direct description of the cosmic inflation in such forcing-based model of the universe. In particular, we want to give a forcing mechanism for emerging exotic $\mathbb{R}^4$ in the early universe (hence the inflation).
- ⋆ Attempt to find the impact of meager ($\sim$ rarefied) real line on the various distortions of CMB
- ⋆ Description of quantum entanglement in the forcing language

Thank You for a Patient Hearing