Post-Newtonian Cosmological Modelling

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VAAS and T. Clifton, Phys. Rev. D 91, 103532

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Motivation

- We want to study the effect of non-linear structure on the large-scale expansion of the universe.
- General relativity is known to be valid locally and there is no unique way to average tensors.
- There is no perturbative framework that works consistently on all scales, in the presence of non-linear structure.
- We construct a bottom-up approach using the post-Newtonian approximation to gravity.
- This could be important to interpret data from large-scale surveys such as *Euclid and SKA (Square Kilometre Array).*

Building a post-Newtonian cosmology

- We put a large numbers of cells next to each other to form a periodic lattice structure.
- Cell shape - regular polyhedra.

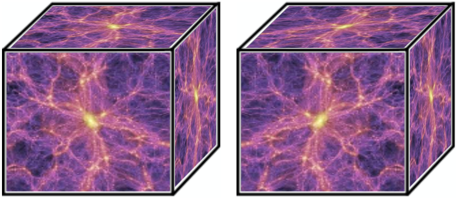
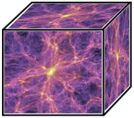


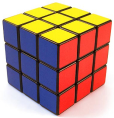
Figure: This figure was produced using an image from D. J. Croton *et al.*, 2005

Building a post-Newtonian cosmology

Lattice Structure	**Lattice Curvature**	**Cell Shape**	**Cells per Lattice**
{333}	+	Tetrahedron	5
{433}	+	Cube	8
{334}	+	Tetrahedron	16
{343}	+	Octahedron	24
{533}	+	Dodecahedron	120
{335}	+	Tetrahedron	600
{434}	0	Cube	∞
{435}	-	Cube	∞
{534}	-	Dodecahedron	∞
{535}	-	Dodecahedron	∞
{353}	-	Icosahedron	∞

- For Eg, for cubic cells





Building a post-Newtonian cosmology

- We put a large numbers of cells next to each other to form a periodic lattice structure.
- Cell shape - regular polyhedra.
- The geometry of each cell is given by a perturbed Minkowski metric.
- Interior of each cell satisfies the post-Newtonian formalism.
- Cell size $<< H_0^{-1}$.
- We assume reflective symmetry across the boundary of the cells.

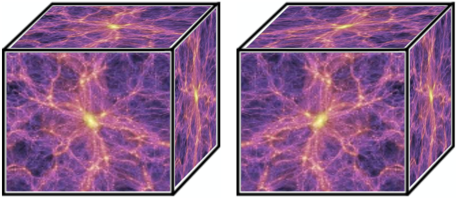


Figure: This figure was produced using an image from D. J. Croton *et al.*, 2005

- We join these perturbed Minkowski patches together to construct a global spacetime.

Building a post-Newtonian cosmology

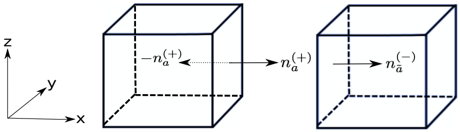
We match these cells together using Israel junction conditions,

$$\gamma_{ij}^{(+)} = \gamma_{ij}^{(-)},$$
$$K_{ij}^{(+)} = K_{ij}^{(-)},$$

where γ_{ij} is the induced metric, and K_{ij} is the extrinsic curvature of the boundary, defined by

$$K_{ij} \equiv \frac{\partial x^a}{\partial \xi^i}\frac{\partial x^b}{\partial \xi^j} n_{a;b} \ ,$$

where ξ^i denotes the coordinates on the boundary, and n^a is the space-like unit vector normal to the boundary.



Now, mirror symmetry implies that $n_{\tilde{a}}^{(-)} = -n_a^{(+)}$. Symmetry therefore demands that

$$\frac{\partial x^a}{\partial \xi^i}\frac{\partial x^b}{\partial \xi^j} n_{a;b}^{(+)} = -\frac{\partial x^{\tilde{a}}}{\partial \xi^i}\frac{\partial x^{\tilde{b}}}{\partial \xi^j} n_{\tilde{a};\tilde{b}}^{(+)} \ .$$

This implies that $K_{ij} = -K_{ij}$, or, in other words, $K_{ij} = 0$.

Post-Newtonian formalism

In the limit of slow motions ($v \ll c$) and weak gravitational fields ($\Phi \ll 1$), we can find relativistic corrections to Newtonian gravity using Einstein's field equations given by

$$R_{ab} = 8\pi G \left(T_{ab} - \frac{1}{2} T g_{ab} \right) .$$

We treat this equation perturbatively, with an expansion parameter

$$\epsilon \equiv \frac{|\boldsymbol{v}|}{c} \ll 1,$$

where $\boldsymbol{v} = v^{\alpha}$ is the 3-velocity associated with the matter fields.

Energy-momentum tensor:

$$T^{ab} = \mu u^a u^b + p(g^{ab} + u^a u^b)$$

where,

$$\mu = \rho + \rho\Pi \sim \epsilon^2 + \epsilon^4$$
$$p \sim \epsilon^4 \ .$$

Perturbed Minkowski Metric:

$$g_{ab} = \eta_{ab} + h_{ab},$$

where,

$$h_{tt} \sim \epsilon^2 + \epsilon^4,$$
$$h_{t\mu} \sim \epsilon^3,$$
$$h_{\mu\nu} \sim \epsilon^2.$$

Post-Newtonian formalism

- We can define Newtonian and post-Newtonian gravitational potentials to relate the metric perturbations to matter. It is useful to make the following implicit definitions for new gravitational potentials

$$
\begin{aligned}
\nabla^2\Phi &\equiv -4\pi G\rho\,, \\
\nabla^2 V_\mu &\equiv -4\pi G\rho v_\mu\,, \\
\nabla^2\Phi_1 &\equiv -4\pi G\rho v^2\,, \\
\nabla^2\Phi_2 &\equiv -4\pi G\rho\Phi\,, \\
\nabla^2\Phi_3 &\equiv -4\pi G\rho\Pi\,, \\
\nabla^2\Phi_4 &\equiv -4\pi G p\,,
\end{aligned}
$$

where $h^{(2)}_{\mu\nu} = h^{(2)}_{tt}\delta_{\mu\nu} = 2\Phi\delta_{\mu\nu}$, $v^2 = v^\alpha v_\alpha$, $V_\mu \sim \epsilon^3$ and $\Phi_1 \sim \Phi_2 \sim \Phi_3 \sim \Phi_4 \sim \epsilon^4$. In what follows, we will not require any potentials of order higher than ϵ^4.

Green's Functions

- We do not assume asymptotic flatness, as one may do in the case of isolated systems. For example,

$$\Phi(\mathbf{x}, t) \neq -\frac{1}{4\pi G} \int_{\Omega} \frac{\rho(\mathbf{x}, t)}{|\mathbf{x} - \mathbf{x}'|} d^3x'$$

- We use a Green's function formalism

$$\Phi = \bar{\Phi} + 4\pi G \int_{\Omega} \mathcal{G} \rho \, dV + \int_{\partial\Omega} \mathcal{G} \boldsymbol{n} \cdot \nabla\Phi \, dA \, ,$$

  where Ω is the spatial volume, $\mathcal{G}$ is the Green's function and $\boldsymbol{n}$ is the normal to the centre of the cell face.

- We can derive this equation for any potential that satisfies a Poisson-like equation using Gauss' theorem.

Results

Newtonian order - Friedmann-like equation

In general, for cubic cells,

$$\frac{(X_{,t})^2}{X^2} = \frac{\pi GM}{3X^3} - \frac{C}{X^2} + O(\epsilon^4)\,,$$

where $X(t, y, z)$ is the distance from the centre of the cell to the centre of the cell face and behaves like the scale factor, M is the mass of matter within a cell, C is an integration constant and time derivatives are taken with respect to coordinate time.

Post-Newtonian Order - Friedmann-like equation

For regularly arranged point masses,

$$\frac{(X_{,t})^2}{X^2} = \frac{2N}{X^3} - \frac{J}{X^4} - \frac{C}{X^2} + O(\epsilon^6)\,,$$

where N and J are known positive constants.

Discussion

- The size of the post-Newtonian correction depends on the size of the cell.
- We did not need to average to obtain the large-scale expansion of our model universe.
- We have obtained similar results in terms of proper time and proper lengths.
- The Friedmann-like behaviour at Newtonian order is an emergent phenomena.
- The existence of a radiation-like term has been found previously. (T. Clifton, D. Gregoris and K. Rosquist, 2014 and S. R. Green and R. M. Wald, 2015)

Summary and Future Work

- We have constructed a perturbative framework that consistently tracks non-linear effects of small-scale structure on the large-scale expansion.
- Calculate observables in these type of models.
- Generalize this model.
- With future large-scale surveys, such as Euclid and SKA (Square Kilometre Array), we will have more data to help understand the large-scale expansion of the universe.
- Inhomogeneous models may help us include any non-linear effects that we have not already considered.

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