Evolution of density perturbations in inflationary models with two scalar fields and with different sound speeds

Lukasz Dulny lukasz.dulny@fuw.edu.pl
Faculty of Physics,
University of Warsaw,
Hoża 69, 00-681 Warsaw, Poland

Abstract

Equations of motion in slow roll inflationary models with two scalar fields for scalar perturbations metrics are analysed for an example of turning valley in scalar potential with non-canonical kinetic term. The impact of the multi-scale dynamics on the sub-horizon evolution of the perturbations is studied with a special emphasis on the (lack of) decoupling of the large scale modes and features of the background evolution.

Introduction

- Slow roll inflation means that the field must slowly change in time
- Two scalar fields – curvature perturbations (pararell to the direction of motion in the field space) v_σ and isocurvature perturbations v_s (perpendicular to the direction of motion in the field space, observable through their impact on the former ones)[2][3]
  - For more than 2 fields – more isocurvature perturbations
- Non-canonical kinetic term $S = \int d^4x\sqrt{-g}\left(\frac{R}{2} + P(X, \phi_1, \phi_2)\right)$ for $X = -\frac{1}{2}g^{\mu\nu}(\nabla_\mu\phi_1\nabla_\nu\phi_1 + \nabla_\mu\phi_1\nabla_\nu\phi_1)$ (simplified from [3]).
- Turns are possible – what happens then to the perturbations? With non-canonical kinetic term, this problem has not been thoroughly analysed yet.

Behaviour of perturbations at sharp turn

- Deep under horizon, when we can ignore that the potential is not constant, perturbations with wave number k (in comoving coordinates) depend on conformal time τ like $v = \frac{e^{-ik\tau}}{\sqrt{2k}}$.[1]
- Generally, we should solve[3]

$$v_\sigma'' - \xi v_s' + (c_s^2k^2 - \frac{z''}{z})v_\sigma - \frac{(z\xi)'}{z}v_s = 0$$

$$v_s'' - \xi v_\sigma' + (k^2 - \frac{\alpha''}{\alpha} + a^2\mu_s^2)v_s - \frac{z'}{z}\xi v_\sigma = 0$$

where:
$0 < c_s \leq 1$ is the speed of sound given by

$$c_s^2 = \frac{P_{,X}}{P_{,X} + 2XP_{,XX}}$$

$\xi \sim \theta'$ (change of direction, for sharp turn approximates the Dirac delta), μ_s is the mass of isocurvature perturbations and for sharp turn under horizon $(k/a_{turn} \gg \xi \gg other\ scales)$ can be reduced to

$$a^2\mu_s^2 = -\frac{\xi^2}{(c_s^2+1)^2}$$

and z and α are proportional to a, but z''/z probably (they appear for the canonical kinetic term, for the non-canonical one calculations are in progress) includes term proportional to ξ^2. The effects of ξ' (derivative of Dirac delta) also need explanation.

- By now, it is known[5] that for canonical kinetic term effects of the turn on modes under horizon is almost invisible. For non-canonical term, lack of effects under horizon is still not confirmed.

Getting the turn

There are a few possibilities:

- Standard[4]

$$V = \frac{1}{2}m^2\phi_1^2 + \frac{1}{2}M^2\cos^2\left(\frac{\Delta\theta}{2}\right)[\phi_2 - (\phi_1 - \phi_{10})\tan\Xi(\phi_1)]^2 \quad (1)$$

where

$$\Xi(\phi_1) = \Delta\theta\arctan[s(\phi_1 - \phi_{10})]$$

- Simplest (polynomial), but there are two tracks and mass of v_s is strongly variable

$$V = \frac{m^2}{2}\phi_1^2 + \lambda[a^2(\phi_1 - \phi_{10})^2 - \phi_2^2 + \phi_{20}^2]^2 \quad (2)$$

- Medium simplicity, one track

$$V = \frac{m^2}{2}\phi_1^2 + \frac{M^2}{2}\left[\phi_2 - \sqrt{a^2(\phi_1 - \phi_{10})^2 + \phi_{20}^2}\right]^2 \quad (3)$$

It is noteworthy that the values of m^2 and ϕ_{10} influence the value of Hubble parameter and decide how much time (and steps in numerical calculation) does given number of e-folds require.
Besides, canonical $P = X - V$ or non-canonical $P = X^2 - V$ kinetic term is used. After the numerical method will be checked, the use of other forms of P in calculations will also be possible.

The track in the field space

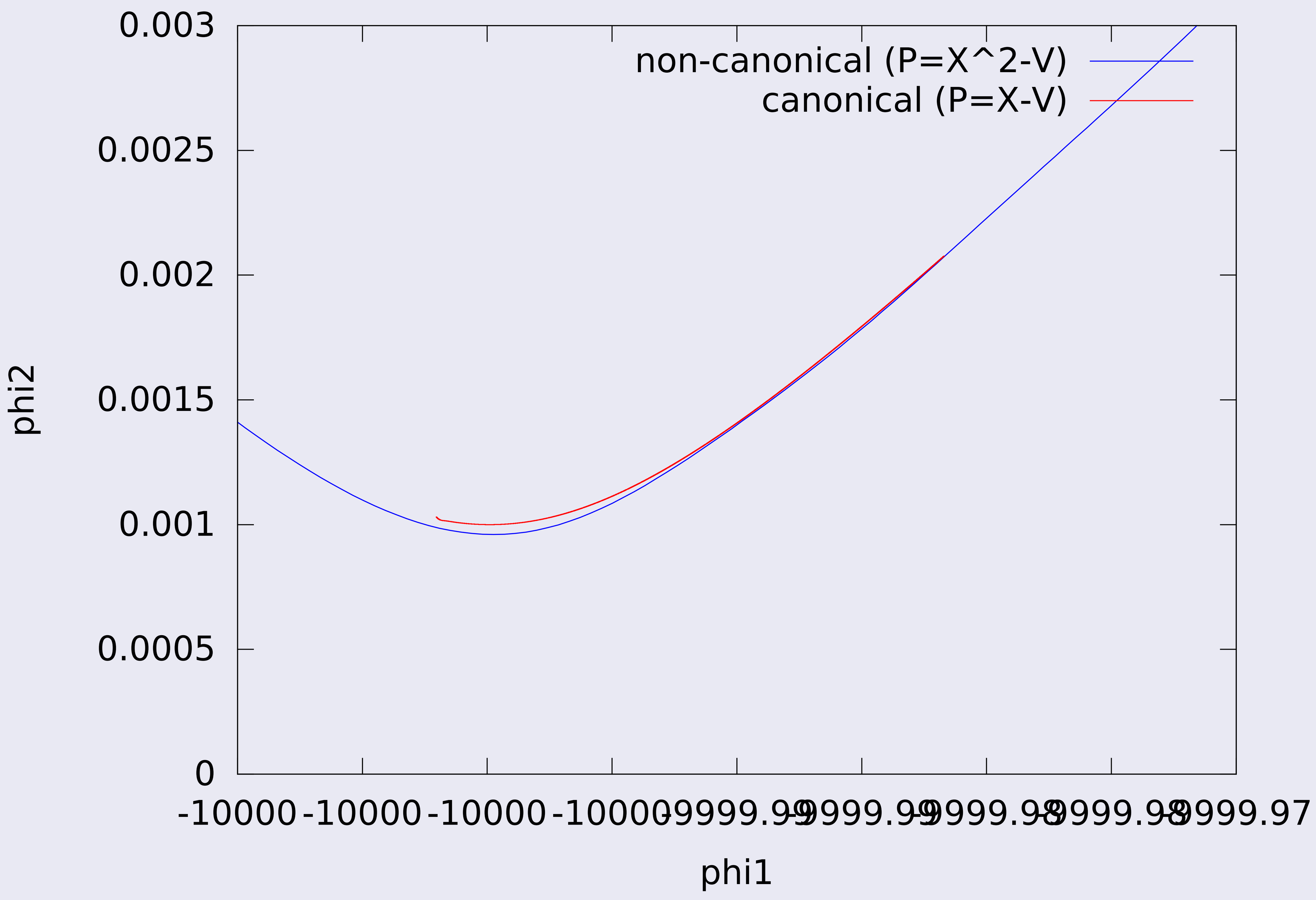

Figure: $\phi_2(\phi_1)$ for eq. 3 with $m = 10^{-4}M_{Pl}, M = 1M_{Pl}, a = 0.1, \phi_{10} = -10^4M_{Pl}, \phi_{20} = 10^{-3}M_{Pl}$. The track (and the value of Hubble parameter) does not depend strongly on canonicity, but field change in the $P = X^2 - V$ case is much quicker. One can see that the track for $P = X - V$ begins during the turn and is pararell to the asymptote at the beginning due to not exact method of finding initial conditions.

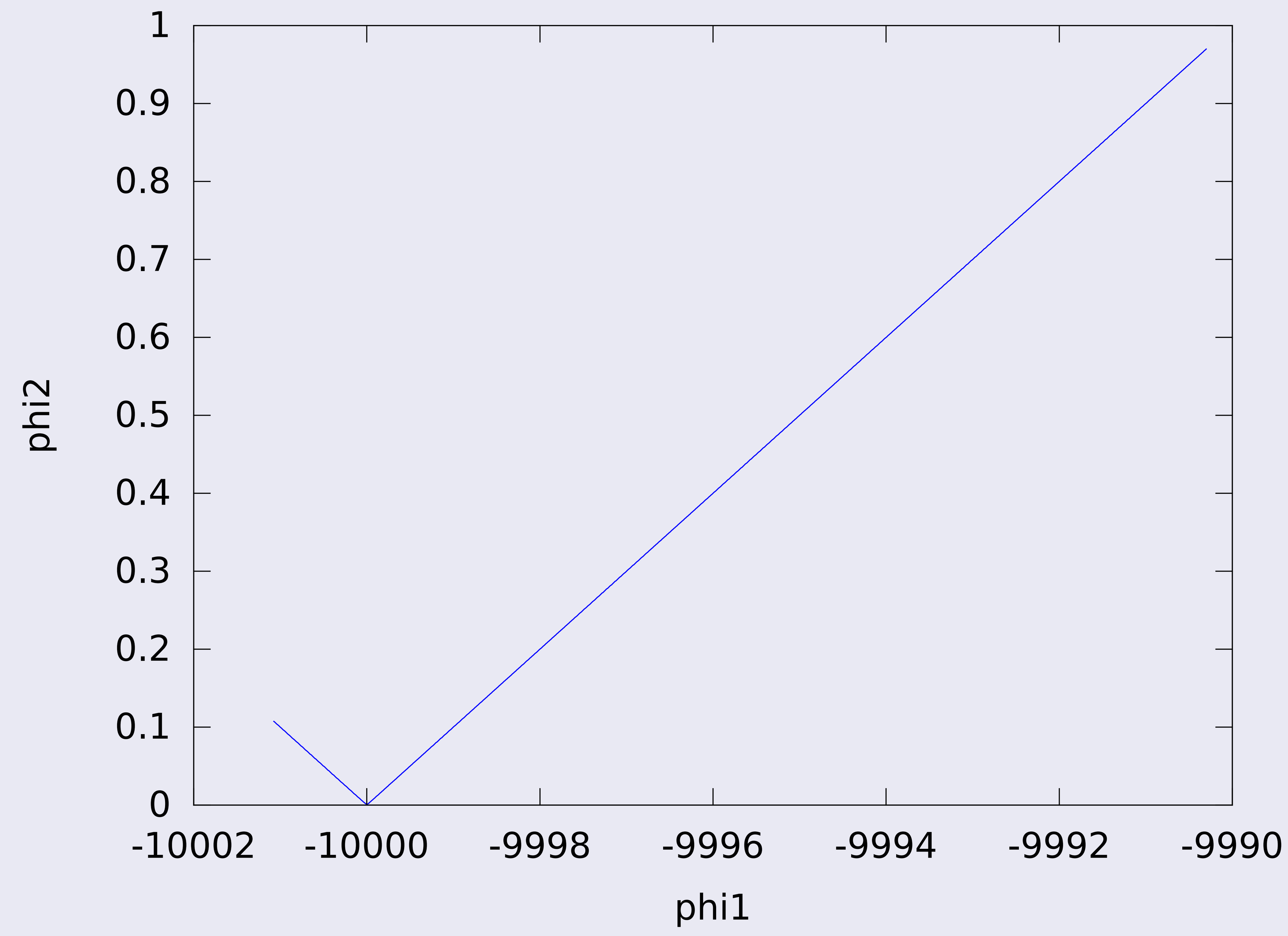

Figure: The same for non-canonical kinetic term and about 100 e-folds (canonical term gives much shorter track for 100 e-folds, as visible above). One can see the scale of variation of the fields. In this scale the turn is sharp.

Summary

- I work on perturbations in a subclass of models with non-canonical kinetic term described in [3].
- The effect of turns in the field space on the spectrum of perturbations will probably be possible to compare with the Planck data.
- The numerical calculation algorithm and analytical approximation require last checks.
- Models that are easy to simulate numerically can save some time.

Bibliography

[1] Zygmunt Lalak, David Langlois, Stefan Pokorski, Krzysztof Turzynski, ‘Curvature and isocurvature perturbations in two-field inflation’, http://arxiv.org/pdf/0704.0212.pdf, 2007-07-13

[2] Min-xin Huang, Gary Shiu, Bret Underwood, ‘Multifield DBI Inflation and Non-Gaussianities’, http://arxiv.org/pdf/0709.3299.pdf, 2008-01-15

[3] David Langlois, Sebastien Renaux-Petel, ‘Perturbations in generalized multi-field inflation’, http://arxiv.org/pdf/0801.1085.pdf, 2008-05-31

[4] Xian Gao, David Langlois, Shuntaro Mizuno, ‘Influence of heavy modes on perturbations in multiple field inflation’, http://arxiv.org/pdf/1205.5275v3.pdf, 2013-07-25

[5] Maciej Konieczka, Raquel H. Ribeiro, Krzysztof Turzynski, ‘The effects of a fast-turning trajectory in multiple-field inflation’, http://arxiv.org/pdf/1401.6163.pdf, 2014-08-27