# Warm quartic inflation in light of Planck 2015 results

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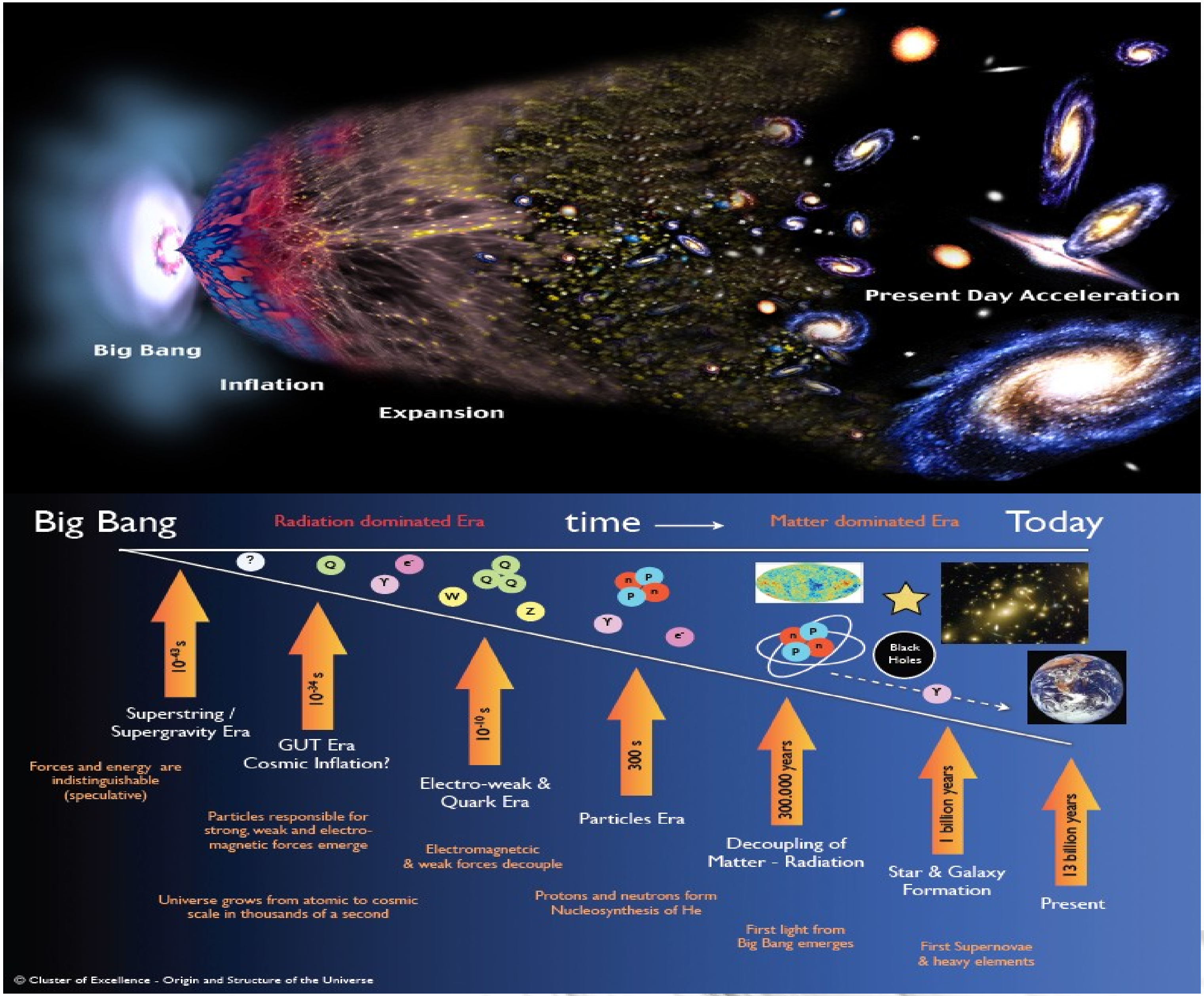
# Outline

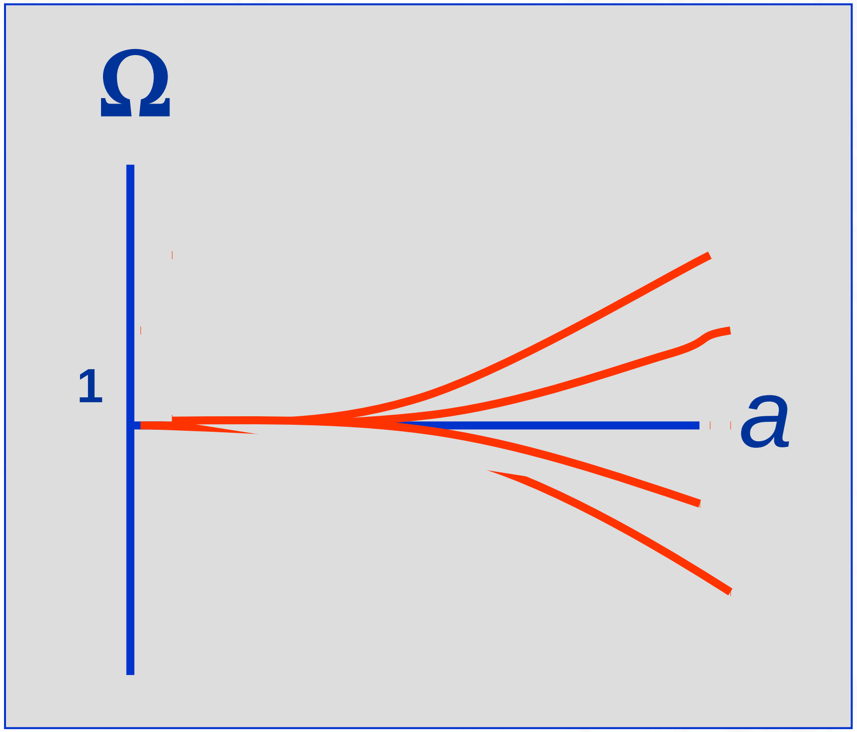
- Motivation
- Basics of standard inflation
- Warm inflationary model
- Numerical results
- Conclusions

# Evolution of the universe



# Flatness problem

$$\Omega = 1 + \frac{k}{H^2 a^2}$$



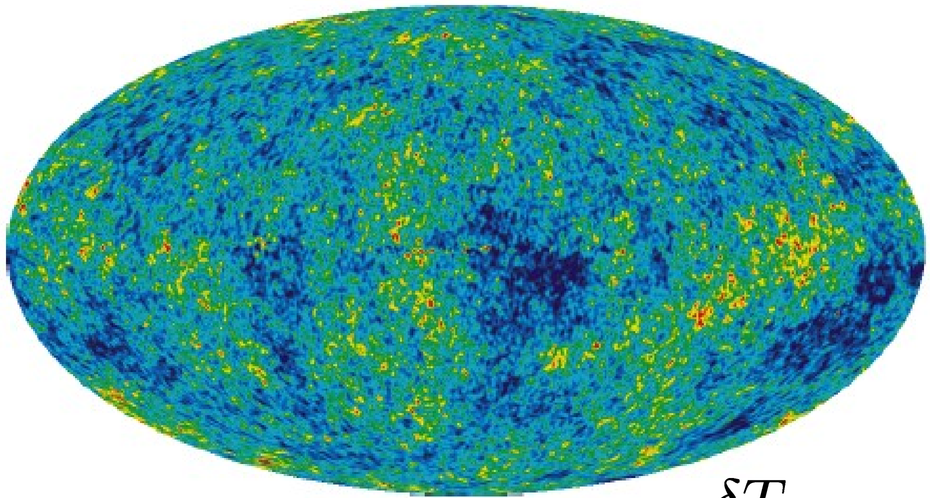

$\Omega = 1$ is unstable fixed point for matter and radiation

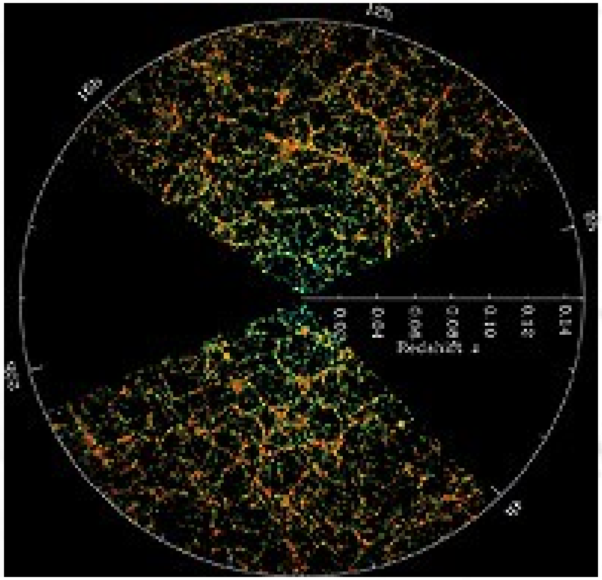
a) Universe is very old

b) Omega today is 0.1-2

Unnatural initial conditions: started extremely close to 1

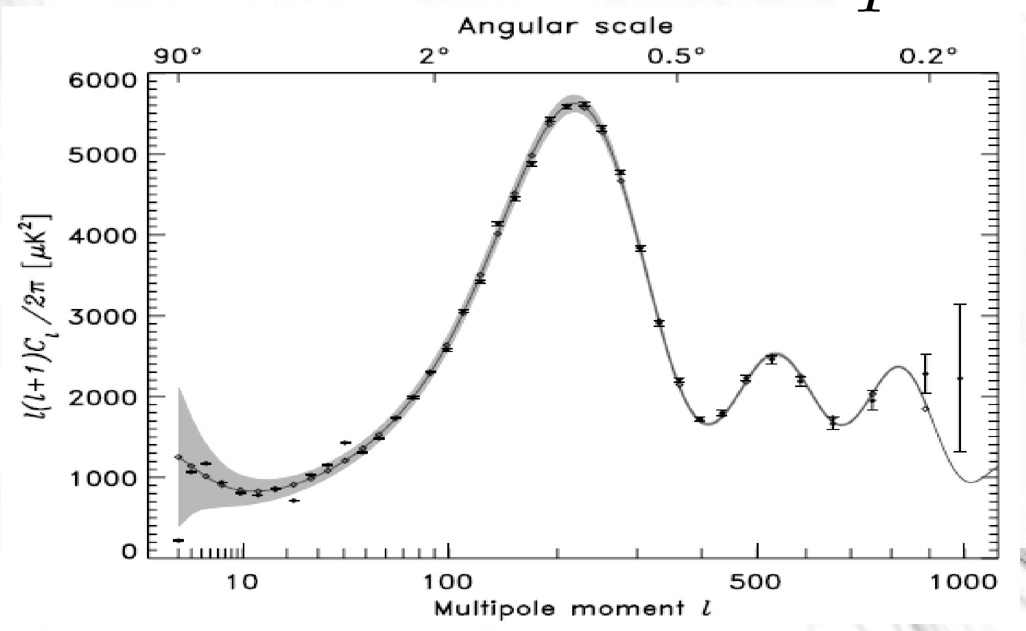
# Origin of primordial fluctuations

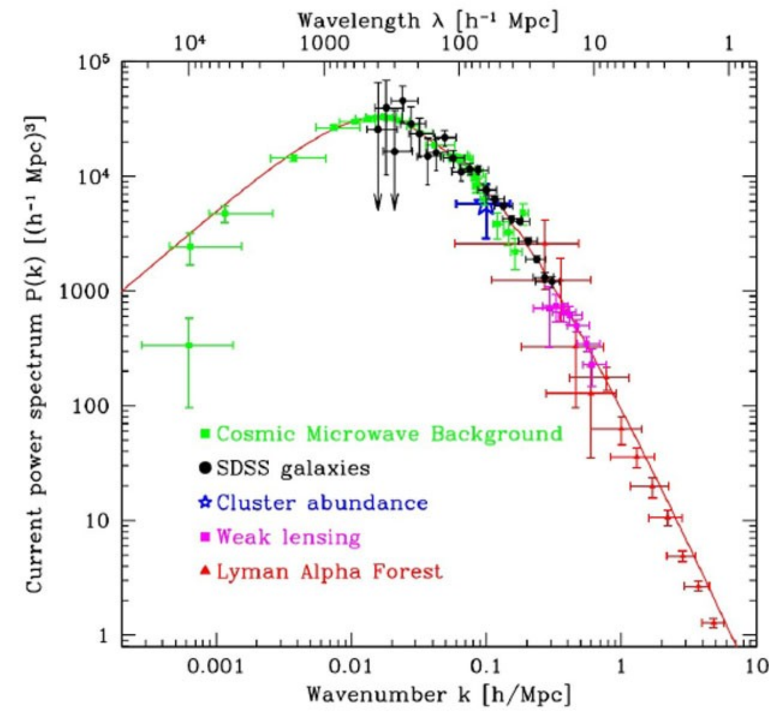




$T = 2.725\ \mathrm{K}$

$$\frac{\delta T}{T} \sim 10^{-5}$$



# Scalar field energy density and pressure

- Both consist of two parts: Potential and kinetic

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi)$$
$$p_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi)\,.$$

- Eqn of state shows small kin. energy

$$w = \frac{\frac{1}{2}\dot{\phi}^2 - V(\phi)}{\frac{1}{2}\dot{\phi}^2 + V(\phi)},$$

# Dynamics in scalar field cosmology

$$\begin{aligned} H^2 &= \frac{8\pi}{3m_{\rm Pl}^2}\left[V(\phi)+\frac{1}{2}\dot{\phi}^2\right], \\ \ddot{\phi}+3H\dot{\phi} &= -V'(\phi), \end{aligned}$$

- Subtitute energy density/pressure in Friedmann and fluid eqns
- Accelerating expansion when energy density dominates

$$\ddot{a}>0 \Longleftrightarrow p<-\frac{\rho}{3} \Longleftrightarrow \dot{\phi}^2<V(\phi)$$

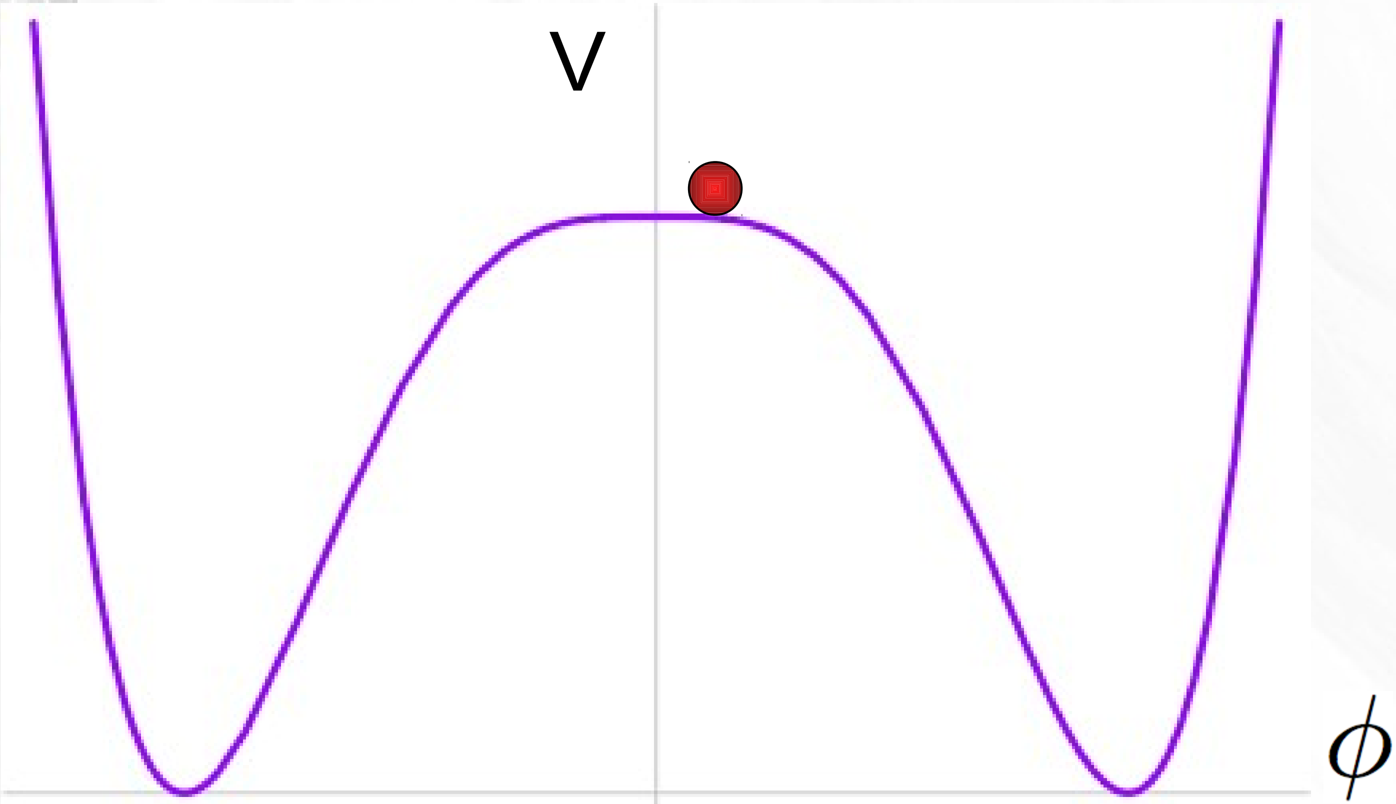
# Slow-roll inflation: A paradigm for the early universe

$$\epsilon = \frac{m_{pl}^2}{16\pi}\left(\frac{V'}{V}\right)^2 \qquad \eta = \frac{m_{pl}^2}{16\pi}\left(\frac{V''}{V}\right)$$

$$H^2 \simeq \frac{8\pi V}{3m_{pl}^2}$$

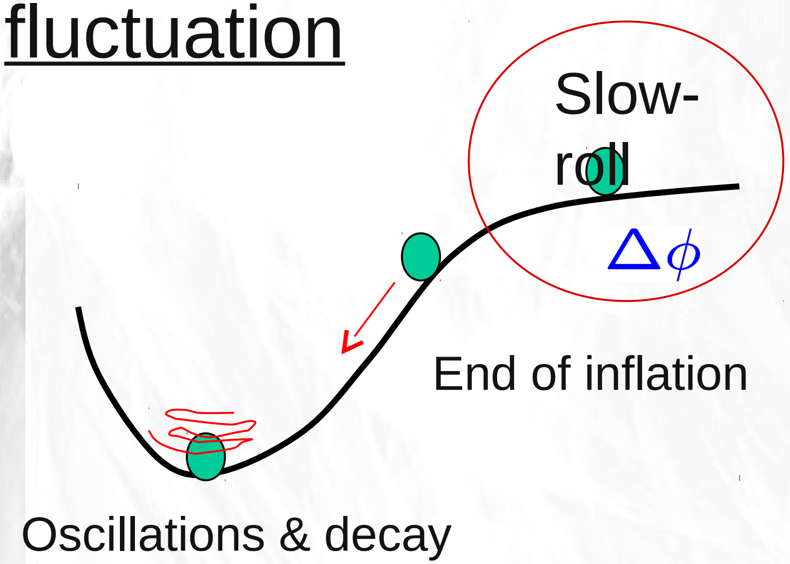
$$\dot{\phi} \simeq -\frac{V'}{3H}$$

$$\Omega - 1 = \frac{k}{a^2 H^2}$$




**Basic prediction of inflation: Universe is flat**

# Primordial density fluctuation




During inflation era, quantum fluctuation of inflaton is enlarged by exp. expansion

$$a \propto e^{H_I t}$$

Inflaton fluctuation → curvature fluctuation
→ structure formation, CMB anisotropy

Inflaton fluctuation ← inflaton potential, initial condition
CMB anisotropy ← precision measurement by observation

# Inflationary Predictions VS. CMB (cont'd)

## Conditions to fix parameters in inflation model

### Power spectrum

$$\mathcal{P}_S(k) = \frac{128\pi}{3M_P^6}\left(\frac{V^3}{V'^2}\right)_{k=aH} = 2.42 \times 10^{-9}$$

### e-foldings

$$N(\phi) = \int_{\phi_e}^{\phi} \frac{1}{m_P^2}\frac{V}{V'}d\phi = 50 - 60$$

By these conditions, the slow-roll parameters are fixed

**Predictions**

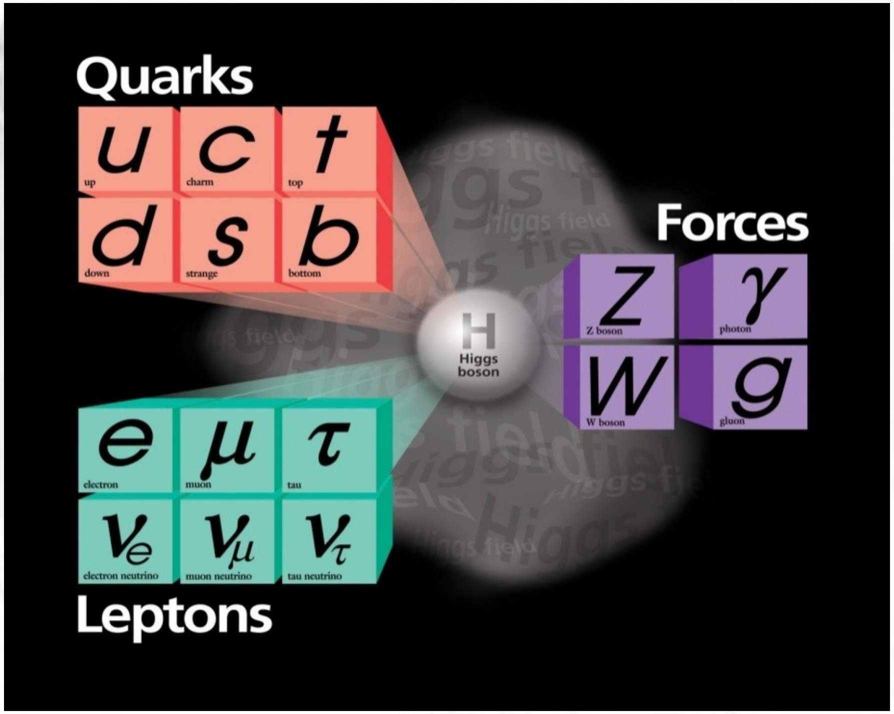
Spectral index:

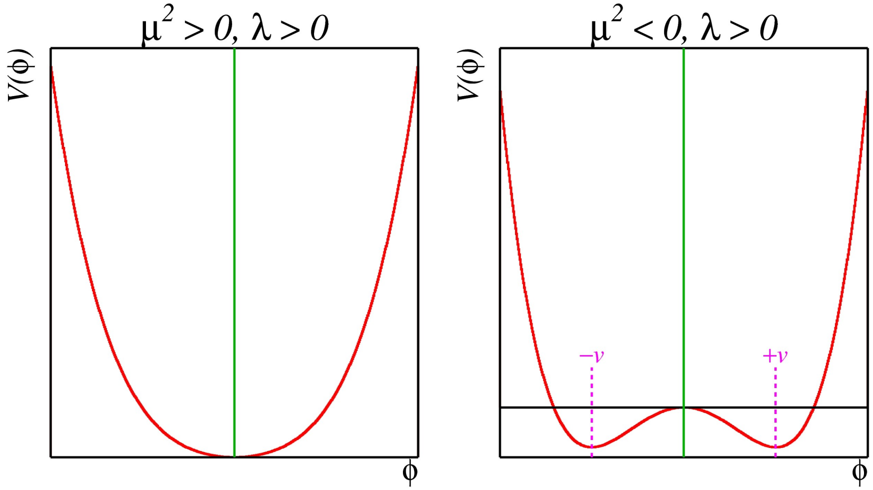
$$n_S(k) - 1 \equiv \frac{d\ln \mathcal{P}_S(k)}{d\ln k} = -6\varepsilon + 2\eta$$

Tensor-to-scalar ratio:

$$r = \frac{P_T}{P_s} = 16\epsilon$$

# The Higgs boson



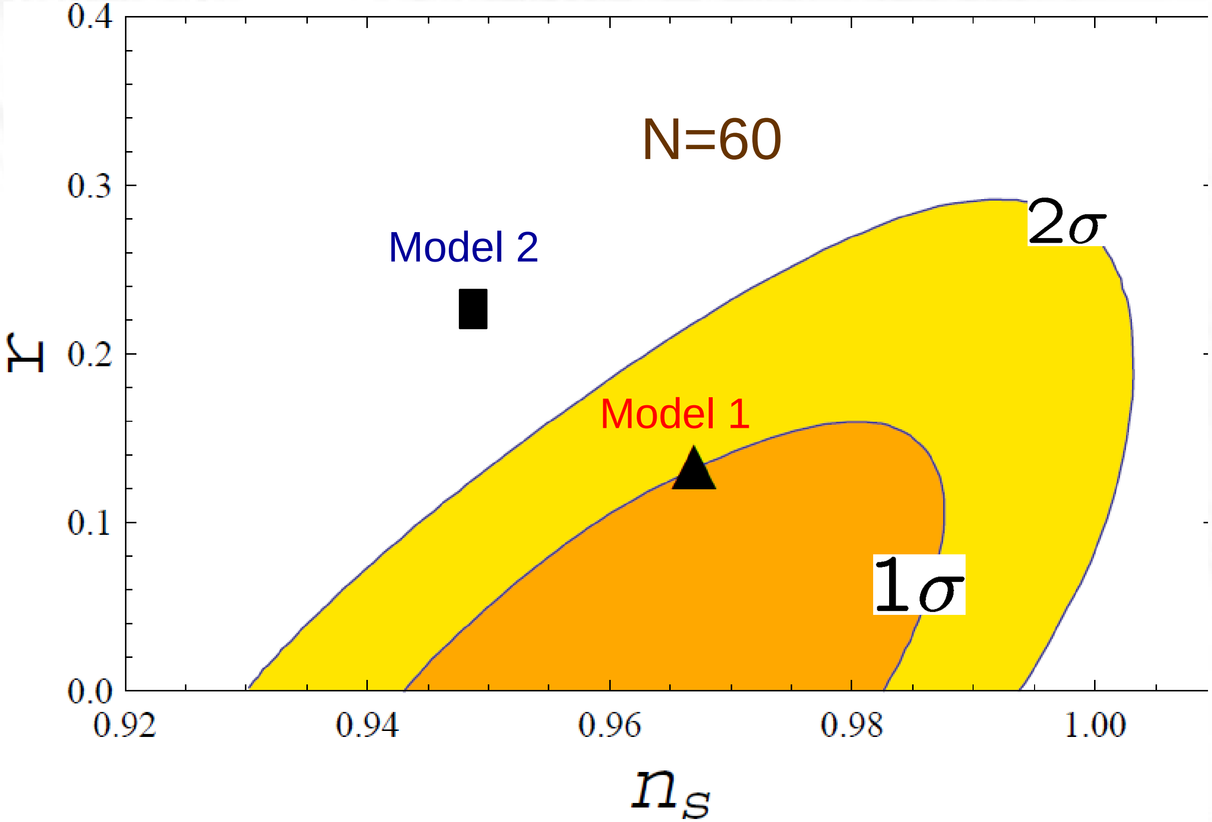

$$V(\phi) = m^2|\phi|^2 + (1/4)\lambda|\phi|^4$$

# Example models

Model 1: $V = \frac{1}{2}m^2\phi^2$

Model 2: $V = \frac{\lambda}{4!}\phi^4$

We calculate the slow-roll parameters for each model and find predictions



# Warm inflation general framework

$$H^2 = \frac{1}{3M_p^2}(\rho_\phi + \rho_\gamma)$$

$$\Gamma(T,\phi) = a\,\frac{T^m}{\phi^{m-1}}$$

$$\dot{\rho}_\gamma + 4H\rho_\gamma = \Gamma\dot{\phi}^2$$

$$\rho_\gamma = \frac{g_*\pi^2 T^4}{30}$$

$$\dot{\rho_\phi} + 3\,H\,(\rho_\phi + P_\phi) = -\Gamma\,\dot{\phi}^2$$

$$R \equiv \frac{\Gamma}{3H}$$

condition for warm inflation $T > H$

# Warm inflation in slow-roll

$$H^2 \simeq \frac{V}{3M_p^2}$$

$$T = \left[\frac{\Gamma V_{,\phi}^2}{36 C_\gamma H^3 (1+R)^2}\right]^{1/4}$$

$$3\,H\,(1+R)\dot{\phi} \simeq -V_{,\phi}$$

$$C_\gamma = \frac{g_* \pi^2}{30}$$

$$4H\rho_\gamma \simeq \Gamma\,\dot{\phi}^2$$

slow-roll conditions

$$\epsilon \ll 1+R, \;\; \eta \ll 1+R \qquad \sigma = M_p^2 \left(\frac{V_{,\phi}}{\phi V}\right) \ll 1+R$$

# Perturbations-Observables

$$\mathcal{P}_{\mathcal{R}}{}^{1/2} \simeq \left(\frac{H}{2\pi}\right)\left(\frac{3H^2}{V_\phi}\right)(1+R)^{5/4}\left(\frac{T}{H}\right)^{1/2}$$ power spectrum

$$n_s = 1 + \frac{d\mathcal{P}_{\mathcal{R}}}{d\ln k} \simeq 1 + \frac{1}{1+R}\left[-(2-5A_R)\epsilon - 3A_R\eta + (2+4A_R)\sigma\right]$$

scalar index

$$r \simeq \left(\frac{H}{T}\right)\frac{16\epsilon}{(1+R)^{5/2}}$$ tensor-to-scalar ratio

# Specify inflaton decay rate and potential

$$V(\phi) = \frac{\lambda\phi^4}{4}$$

$$\Gamma(T) = aT \quad (m = 1)$$

Weak dissipative regime

$$R \ll 1$$

Strong dissipative regime

$$R \gg 1$$

# A. Weak regime ( $R \ll 1$ )

$$T \simeq \left( \frac{aV_{,\phi}^2}{36C_\gamma H^3} \right)^{1/3}$$

end of inflation

$$\eta = 1$$

$$\phi_{end} = 2\sqrt{3}M_p$$

number of e-folds

$$N = \int_{t_*}^{t_{end}} H dt \simeq \frac{1}{4}\left(\frac{\phi_*}{M_p}\right)^2$$

$$\phi_* \gg \phi_{end}$$

# Weak regime continued

$$n_s \simeq 1 - 2\epsilon + 2\sigma$$

$$\mathcal{P}_{\mathcal{R}}{}^{1/2} \simeq \left[\left(\frac{H}{2\pi}\right)\left(\frac{3H^2}{V_{,\phi}}\right)\left(\frac{H}{T}\right)\right]^{1/2}$$

$$\epsilon = \frac{8M_p^2}{\phi^2}, \quad \sigma = \frac{4M_p^2}{\phi^2}$$

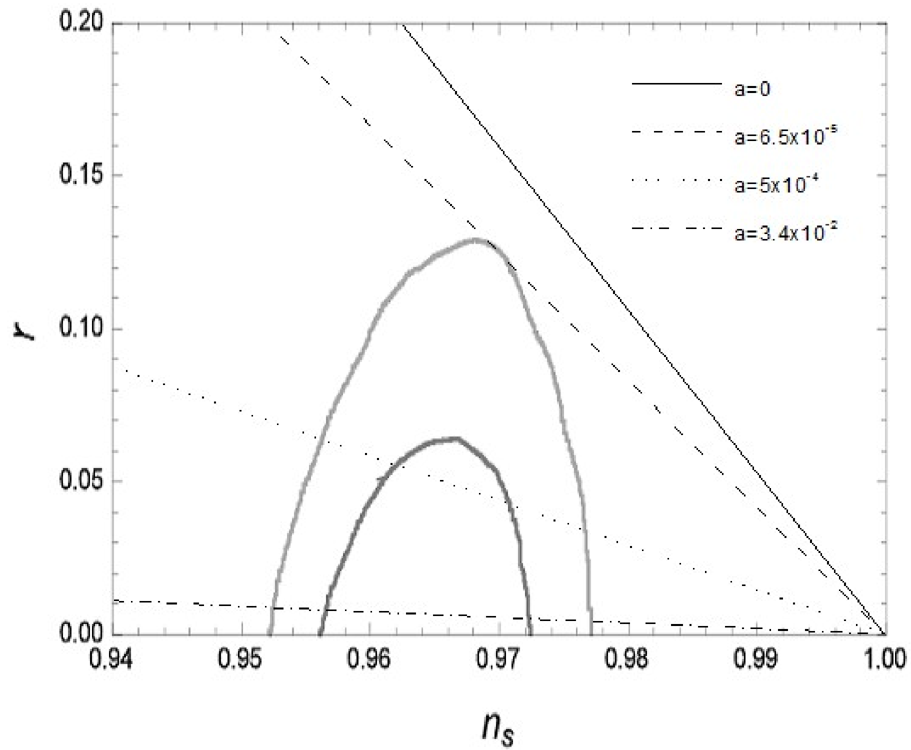
$$\mathcal{P}_{\mathcal{R}}{}^{1/2} \simeq \left(\frac{\lambda\sqrt{a}N^3}{6\sqrt{70}\pi^3}\right)^{1/3} \sim 10^{-5}$$

$$n_s = 1 - \frac{1}{N}$$

$$r \simeq \left(\frac{H}{T}\right) 16\epsilon$$

$$r = \frac{4\sqrt{14}}{625\sqrt{5}\, a^{1/2}}(1 - n_s)$$

# Numerics I: The r-ns plane for weak regime






$$10^{-15} < \lambda < 10^{-13} \qquad 6.5\times10^{-5} < a < 3.4\times10^{-2}$$

# B. Strong regime $(R \gg 1)$

$$T \simeq \left( \frac{V_{,\phi}^2}{36 a C_\gamma H} \right)^{1/5}$$

end of inflation

$$\eta = R$$

$$\phi_{\rm end} = \frac{(6^7 35)^{1/4}}{a} \lambda^{1/4} M_p$$

number of e-folds

$$N = \simeq \frac{1}{8} \left( \frac{a\, 5^4}{7\, \lambda 6^2} \right)^{1/5} \left( \frac{\phi_*}{M_p} \right)^{4/5}$$

# Strong regime continued

$$\mathcal{P}_{\mathcal{R}}^{1/2} \simeq \left(\frac{H}{2\pi}\right)\left(\frac{3H^2}{V_\phi}\right)\left(\frac{T}{H}\right)^{1/2} R^{5/4}$$

$$n_s \simeq 1 + \frac{1}{7R}(-3\eta + 18\sigma - 9\epsilon)$$

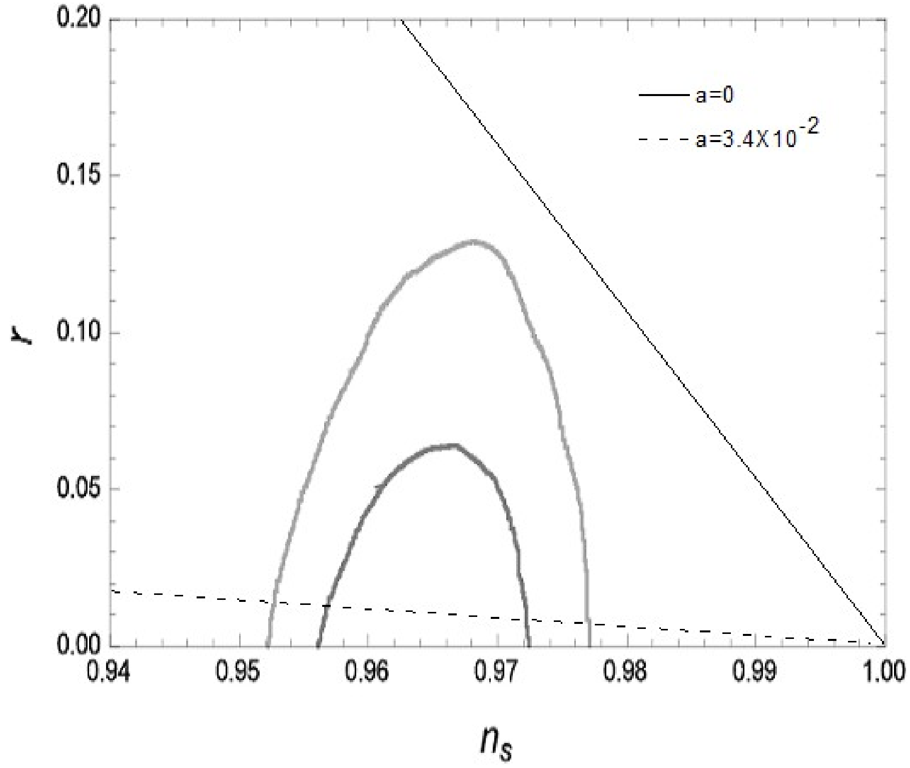
$$\mathcal{P}_{\mathcal{R}}^{1/2} \simeq \left[\frac{4N^3\lambda}{125\pi^{8/3}}\left(\frac{2}{315}\right)^{1/3}\right]^{3/8} \sim 10^{-5}$$

$$n_s = 1 - \frac{45}{28N}$$

$$r \simeq \left(\frac{H}{T}\right)\frac{16\,\epsilon}{R^{5/2}}$$

$$r \simeq 8.5 \times 10^{-9}\,\frac{\pi^{10/3}}{a^4}(1 - n_s)$$

# Numerics II: The r-ns plane for strong regime






$a > 3.4 \times 10^{-2}$ $\lambda \sim 10^{-15}$

# Conclusions

- Inflation is the standard paradigm of the early universe
- Natural candidate for inflaton: Higgs boson in particle physics
- Simplest inflaton potential (quartic) is ruled out in standard inflation
- Warm inflation: Radiation coupled to inflaton, alternative to inflation, no reheating needed
- Quartic potential in warm inflation is viable