Gravitational Effect on Inflaton Decay

Ryusuke Jinno (Univ. of Tokyo)
in collaboration with
Yohei Ema, Kyohei Mukaida and Kazunori Nakayama

Based on
JCAP 1505(2015) (arXiv:1502.02475)
arXiv:1505.04670

COSMO 2015, Warsaw
September 11th, 2015

Inflation & Reheating

- Inflation

  - solves flatness / homogeneity / monopole problems

  - predicts small density fluctuations → seeds of galaxies

- Reheating

  - Vaccum energy of the inflaton must be converted to radiation at the end of inflation

  (and the radiation must be thermalized before BBN)

Gravitational particle production

- Inflaton must be coupled at least gravitatonally to other particles.

  Other particles are produced through the oscillation of the scale factor,

  though such interactions are Planck-suppressed.

  e.g. $$S \sim \int \sqrt{-g}\left[\frac{M_P^2}{2}R - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi) - \frac{1}{2}g^{\mu\nu}\partial_\mu\chi\partial_\nu\chi\right]$$

- Initially studied in the context of Starobinsky type inflation

  [A.A.Starobinsky "Quantum Gravity"(Plemun Press, New York, 1982),
  A. Vilenkin, PRD32(1985) / M.B.Mijic et al., PRD34(1986)...]

- In this talk I focus on gravitational particle production in

  1. R (Einstein gravity) 2. $f(\phi)R$

Gravitational particle production

- Motivations

R (Einstein gravity)

- Standard theory of gravity

- Even in this simplest example, gravitational effects give nonnegligible consequences in the present universe

$f(\phi)R$

- Naively, efficient particle production (especially graviton prod.) is expected

ϕ oscillation → R oscillation → H oscillation → particle production

↘ graviton production

- However, graviton production seems inefficient in the Einstein frame

- Prediction differs among the literature [G. Segre et al., PRL62 (1989)]
[Y. Watanabe, E. Komatsu, PRD75(2007)061301]

Gravitational effect in Einstein gravity

- Setup : Einstein gravity + minimal scalar + matter

$$S = \int d^4x \sqrt{-g}\left[\frac{1}{2}M_P^2 R - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V\right] + S_M$$

$$a^3\left[-3H^2 + \frac{1}{2}\dot{\phi}^2 - \frac{\lambda}{n}\phi^n\right]$$ (especially we focus on n=2)

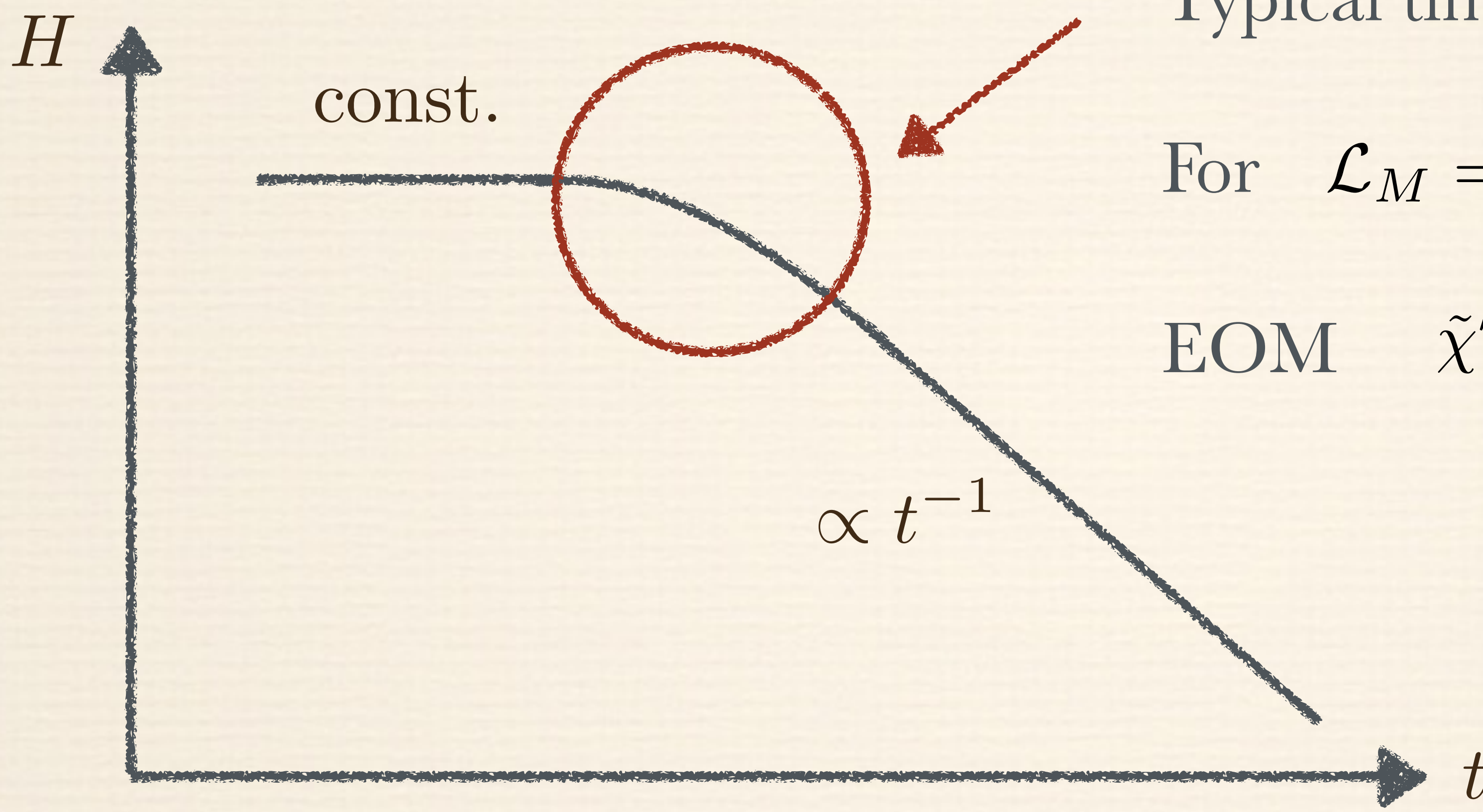
- Background EOM

  - ϕ EOM : $\ddot{\phi} + 3H\dot{\phi} + V' = 0$
  - Friedmann eq. : $3M_P^2 H^2 = \frac{1}{2}\dot{\phi}^2 + V$

Gravitational effect in Einstein gravity

- Averaged evolution of the Hubble parameter

[L.H. Ford, PRD35 (1987) 2955]



Typical timescale $\sim H^{-1}$

For $\mathcal{L}_M = -\frac{1}{2}(\partial\chi)^2$,

EOM $\tilde{\chi}'' + 2\mathcal{H}\tilde{\chi}' + \left(k^2 - \frac{a''}{a}\right)\tilde{\chi} = 0$

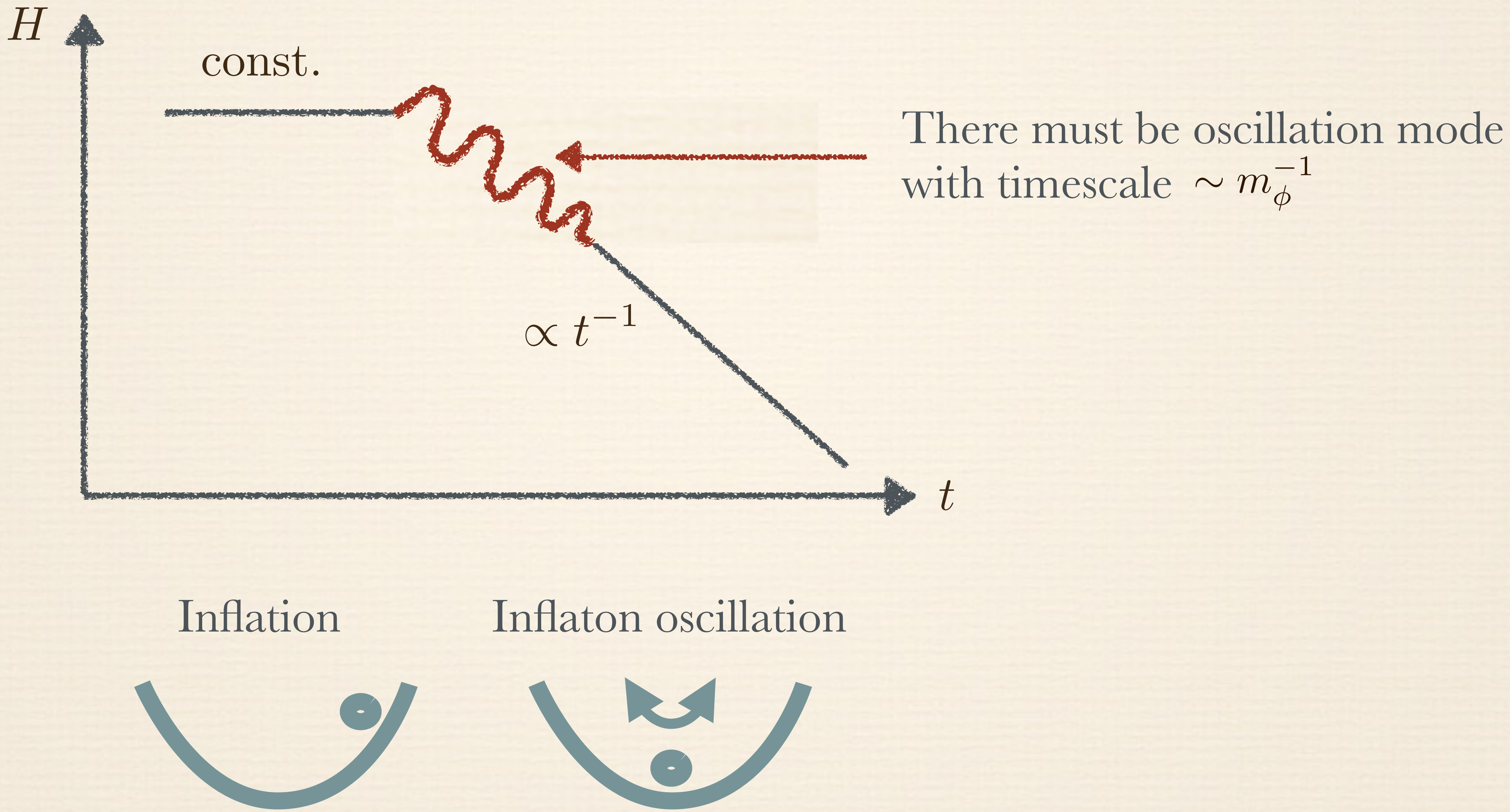
Time-dependent mass
↓
Particle production

$(\tilde{\chi} = a^{-1}\chi,\ d\tau = a^{-1}dt,\ \mathcal{H} = a'/a)$

Inflation Inflaton oscillation

Gravitational effect in Einstein gravity

- Non-averaged evolution of the Hubble parameter



Gravitational effect in Einstein gravity

- Extraction of the oscillation mode

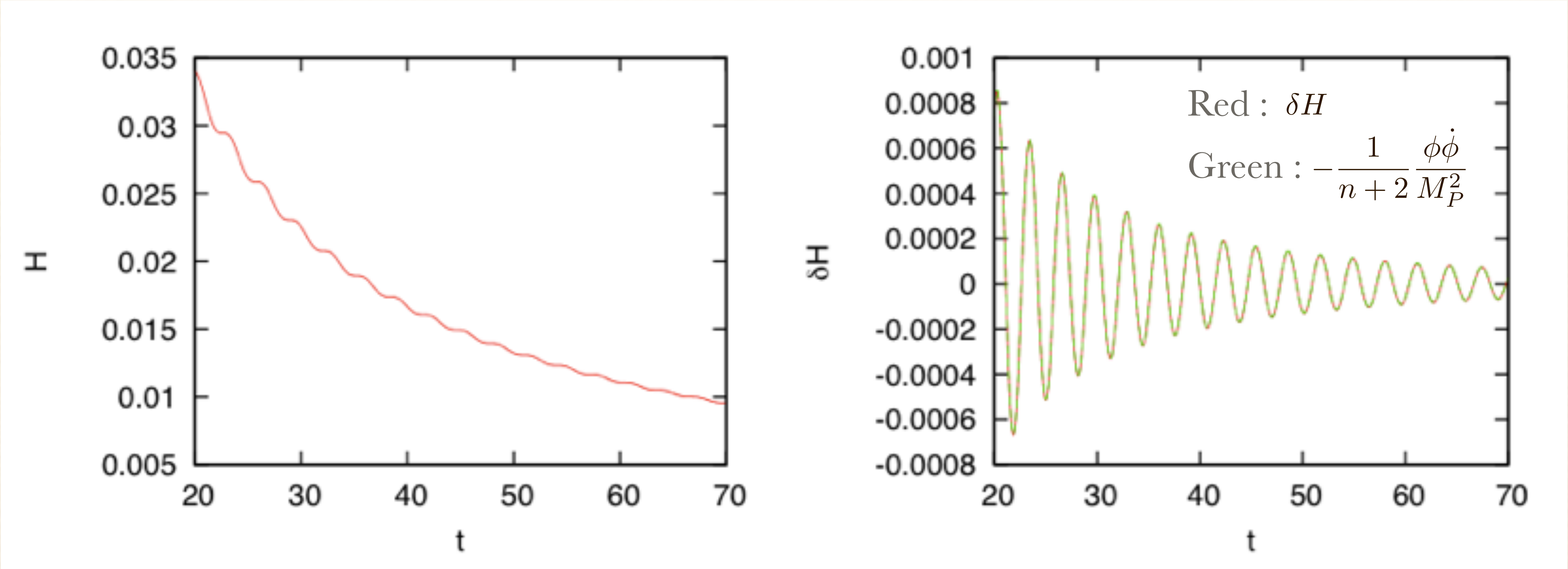
[Y. Ema, RJ, K. Mukaida, K. Nakayama, JCAP1505(2015) / 1502.02475]

See also [B. A. Bassett et al., Phys.Rev. D58 (1998) 021302, Phys.Rev. D60 (1999) 049902]

- ϕ EOM : $\ddot{\phi} + 3H\dot{\phi} + V' = 0$

  Friedmann eq. : $3M_P^2 H^2 = \frac{1}{2}\dot{\phi}^2 + V$

- Decomposition $H = \bar{H} + \delta H \quad \rightarrow \quad \delta H \simeq -\frac{1}{n+2}\frac{\phi\dot{\phi}}{M_P^2}$



Gravitational effect in Einstein gravity

- Extraction of the oscillation mode

[Y. Ema, RJ, K. Mukaida, K. Nakayama, JCAP1505(2015) / 1502.02475]

See also [B. A. Bassett et al., Phys.Rev. D58 (1998) 021302, Phys.Rev. D60 (1999) 049902]

- ϕ EOM : $\ddot{\phi} + 3H\dot{\phi} + V' = 0$

  Friedmann eq. : $3M_P^2 H^2 = \frac{1}{2}\dot{\phi}^2 + V$

- Decomposition $H = \bar{H} + \delta H \quad \rightarrow \quad \delta H \simeq -\frac{1}{n+2}\frac{\phi\dot{\phi}}{M_P^2}$

- $\delta H \simeq -\frac{1}{n+2}\frac{\phi\dot{\phi}}{M_P^2} \quad \rightarrow \quad a \simeq \langle a \rangle \left(1 - \frac{1}{2(n+2)}\frac{\phi^2}{M_P^2}\right)$

- Matter action (for canonical scalar field)

$$S_M = \int d\tau d^3x \, \langle a(t) \rangle^2 \left(1 - \frac{1}{n+2}\frac{\phi^2}{M_P^2}\right) \frac{1}{2}\left[\chi'^2 - (\partial_i \chi)^2\right]$$

"Gravitational annihilation" $\Gamma \sim \frac{\Phi^2 m_\phi^3}{M_P^4}$ (Φ : amplitude of ϕ)

Gravitational effect in Einstein gravity

- Cosmological consequences [Y. Ema, RJ, K. Mukaida, K. Nakayama, JCAP1505(2015) / 1502.02475]

  - Massive scalar production :

$$\frac{\rho_\chi}{s} \simeq 8 \times 10^{-9} \left(\frac{m_\chi}{10^6\mathrm{GeV}}\right)\left(\frac{T_R}{10^{10}\mathrm{GeV}}\right)\left(\frac{H_{\mathrm{inf}}}{10^{14}\mathrm{GeV}}\right) \ [\mathrm{GeV}]$$

Can be DM / Lower bound on χ abundance

  - Massless scalar & graviton production

$$\frac{\rho_\chi}{\rho_{\mathrm{rad}}} \simeq 3 \times 10^{-19} \left(\frac{m_\phi}{10^{13}\mathrm{GeV}}\right)\left(\frac{T_R}{10^{10}\mathrm{GeV}}\right)^{4/3}\left(\frac{H_{\mathrm{inf}}}{10^{14}\mathrm{GeV}}\right)$$

Negligible contribution to present (dark) radiation

Gravitational effect in $f(\phi)R$ theory

- Setup

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{2}f(\phi)R - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V\right] + S_M$$

Since we need Einstein gravity after ϕ settles down, we require

$$f(\phi) = M_P^2\left(1 + f_1\frac{\phi}{M_P} + \ldots\right)$$

- Extraction of the oscillation mode

"Adiabatic invariant" $\mathcal{L}_H = \frac{\partial \mathcal{L}}{\partial H}$ is useful in extracting the oscillation mode

($\rightarrow$ next slide)

Gravitational effect in $f(\phi)R$ theory

- Adiabatic invariant [Y. Ema, RJ, K. Mukaida, K. Nakayama, 1505.04670]

  - In models of extended gravity (e.g. Generalized Galileon theories),

    various quentities (denoted by Q here / including the Hubble parameter)

    oscillate with the timescale of the inflaton oscillation : $\dot{Q} \sim m_\phi Q$

  - However, there exists "adiabatic invariant" $\dot{Q} \sim HQ$.

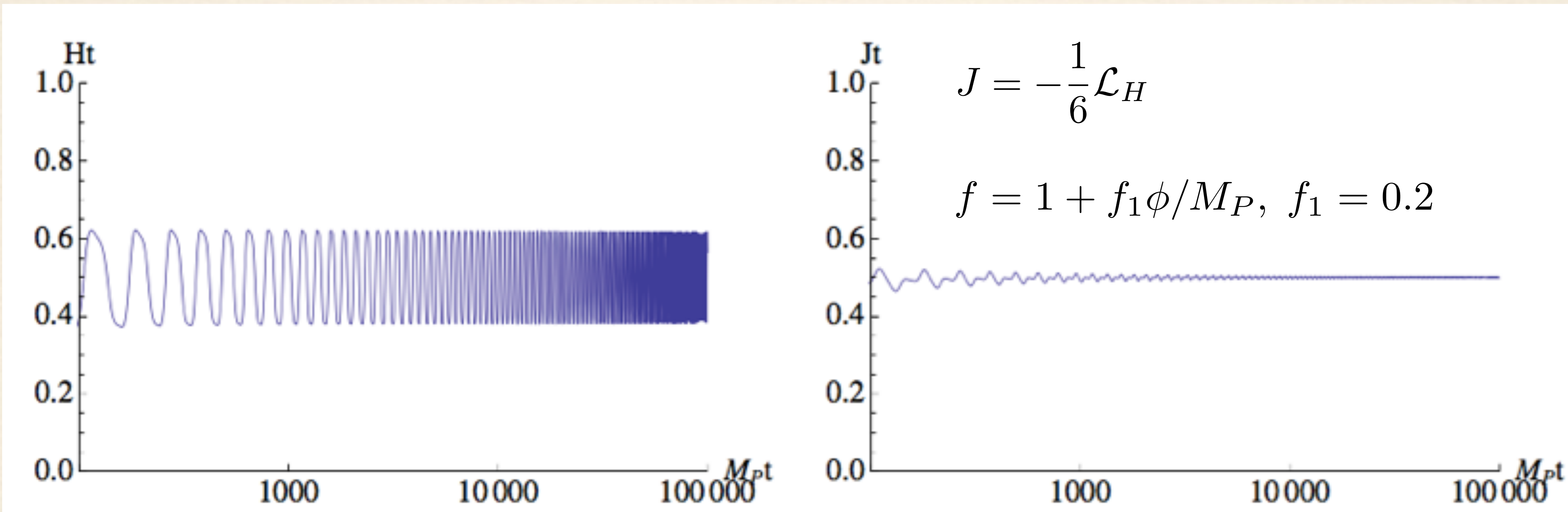
    For a Lagrangian $\mathcal{L} = \mathcal{L}(H, \phi, \dot{\phi})$, the invariant is given by $\mathcal{L}_H = \dfrac{\partial \mathcal{L}}{\partial H}$

  - Example : $\mathcal{L} = \frac{1}{2} f(\phi) R - \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V \ \sim \ -3fH^2 - 3f' \dot{\phi} H + \frac{1}{2} \dot{\phi}^2 - V$

    Then $\mathcal{L}_H = -6fH - 3f' \dot{\phi}$

Gravitational effect in $f(\phi)R$ theory

- Adiabatic invariant



For a Lagrangian $\mathcal{L} = \mathcal{L}(H, \phi, \dot{\phi})$, the invariant is given by $\mathcal{L}_H = \dfrac{\partial \mathcal{L}}{\partial H}$

- Example : $\mathcal{L} = \frac{1}{2}f(\phi)R - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V \ \sim \ -3fH^2 - 3f'\dot{\phi}H + \frac{1}{2}\dot{\phi}^2 - V$

Then $\mathcal{L}_H = -6fH - 3f'\dot{\phi}$

Gravitational effect in $f(\phi)R$ theory

- Extraction of the oscillation mode (cont.)

  - The adiabatic invariant has almost no oscillation

  [Ema, RJ, Mukaida, Nakayama, JCAP1505(2015) / 1502.02475]

$$\mathcal{L}_H = -6fH - 3f'\dot{\phi} \simeq \text{no oscillation} \qquad (\, f = 1 + f_1\phi/M_P \,)$$

Decomposing $H = \bar{H} + \delta H$ we have

$$\delta H \simeq -\frac{f_1}{2M_P}\dot{\phi} \quad \rightarrow \quad a \simeq \langle a(t) \rangle \left(1 - \frac{f_1}{2}\frac{\phi}{M_P} \right)$$

  - We can derive the inflaton-matter coupling through gravity

For $S_M = \int \sqrt{-g} d^4x \left[-\frac{1}{2}h(\phi)g^{\mu\nu}\partial_\mu\chi\partial_\nu\chi \right] \qquad (h = 1 + h_1\phi/M_P)$

we have $S_M = \int d\tau d^3x \, \langle a(t) \rangle^2 \left[1 + (h_1 - f_1)\frac{\phi}{M_P} \right] \frac{1}{2} \left[\chi'^2 - (\partial_i\chi)^2 \right]$

Gravitational effect in $f(\phi)R$ theory

- Extraction of the oscillation mode (cont.)

  - The adiabatic invariant has almost no oscillation

[Ema, RJ, Mukaida, Nakayama, JCAP1505(2015) / 1502.02475]

$$\mathcal{L}_H = -6fH - 3f'\dot{\phi} \simeq \text{no oscillation} \qquad (\, f = 1 + f_1\phi/M_P \,)$$

Decomposing $H = \bar{H} + \delta H$ we have

$$\delta H \simeq -\frac{f_1}{2M_P}\dot{\phi} \quad \rightarrow \quad a \simeq \langle a(t)\rangle \left(1 - \frac{f_1}{2}\frac{\phi}{M_P}\right)$$

  - We can derive the inflaton-matter coupling through gravity

For $S_M = \int \sqrt{-g}d^4x \left[-\frac{1}{2}h(\phi)g^{\mu\nu}\partial_\mu\chi\partial_\nu\chi\right]$

"Gravitational decay"

we have $S_M = \int d\tau d^3x \, \langle a(t)\rangle^2 \left[1 + (h_1 - f_1)\frac{\phi}{M_P}\right]\frac{1}{2}\left[\chi'^2 - (\partial_i\chi)^2\right]$

Gravitational effect in $f(\phi)R$ theory

- Cosmological consequences

  - Scalar $(m_\chi \ll m_\phi/2)$

$$S_M = \int d\tau d^3x \, \langle a(t)\rangle^2 \left[1 + (h_1 - f_1)\frac{\phi}{M_P}\right] \frac{1}{2}\left[\chi'^2 - (\partial_i\chi)^2\right]$$

$$\Gamma(\phi \to \chi\chi) \sim (h_1 - f_1)^2 \frac{m_\phi^3}{M_P^2}$$

[see also Y. Watanabe, E. Komatsu PRD75(2007)061301]

  - Graviton

$$S_h \simeq \int d\tau d^3x \, \langle a(t)\rangle^2 f(\phi)\frac{M_P^2}{8}\left[h_{ij}'^2 - (\partial_k h_{ij})^2\right]$$

(corresponds to $h = f$ in scalar case)

$$\Gamma(\phi \to hh) \sim 0$$

(except for the decay rate already existed in Einstein case)

[see also Y. Watanabe, E. Komatsu PRD75(2007)061301]

Summary

- In Einstein gravity

  - Gravitational particle production is nonnegligible

    (Produced massive scalar particles can be DM)

- In $f(\phi)R$ theory

  - Scalar production can be efficient (to complete reheating)

  - However, graviton production is inefficient

- In addition

  - "Adiabatic invariant" may be useful in extracting the oscillation mode

    of the Hubble parameter and in estimating particle production

Backup

Gravitational effect in Einstein gravity

- Derivation of $\delta H \simeq -\frac{1}{n+2}\frac{\phi\dot{\phi}}{M_P^2}$

$$\dot{H} = -\frac{\dot{\phi}^2}{2M_P^2}$$

$$\delta\dot{H} = -\frac{\dot{\phi}^2}{2M_P^2} - \dot{\bar{H}} = -\frac{\dot{\phi}^2}{2M_P^2} + \frac{3n}{n+2}\bar{H}^2$$

$$\simeq -\frac{\dot{\phi}^2}{2M_P^2} + \frac{3n}{n+2}(H^2 - 2\bar{H}\delta H)$$

$$\simeq -\frac{\dot{\phi}^2}{2M_P^2} + \frac{3n}{n+2}\frac{1}{3M_P^2}\left(\frac{1}{2}\dot{\phi}^2 + V\right) = \frac{3n}{n+2}\frac{1}{3M_P^2}\left(-\frac{1}{n}\dot{\phi}^2 + V\right)$$

$$= \frac{3n}{n+2}\frac{1}{3M_P^2}\left(-\frac{1}{n}\dot{\phi}^2 + \frac{V'\phi}{n}\right) \simeq \frac{3n}{n+2}\frac{1}{3M_P^2}\left(-\frac{1}{n}\dot{\phi}^2 + \frac{\phi}{n}(-\ddot{\phi})\right)$$

$$= -\frac{1}{n+2}\left(\frac{\phi\dot{\phi}}{M_P^2}\right)^{\cdot}$$

Decay rate

- Rough estimation of the decay rate

  - Assume daughter mass oscillates with ratio q compared to parent mass

$$|\Delta m_\chi^2| = q m_\phi^2$$

  - This causes interaction term in Lagrangian

$$\mathcal{L}_{\rm int} \sim \Delta m_\chi^2 \chi_c^2 \sim q m_\phi^2 \left(\frac{\phi_c}{\Phi_c}\right)^m \chi_c^2 \sim q m_\phi^2 \frac{\phi_c}{\Phi_c} \chi_c^2$$

c : canonically normalized

  - Then, decay rate is

$$\Gamma \sim \frac{q^2 m_\phi^3}{\Phi_c^2}$$

Decay rate

- Rough estimation of the decay rate

1. $|\Delta m_\chi^2| = q m_\phi^2$
2. $\Gamma \sim \dfrac{q^2 m_\phi^3}{\Phi_c^2}$

  - Example

$$\mathcal{L}_{\rm int} = -\frac{1}{2} F^2(\phi)(\partial\chi)^2$$

  - Canonically normalizing $\chi_c = F(\phi)\chi$,

$$\partial^2\chi_c - \frac{\partial^2 F}{F}\chi_c = 0 \quad \rightarrow \quad \Delta m_\chi^2 = \frac{F''}{F}$$

  - Decay rate

$$\Gamma \sim \left(\frac{F''}{F}\frac{1}{m_\phi}\right)^2 \frac{m_\phi^3}{\Phi_c^2}$$

Rough sketch of the proof of adiabaticity

- Action

$$S_G = \int d^4x\, a^3 \mathcal{L}(H, \dot{\phi}, \phi)$$

- EOM

$$(\mathcal{L}_{\dot{\phi}})^{\cdot} + 3H\mathcal{L}_{\dot{\phi}} - \mathcal{L}_{\phi} = 0,$$: Inflaton EOM

$$\mathcal{L} - \dot{\phi}\mathcal{L}_{\dot{\phi}} - H\mathcal{L}_H = 0,$$: Friedmann eq.

$$(\mathcal{L}_H)^{\cdot} + 3H\mathcal{L}_H - 3\mathcal{L} = 0,$$: 2nd. Friedmann eq.

If $H\mathcal{L}_H \sim \mathcal{L}$ holds (and in fact this holds in most cases),

$\mathcal{L}_H$ becomes an adiabatic invariant since $(\mathcal{L}_H)^{\cdot} \sim H\mathcal{L}_H$

Figures

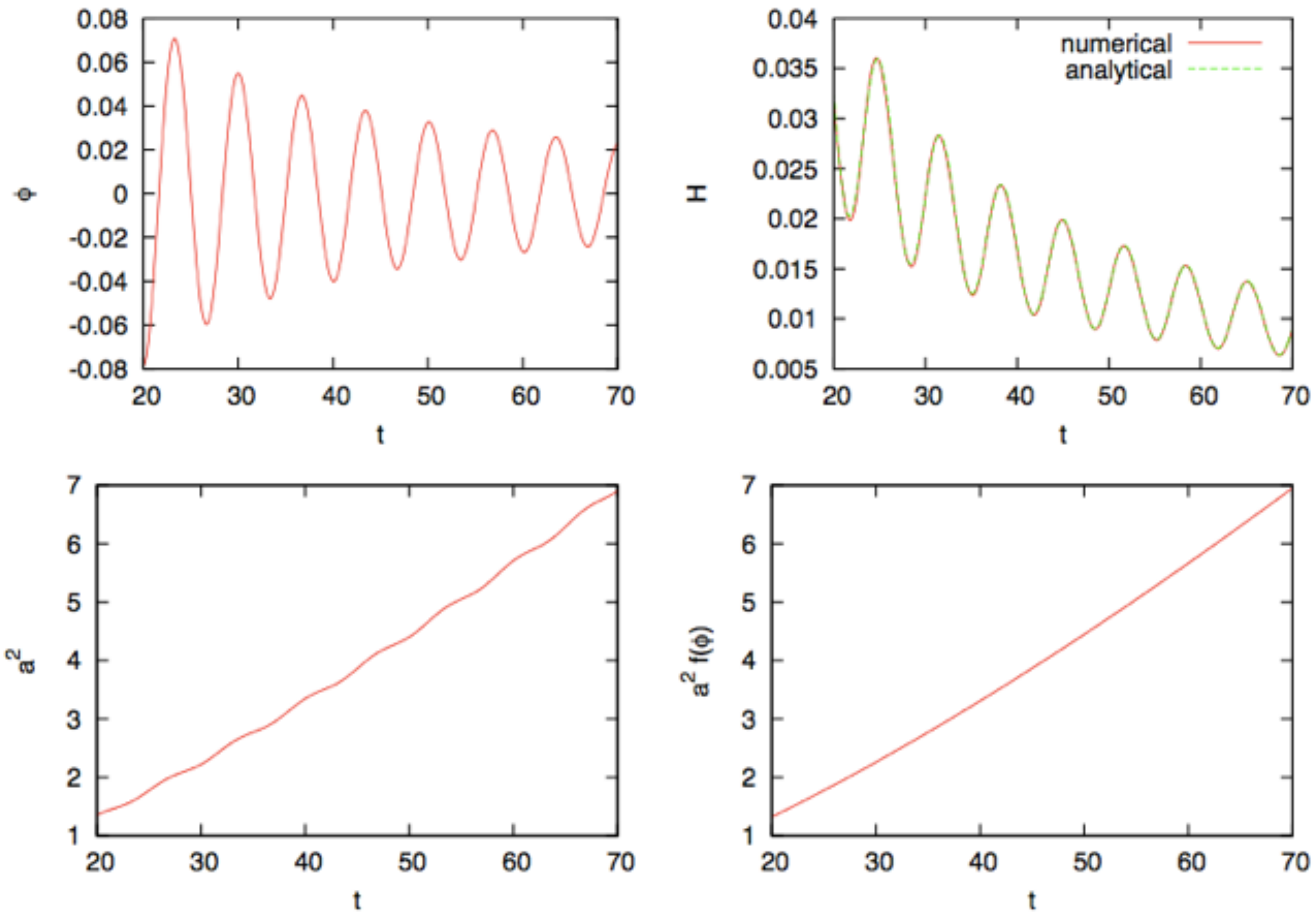


Figure 2: Time evolution of $\phi(t)$ (top left), H (top right), a^2 (bottom left) and $a^2 f(\phi)$ (bottom right). We have compared numerical results and approximate analytic formula (3.10) for H. We have taken $c_1 = 0.3$ and $m_\phi = 1$ in Planck unit.