Non-local bias in the halo bispectrum with primordial non-Gaussianity

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in collaboration with D. Wands, A. J. Ross, G. Tasinato

based on JCAP $07(2015)004$ (ArXiv:1504.00324)

Overview: Inflation

- Inflation:
  - Phase of accelerated expansion
  - Different models make different predictions for the statistics of primordial potential Φ

$$\langle \Phi(\mathbf{k}_1) \dots \Phi(\mathbf{k}_n) \rangle \neq 0 \quad \text{for} \quad n \geq 2$$

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- In this talk: focus on local type models, whose perturbations are sourced by a single, Gaussian field φ

$$\Phi(\mathbf{x}) = \varphi(\mathbf{x}) + f_{\mathrm{NL}} \left(\varphi^2(\mathbf{x}) + \langle \varphi^2(\mathbf{x}) \rangle \right) + \ldots$$

e.g, inflaton, curvaton

Overview: Constraining $f_{\rm NL}$

- Cosmic Microwave Background constraints, from bispectrum of the 2D temperature anisotropies map at $z = 1100$

$$\text{WMAP9}: \quad -3 < f_{\rm NL} < 77 \quad \text{at} \quad 95\%\,\text{CL}$$
$$\text{Planck2015}: \; -9.2 < f_{\rm NL} < 10.8 \quad \text{at} \quad 95\%\,\text{CL}$$

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  - Biased objects respect to the underlying density field
  - Non-linear evolution: can a primordial signal survive?

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$$\overbrace{\Phi(\mathbf{x},0)}^{\text{Inflationary perturbations}} \xrightarrow[\text{N-body simulations}]{\text{Perturbation theory}} \overbrace{\delta(\mathbf{x},\tau)}^{\text{Density contrast}}$$

$$\delta(\mathbf{x},\tau) \;\big\downarrow\; \text{halo biasing}$$

$$\delta_h(\mathbf{x},\tau)$$

$$\text{galaxy biasing} \;\big\downarrow$$

$$\delta_g(\mathbf{x},\tau) \xrightarrow{\text{Observations}} \delta_g(\theta,z)$$

from Assassi et al. arXiv:1402.5916

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Density contrast

$\delta(\mathbf{x},\tau)$ ↓ Scale-dependent halo bias

$\delta_h(\mathbf{x},\tau)$

galaxy biasing ↓

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The mass function acquires a dependence on the local value $\sigma_{\rm eff}$ …

$$n_h = n_h(\delta_L, \sigma_{\rm eff}\,; M, z) \Longrightarrow \delta_h = \frac{n_h - \bar{n_h}}{\bar{n_h}} = \left(b_{10} + \frac{b_{01}}{\alpha(k,z)}\right)\delta_L$$

…producing a large correction to the halo overdensity on large scales

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  - Constraint $\sigma_{f_{\rm NL}} \sim 1$ expected, in principle...

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  - ...but, in practice, complications exist!

    (Redshift space distortions, complicated mask geometry, non-linearities in halo populations, ...)

Aim of our work:

- Improve the bias model, $\delta_h = \mathcal{F}(\delta)$
- Focus on the second-order, non-local and non-Gaussian effects
- Investigate the signature in the tree-level bispectrum

Local Lagrangian Biasing

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- Local relation between the initial number density of objects and the initial density field

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- Given that the number of objects is conserved in a given volume element,

$$1 + \delta_h^{\mathrm{E}}(\mathbf{x}, z) = \left[1 + \delta(\mathbf{x}, z)\right] \left[1 + \delta_h^{\mathrm{L}}(\mathbf{q}, z)\right]$$

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Local Eulerian model but not accurate enough

[Roth & Porciani (ArXiv:1101.1520), Chan et al. (ArXiv:1201.3614), Baldauf et al. (ArXiv:1201.4827), Pollack et al. (ArXiv:1309.0504), Saito et al. (ArXiv:1405.1447),...]

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Tidal term introduces non locality in the Eulerian halo overdensity

[Catelan et al. (astro-ph/0005544), McDonald & Roy (ArXiv:0902.0991), Baldauf et al. (ArXiv:1201.4827)]

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Tidal and non-Gaussian shift terms introduce non locality in the Eulerian bivariate model

Halo overdensity

Fourier transform of halo overdensity in Eulerian frame

$$\delta_h^{\mathrm{E}}(\mathbf{k}) = b_{10}\delta + b_{01}\varphi + b_{20}\delta * \delta + b_{11}\delta * \varphi + b_{02}\varphi * \varphi - \frac{2}{7}b_{10}^{\mathrm{L}}s^2 - b_{01}^{\mathrm{L}}n^2$$

where the density contrast, tidal and non-Gaussian shift terms are

$$\delta(\mathbf{k}) = \delta_{\mathrm{lin}}(\mathbf{k}) + \int \frac{d\mathbf{q}}{(2\pi)^3}\left[\mathcal{F}_2(\mathbf{q},\mathbf{k}-\mathbf{q}) + f_{\mathrm{NL}}\frac{\alpha(k)}{\alpha(q)\alpha(|\mathbf{k}-\mathbf{q}|)}\right]\delta_{\mathrm{lin}}(\mathbf{k})\delta_{\mathrm{lin}}(\mathbf{k}-\mathbf{q})$$

$$\mathcal{F}_2(\mathbf{k}_1,\mathbf{k}_2) = \frac{5}{7} + \frac{1}{2}\frac{\mathbf{k}_1\cdot\mathbf{k}_2}{k_1 k_2}\left(\frac{k_1}{k_2}+\frac{k_2}{k_1}\right) + \frac{2}{7}\frac{(\mathbf{k}_1\cdot\mathbf{k}_2)^2}{k_1^2 k_2^2}$$

$$s^2(\mathbf{k}) = \int \frac{d\mathbf{q}}{(2\pi)^3}\mathcal{S}_2(\mathbf{q},\mathbf{k}-\mathbf{q})\delta_{\mathrm{lin}}(\mathbf{q})\delta_{\mathrm{lin}}(\mathbf{k}-\mathbf{q})$$

$$\mathcal{S}_2(\mathbf{k}_1,\mathbf{k}_2) = \frac{(\mathbf{k}_1\cdot\mathbf{k}_2)^2}{k_1^2 k_2^2} - \frac{1}{3}$$

$$n^2(\mathbf{k}) = 2\int \frac{d\mathbf{q}}{(2\pi)^3}\mathcal{N}_2(\mathbf{q},\mathbf{k}-\mathbf{q})\frac{\delta_{\mathrm{lin}}(\mathbf{q})\delta_{\mathrm{lin}}(\mathbf{k}-\mathbf{q})}{\alpha(|\mathbf{k}-\mathbf{q}|)}$$

$$\mathcal{N}_2(\mathbf{k}_1,\mathbf{k}_2) = \frac{\mathbf{k}_1\cdot\mathbf{k}_2}{2k_1^2}$$

Halo bispectrum

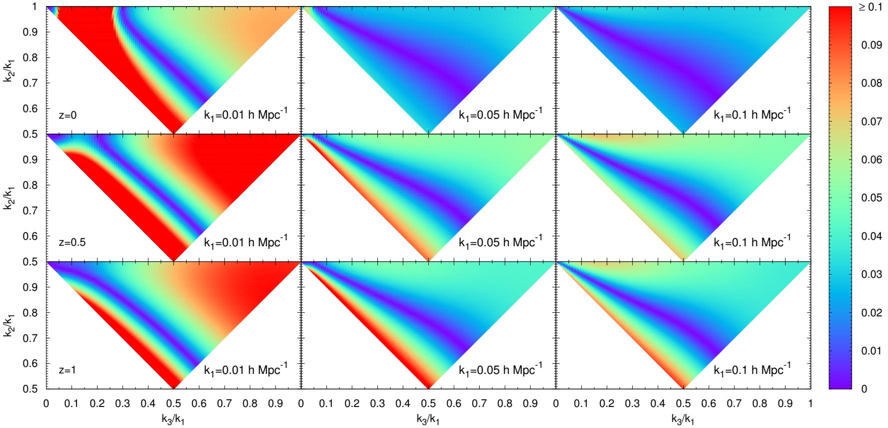
$$\langle \delta_h^E(\mathbf{k}_1)\delta_h^E(\mathbf{k}_2)\delta_h^E(\mathbf{k}_3)\rangle = (2\pi)^3\delta^D(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)B_{hhh}(\mathbf{k}_1,\mathbf{k}_2)$$

$$\begin{aligned}
B_{\mathsf{hhh}}(\mathbf{k}_1,\mathbf{k}_2) = {} & B_{\mathsf{hhh}}^{(\mathsf{A}\to\mathsf{L})}(\mathbf{k}_1,\mathbf{k}_2) \\
& -\frac{2}{7}b_{10}^2 b_{10}^{\rm L}\left(2P(k_1)P(k_2)\mathcal{S}_2(\mathbf{k}_1,\mathbf{k}_2)+2\ \text{cyc.}\right)_{\mathsf{M}} \\
& -\frac{2}{7}b_{10}b_{01}b_{10}^{\rm L}\left(2P(k_1)P(k_2)\left(\frac{1}{\alpha(k_1)}+\frac{1}{\alpha(k_2)}\right)\mathcal{S}_2(\mathbf{k}_1,\mathbf{k}_2)+2\ \text{cyc.}\right)_{\mathsf{N}} \\
& -\frac{2}{7}b_{01}^2 b_{10}^{\rm L}\left(2\frac{P(k_1)P(k_2)}{\alpha(k_1)\alpha(k_2)}\mathcal{S}_2(\mathbf{k}_1,\mathbf{k}_2)+2\ \text{cyc.}\right)_{\mathsf{O}} \\
& -b_{10}^2 b_{01}^{\rm L}\left(2P(k_1)P(k_2)\left(\frac{\mathcal{N}_2(\mathbf{k}_1,\mathbf{k}_2)}{\alpha(k_2)}+\frac{\mathcal{N}_2(\mathbf{k}_2,\mathbf{k}_1)}{\alpha(k_1)}\right)+2\ \text{cyc.}\right)_{\mathsf{P}} \\
& -b_{10}b_{01}b_{01}^{\rm L}\left(2P(k_1)P(k_2)\left(\frac{\mathcal{N}_2(\mathbf{k}_1,\mathbf{k}_2)}{\alpha(k_2)}+\frac{\mathcal{N}_2(\mathbf{k}_2,\mathbf{k}_1)}{\alpha(k_1)}\right)\left(\frac{1}{\alpha(k_1)}+\frac{1}{\alpha(k_2)}\right)+2\ \text{cyc.}\right)_{\mathsf{Q}} \\
& -b_{01}^2 b_{01}^{\rm L}\left(2\frac{P(k_1)P(k_2)}{\alpha(k_1)\alpha(k_2)}\left(\frac{\mathcal{N}_2(\mathbf{k}_1,\mathbf{k}_2)}{\alpha(k_2)}+\frac{\mathcal{N}_2(\mathbf{k}_2,\mathbf{k}_1)}{\alpha(k_1)}\right)+2\ \text{cyc.}\right)_{\mathsf{R}}
\end{aligned}$$

Baldauf et al. (ArXiv:1011.1513) model for the bispectrum, not including s^2 and n^2. Good fit against simulations within 10% error bars.

Halo bispectrum

The relative difference in absolute value between our model and the one by Baldauf et al., assuming $M = 10^{13} h^{-1} M_\odot$ and $f_{\rm NL} = 10$.



Conclusions

Main result:

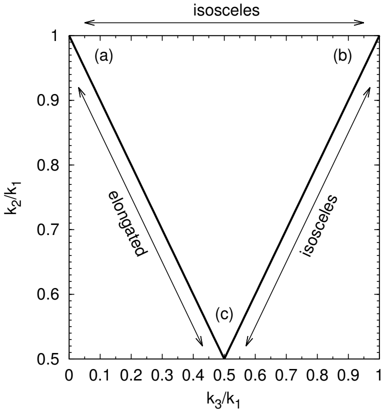
- Found the presence of a novel non-Gaussian shift term, n^2, in the halo overdensity
- Extra contributions to the halo bispectrum from the tides s^2 and the shift term n^2
- Numerical comparison with the model by Baldauf et al. revealed corrections mostly within 10%, though on large scales and/or high redshift larger differences can arise in some configurations

Future work:

- Redshift-space galaxy bispectrum with PNG (Tellarini et al. in prep.)
- Test against simulations
- Other types of primordial non-Gaussianity, including relativistic effects

Thank you for your attention!

Halo bispectrum: graphical representation



Condition: $k_3 \leq k_2 \leq k_1$

Point (a) is for the squeezed limit, $k_1 \simeq k_2 \gg k_3$

Point (b) for the equilateral configuration, $k_1 = k_2 = k_3$

Point (c) for the folded configuration $k_1 = 2k_2 = 2k_3$

Halo bispectrum

$$\langle \delta_h^E(\mathbf{k}_1)\delta_h^E(\mathbf{k}_2)\delta_h^E(\mathbf{k}_3)\rangle = (2\pi)^3\delta^D(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)B_{hhh}(\mathbf{k}_1,\mathbf{k}_2)$$

$$\begin{aligned}
B_{\mathsf{hhh}}(\mathbf{k}_1,\mathbf{k}_2) =& B_{\mathsf{hhh}}^{(\mathsf{A}\to\mathsf{L})}(\mathbf{k}_1,\mathbf{k}_2) \\
& -\frac{2}{7}b_{10}^2 b_{10}^{\mathrm{L}}\left(2P(k_1)P(k_2)\mathcal{S}_2(\mathbf{k}_1,\mathbf{k}_2) + 2\ \text{cyc.}\right)_{\mathsf{M}} \\
& -\frac{2}{7}b_{10}b_{01}b_{10}^{\mathrm{L}}\left(2P(k_1)P(k_2)\left(\frac{1}{\alpha(k_1)}+\frac{1}{\alpha(k_2)}\right)\mathcal{S}_2(\mathbf{k}_1,\mathbf{k}_2) + 2\ \text{cyc.}\right)_{\mathsf{N}} \\
& -\frac{2}{7}b_{01}^2 b_{10}^{\mathrm{L}}\left(2\frac{P(k_1)P(k_2)}{\alpha(k_1)\alpha(k_2)}\mathcal{S}_2(\mathbf{k}_1,\mathbf{k}_2) + 2\ \text{cyc.}\right)_{\mathsf{O}} \\
& -b_{10}^2 b_{01}^{\mathrm{L}}\left(2P(k_1)P(k_2)\left(\frac{\mathcal{N}_2(\mathbf{k}_1,\mathbf{k}_2)}{\alpha(k_2)}+\frac{\mathcal{N}_2(\mathbf{k}_2,\mathbf{k}_1)}{\alpha(k_1)}\right) + 2\ \text{cyc.}\right)_{\mathsf{P}} \\
& -b_{10}b_{01}b_{01}^{\mathrm{L}}\left(2P(k_1)P(k_2)\left(\frac{\mathcal{N}_2(\mathbf{k}_1,\mathbf{k}_2)}{\alpha(k_2)}+\frac{\mathcal{N}_2(\mathbf{k}_2,\mathbf{k}_1)}{\alpha(k_1)}\right)\left(\frac{1}{\alpha(k_1)}+\frac{1}{\alpha(k_2)}\right) + 2\ \text{cyc.}\right)_{\mathsf{Q}} \\
& -b_{01}^2 b_{01}^{\mathrm{L}}\left(2\frac{P(k_1)P(k_2)}{\alpha(k_1)\alpha(k_2)}\left(\frac{\mathcal{N}_2(\mathbf{k}_1,\mathbf{k}_2)}{\alpha(k_2)}+\frac{\mathcal{N}_2(\mathbf{k}_2,\mathbf{k}_1)}{\alpha(k_1)}\right) + 2\ \text{cyc.}\right)_{\mathsf{R}}
\end{aligned}$$

Sourced by $\langle s^2\delta\delta\rangle + \langle\delta s^2\delta\rangle + \langle\delta\delta s^2\rangle$

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\end{aligned}$$

Sourced by $\langle s^2\delta\varphi\rangle + \langle s^2\varphi\delta\rangle + \langle \delta s^2\varphi\rangle + \langle \varphi s^2\delta\rangle + \langle \delta\varphi s^2\rangle + \langle \varphi\delta s^2\rangle$

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