The bispectrum of relativistic galaxy number counts

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Mainly based on: E. Di Dio, R. Durrer, GM, F. Montanari, JCAP 12 (2014) 017;
E. Di Dio, R. Durrer, GM, F. Montanari, to appear.

What are LSS surveys really observing?

- All observations are made on our past light-cone $\Rightarrow$ We do not observe 3 spatial dimensions, but 2 spatial and one light-like.
- The measured redshift is perturbed by peculiar velocities and by the gravitational potential.
- The observed volume is distorted.
- Direction of observation $\neq$ angular direction of the source.
- We have distorted observable coordinates (redshift and incoming photons direction) $\Rightarrow$ Relativistic effects!

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For 3-point correlation functions $\Rightarrow$ We have to go beyond Newtonian gravity and beyond linear theory!

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Geodesic light-cone coordinates

An adapted light-cone coordinate system $x^\mu = (w, \tau, \tilde{\theta}^a)$, $a = 1, 2$ can be defined by the following metric (Gasperini, GM, Nugier, Veneziano (2011)):

$$ds^2 = \Upsilon^2 dw^2 - 2\Upsilon dw d\tau + \gamma_{ab}(d\tilde{\theta}^a - U^a dw)(d\tilde{\theta}^b - U^b dw) \; ; \qquad a, b = 1, 2 \; .$$

This metric depends on six arbitrary functions (Υ, the two-dimensional vector U^a and the symmetric tensor γ_{ab}) and is completely gauge fixed.

w is a null coordinate , $\partial_\mu \tau$ defines a geodesic flow

$k^\mu = g^{\mu\nu}\partial_\nu w = g^{\mu w} = -\delta^\mu_\tau \Upsilon^{-1}$ null geodesics connecting sources and observer

$\Downarrow$

Photons travel at constant w and $\tilde{\theta}^a$

The exact non-perturbative redshift is given by

$$1 + z_s = \frac{(k^\mu u_\mu)_s}{(k^\mu u_\mu)_o} = \frac{(\partial^\mu w \partial_\mu \tau)_s}{(\partial^\mu w \partial_\mu \tau)_o} = \frac{\Upsilon(w_o, \tau_o, \tilde{\theta}^a)}{\Upsilon(w_o, \tau_s, \tilde{\theta}^a)}$$

where the subscripts “o” and “s” denote, respectively, a quantity evaluated at the observer and source space-time position.

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where the subscripts “o” and “s” denote, respectively, a quantity evaluated at the observer and source space-time position.

Galaxy Number Counts

Galaxy Number Counts= number N of sources (galaxies) per solid angle and redshift.
The fluctuation of the galaxy number counts in function of observed redshift and direction is given by

$$\Delta(\mathbf{n}, z) \equiv \frac{N(\mathbf{n}, z) - \langle N \rangle (z)}{\langle N \rangle (z)} ,$$

where

$$N(\mathbf{n}, z) = \rho(\mathbf{n}, z) V(\mathbf{n}, z) .$$

Considering the density and volume fluctuations per redshift bin dz and per solid angle $d\Omega$

$$V(\mathbf{n}, z) = \bar{V}(z) \left(1 + \frac{\delta V^{(1)}}{\bar{V}} + \frac{\delta V^{(2)}}{\bar{V}}\right)$$
$$\rho(\mathbf{n}, z) = \bar{\rho}(z) \left(1 + \delta^{(1)} + \delta^{(2)}\right) ,$$

we can give the directly observed number fluctuations

$$\Delta(\mathbf{n}, z) = \left[\delta^{(1)} + \frac{\delta V^{(1)}}{\bar{V}} + \delta^{(1)} \frac{\delta V^{(1)}}{\bar{V}} + \delta^{(2)} + \frac{\delta V^{(2)}}{\bar{V}} - \langle \delta^{(1)} \frac{\delta V^{(1)}}{\bar{V}} \rangle - \langle \delta^{(2)} \rangle - \langle \frac{\delta V^{(2)}}{\bar{V}} \rangle \right]$$

Volume Perturbation

The 3-dimensional volume element dV seen by a source with 4-velocity u^μ is

$$dV = \sqrt{-g}\epsilon_{\mu\nu\alpha\beta}u^\mu dx^\nu dx^\alpha dx^\beta \,.$$

In terms of the observed quantities (z, θ_o, ϕ_o)

$$dV = \sqrt{-g}\epsilon_{\mu\nu\alpha\beta}u^\mu \frac{\partial x^\nu}{\partial z}\frac{\partial x^\alpha}{\partial \theta_s}\frac{\partial x^\beta}{\partial \phi_s}\left|\frac{\partial\left(\theta_s, \phi_s\right)}{\partial\left(\theta_o \phi_o\right)}\right| dz d\theta_o d\phi_o \equiv v\left(z, \theta_o, \phi_o\right) dz d\theta_o d\phi_o \,.$$

Going to GLC we then have

$$dV = -\sqrt{-g}u^w \frac{\partial \tau}{\partial z} dz d\theta_o d\phi_o \,.$$

and

$$dV = \sqrt{|\gamma|}\left(-\frac{d\tau}{dz}\right) dz d\theta_o d\phi_o \,, \qquad \text{or} \qquad v = \sqrt{|\gamma|}\left(-\frac{d\tau}{dz}\right)$$

This is a non-perturbative expression for the volume element at the source in terms of the observed redshift and the observation angles in GLC gauge.

If we would know $\rho(\mathbf{n}, z)$ non-perturbatively we could write the number counts in an exact way in GLC.

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Coordinates Trasformation

Let us consider a stochastic background of scalar perturbations on a conformally flat FLRW space-time to describe the inhomogeneities of our Universe at large scale.

Using spherical coordinates ($y^\mu = (\eta, r, \theta, \phi)$) in the Poisson gauge (PG) we have

$$g_{NG}^{\mu\nu} = a^{-2}(\eta)\,\mathrm{diag}\left(-1 + 2\Phi, 1 + 2\Psi, (1 + 2\Psi)\gamma_0^{ab}\right)$$

where $\gamma_0^{ab} = \mathrm{diag}\left(r^{-2}, r^{-2}\sin^{-2}\theta\right)$, $\Phi = \psi + \frac{1}{2}\phi^{(2)} - 2\psi^2$ and $\Psi = \psi + \frac{1}{2}\psi^{(2)} + 2\psi^2$.

To use the previous results we have to re-express this metric in GLC form. We define the coordinates transformation using

$$g_{GLC}^{\rho\sigma}(x) = \frac{\partial x^\rho}{\partial y^\mu}\frac{\partial x^\sigma}{\partial y^\nu} g_{NG}^{\mu\nu}(y)$$

and imposing the following boundary conditions

- Non-singular transformation around the observer position at $r = 0$.
- The two-dimensional spatial section $r =$ const is locally parametrized at the observer position by standard spherical coordinates.

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Cosmological Observables: redshift

The redshift up to second order in perturbation theory is

$$1 + z = \frac{a(\eta_o)}{a(\eta_s)} \left[1 + \delta^{(1)} z + \delta^{(2)} z \right]$$

with

$$\delta z^{(1)} = -v_{||s} - \psi_s - 2 \int_{\eta_s}^{\eta_o} d\eta' \partial_{\eta'} \psi \left(\eta' \right)$$

$$\delta z^{(2)} = -v_{||s}^{(2)} - \frac{1}{2} \phi_s^{(2)} - \frac{1}{2} \int_{\eta_s}^{\eta_0} d\eta' \partial_{\eta'} \left[\phi^{(2)} + \psi^{(2)} \right] \left(\eta' \right) + \frac{1}{2} \left(v_{||s} \right)^2 + \frac{1}{2} \left(\psi_s \right)^2$$

$$+ \left(-v_{||s} - \psi_s \right) \left(-\psi_s - 2 \int_{\eta_s}^{\eta_o} d\eta' \partial_{\eta'} \psi \left(\eta' \right) \right) + \frac{1}{2} v_{\perp s}^a v_{\perp a s} + 2a\, v_{\perp s}^a \partial_a \int_{\eta_s}^{\eta_o} d\eta' \psi \left(\eta' \right)$$

$$+4 \int_{\eta_s}^{\eta_0} d\eta' \left[\psi \left(\eta' \right) \partial_{\eta'} \psi \left(\eta' \right) + \partial_{\eta'} \psi \left(\eta' \right) \int_{\eta'}^{\eta_o} d\eta'' \partial_{\eta''} \psi \left(\eta'' \right) \right.$$

$$\left. +\psi \left(\eta' \right) \int_{\eta'}^{\eta_o} d\eta'' \partial^2_{\eta''} \psi \left(\eta'' \right) - \gamma_0^{ab} \partial_a \left(\int_{\eta'}^{\eta_o} d\eta'' \psi \left(\eta'' \right) \right) \partial_b \left(\int_{\eta'}^{\eta_o} d\eta'' \partial_{\eta''} \psi \left(\eta'' \right) \right) \right]$$

$$+2\partial_a \left(v_{||s} + \psi_s \right) \int_{\eta_s}^{\eta_o} d\eta' \gamma_0^{ab} \partial_b \int_{\eta'}^{\eta_o} d\eta'' \psi \left(\eta'' \right)$$

$$+4 \int_{\eta_s}^{\eta_o} d\eta' \partial_a \left(\partial_{\eta'} \psi \left(\eta' \right) \right) \int_{\eta'}^{\eta_o} d\eta'' \gamma_0^{ab} \partial_b \int_{\eta''}^{\eta_o} d\eta''' \psi \left(\eta''' \right)$$

Ben-Dayan, GM, Nugier, Veneziano (2012), Fanizza, Gasperini, GM, Veneziano (2013) and GM (2015)
(see also Umeh, Clarkson, Maartens (2014))

Cosmological Observables

To obtain Δ in the PG, in function of the observed redshift and of the direction of observation (θ_o, φ_o), we have:

Step 1 $\rightarrow$ Expand the exact expression of Δ in function of the PG coordinate using the coordinate transformation.

Step 2 $\rightarrow$ Expand conformal time and radial PG coordinates around a fiducial model as $\eta_s = \eta_s^{(0)} + \eta_s^{(1)} + \eta_s^{(2)}$ and $r_s = r_s^{(0)} + r_s^{(1)} + r_s^{(2)}$ perturbatively solving

$$1+z_s = \frac{a(\eta_o)}{a(\eta_s^{(0)})} = \frac{a(\eta_o)}{a(\eta_s)}\left[1 + \delta^{(1)}z + \delta^{(2)}z\right] \quad , \quad w_o = \eta_s^{(0)}+r_s^{(0)} = w^{(0)}+w^{(1)}+w^{(2)}$$

Step 3 $\rightarrow$ Taylor expand the solution of Step 1 around the fiducial model using Step 2, and around the direction of observation using the fact that $\tilde{\theta}^a = \theta_o^a$ are constant along the line-of-sight and therefore

$$\theta^a = \theta^{a(0)} + \theta^{a(1)} = \theta_o^a - 2\int_{\eta_s^{(0)}}^{\eta_o} d\eta' \; \gamma_0^{ab}\partial_b \int_{\eta'}^{\eta_o} d\eta'' \; \psi(\eta'', \eta_o - \eta'', \theta_o^a) \, .$$

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Galaxy Number Counts

The (second-order, non-homogeneous, non-averaged) expression of Δ in our perturbed background is so given (in a concise form) by

$$\Delta = \Delta^{(1)}(\mathbf{n}, z_s) + \Delta^{(2)}(\mathbf{n}, z_s)$$

To first order we have (Yoo, Fitzpatrick, Zaldarriaga (2009), Yoo (2010), Bonvin, Durrer (2011), Challinor, Lewis (2011))

$$\begin{aligned}
\Delta^{(1)}(\mathbf{n}, z) &= \left(\frac{2}{\mathcal{H} r(z)} + \frac{\mathcal{H}'}{\mathcal{H}^2}\right)\left(\partial_r v^{(1)} + \Psi^{(1)} + 2\int_0^{r(z)} dr \partial_\eta \Psi^{(1)}\right) - \Psi^{(1)} \\
&\quad + 4\Psi_1 - 2\kappa + \frac{1}{\mathcal{H}}\left(\partial_\eta \Psi^{(1)} + \partial_r^2 v^{(1)}\right) + \delta^{(1)}
\end{aligned}$$

with

$$\Psi_1(\mathbf{n}, z) = \frac{2}{r(z)} \int_0^{r(z)} dr \Psi^{(1)}(r) \quad , \quad 2\kappa = -\Delta_2 \psi = 2\int_0^{r(z)} dr \frac{r(z) - r}{r(z) r} \Delta_2 \Psi^{(1)}(r)$$

Galaxy Number Counts

Keeping only the leading (potentially observables) terms the number counts to second order turns to be

$$\Delta^{(2)} = \Sigma^{(2)} - \langle \Sigma^{(2)} \rangle$$

where

$$\begin{aligned}
\Sigma^{(2)}(\mathbf{n}, z) \quad = \quad & \delta^{(2)} + \mathcal{H}^{-1}\partial_r^2 v^{(2)} - 2\kappa^{(2)} + \mathcal{H}^{-2}\left(\partial_r^2 v\right)^2 + \mathcal{H}^{-2}\partial_r v \partial_r^3 v \\
& + \mathcal{H}^{-1}\left(\partial_r v \partial_r \delta + \partial_r^2 v \, \delta\right) - 2\delta\kappa + \nabla_a \delta \nabla^a \psi \\
& + \mathcal{H}^{-1}\left[-2(\partial_r^2 v)\kappa + \nabla_a(\partial_r^2 v)\nabla^a \psi\right] + 2\kappa^2 - 2\nabla_b \kappa \nabla^b \psi \\
& - \frac{1}{2r(z)} \int_0^{r(z)} dr \frac{r(z) - r}{r} \Delta_2 \left(\nabla^b \Psi_1 \nabla_b \Psi_1\right) - 2 \int_0^{r(z)} \frac{dr}{r} \nabla^a \Psi_1 \nabla_a \kappa \, .
\end{aligned}$$

with

$$\kappa^{(2)} = \frac{1}{2} \int_0^{r(z)} dr \frac{r(z) - r}{r(z) r} \Delta_2 (\Psi + \Phi)^{(2)}(-r\mathbf{n}, \eta_0 - r) \, .$$

Galaxy Number Counts: some leading terms

Among the leading (potentially observable) terms, we can isolate the following three contributions

$\delta_\rho^{(2)}$	Second order density
$\left(\frac{1}{\mathcal{H}_s}\partial_r^2 v\right)\delta_\rho^{(1)}$	Redshift space distortion - density
$\left[-\frac{2}{r_s}\int_{\eta_s}^{\eta_o} d\eta' \frac{\eta'-\eta_s}{\eta_o-\eta'}\Delta_2\psi\left(\eta'\right)\right]\delta_\rho^{(1)}$	Lensing - density

These are some of the most relevant terms in the galaxy number counts reduced bispectrum.

Reduced bispectrum number counts

We define the bispectrum in real space as

$$B(\mathbf{n}_1, \mathbf{n}_2, \mathbf{n}_3, z_1, z_2, z_3) = \langle \Delta(\mathbf{n}_1, z_1)\, \Delta(\mathbf{n}_2, z_2)\, \Delta(\mathbf{n}_3, z_3) \rangle$$

Expanding the direction dependence of Δ in spherical harmonics

$$B(\mathbf{n}_1, \mathbf{n}_2, \mathbf{n}_3, z_1, z_2, z_3) = \sum_{\ell_i, m_i} B^{m_1 m_2 m_3}_{\ell_1 \ell_2 \ell_3}(z_1, z_2, z_3)\, Y_{\ell_1 m_1}(\mathbf{n}_1)\, Y_{\ell_2 m_2}(\mathbf{n}_2)\, Y_{\ell_3 m_3}(\mathbf{n}_3)$$

and

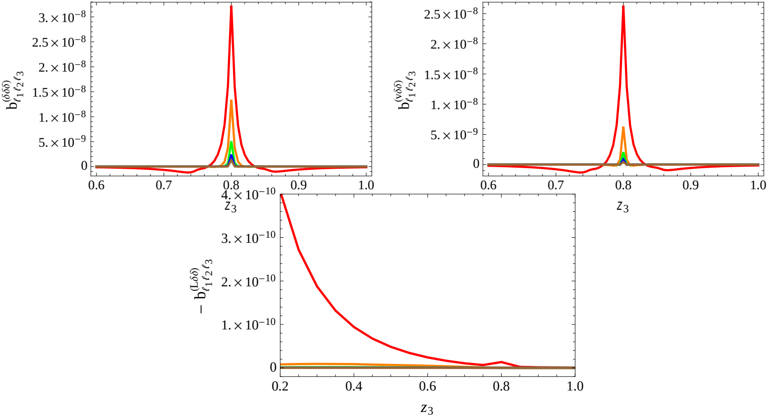
$$B^{m_1 m_2 m_3}_{\ell_1 \ell_2 \ell_3}(z_1, z_2, z_3) = \mathcal{G}^{m_1, m_2, m_3}_{\ell_1, \ell_2, \ell_3} \; b_{\ell_1, \ell_2, \ell_3}(z_1, z_2, z_3)$$

with

$\mathcal{G}^{m_1, m_2, m_3}_{\ell_1, \ell_2, \ell_3}$ Gaunt integral

$b_{\ell_1, \ell_2, \ell_3}(z_1, z_2, z_3)$ Reduced bispectrum

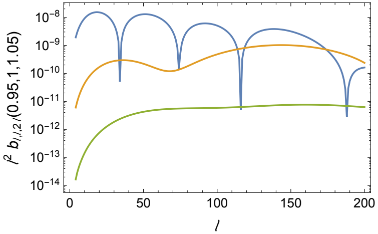
Reduced bispectrum number counts



We plot the contribution $b^{\delta\delta\delta}_{\ell_1\ell_2\ell_3}$ (upper left panel), $b^{v\delta\delta}_{\ell_1\ell_2\ell_3}$ (upper right panel), $-b^{L\delta\delta}_{\ell_1\ell_2\ell_3}$ (bottom panel) to the bispectrum for $z_1 = z_2 = 0.8$ as a function of z_3 for different values of $\ell_1 = \ell_2 = \ell_3/2$ (3 red, 103 orange, etc.).

Reduced bispectrum number counts

Preliminary!



We show the contributions to the reduced bispectrum from the Newtonian terms (blue), the Newtonian$\times$lensing terms (yellow) and the pure lensing terms (green) for $z_1 = 0.95$, $z_2 = 1$ and $z_3 = 1.05$ (left panel), and for $z_1 = 0.9$, $z_2 = 1$ and $z_3 = 1.1$ (right panel), as a function of $\ell = \ell_1 = \ell_2 = \ell_3/2$.

Conclusions

- We have presented the geodesic light-cone coordinates, a coordinate system adapted to an observer and his past light-cone.
- In the framework of the GLC we can write LSS observables in an exact, non-perturbative way.
- We have show the leading perturbative expressions for the number counts at second order as a function of the observed redshift and the direction of the observation.
- We have defined the number counts reduced bispectrum in the directly observable spherical-harmonics-redshift space.
- In particular configurations the integrated relativistic terms can dominate the signal/be not negligible
  - Well separated redshifts.
  - Broad window functions.

Outlook: Evaluation of the signal-to-noise to investigate whether planed surveys can detect the lensing signal when it dominates.

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- In the framework of the GLC we can write LSS observables in an exact, non-perturbative way.
- We have show the leading perturbative expressions for the number counts at second order as a function of the observed redshift and the direction of the observation.
- We have defined the number counts reduced bispectrum in the directly observable spherical-harmonics-redshift space.
- In particular configurations the integrated relativistic terms can dominate the signal/be not negligible
  - Well separated redshifts.
  - Broad window functions.

Outlook: Evaluation of the signal-to-noise to investigate whether planed surveys can detect the lensing signal when it dominates.

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THANKS FOR THE ATTENTION!

Coordinates Trasformation

Les us introduce the following auxiliary quantities:

$$P(\eta, r, \theta^a) = \int_{\eta_{in}}^{\eta} d\eta' \frac{a(\eta')}{a(\eta)} \psi(\eta', r, \theta^a) \quad , \quad Q(\eta_+, \eta_-, \theta^a) = \int_{\eta_o}^{\eta_-} dx\, \hat{\psi}(\eta_+, x, \theta^a) \ ,$$

where $\eta_\pm = \eta \pm r$. We then obtain

$$\begin{aligned}
\tau &= \tau^{(0)} + \tau^{(1)} + \tau^{(2)} \\
&\equiv \left(\int_{\eta_{in}}^{\eta} d\eta' a(\eta') \right) + a(\eta) P + \int_{\eta_{in}}^{\eta} d\eta' \frac{a(\eta')}{2} \left[\phi^{(2)} - \psi^2 + (\partial_r P)^2 + \gamma_0^{ab}\, \partial_a P\, \partial_b P \right] (\eta', r, \theta^a) \\
w &= w^{(0)} + w^{(1)} + w^{(2)} \\
&\equiv \eta_+ + Q + \frac{1}{4} \int_{\eta_o}^{\eta_-} dx \left[\hat{\psi}^{(2)} + \hat{\phi}^{(2)} + 4 \hat{\psi} \partial_+ Q + \hat{\gamma}_0^{ab}\, \partial_a Q\, \partial_b Q \right] (\eta_+, x, \theta^a) \\
\tilde{\theta}^a &= \tilde{\theta}^{a(0)} + \tilde{\theta}^{a(1)} + \tilde{\theta}^{a(2)} \equiv \theta^a + \frac{1}{2} \int_{\eta_o}^{\eta_-} dx \left[\hat{\gamma}_0^{ab} \partial_b Q \right] (\eta_+, x, \theta^a) + \int_{\eta_o}^{\eta_-} dx \left[\frac{1}{2} \gamma_o^{ac} \partial_c w^{(2)} \right. \\
&\left. + \hat{\psi} \left(\gamma_o^{ac} \partial_c w^{(1)} + \partial_+ \tilde{\theta}^{a(1)} \right) - \partial_+ w^{(1)} \partial_- \tilde{\theta}^{a(1)} + \frac{1}{2} \gamma_o^{dc} \partial_d w^{(1)} \partial_c \tilde{\theta}^{a(1)} \right] (\eta_+, x, \theta^a)
\end{aligned}$$

Coordinates Trasformation

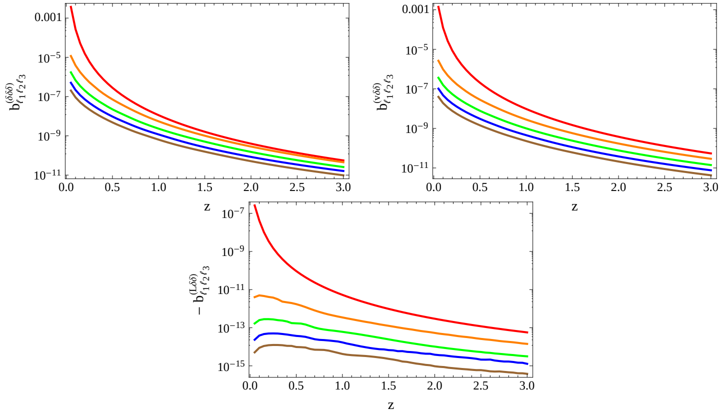
The non-trivial entries of the GLC metric are then given by

$$\Upsilon^{-1} = \frac{1}{a(\eta)}\Big(1 + \partial_+ Q - \partial_r P + \partial_\eta w^{(2)} + \frac{1}{a}(\partial_\eta - \partial_r)\tau^{(2)} - \psi\partial_\eta Q - \phi^{(2)}$$
$$+2\psi^2 - \partial_r P \partial_r Q - 2\psi\partial_r P - \gamma_0^{ab}\partial_a P \partial_b Q\Big)$$

$$U^a = \partial_\eta \tilde{\theta}^{a(1)} - \frac{1}{a}\gamma_0^{ab}\partial_b\tau^{(1)} + \partial_\eta\tilde{\theta}^{a(2)} - \frac{1}{a}\gamma_0^{ab}\partial_b\tau^{(2)} - \frac{1}{a}\partial_r\tau^{(1)}\partial_r\tilde{\theta}^{a(1)} - \psi\Big(\partial_\eta\tilde{\theta}^{a(1)}$$
$$+\frac{2}{a}\gamma_0^{ab}\partial_b\tau^{(1)}\Big) - \frac{1}{a}\gamma_0^{cd}\partial_c\tau^{(1)}\partial_d\tilde{\theta}^{a(1)} + (\partial_+ Q - \partial_r P)\Big(-\partial_\eta\tilde{\theta}^{a(1)} + \frac{1}{a}\gamma_0^{ab}\partial_b\tau^{(1)}\Big) \ ,$$

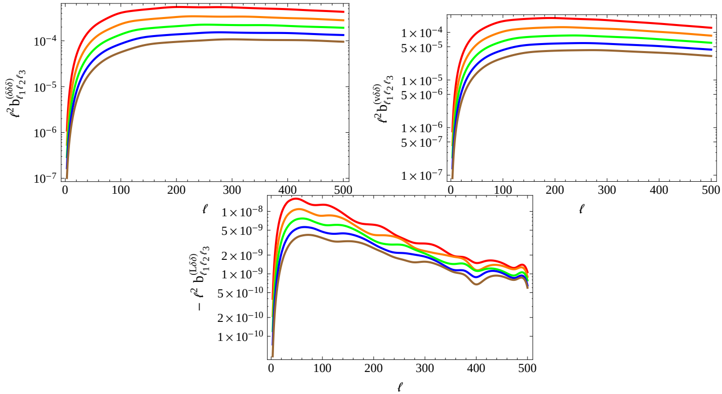
$$a(\eta)^2\gamma^{ab} = \gamma_0^{ab}(1+2\psi) + \left[\gamma_0^{ac}\partial_c\tilde{\theta}^{b(1)} + (a \leftrightarrow b)\right] + \gamma_0^{ab}\left(\psi^{(2)} + 4\psi^2\right) - \partial_\eta\tilde{\theta}^{a(1)}\partial_\eta\tilde{\theta}^{b(1)}$$
$$+\partial_r\tilde{\theta}^{a(1)}\partial_r\tilde{\theta}^{b(1)} + 2\psi\left[\gamma_0^{ac}\partial_c\tilde{\theta}^{b(1)} + (a \leftrightarrow b)\right] + \gamma_0^{cd}\partial_c\tilde{\theta}^{a(1)}\partial_d\tilde{\theta}^{b(1)}$$
$$+\left[\gamma_0^{ac}\partial_c\tilde{\theta}^{b(2)} + (a \leftrightarrow b)\right] .$$

Reduced bispectrum number counts



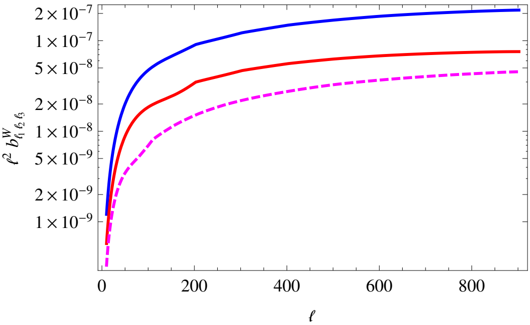
We plot the contributions of $b^{\delta\delta\delta}_{\ell_1\ell_2\ell_3}$ (up left), $b^{v\delta\delta}_{\ell_1\ell_2\ell_3}$ (up right), $-b^{L\delta\delta}_{\ell_1\ell_2\ell_3}$ (bottom) to the bispectrum for $z_1 = z_2 = z_3 = z$ as a function of z for different $\ell = \ell_1 = \ell_2 = \ell_3/2$ (3 red, 103 orange, 203 green, 303 blue, 403 brown).

Reduced bispectrum number counts



We plot the contributions of $\ell^2 b^{\delta\delta\delta}_{\ell_1\ell_2\ell_3}$ (up left), $\ell^2 b^{v\delta\delta}_{\ell_1\ell_2\ell_3}$ (up right), $-\ell^2 b^{L\delta\delta}_{\ell_1\ell_2\ell_3}$ (bottom) to the bispectrum for different $z_1 = z_2 = z_3 = z$ ($z = 0.6$ red, $z = 0.7$ orange, $z = 0.8$ green, $z = 0.9$ blue, $z = 1$ brown) as a function of $\ell_1 = \ell_2 = \ell_3/2 = \ell$.

Reduced bispectrum number counts: redshift bin



We show the effect of a very large redshift bin, for which the bispectrum is integrated from $z_{\rm min} = 0.2$ to $z_{\rm max} = 3$, for a fix $\ell_3 = 3$ while varying $\ell = \ell_1 = \ell_2$. We plot the density (blue), redshift space distortions (red) and lensing (magenta) contributions. Dashed lines correspond to negative values.