Brans-Dicke supergravity and the Λ naturalness problem

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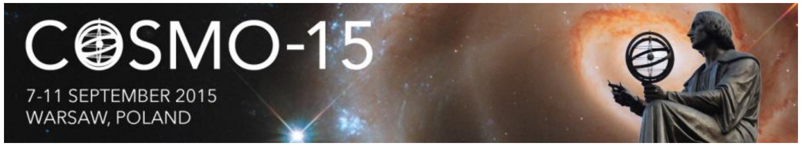


The vacuum has 2 intrinsic properties independent of the location, epoch and velocity of the observer:

c and Λ (cosmological constant)

Λ ~ 0.6 nJ/m^3, -0.6 nPa, estimated from observation.

Λ breaks geometric conformal symmetry (scale invariance of the geometric properties of empty space-time). However, the Standard Model has conformal anomalies, and needs these to work!

Significance of a finite theory without external regulator (e.g. superstrings): in the Einstein equations, Λ is calculable and finite (with no arbitrary extra part from a regulator needed).

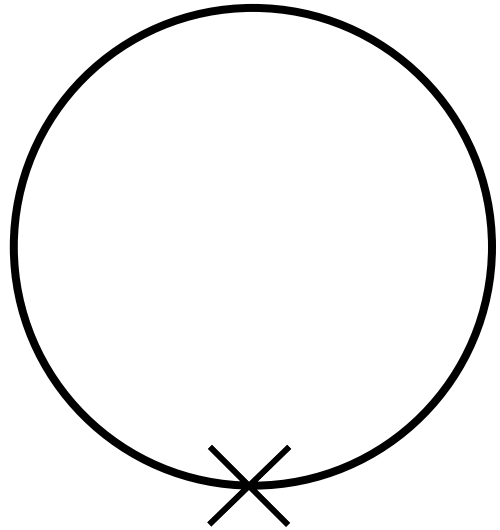
Λ is very small compared to quantum gravity scales: 10^{-120}compared to the Planck density, 10^{-60}relative to the supersymmetry breaking scale (~ TeV^4).

Witten’s paradox – Λ is extremely small, but cannot be forced to be 0 by some principle as it is actually nonzero.

However – this value may not be so unnatural in string theory (even without invoking the landscape, anthropic principle etc.)

The vacuum fluctuation energy (open Casimir effect) diverges, or is limited by a cutoff (e.g. at the Planck scale) in general, without supersymmetry.

Cancels in exact supersymmetry (Bose vs Fermi degeneracy), is finite & calculable in softly broken susy, $O(m_{SB}^4)$ ~ 10^{-60} in Planck units.



g coupling insertion gives the contribution to the energy density energy density from susy breaking.

Highest powers of momentum from insertion and propagator cancel.

Susy breaking scale acts as effective cut-off.

In spontaneously broken local susy, the Volkov fermion $Q_\alpha|0> \sim |v_\alpha>$ is eaten by the gravitino ψ_α, and has a bosonic partner v .

This multiplet takes the role of the source of susy breaking, and thus of conformal symmetry breaking (from flat classical background solution).

Principle of delegation: the variation of the superpotential G against the Volkov boson

$\frac{\partial G}{\partial v}$ gives the susy breaking part of the vacuum energy.

G has its slope in the direction of the Volkov boson v (among positive norm multiplets).

Susy breaking always gives a positive contribution to Λ.

G is at a minimum along other directions, which therefore do not contribute to susy breaking or Λ.

E.g. contribution from Higgs multiplet due to SSB = 0.

Λ can only be cancelled by the gravity multiplet contribution. This comes from a scalar auxiliary field S, with a negative norm. The dimension1 ghost mode ψ of this multiplet is gauged away by conformal transformations.

Origin of the compensator multiplet – in conformal supersymmetry, a negative norm conformal compensator is needed to give a positive sign for the Einstein-Hilbert action.

Unbroken susy is possible in an AdS background – the equivalent of v is gauged away by superconformal transformations. Spacetime can then be flattened by susy breaking.

$V = m_{Pl}^2 m_{3/2}^2 \left(\frac{G_{,v} G_{,\bar{v}}}{G_{,v\bar{v}}} - 3 \right)$ gives the potential in the vacuum

Tev scale susy breaking – the magnitude of the cancelling terms (breaking and non-breaking) can be estimated ~ 10^{-30} in Planck units.

The Casimir correction is very small in comparison – even if it is negative, the overall susy breaking part is easily > 0.

Λ small → the overall slope of G is in an almost null direction in field space (including compensator ψ) with Kahler metric $G_{,y\bar{y}}$. A mirror symmetry between the Volkov v and compensator ψ multiplets is almost exact at the vacuum, even for $\Lambda \sim m_{3/2}^4$.

Idea: v represents an approximate geometric conformal symmetry generated by $\frac{\partial}{\partial v}$ – so can be treated as approximately a Brans Dicke scalar graviton near the vacuum.

Problem of getting a mechanism to suppress Λ in Einstein gravity – compare this to the Peccei Quinn axion mechanism for solving the strong CP problem.

Potential can have minimum where the effective θ_{QCD} = 0. But the graph of vacuum energy vs vacuum energy is necessarily a straight line, so a direct analogy will not work.

Try generalising using a Brans Dicke action –

$$L = \sqrt{-g}\,(T + \phi^2\, R + K(\phi))$$

An exact goldstone mode ϕ is problematic e.g. restricted by tests of GR.

A generalised form with a potential will be used (The scalar graviton becomes a pseudo Goldstone mode): $T = K + V(\phi)$.

The field equations in vacuo: (ϕ constant), using a low energy unit (Brans – Dicke) metric are:

$$\frac{\partial T}{\partial \phi} + 2\phi R = 0$$

$$T + \phi^2 \mathrm{R} = 0$$

So $$\delta\left(\frac{T}{\phi^2}\right) = 0$$

This leads to geometric stability at the vacuum point – the BD frame geometry is unchanged under small variations in the BD mode ϕ i.e. $\delta R = 0$.

Scalar gravitons are natural in string models (heterotic strings on Calabi-Yau, type II strings on elliptic fibrations, Randall-Sundrum models…etc….)

Combine the Fradkin-Tseytlin dilaton with the compactification breathing mode to get a mode which leaves 4 dimensional couplings unchanged – candidate BD mode.

This mode will control the scale of low-energy physics in 2 ways:

1. Through setting the scale from which couplings run through the renormalisation group.

2. Through setting the scale for soft susy breaking via the gaugino condensation scale.

If these vary together near the vacuum, there will be an approximate conformal symmetry.

The Brans –Dicke model is only a good approximation near a ‘chase point’ – where $m_{3/2}$ scales with the compactification scale without an anomalous dimension i.e. $\frac{d\, m_{3/2}}{d\phi} = \frac{m_{3/2}}{\phi}$.

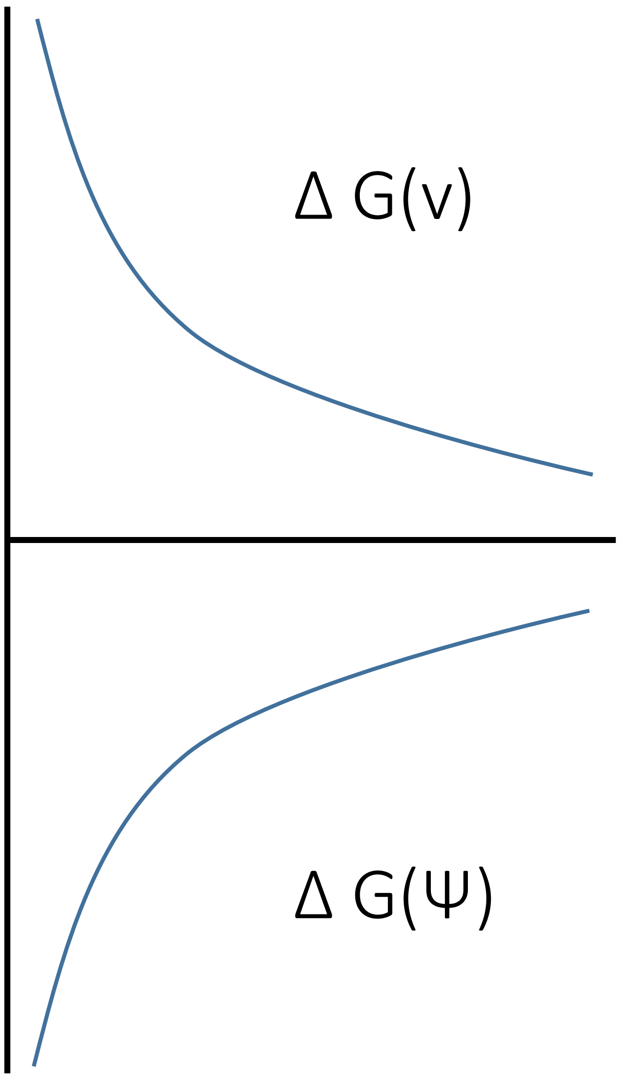
At a chase point, the Volkov mode v may be identified with a Brans-Dicke mode ϕ.

The existence of a chase point is a generic property (two lines cross), is stable against small corrections so does not require fine tuning.

A mirror symmetry between v and ψ can be understood in the context of the superconformal group $SO(4,2;1)$.

The v and ψ multiplets break Q_α and S_α respectively. Conformal mirror exchanges P, Q_α and K, S_α with opposite dilation charges ($SO(1,1)$ in $SO(4,2;1)$)

In a mirror symmetric region the graph of G against v is that against ψ upside down. Combined mode leaves V invariant.



Δ G(v)
Δ G(Ψ)

Mirror symmetry – reverses the sign of G (superpotential): reverses the sign of the propagator (from Kahler metric) and potential.

Mirror principle – Volkov mode (compactification mode) ~ Brans Dicke scalar: almost a mirror of the conformal compensator near the vacuum point:

Behaviour of G against v approximates reverse of the conformal behaviour of G against ψ.

This approach should evade Witten's paradox, as gravity does have a measurable effect.

3 scales – compensator ψ, compactification ϕ and susy breaking ($m_{3/2}$) scales. At the chase point ϕ and $m_{3/2}$ scale together so v may be identified with ϕ and approximate conformal mirror symmetry is realised as

$$\frac{\partial}{\partial\phi}\,|0\rangle \sim \frac{\partial}{\partial\psi}\,|0\rangle$$

Mirror symmetry near the vacuum comes from an approximate decoupling of low energy physics from gravity, i.e. approximate independence of physics in the low energy frame from ϕ.

Exact mirror symmetry is realised by the Polonyi form for G :

$$G = -3\log(z + \bar{z})$$

Giving potential

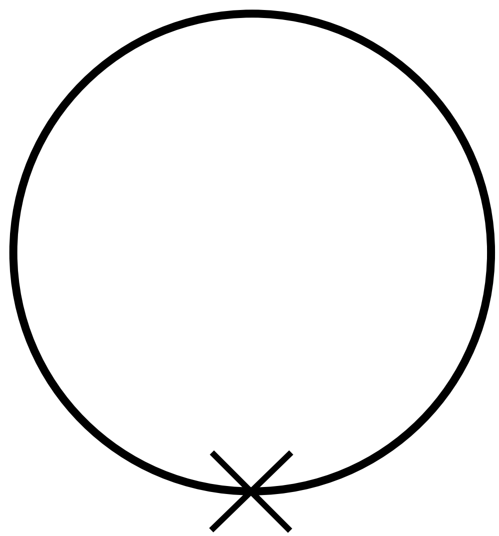
$$V = e^{-G}\left(\frac{G_{,z}G_{,\bar{z}}}{G_{,z\bar{z}}} - 3\right) = 0$$

as the slope of $m_{3/2}$ against z cancels that against ψ (exact mirror symmetry).

G is stable against the coupling varying mode, which is used in the gaugino condensation mechanism.

However, the magnitude of susy breaking will depend on the compactification scale ϕ through the scale of the gaugino condensate.

1 loop graph – the basic open Casimir effect.

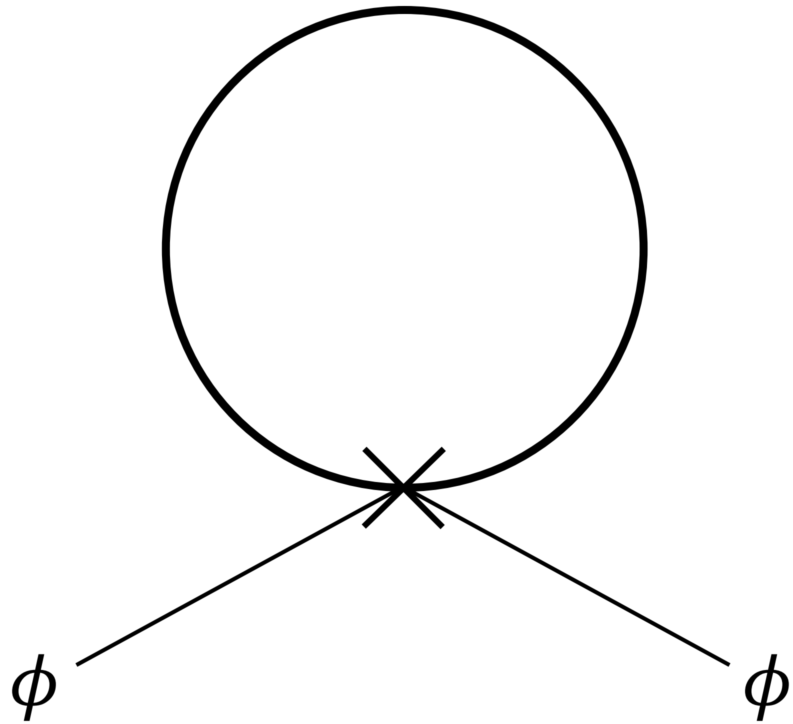


This is independent of gravity, so does not break mirror symmetry. This contribution to the vacuum energy adjusts the susy breaking energy by delegation, but the compactification mode stays on the Polonyi curve by mirror symmetry.

This adjustment is realised as a re-parametrization of compactification ϕ vs Polonyi z coordinates – or in physical terms of $m_{3/2}$ vs the compactification size.

$\Delta m_{3/2} \sim O(m_{3/2}^2/m_{Pl})$

The 1 loop contribution to the ϕ potential gives a quadratic effect around the chase point:

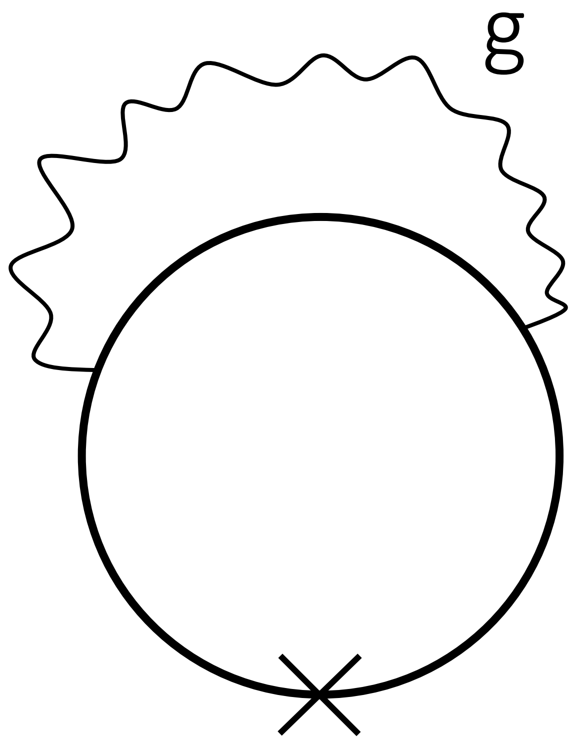


This leads to a mass for an excitation in the compactification mode ϕ of order $m_{3/2}$.

This means that the scalar graviton of the Brans-Dicke description has a microscopic range, and does not conflict with tests of GR.

The potential well for ϕ has $\Lambda = 0$ so far, with mass at the bottom ~ $m_{3/2}$, width ~ $m_{3/2}$ and height ~ $m_{3/2}^4$

1 graviton handle (2 loop)contribution:



In Planck units this gives an order $m_{3/2}^6/m_{Pl}^2$ correction, still too big for naturalness (10^{-90} in Planck units) if it were un-cancelled.

This effect pushes the solution away from the chase point by $O(m_{3/2}^2 \,/\, m_{Pl})$.

At the solution we must still have geometric stability, from the field equations.

Without assuming that Λ is zero, the gravitational effect on the Casimir energy is independent of a small change in the compactification scale, so mirror symmetry remains unbroken.

Also, the factor m_{PL}^2 in V applies to the whole of V, so geometric stability also applies off-shell.

The 1 graviton handle correction MAY therefore also be absorbed in a change to G_0 the vacuum value of G, and Λ remain zero in this approximation (mirror cancellation).

However, this is only the mean field approximation – there will be a correction to this from fluctuations in ϕ.

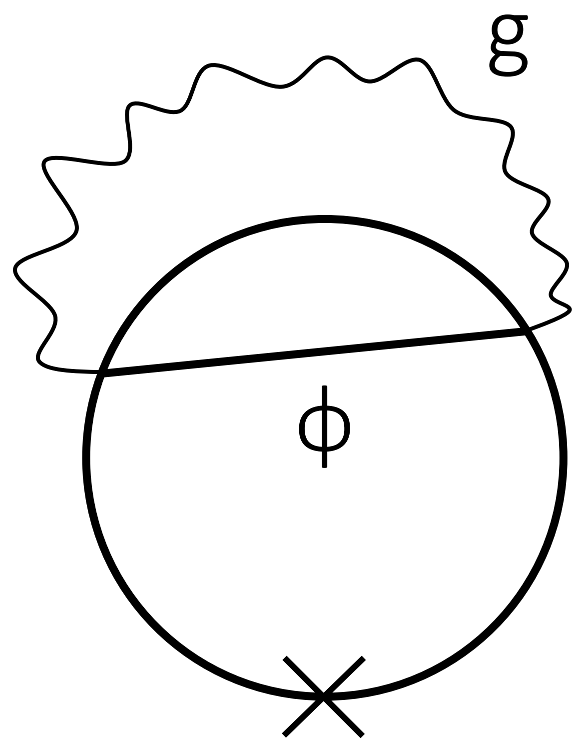
These will break the mirror symmetry and lead to a nonzero Λ.

Mirror symmetric diagrams make equal contributions to the conformal charges for $\frac{\partial}{\partial\phi}$ and $\frac{\partial}{\partial\psi}$,

i.e. to low energy frame conformal transformations with and without changing the strength of gravity, and the corresponding contributions to Λ via ϕ and ψ cancel.

Note that this cancellation mechanism only applies to Λ rather than to $Tr(T_{\mu\nu})$ for a
general low energy configuration - ψ only has a zero mode after gauge fixing, so can only cancel against the zero mode of ϕ.

The symbols $\frac{\partial}{\partial\phi}$ and $\frac{\partial}{\partial\psi}$ represent generators of GLOBAL conformal transformations of 2 kinds – with and without alteration to the strength of gravity.



This gives a contribution $O(m_{3/2}^8/m_{Pl}^4)$ to Λ
which is un-cancelled as it breaks mirror symmetry – there is no corresponding contribution involving the compensator ψ, which has only the zero mode surviving superconformal gauging.

At the vacuum solution we are near the chase point – this gives approximately a generalised Polonyi form for G :

$$G = -(3+\epsilon)\log(z+\bar{z})$$

giving $V \sim \epsilon m_{3/2}^2 m_{Pl}^2 \sim \epsilon m_{3/2}^2 \phi^2$ in the low-energy frame, consistent with a Brans-Dicke vacuum solution with non-zero $\Lambda \sim \epsilon$.

Conclusions

- TeV scale susy breaking in this scheme gives a reasonable agreement for Λ.

- This scenario may be testable – from the pattern of susy breaking, and comparison of Λ from specific models.

- The massive scalar graviton that appears here has short range only and is consistent with GR tests.