Constraints on hybrid metric-Palatini models from background evolution

Vanessa Smer - Barreto in collaboration with Nelson A. Lima
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Supervisor: Prof. Andrew R. Liddle
Royal Observatory of Edinburgh at University of Edinburgh

vsm@roe.ac.uk

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Overview

Characteristics of hybrid metric-Palatini gravity

It is a type of $f(R)$ theory.

These can be approached from two directions:

- Metric variation.
- Connection independent from the metric.

In hybrid metric-Palatini theries

The Lagrangian is a function of the Ricci scalars of both the connection and the metric.

Cosmology of hybrid metric-Palatini gravity

The action describing the hybrid metric-Palatini gravity is given by

$$S = \frac{1}{2\kappa^2} \int d^4x \sqrt{-g} \left[R + f(\mathcal{R}) \right] + S_{\rm m}, \tag{1}$$

where $\mathcal{R} = g^{\mu\nu}\mathcal{R}_{\mu\nu}$ is the Palatini curvature.

The modified Friedmann equations are

$$3H^2 = \frac{1}{1+f_{\mathcal{R}}}\left[\kappa^2\rho - 3H\dot{f_{\mathcal{R}}} - \frac{3\dot{f_{\mathcal{R}}}^2}{4f_{\mathcal{R}}} + \frac{\mathcal{R}F(\mathcal{R}) - f(\mathcal{R})}{2}\right], \qquad (2)$$

and,

$$2\dot{H} = \frac{1}{1+f_{\mathcal{R}}}\left[-\kappa^2\rho + p + H\dot{f_{\mathcal{R}}} - \ddot{f_{\mathcal{R}}} + \frac{3\dot{f_{\mathcal{R}}}^2}{2f_{\mathcal{R}}}\right], \qquad (3)$$

Identifying $\phi = F(\mathcal{R})$ the effective Klein-Gordon equation for the scalar field (by T. Harko, T. S. Koivisto et al, Phys. Rev. D 85, 084016, 2012)

$$\ddot{\phi} + 3H\dot{\phi} - \frac{\dot{\phi}^2}{2\phi} + \frac{\phi}{3}\left[2V - 1 + \phi\frac{dV}{d\phi}\right] = \frac{\kappa^2\phi}{3}T, \tag{4}$$

Recalling $\phi \equiv F(\mathcal{R}) \equiv f_{\mathcal{R}}$ Eq.4 may be rewritten as

$$\ddot{\mathcal{R}} = -\frac{1}{f_{\mathcal{R}\mathcal{R}}}\left[\dot{\mathcal{R}}^2\left[f_{\mathcal{R}\mathcal{R}\mathcal{R}} - \frac{f^2_{\mathcal{R}\mathcal{R}}}{2f_{\mathcal{R}}}\right] + 3H\dot{\mathcal{R}}f_{\mathcal{R}\mathcal{R}} + \frac{f_{\mathcal{R}}}{3}\left[\mathcal{R}\left[f_{\mathcal{R}} - 1\right] - 2f(\mathcal{R})\right] - \kappa^2\frac{f_{\mathcal{R}}}{3}T\right] \tag{5}$$

The effective potential is

$$\frac{dV_{\rm eff}}{d\mathcal{R}} = \frac{f_\mathcal{R}}{3f_{\mathcal{R}\mathcal{R}}} \left(\mathcal{R}\left[f_\mathcal{R} - 1\right) - 2f(\mathcal{R})\right], \tag{6}$$

meaning $V(\mathcal{R})$ will be given by the indefinite integral

$$V(\mathcal{R}) = \int^{\mathcal{R}} \frac{f_\mathcal{R}}{3f_{\mathcal{R}\mathcal{R}}} \left[\mathcal{R}\left(f_\mathcal{R} - 1\right) - 2f(\mathcal{R})\right] d\mathcal{R}. \tag{7}$$

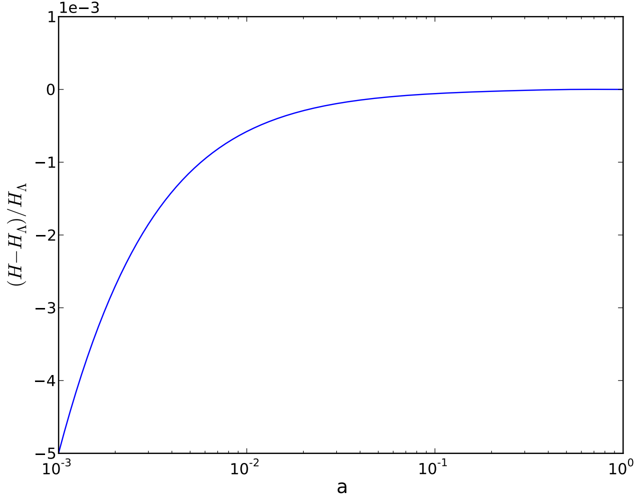
Exponential model

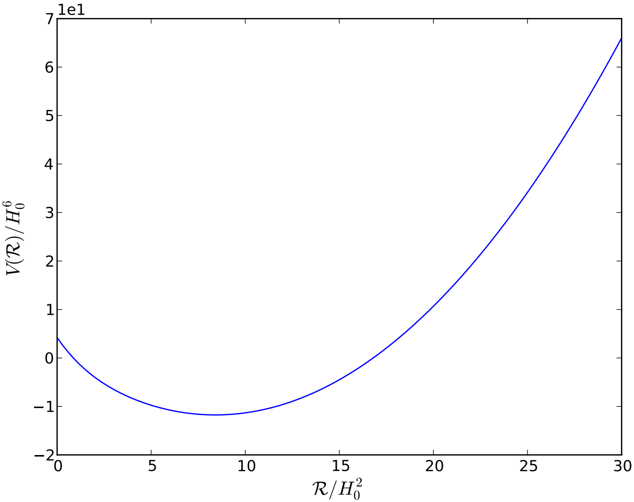
It is defined by the exponential function

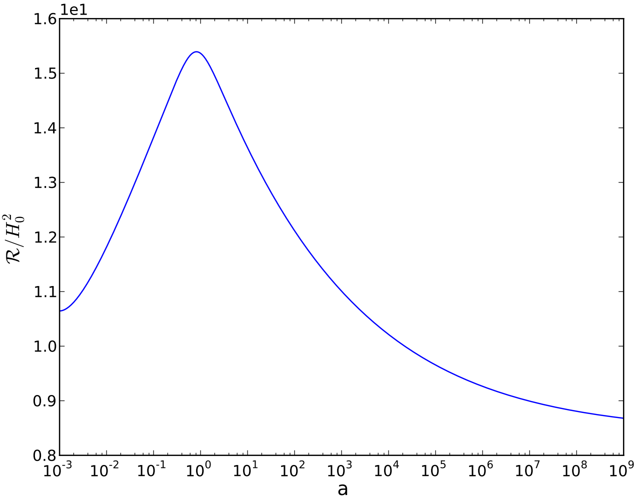
$$f(\mathcal{R}) = \Lambda_\star \left[1 + e^{-\mathcal{R}/\mathcal{R}_\star}\right], \tag{8}$$

with effective potential of the form

$$V(\mathcal{R}) = \frac{\mathcal{R}_\star}{3}\left[2\Lambda_\star\mathcal{R} + \frac{1}{2}\mathcal{R}^2 - 2\Lambda_\star\mathcal{R}_\star e^{-\mathcal{R}/\mathcal{R}_\star} - e^{-\mathcal{R}/\mathcal{R}_\star}\left(\Lambda_\star\mathcal{R} + \Lambda_\star\mathcal{R}_\star\right)\right]. \tag{9}$$


1e−3
1
0
−1
−2
−3
−4
−5
$(H - H_\Lambda)/H_\Lambda$
10^{-3}
10^{-2}
10^{-1}
10^{0}
a


1e1
7
6
5
4
3
2
1
0
−1
−2
0
5
10
15
20
25
30
$V(\mathcal{R})/H_0^6$
$\mathcal{R}/H_0^2$


1e1
1.6
1.5
1.4
1.3
1.2
1.1
1.0
0.9
0.8
$\mathcal{R}/H_0^2$
10^{-3} 10^{-2} 10^{-1} 10^{0} 10^{1} 10^{2} 10^{3} 10^{4} 10^{5} 10^{6} 10^{7} 10^{8} 10^{9}
a

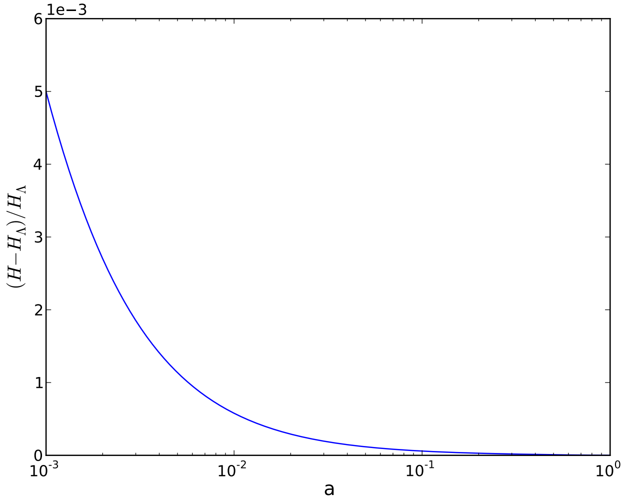
Quadratic model

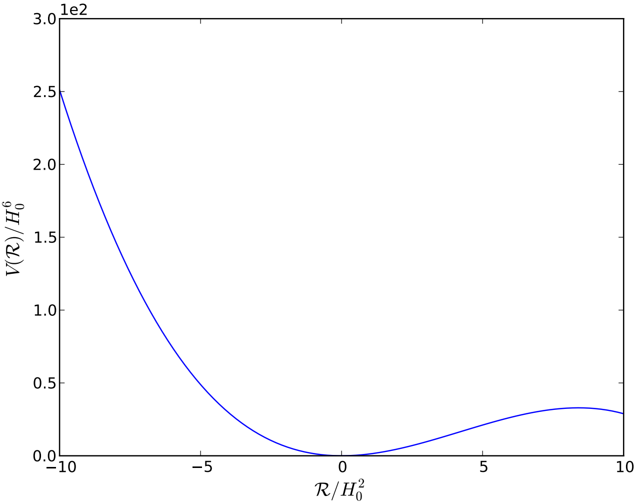
It is defined by the quadratic function

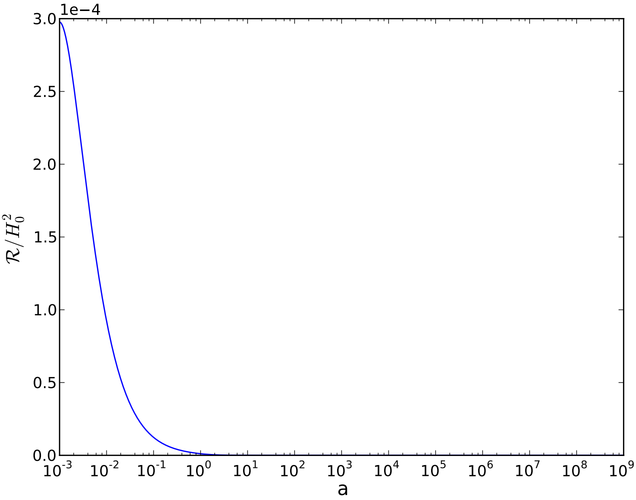
$$f(\mathcal{R}) = \Lambda_\star \left[1 + \mathcal{R}^2/\mathcal{R}_\star^2\right], \tag{10}$$

with the associated potential

$$V(\mathcal{R}) = -\frac{1}{3}\Lambda_\star \mathcal{R}^2 - \frac{1}{9}\mathcal{R}^3, \tag{11}$$


1e−3
6
5
4
3
2
1
0
$(H - H_\Lambda)/H_\Lambda$
10^{-3}
10^{-2}
10^{-1}
10^{0}
a


1e2
3.0
2.5
2.0
1.5
1.0
0.5
0.0
−10
−5
0
5
10
$V(\mathcal{R})/H_0^6$
$\mathcal{R}/H_0^2$


1e−4
3.0
2.5
2.0
1.5
1.0
0.5
0.0
$\mathcal{R}/H_0^2$
10^{-3} 10^{-2} 10^{-1} 10^{0} 10^{1} 10^{2} 10^{3} 10^{4} 10^{5} 10^{6} 10^{7} 10^{8} 10^{9}
a

Results using Union2.1, BAO, Planck 2013

General priors are

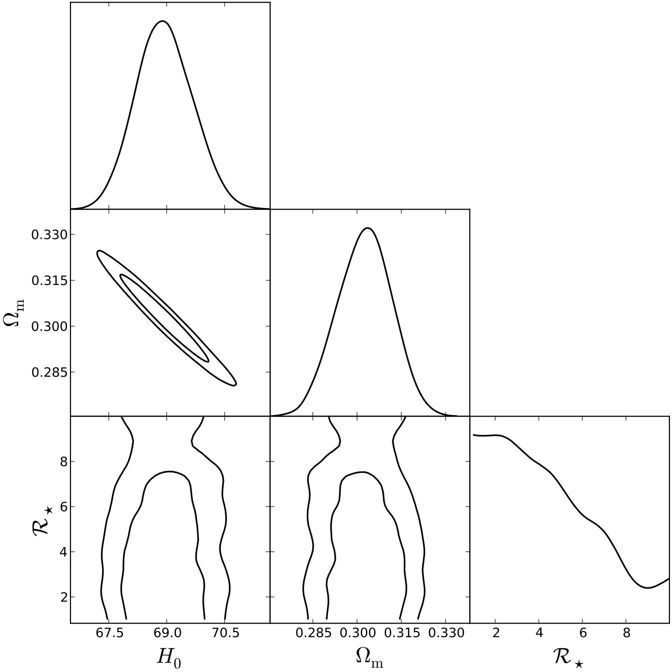
- $\Omega_{\rm m}$: $[0.01, 0.99]$.
- $\Omega_{\rm b}$: $[0.001, 0.080]$.
- H_0: $[40.0, 100.0]$.

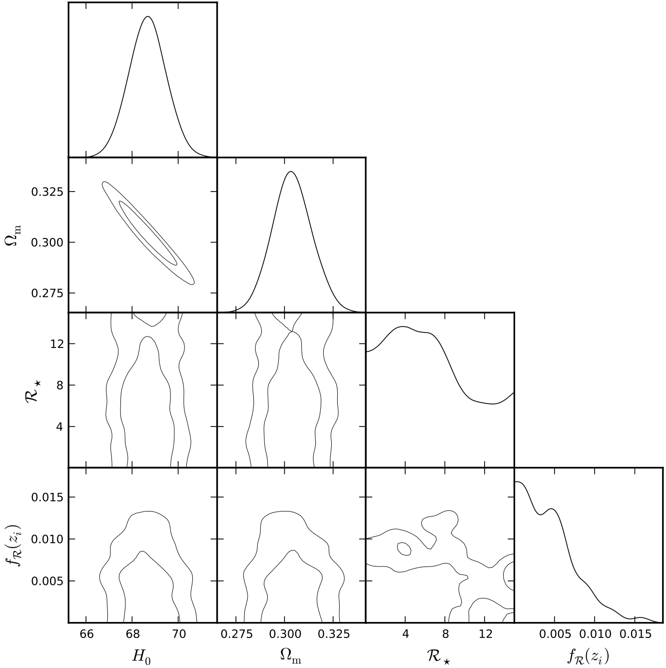
Exponential priors

$f_{\mathcal{R}}$ at $z_{\rm i} = 10^{-4}$, $\mathcal{R}_\star$ from $[1.0, 10.0]$.
$f_{\mathcal{R}}(z_{\rm i})$ from $[1 \times 10^{-6}, 0.1]$ and $\mathcal{R}_\star$ from$[0.01, 15.0]$.

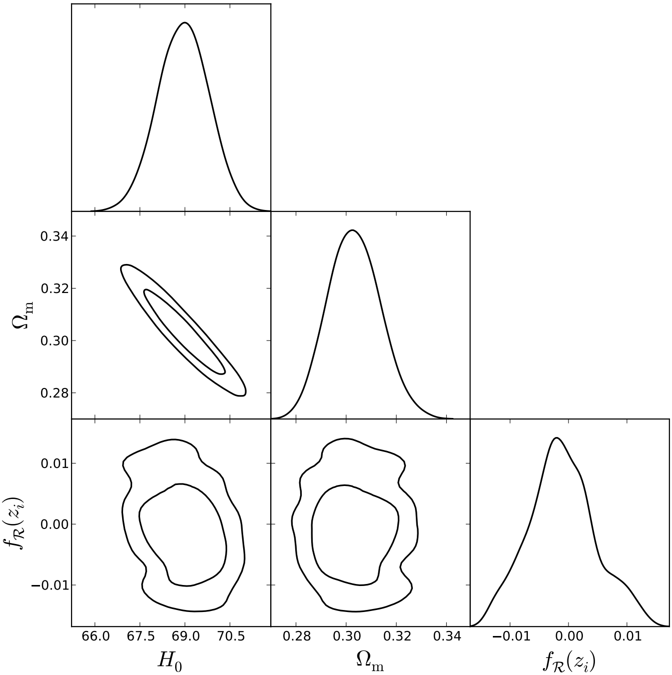
Quadratic priors

$\mathcal{R}_\star$ is fix and $f_{\mathcal{R}}$ at z_i from $[-0.1, 0.1]$.


0.330
0.315
0.300
0.285
Ω_m
8
6
4
2
$\mathcal{R}_\star$
67.5
69.0
70.5
H_0
0.285
0.300
0.315
0.330
Ω_m
2
4
6
8
$\mathcal{R}_\star$



0.325
0.300
0.275
$\Omega_{\rm m}$
12
8
4
$\mathcal{R}_\star$
0.015
0.010
0.005
$f_{\mathcal{R}}(z_i)$
66
68
70
H_0
0.275
0.300
0.325
$\Omega_{\rm m}$
4
8
12
$\mathcal{R}_\star$
0.005
0.010
0.015
$f_{\mathcal{R}}(z_i)$


0.34
0.32
0.30
0.28
Ω_m
0.01
0.00
−0.01
$f_{\mathcal{R}}(z_i)$
66.0 67.5 69.0 70.5
H_0
0.28 0.30 0.32 0.34
Ω_m
−0.01 0.00 0.01
$f_{\mathcal{R}}(z_i)$

Conclusions

- Small deviation from ΛCDM in both scenarios.
- $\mathcal{R}$ tends to the minimum at late times.
- Constraints on parameters after imposing observations available.
- Interesting enough theory to add perturbative analysis.

Thank you for listening.