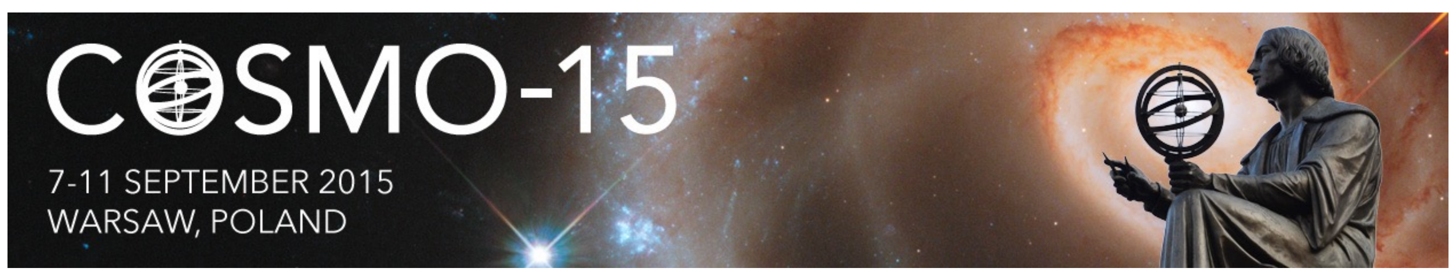
Quasi-static solutions for compact objects in Chameleon models

Ilia Musco
(CNRS, Observatoire de Paris/Meudon - LUTH)



DE session 10/09/2015

Collaborators:
Pier Stefano Corasaniti (Meudon)
Pedro Ferreira (Oxford)
David Mota (Oslo)

Outline

- Introduction: What’s/Why Chameleon?
- TOV-KG equations in quasi-static approximation
- Results (Incompressible model)
- Conclusions and future perspective

RELATIVISTIC STARS IN SCALAR TENSOR THEORIES: Quasi Static solution of Chameleon mechanism

- In **spherical symmetry** the static structure of a star is obtained solving the **TOV equations** (Euler equation + mass-energy conservation equation).

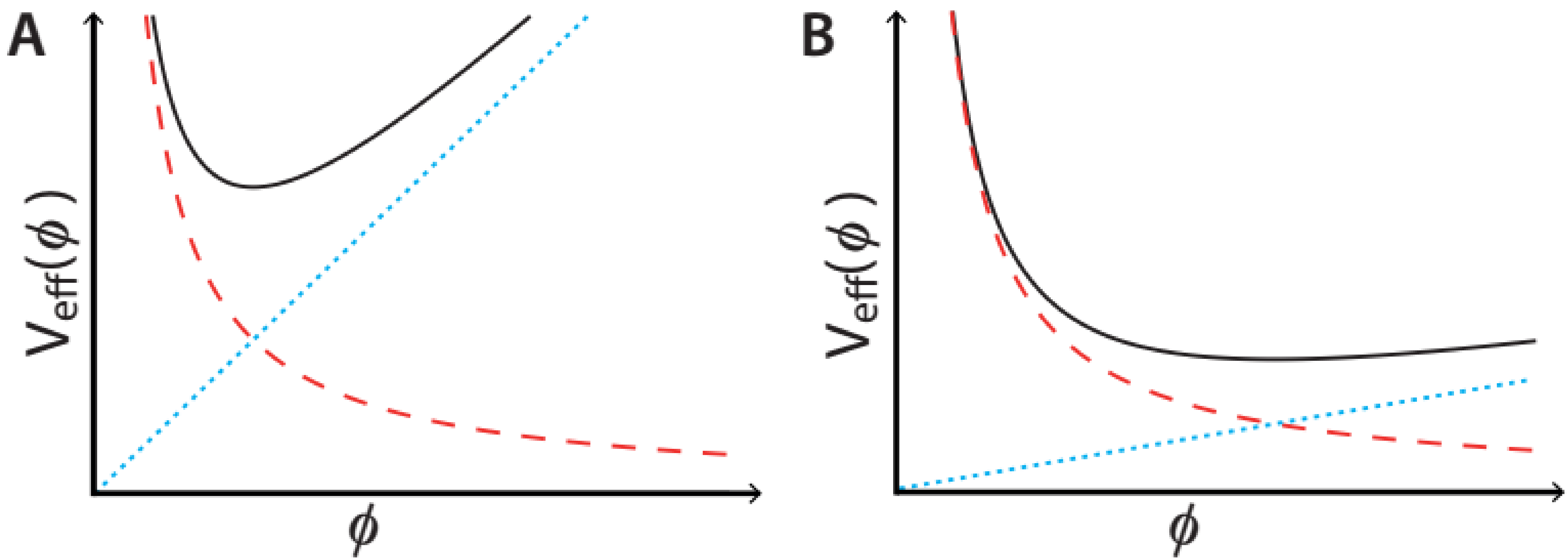
- The presence of a coupled non minimally coupled scalar field introduces the **Klein-Gordon equation** into the system.

- Investigation on the viability of the **chameleon (screening) mechanism** proposed by *Khoury & Weltman* (2004).

- The scalar field ϕ is characterised by a **coupling** $\beta(\phi)$ and a **potential** $V(\phi)$ that at large distances should describe a Dark Energy field.

- The coupling of the field with the matter is screening the field with a transition (**thin/ thick shell**) to an expanding FRW Universe ($R \rightarrow \infty$).

- Private Discussion with *Gia DVALI, Sandrine SCHLOEGEL, Ryo SAITO, Sebastien CLESSE* on Tuesday during Coffe Break: **is chameleon consistent with Astrophysics?**



- To match a static solution with a non static background we need **quasi static (slow roll) regime** ($\dot{\phi}, \ddot{\phi} \sim 0$).

- The **Hubble flow** ($U = HR$) is not negligible.

- Quasi- static approximation keeps into account Universe expansion outside the star (no need of artificial matter outside as in ***Babichev & Langlois*** - 2010)

TOV-KG (QUASI-STATIC) EQUATIONS

$$\frac{dM}{dR} = 4\pi R^2 e$$

$$\frac{1}{a}\frac{da}{dR} = -\frac{1}{e_m + p_m}\left[\frac{dp_m}{dR} + \beta(e_m - 3p_m)\frac{d\phi}{dR}\right]$$

$$\frac{1}{a}\frac{\partial U}{\partial t} = \frac{\Gamma^2}{a}\frac{da}{dR} - \frac{M}{R^2} - 4\pi R p$$

$$\frac{d^2\phi}{dR^2} + \left[\frac{2}{R} + \frac{1}{2\Gamma^2}\frac{d\Gamma^2}{dR^2} + \frac{1}{a}\frac{da}{dR}\right]\frac{d\phi}{dR} = \frac{1}{\Gamma^2}\left[\frac{dV(\phi)}{d\phi} + \beta(e_m - 3p_m)\right]$$

$$\Gamma^2 = 1 + U^2 - \frac{2M}{R}$$

$$e = e_m + e_\phi \ , \qquad e_\phi = \frac{\Gamma^2}{2}\left(\frac{d\phi}{dR}\right)^2 + V(\phi)$$

$$p = p_m + p_\phi \ , \qquad p_\phi = \frac{\Gamma^2}{2}\left(\frac{d\phi}{dR}\right)^2 - V(\phi)$$

INCOMPRESSIBLE STAR MODEL

<u>Inside the star:</u>

$$e = e_0$$
$$p(0) = p_0$$
$$U = 0$$

ϕ_0 shooting parameter

<u>Outside the star:</u>

$$e = e_b$$
$$p = 0$$
$$U = HR$$
$$H^2 = \frac{8\pi}{3} e_b$$

ϕ_∞ Dark Energy

<u>Boundary Condition:</u>

$$\frac{dV(\phi)}{d\phi} + \beta e_b = 0$$

Equilibrum at infinity

$$V(\phi) = \frac{\mu^{n+4}}{\phi^n} \qquad \Omega(\phi) = \exp\left(\frac{\sqrt{8\pi G}}{c^2}\beta\phi\right) \qquad \beta(\phi) \equiv \frac{\Omega'(\phi)}{\Omega(\phi)}$$

$$\mu^{n+4} = \frac{\hbar}{n}\frac{\sqrt{8\pi G}}{c^2}\beta e_b \phi_\infty^{n+1} \implies V(\phi) = \frac{\sqrt{8\pi G}}{c^2}\frac{\beta}{n}e_b\phi_\infty\left(\frac{\phi_\infty}{\phi}\right)^n$$

$$\phi_\infty = \frac{1}{4\sqrt{8\pi}\beta}W\left(4n\frac{1-\Omega_m}{\Omega_m}\right) \qquad W_4 = \frac{1}{4}W\left(4n\frac{1-\Omega_m}{\Omega_m}\right) \simeq 2 \ (\text{for } n=1,\ \Omega_m = 0.3)$$

$$m_\phi^2 \equiv \frac{d^2 V_{\text{eff}}}{d\phi^2}\bigg|_{\phi=\phi_\infty} \qquad \frac{8\pi\beta^2}{W_4} = \frac{m_\phi^2 c^4}{(n+1)Ge_b} = \frac{m_\phi^2}{(n+1)R^2}$$

Normalised Equations (with background):

$$\frac{d\hat{p}_m}{d\hat{R}} = -\frac{(\hat{e}_m + \hat{p}_m)\left(4\pi\hat{p}\hat{R}^3 + \hat{M}\right)}{\Gamma^2\hat{R}^2} - W_4(\hat{e}_m - 3\hat{p}_m)\hat{\lambda}$$

$$\frac{d\hat{M}}{d\hat{R}} = 4\pi\hat{R}^2\hat{e}$$

$$\hat{\lambda} = \frac{d\hat{\phi}}{d\hat{R}}$$

$$\frac{d\hat{\lambda}}{d\hat{R}} = -\left[\frac{2}{\hat{R}} - \frac{4\pi\hat{R}^3(\hat{e}-\hat{p}) - 2\hat{M}}{\hat{\Gamma}^2\hat{R}^2}\right]\hat{\lambda} + \frac{\hat{\beta}^2}{W_4\Gamma^2}\left[\hat{e}_m - 3\hat{p}_m - \hat{\phi}^{-(n+1)}\right]$$

$$\hat{\beta} = \sqrt{8\pi}\beta \qquad \Omega(\hat{\phi}) = \exp\left(W_4\hat{\phi}\right) \qquad \hat{V}(\hat{\phi}) = \frac{W_4}{n}\hat{\phi}^{-n}$$

$$\hat{e}_\phi = \frac{W_4^2\Gamma^2}{2\hat{\beta}^2}\hat{\lambda}^2 + \frac{W_4}{n}\hat{\phi}^{-n} \qquad \hat{e} = \hat{e}_m + \hat{e}_\phi$$

$$\hat{p}_\phi = \frac{W_4^2\Gamma^2}{2\hat{\beta}^2}\hat{\lambda}^2 - \frac{W_4}{n}\hat{\phi}^{-n} \qquad \hat{p} = \hat{p}_m + \hat{p}_\phi$$

Series expansion:

$$\hat{\phi} = \hat{\phi}_0 + \hat{\phi}_1 \hat{R} + \frac{\hat{\phi}_2}{2}\hat{R}^2 + \frac{\hat{\phi}_3}{6}\hat{R}^3$$

$$\hat{\lambda} = \hat{\lambda}_0 + \hat{\lambda}_1 \hat{R} + \frac{\hat{\lambda}_2}{2}\hat{R}^2 + \frac{\hat{\lambda}_3}{6}\hat{R}^3$$

$$\hat{e}_m = \hat{e}_0 + \hat{e}_1 \hat{R} + \frac{\hat{e}_2}{2}\hat{R}^2 + \frac{\hat{e}_3}{6}\hat{R}^3$$

$$\hat{p}_m = \hat{p}_0 + \hat{p}_1 \hat{R} + \frac{\hat{p}_2}{2}\hat{R}^2 + \frac{\hat{p}_3}{6}\hat{R}^3$$

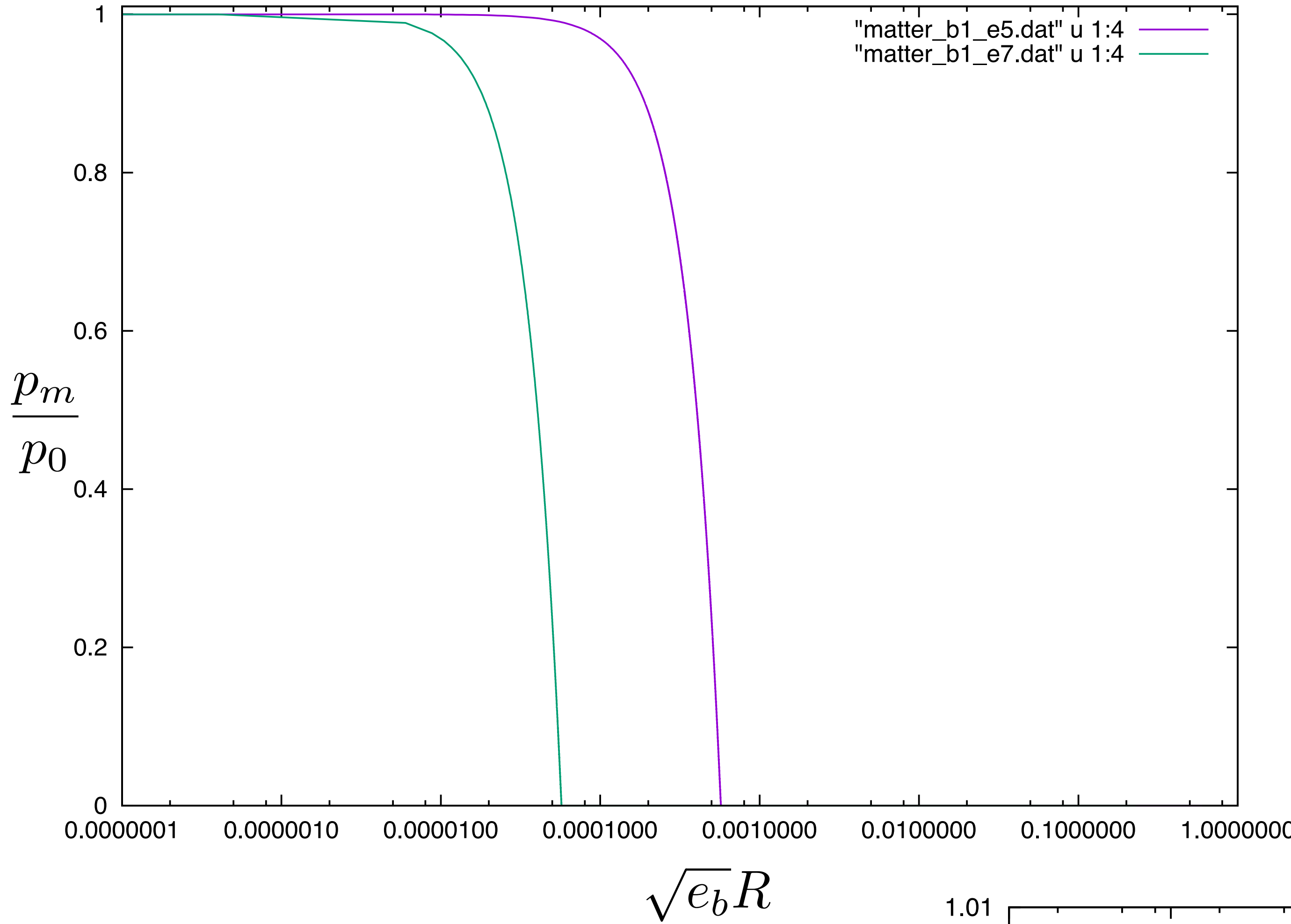
$$\hat{M} = \hat{M}_0 + \hat{M}_1 r + \frac{\hat{M}_2}{2}\hat{R}^2 + \frac{\hat{M}_3}{6}\hat{R}^3$$

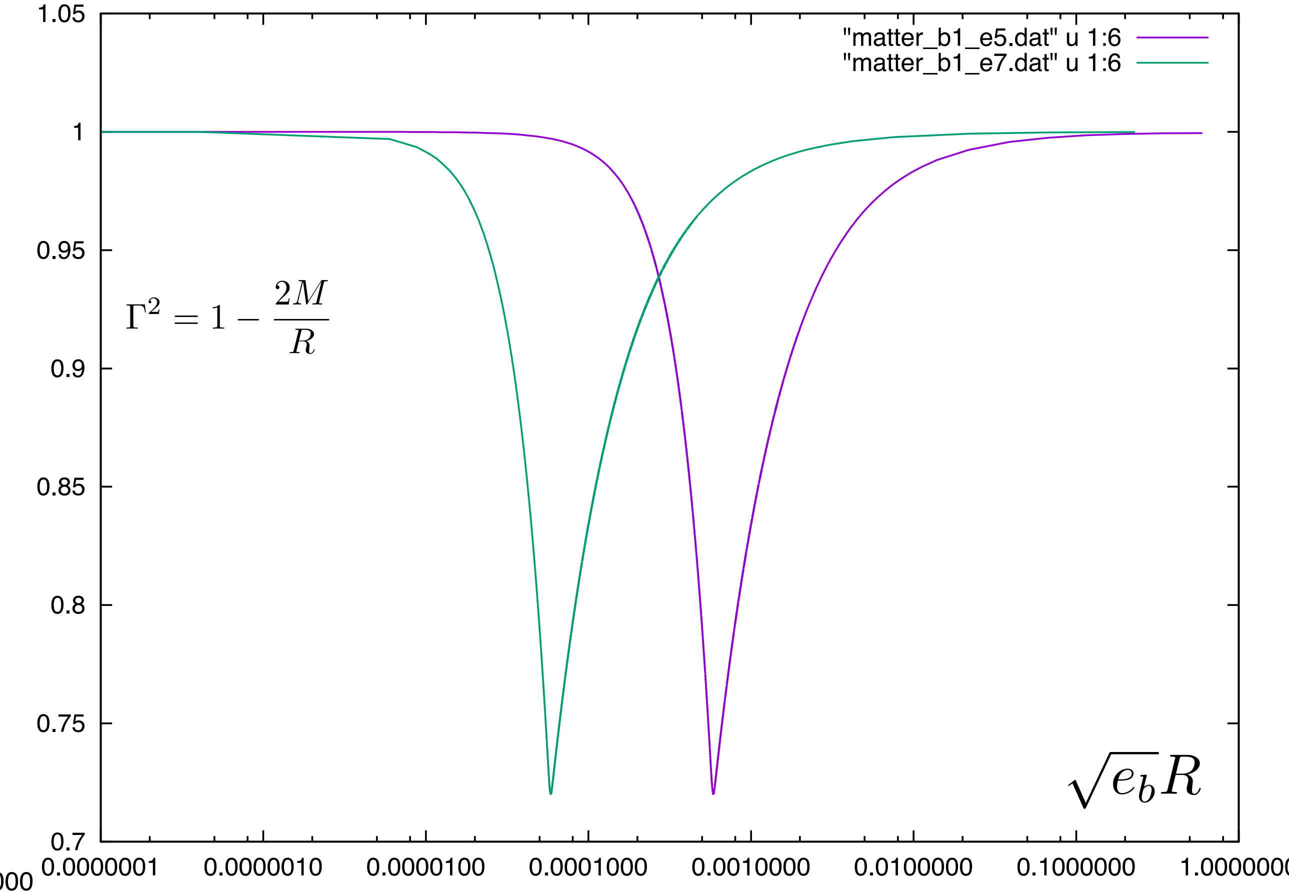
$$\hat{\phi}_2 = \hat{\lambda}_1 = \frac{\hat{\beta}^2}{3W_4}\left[\hat{e}_0 - 3\hat{p}_0 - \hat{\phi}_0^{-(n+1)}\right]$$

$$\hat{p}_2 = -\frac{4\pi}{3}(\hat{e}_0 + \hat{p}_0)\left[\hat{e}_0 + 3\hat{p}_0 - 2\frac{W_4}{n}\hat{\phi}_0^{-n}\right] - W_4(\hat{e}_0 - 3\hat{p}_0)\hat{\phi}_2$$

$$\hat{M}_3 = 8\pi\left[e_0 + \frac{W_4}{n}\hat{\phi}_0^{-n}\right]$$

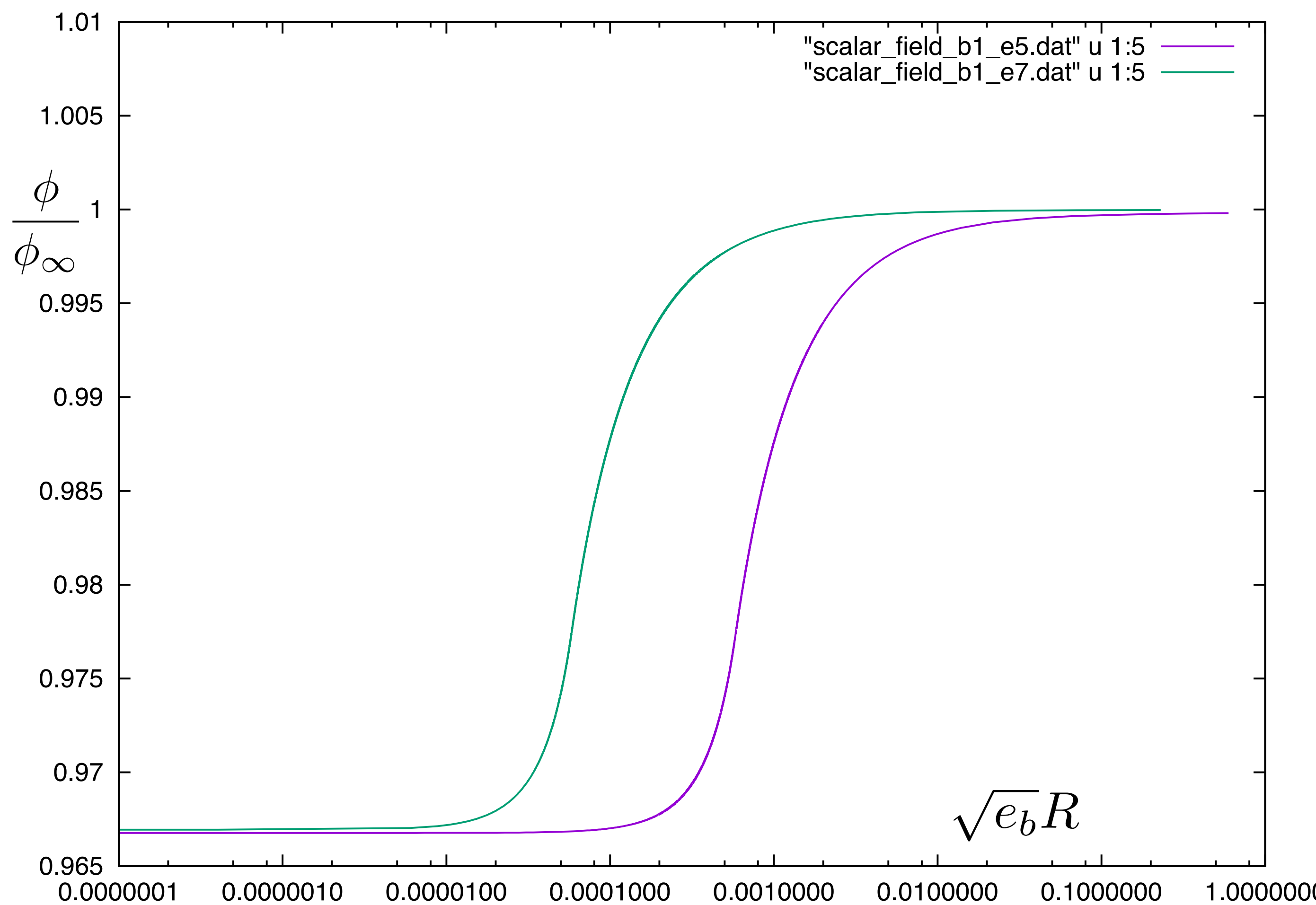
$$\hat{\lambda}_3 = \frac{8\pi}{5}\left[7\hat{e}_0 - 3\hat{p}_0 + 10\frac{W_4}{n}p\hat{h}i_0^{-n}\right]\hat{\phi}_2 + \frac{3\hat{\beta}^2}{5W_4}\left[\hat{e}_2 - 3\hat{p}_2 + (n+1)\hat{\phi}_2\hat{\phi}_0^{-(n+2)}\right]$$

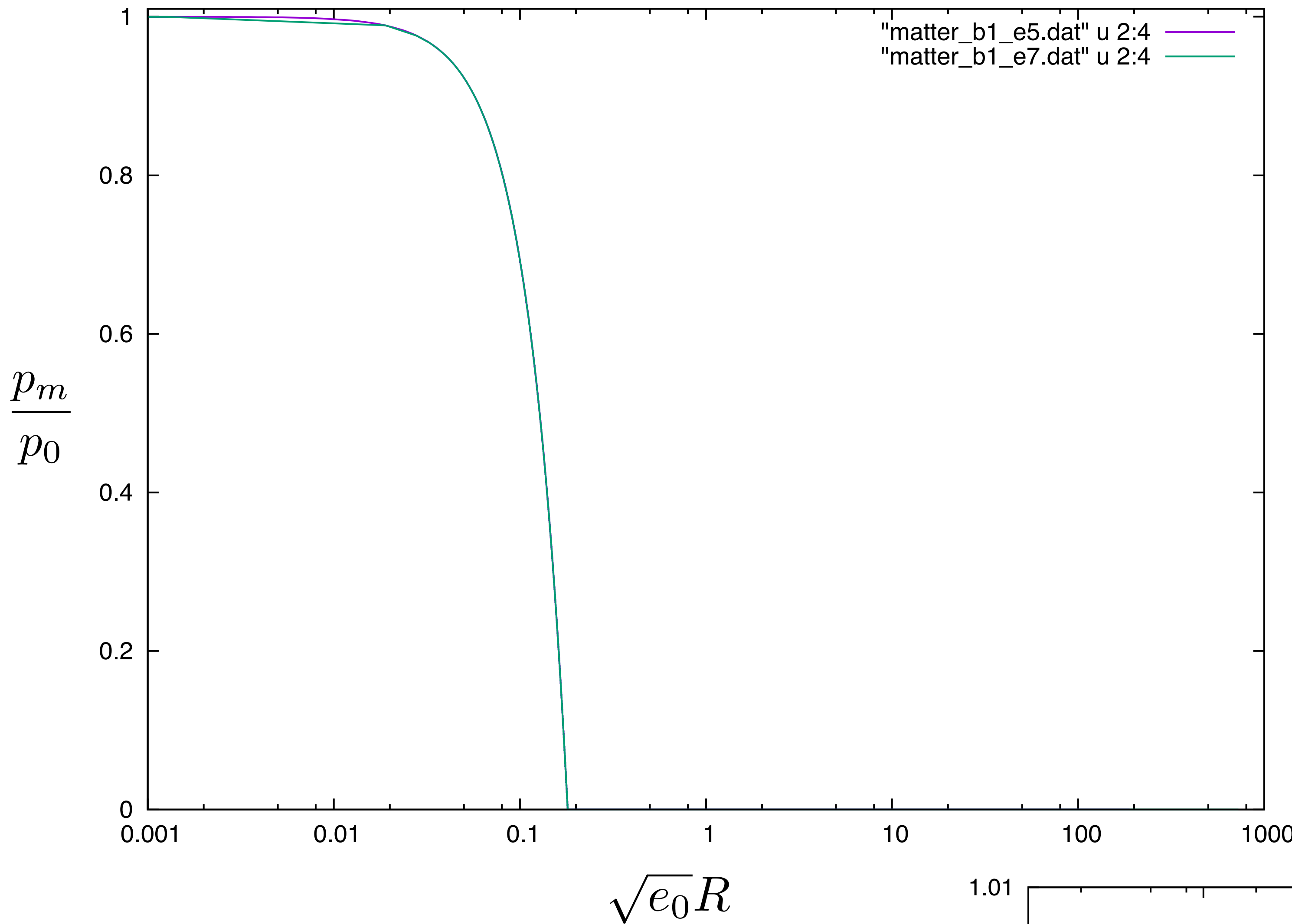




$$\beta = 1$$

$$\frac{e_0}{e_b} = 10^5 - 10^7$$

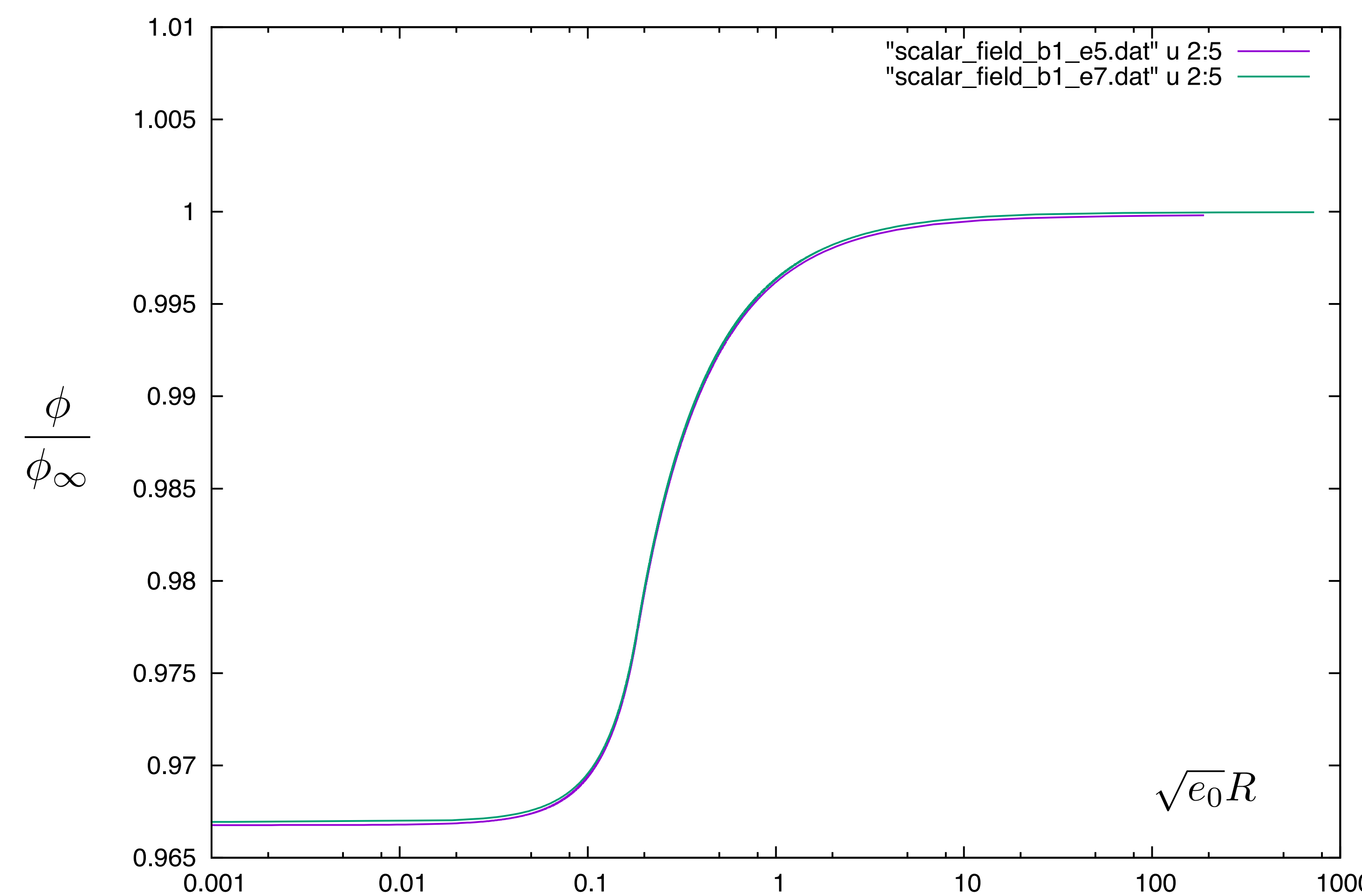


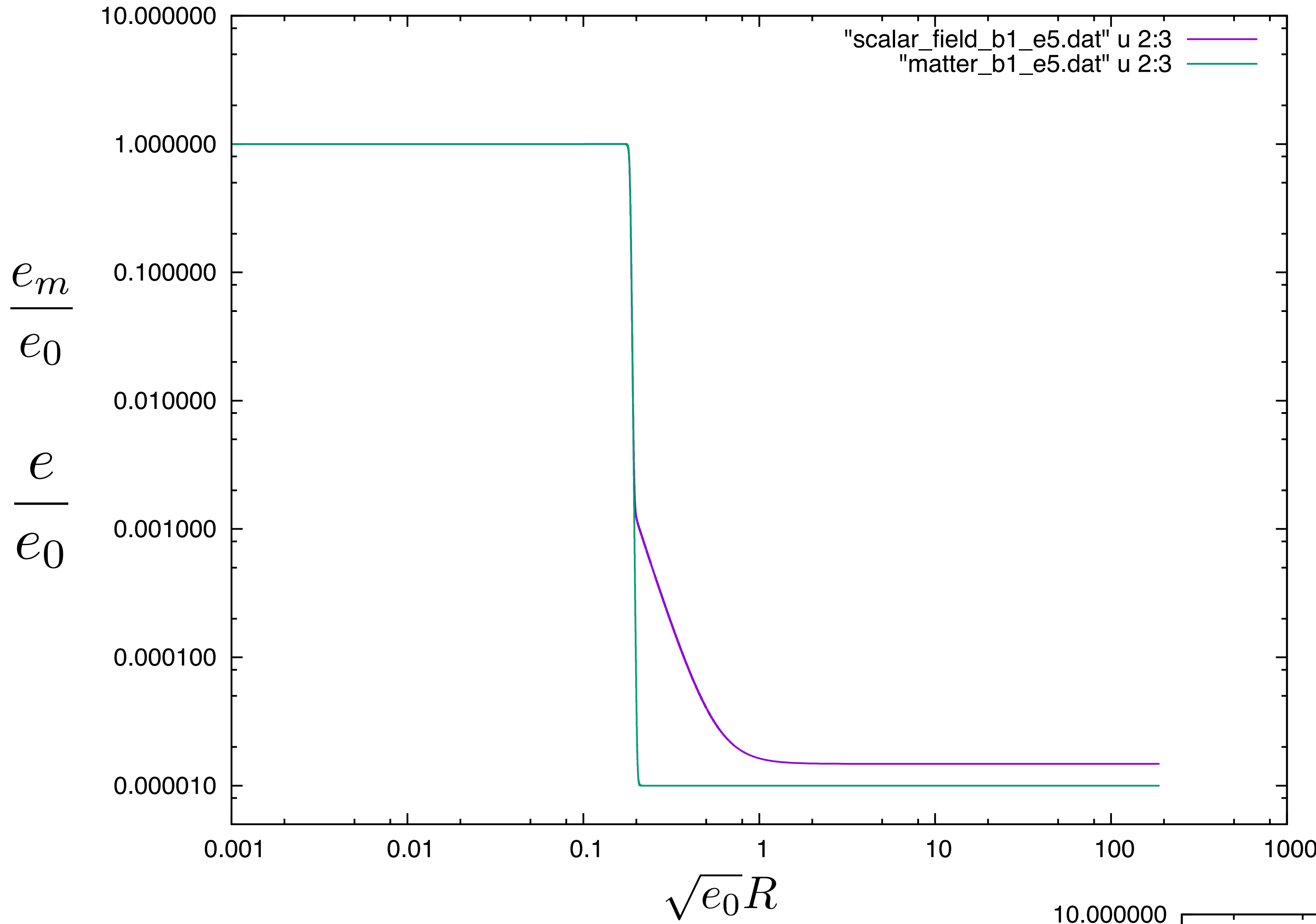


$$\beta = 1$$

$$\frac{e_0}{e_b} = 10^5 - 10^7$$

“Universal” solution:
central value ϕ_0 depends
on compactness M/R
and coupling strength β





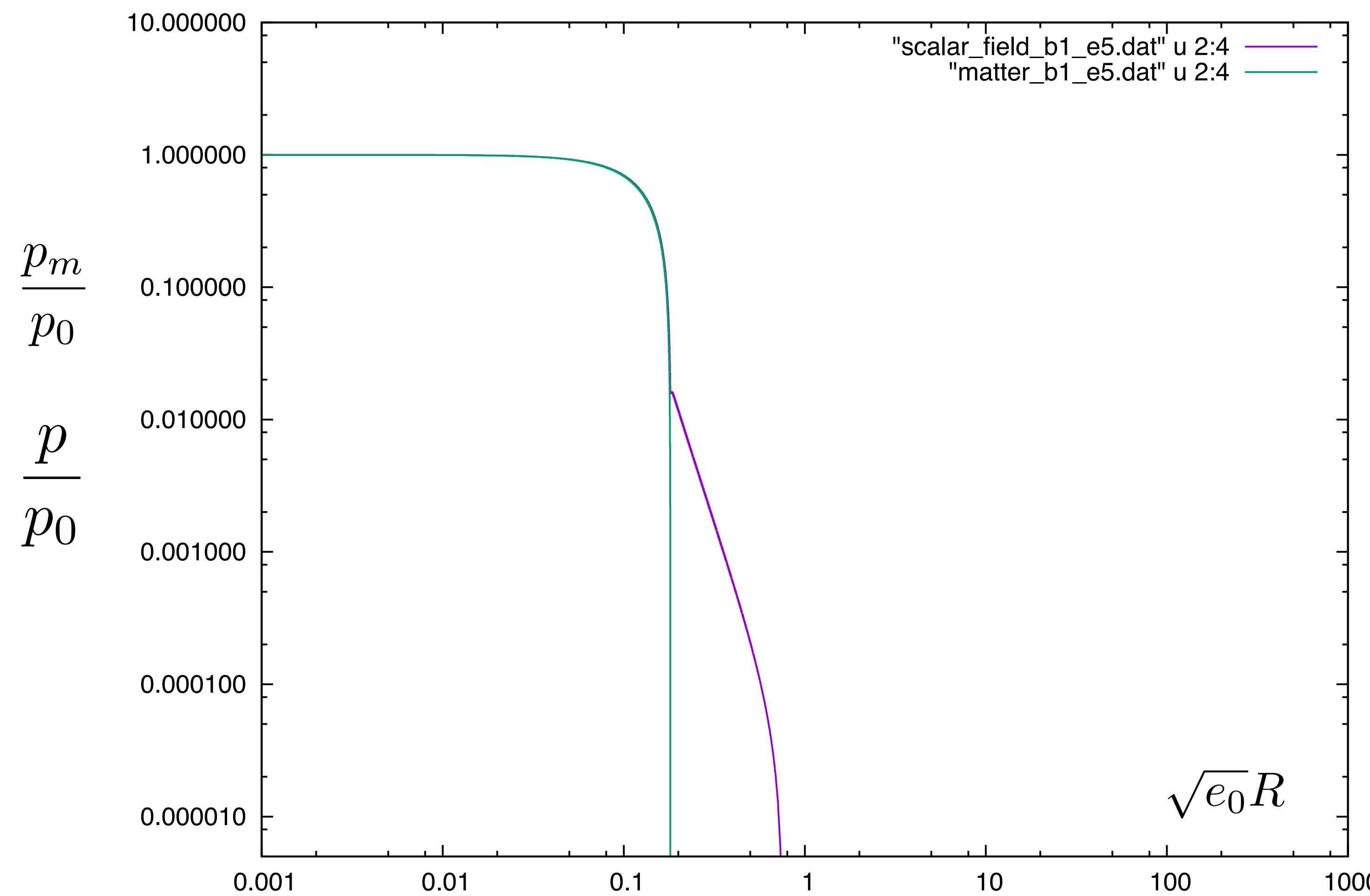
$$\beta = 1$$

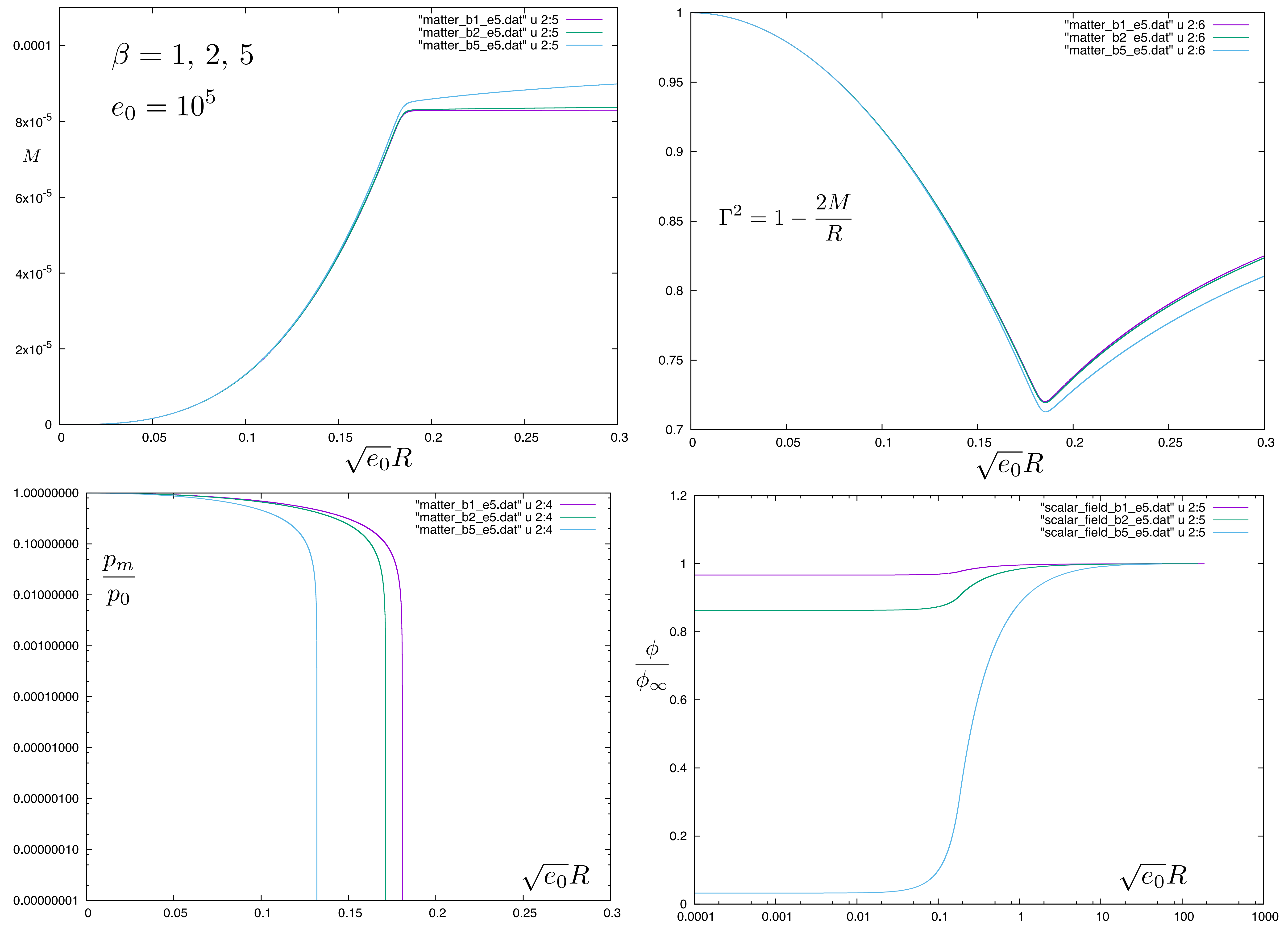
$$\frac{e_0}{e_b} = 10^5 - 10^7$$

$$e = e_m + e_\phi \qquad p = p_m + p_\phi$$

$$e_\phi = \frac{\Gamma^2}{2}\left(\frac{d\phi}{dR}\right)^2 + V(\phi)$$

$$p_\phi = \frac{\Gamma^2}{2}\left(\frac{d\phi}{dR}\right)^2 - V(\phi)$$





$\beta = 1,\ 2,\ 5$
$e_0 = 10^5$
M
"matter_b1_e5.dat" u 2:5
"matter_b2_e5.dat" u 2:5
"matter_b5_e5.dat" u 2:5
0.0001
8x10-5
6x10-5
4x10-5
2x10-5
0
0 0.05 0.1 0.15 0.2 0.25 0.3
$\sqrt{e_0}R$
$\Gamma^2 = 1 - \frac{2M}{R}$
"matter_b1_e5.dat" u 2:6
"matter_b2_e5.dat" u 2:6
"matter_b5_e5.dat" u 2:6
1 0.95 0.9 0.85 0.8 0.75 0.7
0 0.05 0.1 0.15 0.2 0.25 0.3
$\sqrt{e_0}R$
$\frac{p_m}{p_0}$
"matter_b1_e5.dat" u 2:4
"matter_b2_e5.dat" u 2:4
"matter_b5_e5.dat" u 2:4
1.00000000 0.10000000 0.01000000 0.00100000 0.00010000 0.00001000 0.00000100 0.00000010 0.00000001
0 0.05 0.1 0.15 0.2 0.25 0.3
$\sqrt{e_0}R$
$\frac{\phi}{\phi_\infty}$
"scalar_field_b1_e5.dat" u 2:5
"scalar_field_b2_e5.dat" u 2:5
"scalar_field_b5_e5.dat" u 2:5
1.2 1 0.8 0.6 0.4 0.2 0
0.0001 0.001 0.01 0.1 1 10 100 1000
$\sqrt{e_0}R$

Conclusions & Future perpectives

- Chameleon models provide a potentially viable screening mechanism with the scalar field changing with the compactness of the star and with the coupling strength.
- The necessary coupling strength might have a back-reaction effect on the compactness of the star… this could rule out the chameleon model… needing to consider a non constant coupling (fine tuning!).
- More realistic star models (e.g. polytropes) need to be considered to quantify the real effects of the field on stars/compact objects (potential change of compactness?!!). A substantial change of the stellar structure could be observable (constraining chameleon model)!
- The screening from the dark matter halo around the star need to be considered for realistic astrophysical cases.
- Stability of the quasi-static solution?!
- *Musco, Corasaniti, Ferreira, Mota* - in preparation