Generating Luminous and Dark Matter During Inflation

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Matter-antimatter Asymmetry and Dark Matter

The asymmetry is described quantitatively by,

$$\eta = \frac{n_b - n_{\bar{b}}}{s} \simeq 8.5 \times 10^{-11}$$

The Sakharov Conditions

1. Baryon number violation
2. $\mathcal{C}$ and $\mathcal{CP}$ violation
3. Period of non-equilibrium

Standard Model $\rightarrow \eta_{sm} \simeq 10^{-18}$

Dark-to-visible matter mass density ratio:

$$\rho_{DM} \simeq 5.5\rho_B$$

Do they have the same origin?

Why not have both?

- New gauge group: $SU(3) \times SU(2) \times U(1)_Y \times U(1)_X$

New Lagrangian terms:

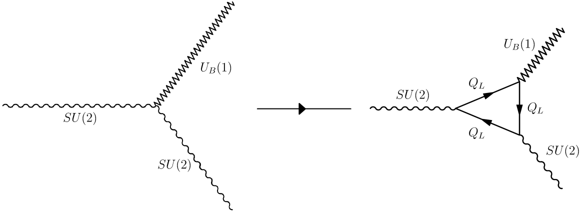
$$\frac{1}{\sqrt{-g}}\mathcal{L}_X = -\frac{1}{4}g^{\mu\alpha}g^{\nu\beta}X_{\mu\nu}X_{\alpha\beta} + \frac{1}{2}f_X^2 g^{\mu\nu}\left(g_X X_\mu - \partial_\mu\theta\right)\left(g_X X_\nu - \partial_\nu\theta\right) - \mathcal{A}_2\frac{g_X^2}{16\pi^2}\theta(x)X_{\mu\nu}\tilde{X}^{\mu\nu}$$

- New sterile fermion ψ.

Satisfying the Sakharov Conditions

1. Anomalous currents,
2. Counter terms in the cosmological setting,
3. Inflationary epoch.

Anomalies and the Green-Schwarz mechanism



- Current corrections:

$$\partial_\mu j^\mu = \mathcal{A}\frac{g_F^2}{16\pi^2}F_{\mu\nu}\tilde{F}^{\mu\nu}$$

- Gauge anomalies $\rightarrow$ breakdown in gauge invariance.
- Non-invariance of the effective action matched by counter terms.
- Introduce ($\mathcal{CP}$ invariant) counter terms of the form:

$$\mathcal{L} = -\frac{g^2}{16\pi^2}\mathcal{A}\theta(x)F_{\mu\nu}\tilde{F}^{\mu\nu}$$

$\mathcal{CP}$ Violation and Particle Production

- Considering a FRW universe $\Rightarrow f_X\theta(x) \to f_X\theta(t) = \phi(t)$
- $\mathcal{CP}$ violation: $\mathcal{CP}(\phi(t)F_{\mu\nu}\tilde{F}^{\mu\nu}) = -\phi(t)F_{\mu\nu}\tilde{F}^{\mu\nu}$

- Expanding spacetime $\to$ time dependent vacuum state.
- Bogoliubov transformation $\to$ accumulated particle number.
- Anomalous currents lead to the generation of X charge.

The Chern-Simons number density,

$$n_{CS} = n_X = n_g \frac{g_X^2}{32\pi^2} \int d^3x \epsilon^{ijk} Tr(X_i \partial_j X_k + \frac{2ig_X}{3} X_i X_j X_k)$$

The n_{CS} accumulated during inflation will be considered.

Calculating $\frac{\rho_{DM}}{\rho_B}$ and η_B

Field Quantisation and Mode Functions

$$\mathcal{L}_X = -\frac{1}{4}\eta^{\mu\alpha}\eta^{\nu\beta}X_{\mu\nu}X_{\alpha\beta} + \frac{1}{2}a(\tau)^2\eta^{\mu\nu}\left(m_X X_\mu - \partial_\mu\phi(\tau)\right)\left(m_X X_\nu - \partial_\nu\phi(\tau)\right)$$
$$- \mathcal{A}_2\frac{g_X^2\phi(\tau)}{32\pi^2 f_X}\epsilon^{\mu\nu\alpha\beta}X_{\mu\nu}X_{\alpha\beta}$$

Solving for circularly polarised wave modes ($\alpha = +, -$),

$$X_i = \int \frac{d^3\vec{k}}{(2\pi)^{3/2}}\sum_\alpha \left[G_\alpha(\tau, k)\epsilon_{i\alpha}\hat{a}_\alpha \mathrm{e}^{i\vec{k}\cdot\vec{x}} + G^*_\alpha(\tau, k)\epsilon^*_{i\alpha}\hat{a}^\dagger_\alpha \mathrm{e}^{-i\vec{k}\cdot\vec{x}}\right]$$

where $a(\mathbf{k})\left|0\right\rangle = 0$ and $a^\dagger(\mathbf{k})\left|0\right\rangle = \left|\mathbf{k}\right\rangle$.

$$G''_\pm + \left(k^2 + \frac{m_X^2}{H^2\tau^2} \mp \kappa_X\tau^2 k\right)G_\pm = 0$$

where $\kappa_X = |\mathcal{A}_2|\frac{m_X^2\phi_0' H_{\mathrm{inf}}^2}{4\pi^2 f_X^3}$

Calculating the Generated X Charge Asymmetry

- No significant entropy production after reheating ($s \simeq \frac{2\pi^2}{45} g^* T_{\rm rh}^3$)
- Considering large scale superhorizon modes ($k|\tau| \approx 0$):

$$\begin{aligned}\eta_X = &\frac{n_X}{s} \approx |\mathcal{A}_2| \frac{30 g_X^2}{\pi^{10} g^*} \Gamma\left(\frac{3}{4}\right)^4 e^{-3N_{\rm e}} \left(\frac{\kappa_X}{\mu T_{\rm rh}^2}\right)^{\frac{3}{2}} \\ &\approx 7 \cdot 10^{-11} |\mathcal{A}_2|^{5/2} \left(\frac{m_X}{10^{12}\ {\rm GeV}}\right)^5 \left(\frac{\phi_0'}{M_p^2}\right)^{\frac{3}{2}} \left(\frac{H}{10^{14}\ {\rm GeV}}\right) \\ &\times \left(\frac{T_{\rm rh}}{10^{16}\ {\rm GeV}}\right)^{-2} \left(\frac{f_X}{10^{14}\ {\rm GeV}}\right)^{-\frac{13}{2}} \left(\frac{\mu}{10^{-42}\ {\rm GeV}}\right)^{-\frac{3}{2}}\end{aligned}$$

IR cut-off: $\mu = H_{BBN} \approx 10^{-25}$ GeV and $\mu = H_0 \approx 10^{-42}$ GeV

General Results

- Initial uniform distribution or Sphaleron redistribution.

Dark matter asymmetry:

$$\eta_D = \frac{\sum_i N_D^i |q_D^i|}{\sum_i N_{SM}^i |q_{SM}^i| + \sum_i N_D^i |q_D^i|} \eta_X^{\text{unmixed}}$$

Matter-antimatter asymmetry:

$$\eta_B = \alpha(T_{\text{rh}}) \left(\eta_X^{\text{mixed}} + \frac{\sum_i N_{SM}^i |q_{SM}^i|}{\sum_i N_{SM}^i |q_{SM}^i| + \sum_i N_D^i |q_D^i|} \eta_X^{\text{unmixed}} \right)$$

where $\alpha(T_{\text{rh}} > T_{\text{c}}) = \frac{28}{79}$ and $\alpha(T_{\text{rh}} < T_{\text{c}}) = 1$.

Dark-to-visible mass density ratio $\rightarrow m_\psi$:

$$m_\psi \simeq 5.5 \text{ GeV} \frac{|q_\psi| \eta_B}{\eta_D}$$

Case 1: $U(1)_{B-L}$ and a sterile fermion ψ

Only contains unmixed anomalies. Dark matter mass:

$$m_\psi \approx 66\alpha \text{ GeV}$$

For $\mu = H_0$,

$$\eta_B \approx 3.5 \times 10^{-18} \text{ GeV}^{-1/2} \, \alpha \frac{|3 + q_\psi^3|^{5/2}}{21 + |q_\psi|} \frac{m_X^5}{f_X^5} \frac{H_{\text{inf}}}{T_{\text{rh}}^2} \left(\frac{\phi_0'}{f_X}\right)^{\frac{3}{2}}$$

For $\mu = H_{BBN}$,

$$\eta_B \approx 10^{-43} \text{ GeV}^{-1/2} \, \epsilon \frac{|\mathcal{A}_1|^{5/2}}{21 + |q_\psi|} \frac{m_X^5}{f_X^5} \frac{H_{\text{inf}}}{T_{\text{rh}}^2} \left(\frac{\phi_0'}{f_X}\right)^{\frac{3}{2}}$$

Case 2: $U(1)_B$ and a sterile fermion ψ

Consists of both mixed and unmixed anomalies.

Dark matter mass:

$$m_\psi \approx (12 + |q_\psi| + 12|q_\psi|^{15/2} g_X^5) \frac{11 \alpha g_X^5}{2|q_\psi|^{15/2}} \text{ GeV}$$

For $\mu = H_0$,

$$\eta_B \approx 3 \times 10^{-19} \text{ GeV}^{-1/2} \alpha \left(\frac{12|q_\psi|^{15/2}}{|q_\psi| + 12} \frac{m_X^5}{f_X^5} + 1 \right) \frac{H_{\text{inf}}}{T_{\text{rh}}^2} \left(\frac{\phi_0'}{f_X} \right)^{\frac{3}{2}}$$

For $\mu = H_{BBN}$,

$$\eta_B \approx 10^{-44} \text{ GeV}^{-1/2} \epsilon \mathcal{A} \frac{H_{\text{inf}}}{T_{\text{rh}}^2} \left(\frac{\phi_0'}{f_X} \right)^{\frac{3}{2}}$$

Allowed Paramater Summary

- For $U(1)_{B-L}$, less constrained due to fixed m_ψ,
- $H_{\rm inf} \sim 10^4$, $m_X < 100 keV$ (using $\mu = H_{\rm BBN}$: $m_X \gtrsim 15$ GeV) with $g_X^2 \sim 10^{-16}$ (using $\mu = H_{\rm BBN}$: $g_X^2 \sim 10^{-6}$),

- For $U(1)_B$, g_X too small in can be hard to satisfy the two required constraints,
- m_X cannot be too small relative to $H_{\rm inf}$,
- $10^{-5} \lesssim g_X^2 \lesssim 0.01$ for $H_{\rm inf} \approx 10^{14}$ GeV $\rightarrow 3 \times 10^{11}$ GeV $\lesssim m_X \lesssim 10^{13}$ GeV. Lower $H_{\rm inf}$ shrinks this window. Well out of range of current colliders.

Conclusion and Future Work

- Gauge boson which couples to X charge, and dark matter candidate.
- Inflationary model for the origin of both luminous and dark matter.
- Can produce the observed η_B and $\frac{\rho_{DM}}{\rho_B}$.
- General framework which can be used for more complicated dark sectors or gauge group structures.

Future work

- Further exploration of the mechanism
- LHC Phenomenology
- Cosmological implications

Wave Mode Functions

$$G_+(\tau,k)=2^{\frac{1+\nu}{2}}e^{-\frac{k^2\tau^2}{2\Omega_k}}\tau^{\frac{1}{2}+\nu}\left[C_1U\left(\frac{1+\nu}{2}-\frac{\Omega_k}{4},1+\nu,\frac{k^2\tau^2}{\Omega_k}\right)+C_2M\left(\frac{1+\nu}{2}-\frac{\Omega_k}{4},1+\nu,\frac{k^2\tau^2}{\Omega_k}\right)\right]\ ,$$

$$G_-(\tau,k)=2^{\frac{1+\nu}{2}}e^{\frac{ik^2\tau^2}{2\Omega_k}}\tau^{\frac{1}{2}+\nu}\left[C_3U\left(\frac{1+\nu}{2}-\frac{i\Omega_k}{4},1+\nu,\frac{k^2\tau^2}{i\Omega_k}\right)+C_4M\left(\frac{1+\nu}{2}-\frac{i\Omega_k}{4},1+\nu,\frac{k^2\tau^2}{i\Omega_k}\right)\right]\ ,$$

where $\Omega_k=\left(\frac{k^3}{\kappa X}\right)^{1/2}$, $\nu=\frac{1}{2}\sqrt{1-4\lambda^2}\sim\frac{1}{2}-\lambda^2$

$$F_+(\tau,k)=\frac{\sqrt{\pi\tau}}{2}H_\nu^{(2)}(k\tau)e^{-i\frac{\pi}{2}(\frac{1}{2}+\nu)}\ \text{and}\ F_-(\tau,k)=\frac{\sqrt{\pi\tau}}{2}H_\nu^{(1)}(k\tau)e^{i\frac{\pi}{2}(\frac{1}{2}+\nu)}$$

Coefficients

Matching to planewaves solutions in the limit $k|\tau| \to 0$:

$$C_1 = \frac{\sqrt{\pi}\Gamma(\frac{1+\nu}{2} - \frac{\Omega_k}{4})}{2^{\frac{1}{2}(3-\nu)}} \left(\frac{k}{\Omega_k}\right)^{\nu} e^{i\frac{\pi}{2}(\frac{1}{2}-\nu)}$$

$$C_3 = \frac{\Gamma(\frac{1+\nu}{2} - \frac{i\Omega_k}{4})}{2^{\frac{1}{2}(3-\nu)}\sqrt{\pi}} \left(\frac{k}{\Omega_k}\right)^{\nu} e^{-i\frac{\pi}{4}}$$

Wronskian normalisation:

$$C_2 = \frac{k^{\nu}}{2^{\frac{1}{2}(1+\nu)}\sqrt{\pi}} e^{-i\frac{\pi}{2}(\frac{1}{2}+\nu)}$$

$$C_4 = \frac{\sqrt{\pi}k^{\nu}e^{i\frac{\pi}{2}(\frac{1}{2}+\nu)}}{2^{\frac{1}{2}(3-\nu)}\Gamma(1+\nu)} \frac{\Gamma\left(\frac{1+\nu}{2}+\frac{i\Omega_k}{4}\right)}{\Gamma\left(\frac{1-\nu}{2}+\frac{i\Omega_k}{4}\right)} \mathrm{Im}\left(\frac{\Gamma\left(\frac{1+\nu}{2}+\frac{i\Omega_k}{4}\right)}{\Gamma\left(\frac{1-\nu}{2}+\frac{i\Omega_k}{4}\right)}\right)^{-1} \left(1 + e^{\frac{\pi\Omega_k}{4}} \frac{|\Gamma(\frac{1+\nu}{2}-\frac{i\Omega_k}{4})|^2}{\pi 2^{1-2\nu}\Omega_k^{\nu}}\right)$$

Bogoliubov Transformations

The two sets of creation and annihilation operators, $\{\hat{a}^a_\alpha, \hat{a}^{a\dagger}_\alpha\}$ and $\{\hat{b}^a_\alpha, \hat{b}^{a\dagger}_\alpha\}$ are related through the Bogoliubov transformations:

$$\hat{b}^a_\alpha(\vec{k}) = \alpha_\alpha a^{a\dagger}_\alpha(\vec{k}) + \beta^*_\alpha \hat{a}^a_\alpha(\vec{k})$$
$$\hat{b}^{a\dagger}_\alpha(\vec{k}) = \alpha^*_\alpha a^a_\alpha(\vec{k}) + \beta_\alpha \hat{a}^{a\dagger}_\alpha(\vec{k})$$

The relevant Bogoliubov coefficients can be computed explicitly:

$$\alpha_\pm = \frac{1}{2^{1-\nu}}\left(2^{1-\nu} - 1 \mp \frac{i\lambda^2}{(k\tau)^{1-2\lambda^2}}\left(1 - \frac{\pi(k\tau)^{1-2\lambda^2}}{2^\nu}\right) \pm \frac{i2^{1-\nu}(k\tau)^{\lambda^2}}{\sqrt{k}} e^{\mp i\pi\lambda^2/2} G'^*_\pm|_{\frac{\kappa\tau^2}{k}, k|\tau|\to 0}\right)$$

$$\beta_\mp = \frac{e^{\pm i\pi\lambda^2}}{2^{1-\nu}}\left(1 \pm \frac{i\lambda^2}{(k\tau)^{1-2\lambda^2}}\left(1 - \frac{\pi(k\tau)^{1-2\lambda^2}}{2^\nu}\right) \mp \frac{i2^{1-\nu}(k\tau)^{\lambda^2}}{\sqrt{k}} e^{\mp i\pi\lambda^2/2} G'^*_\pm|_{\frac{\kappa\tau^2}{k}, k|\tau|\to 0}\right)$$