COSMO 2015

Random Potentials in Cosmology

arxiv: 1203.3941,1304.0461,
1409.5135, 1509.????

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In collaboration with:

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Motivation

<u>CMBR obs. via Planck:</u> Inflation is well described by an (effective) single field model

Single field

Consistent with all experiments

Motivation

<u>Theoretical bias:</u> large number of degrees of freedom appear generic.

Multi-field

Generic in String Theory

Motivation

Given the plethora of models, can we make statistical statements about likely predictions on certain landscapes?

How to create large numbers of ``random'' landscapes efficiently?

Single field
Consistent with all experiments

vs.

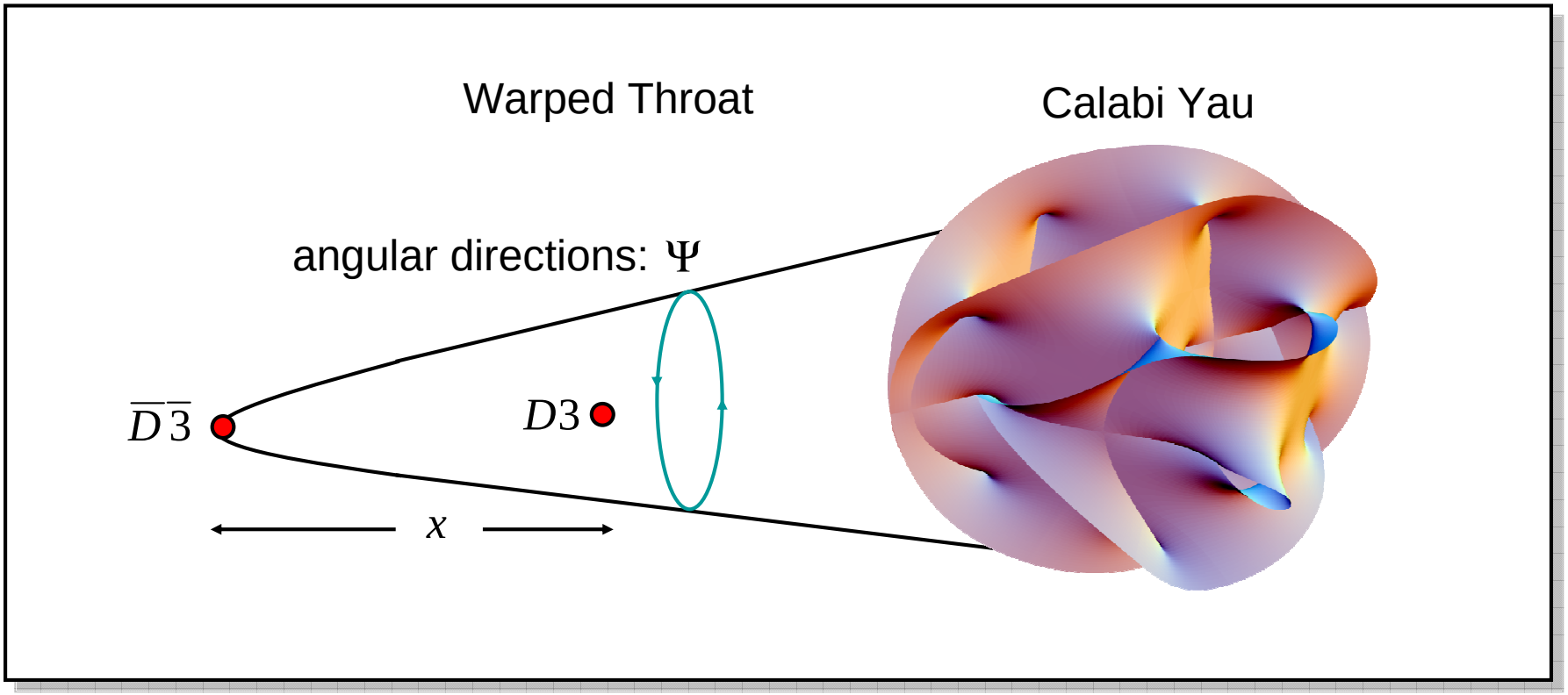
Multi-field
Generic in String Theory

Why Random Multi-field Potentials?

Low energy effective potential is sensitive to the concrete (unknown) stringy construction.

Example: KKLMMT proposal with turned on angular directions and effect of bulk physics:

Agarwal, Bean, McAllister, Xu 11; McAllister, Renaux-Petel, Xu 12
See also: Dias, Frazer, Liddle 12, …



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Coulomb potential between branes

$$V(x,\Psi) = V_0 + V_{\mathcal{C}}(x) + V_{\mathcal{R}}(x) + V_{\text{bulk}}(x,\Psi)$$

Distant sources Curvature Bulk physics

Unknown (from first principles) bulk physics (CY):

$$V_{\text{bulk}}(x,\Psi) = \mu^4 \sum_{LM} c_{LM} x^{\delta(L)} f_{LM}(\Psi)$$

``Random'' coefficients:

$$c_{LM} \sim \mathcal{O}(1)$$

Such random landscapes appear to be generic in string theory.

Identify generic features of inflation on random landscapes to test if the basic idea of a landscape in string theory is consistent with existing experiments or if fine tuning is needed.

Questions

- How to model/parameterize random landscapes?
- Distribution of Minima, Maxima, Saddles? (Important for inflation and vacuum selection).
- How likely is Inflation in a random potential?
- Type of inflation? Small/large field?
- Duration of inflation?
- How does inflation end and where do we end up?
- How do the answers change if the dimensionality of field space ``D'' is increased?
- What are generic predictions? (Curvature? Anomalies? …)
- Predictions of observables in ensambles of potentials, e.g.

  Dias, Frazer, Liddle 12; McAllister, Renaux-Petel, Xu 12, …

Creating Random Potentials

<u>Truncated Fourier Series (TFS):</u> generate potential globally

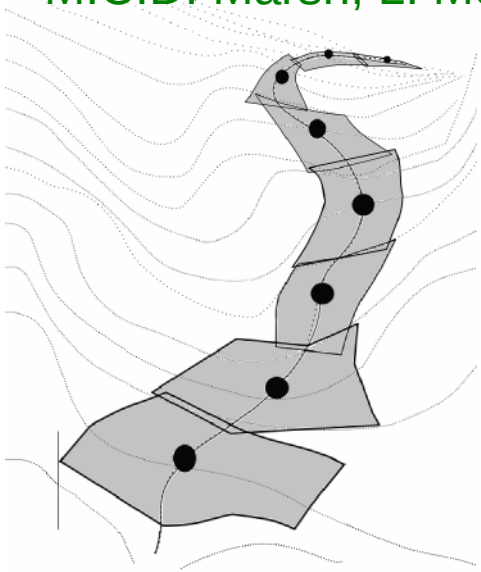
Tegmark 04; Frazer, Liddle 11; T.B., D.B., S.Schulz 12; …

$$V = \sum_{J_1,\ldots,J_D=1}^{n} \left(a_{J_1\ldots J_D} \cos \sum_{i=1}^{D} J_i x_i + b_{J_1\ldots J_D} \sin \sum_{i=1}^{D} J_i x_i \right)$$

Number of coefficients needed to specify potential: $\boxed{\propto n^D}$

<u>Dyson Brownian Motion (DBM):</u> generate potential locally, patch together smoothly

M.C.D. Marsh, L. McAllister, E. Pajer, T. Wrase 13; T.B., C. Modi 14.



$$V = \Lambda_v^4 \sqrt{N} \left[v_0 + v_a \tilde{\phi}^a + \frac{1}{2} v_{ab} \tilde{\phi}^a \tilde{\phi}^b \right]$$

Number of parameters needed to randomly perturb Hessian:

$$\boxed{\propto nD^2}$$

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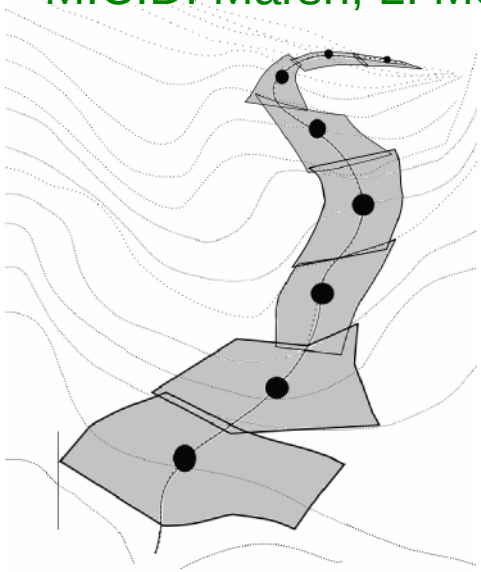

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D ~ 100 is simple to accomplish via DBM, while D~10 is already hard for TFS.

Status and goals:

In M.C.D. Marsh, L. McAllister, E. Pajer, T. Wrase 13:

- Inflationary background solution,
- simple rules for choosing the Hessian (proof of concept),
- several consistency checks with RMT results and comparison of DBM with TFS,
- two instructive case studies (inflation from last patch or saddle),
- application: Gaussian random supergravities, Bachlechner 14,
- see also: small disorder on smooth potential Tye et.al. 08, D.Greene 14.

Goals:

- application to realistic landscapes
  Need: defining properties of landscapes to specify PD of Hessian.

- generalization to the perturbed level:
  Problem: DBM entails jumps in Hessian,
  resulting in artifacts in correlation functions.

- searching for minima
  Problem: DBM entails jumps in Hessian (alternatively Bachlechner 14),
  resulting in sudden turns as minimum is approached.

Our current work:

T.B., C.Modi 14: extending DBM (this talk)

- extend DBM: perturb tensor of higher derivatives instead of Hessian
- generate potentials in any desired differentiability class

T.B. M. Breuhaus apply DBM to non-canonical field space metric (BS thesis, work in progess)

T.B., L.Schmidt: apply DBM to bounded potentials (PhD thesis, to appear)

- apply to similar setups as in T.B., D.B., S.Schulz 12 and T.B., D.B. 13. Proof of concept studies and check consistency.

T.B., Adrian Lux perturbations in extended DBM potentials (MS thesis in progress)

- incorporate DBM into MultiModecode by L.C. Price, J.Frazer, J.Xu, H. V. Peiris, R. Easther 14

T.B., G. Wang: Axionic Landscapes (BS thesis, publication in prep.)

- dynamical vacuum selection in presence of a sharp lower bound.
  (see also M. Dine, S. Paban 15)

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How can we improve on potentials generated via
Dyson Brownian Motion?

DBM Potentials: some technical details

After step (delta s), perturb the Hessian:

$$\boxed{\delta\mathcal{H}_{ab} = \delta A_{ab} - \mathcal{H}_{ab}\frac{\delta s}{\Lambda_h}}$$

If perturbations are chosen to be Gaussian with mean and standard variation:

$$\langle \delta\mathcal{H}_{ab}|_{p_1}\rangle = -\mathcal{H}_{ab}|_{p_0}\frac{\delta s}{\Lambda_h}\,,$$

$$\langle (\delta\mathcal{H}_{ab})^2\rangle = (1+\delta_{ab})\frac{\delta s}{\Lambda_h}\sigma^2$$

the Hessian (at well separated points) is a random matrix in the Gaussian orthogonal ensemble (Wiegner matrix, SD sigma).

What does this entail for the potential?

DBM Potentials: some technical details

Consider Potential:

$$V = \Lambda_v^4 \sqrt{D} \left[v_0 + v_a \tilde{\phi}^a + \frac{1}{2} v_{ab} \tilde{\phi}^a \tilde{\phi}^b + \dots \right]$$

Once trajectory moved fare enough,

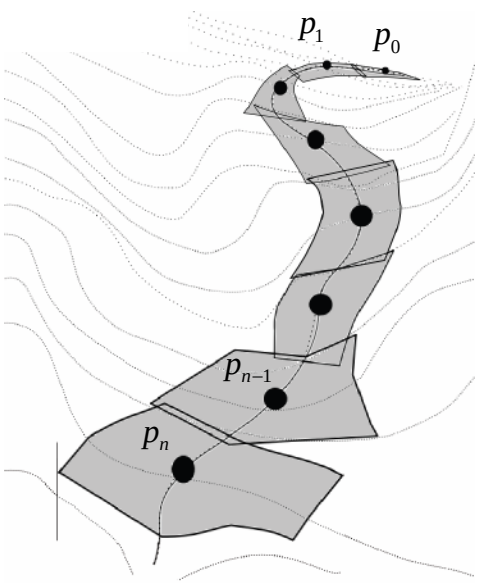
$$\begin{aligned} v_0 |_{p_1} &= v_0 |_{p_0} + v_a |_{p_0} \, \delta\tilde{\phi}^a + \frac{1}{2} v_{ab} |_{p_0} \, \delta\tilde{\phi}^a \delta\tilde{\phi}^b + \dots \\ v_a |_{p_1} &= v_a |_{p_0} + v_{ab} |_{p_0} \, \delta\tilde{\phi}^b + \dots \\ v_{ab} |_{p_1} &= v_{ab} |_{p_0} + \dots \end{aligned}$$

add perturbation to

$$v_{ab} |_{p_1} = v_{ab} |_{p_0} + \delta v_{ab} |_{p_0}$$

with mean and SD:

$$\boxed{\begin{aligned} \langle \delta v_{ab} |_{p_n} \rangle &= -v_{ab} |_{p_{n-1}} \frac{\|\delta\phi^a\|}{\Lambda_h} , \\ \langle (\delta v_{ab} |_{p_n})^2 \rangle &= (1 + \delta_{ab}) \frac{\|\delta\phi^a\|}{\Lambda_h} \sigma^2 \end{aligned}}$$



Creating Random Potentials of High Differentiability:

Delegate perturbations to higher order tensors, while imposing condition on Hessian, e.g. if perturbation is at third order

$$\langle \delta v_{ab}\,|_{p_n}\rangle = \langle v_{abc}\,|_{p_{n-1}}\,\delta\tilde{\phi}^c\rangle \equiv -v_{ab}\,|_{p_{n-1}}\,\frac{\|\delta\phi^a\|}{\Lambda_h}\,,$$

$$\mathrm{Var}(v_{ab}|_{p_n}) = \mathrm{Var}(v_{abc}\,|_{p_{n-1}}\,\delta\tilde{\phi}^c) \equiv \frac{(1+\delta_{ab})\|\delta\phi^a\|\sigma^2}{\Lambda_h} - \left(-\,v_{ab}|_{p_{n-1}}\frac{\|\delta\phi^a\|}{\Lambda_h}\right)^2$$

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<u>Problem:</u> sum in the middle expressions.

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<u>Problem:</u> sum in the middle expressions.

<u>Solution:</u> rotate field space, i.e. to align step with one of the fields; in this coordinate system:

$$< \delta v_{abi}|_{p_{n-2}} > = -v_{ab}|_{p_{n-1}} - v_{abi}|_{p_{n-2}}$$

$$\mathrm{Var}(\delta v_{abi}|_{p_{n-2}}) = (1+\delta_{ab}) \frac{\Lambda_h}{\delta s} \sigma^2 - \left(v_{ab}|_{p_{n-1}} \right)^2$$

In addition to conditions from symmetry.

Creating Random Potentials of High Differentiability:

There are $D(D-1)^2/6$ unspecified means (similar for variances).

Freedom of choice:
Hessians have the same statistical properties,
but potentials differ at higher derivatives.

We investigated to choices:

1. Keep as many entries unchanged (``smoothest potentials'').

2. Employ a different rotation (coordinate system spanned by Eigenvectors of Hessian) and choose ``simple'' rules.

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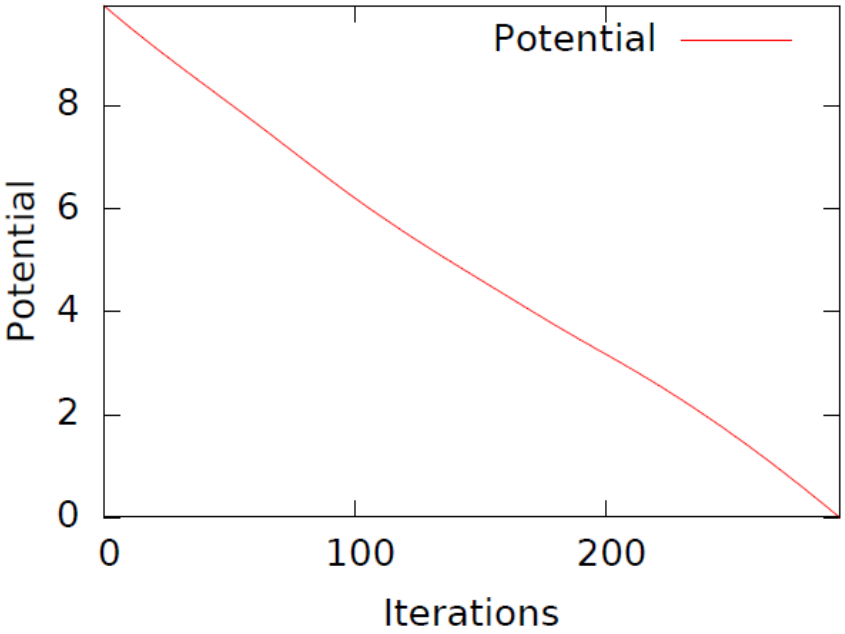
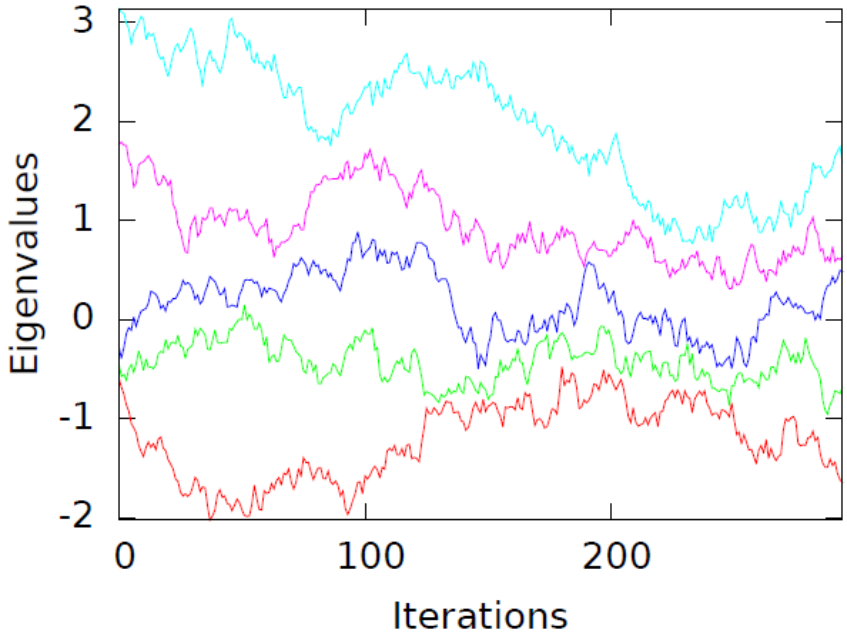
We further generalized procedure 1 to any desired order of differentiability.

Example DBM: 5 fields, potential along path of steepest descent:



(a) $V \in C^1$ via Dyson Brownian Motion



(b) $V \in C^1$ - Eigenvalue Relaxation.

$$\Lambda_v = M_P = 1,\ \Lambda_h = 0.1 M_P = 0.1,\ \delta s = \Lambda_h/100 \text{ and } \sigma^2 = 2/D$$

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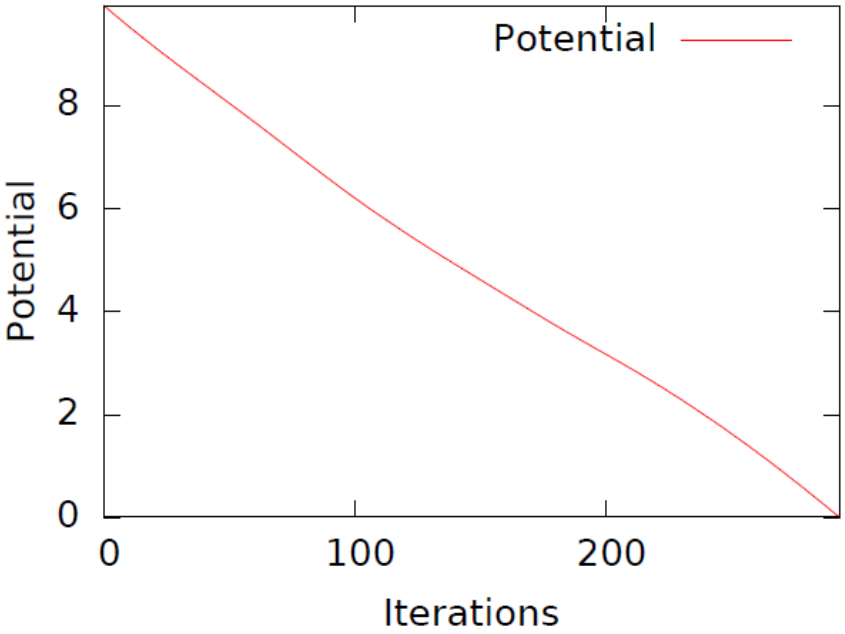

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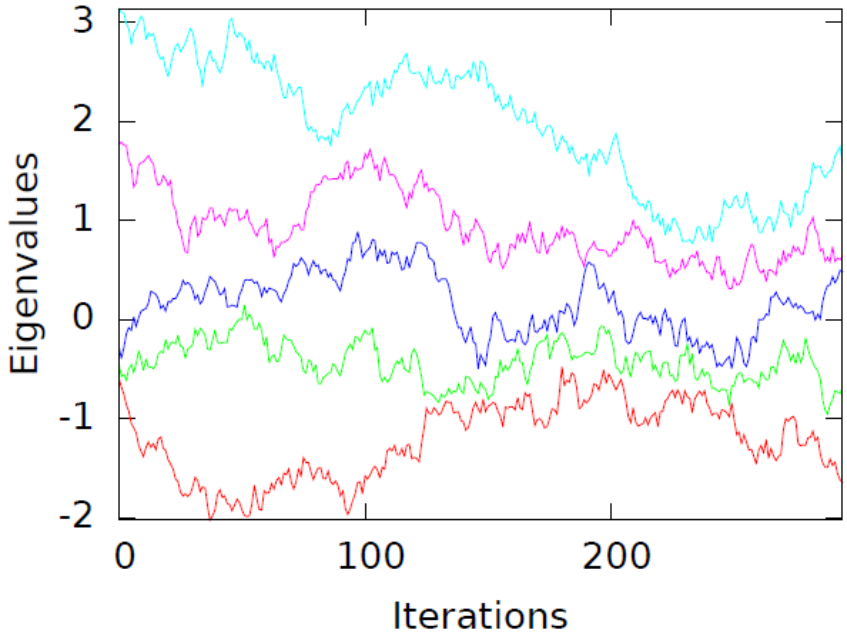

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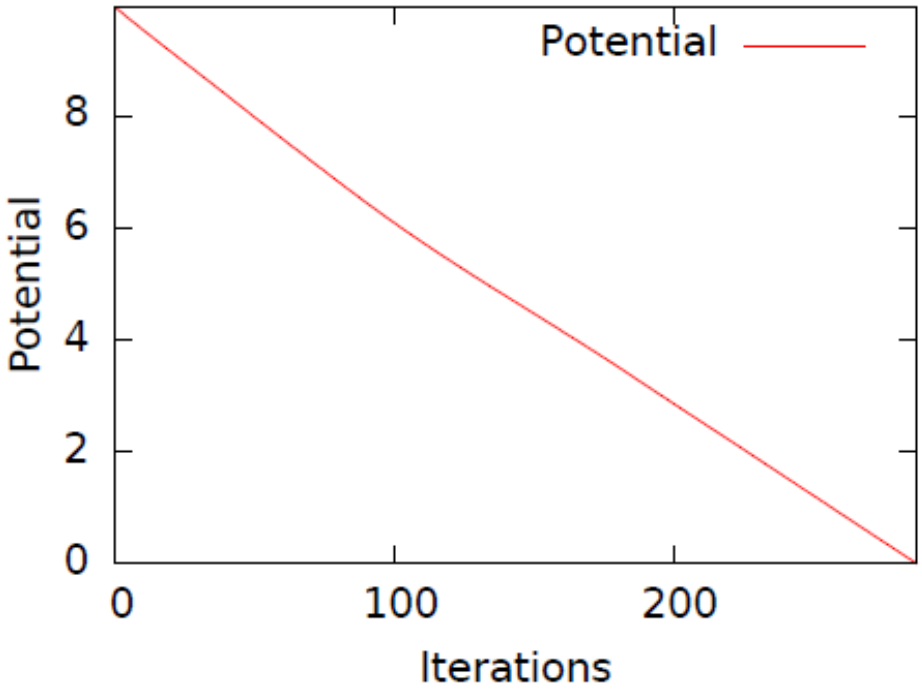
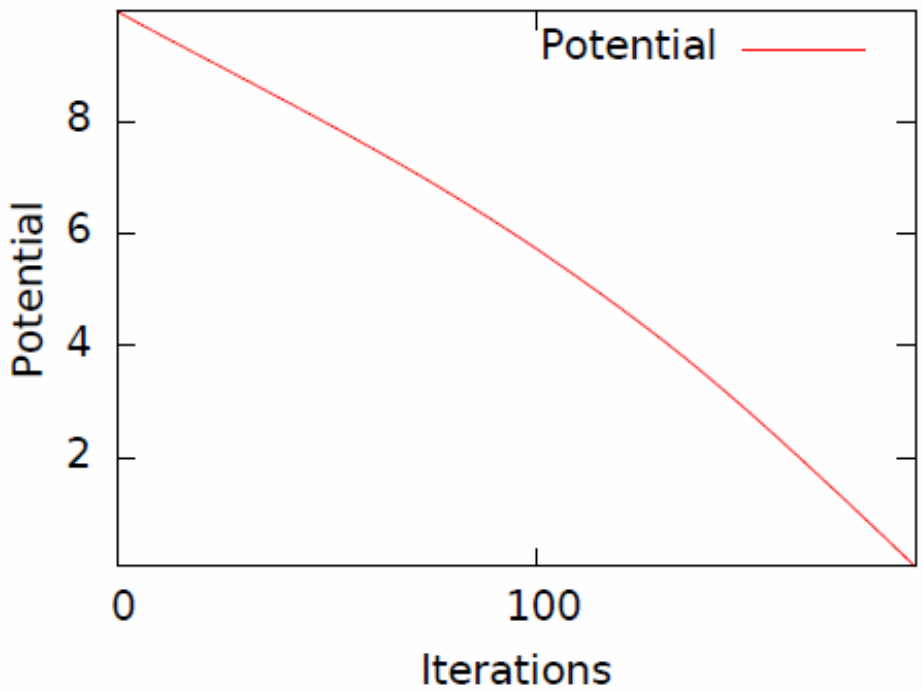
Eigenvalue Relaxation:
Initial eigenvalues are forgotten after the horizontal correlation length is passed (100 steps).

Example C^2: 5 fields, potential along path of steepest descent:



(b) $V \in C^2$ - Rotation to diagonalize Hessian.



(c) $V \in C^2$ - Rotation to align basis vector with $\delta\tilde{\phi}$.

Potentials ``look'' qualitatively indistinguishable, …

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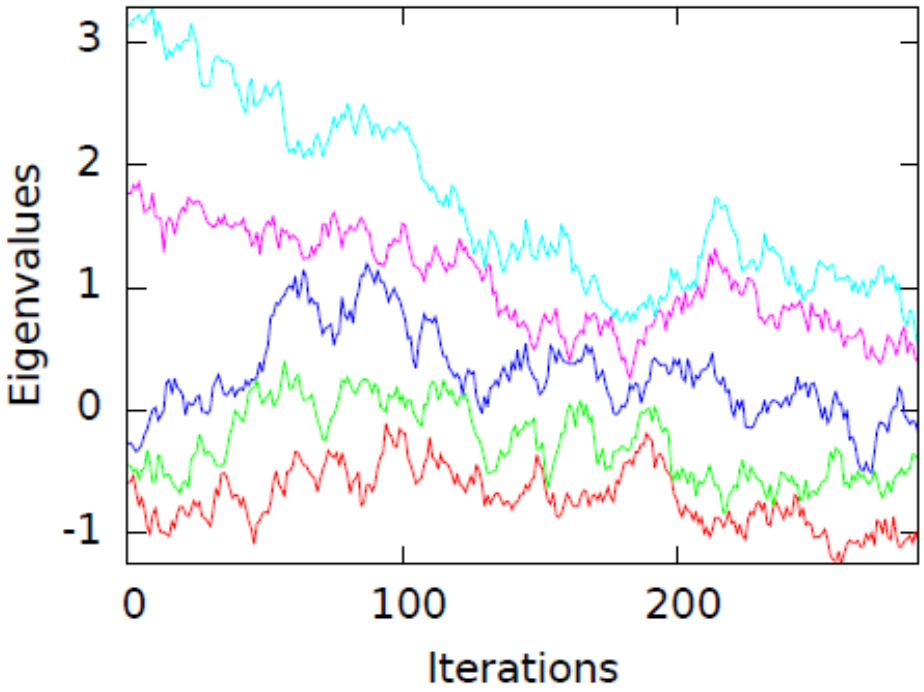

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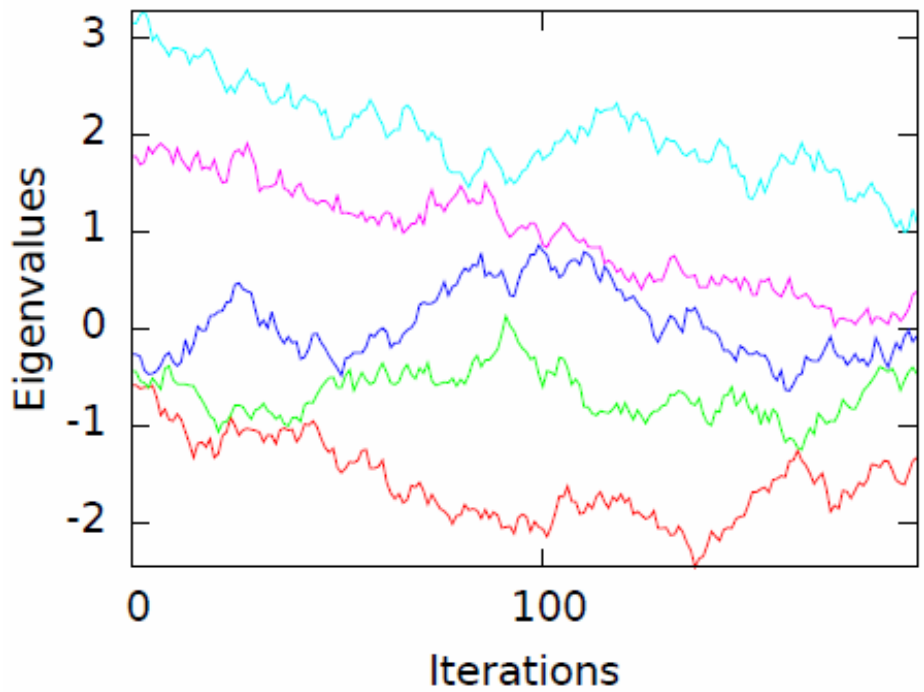

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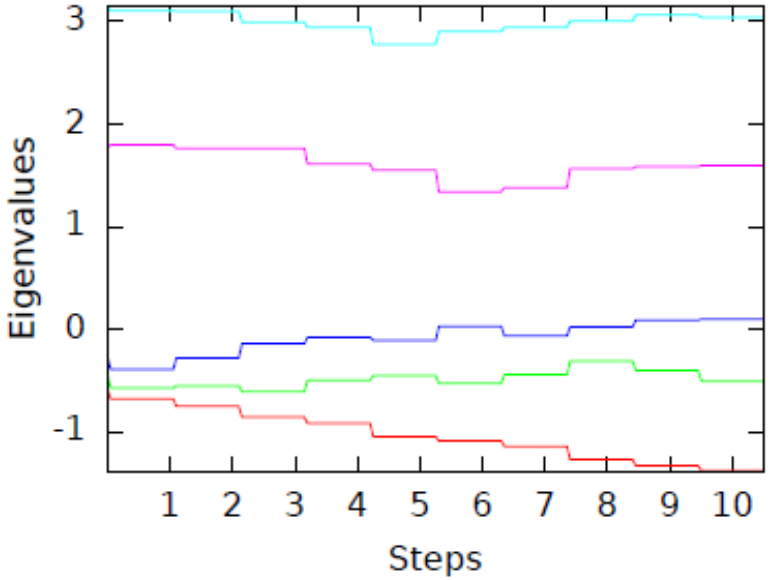
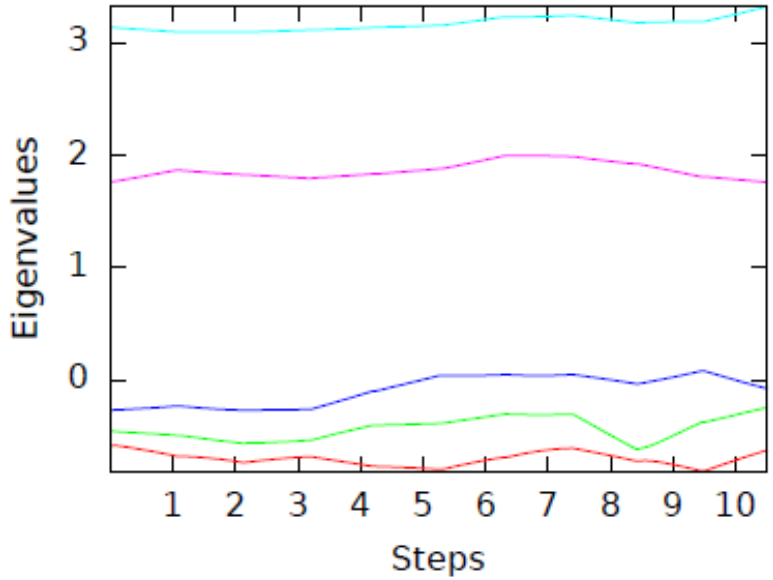
Potentials ``look'' qualitatively indistinguishable, …

The Hessian is smooth:



(a) $V \in C^1$ - via Dyson Brownian Motion.



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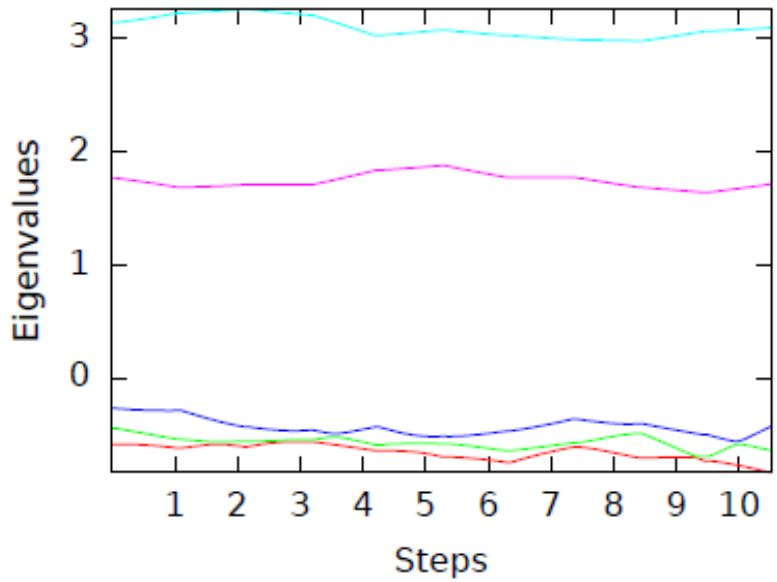

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These potentials can be used to compute cosmological perturbations, compute the powerspectrum, hunt four our vacuum (find minima), etc.

We developed formalism for $V \in C^k$ as well.

Our next steps:

Study cosmological perturbation in extended DBM potentials.

Apply methods to model realistic landscapes.

Predict distribution of observables and quantify the effect of the chosen measure.

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