Cosmology of Massive Gravity

the quest for stable, compelling solutions

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mostly based on work with C. de Rham and A. J. Tolley

Cosmo 2015, Warsaw

“Ask not what massive gravity can do for you...”

Old Cosmological Constant Problem

Why is the c.c. so small? No

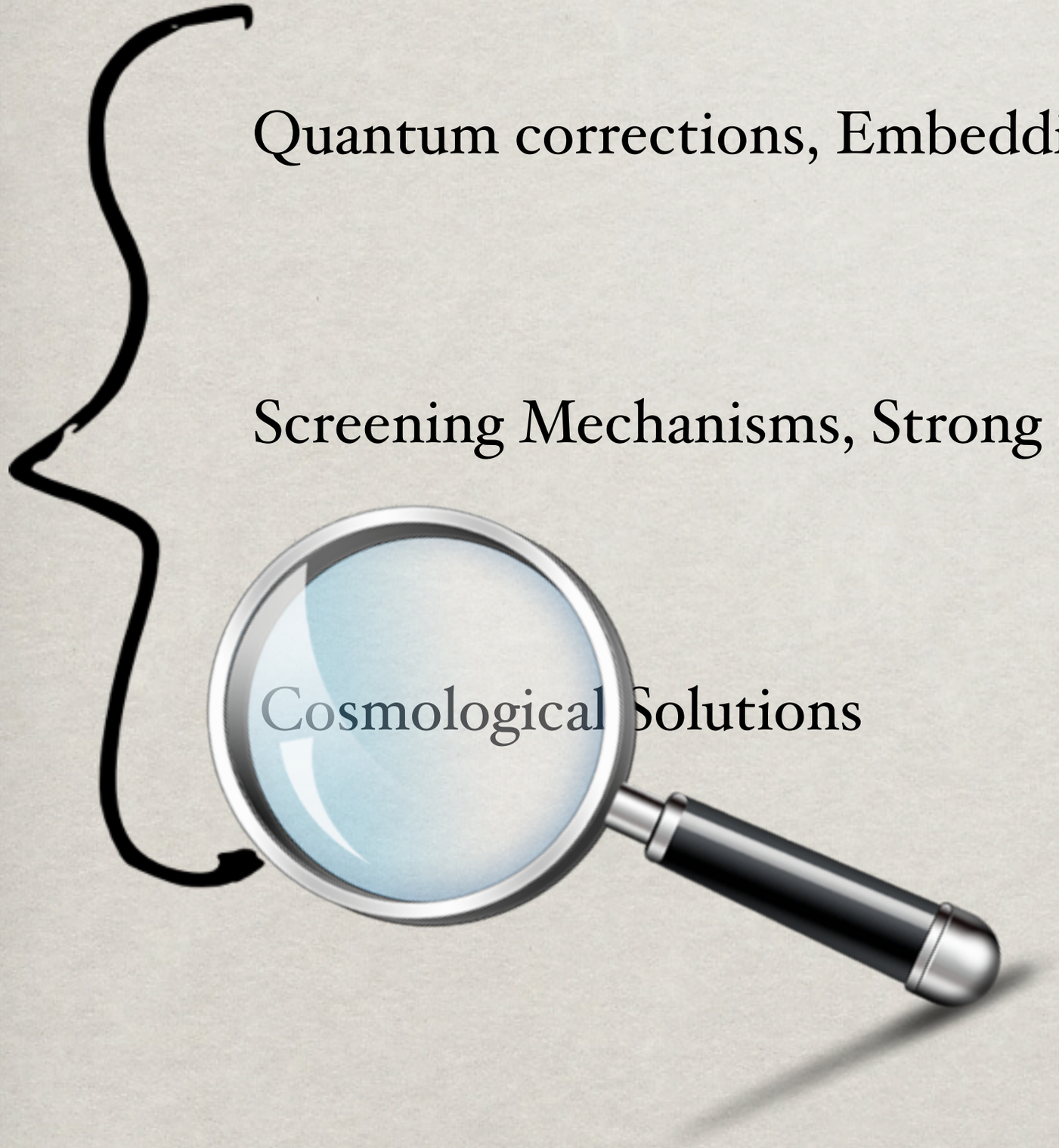
New Cosmological Constant Problem

Technically natural small scale at the origin of late-time acceleration ✓

Current Directions

Quantum corrections, Embedding

Screening Mechanisms, Strong Coupling Scale



Posters/Talks by

Cusin
Yamashita
Koennig
Terrana
Nersisyan
Yoshida

...

- Unexpected applications: use of massive gravity in holography to describe conductivity in strange metals

A little context..

Why massive gravity ? Mechanism for cosmic acceleration

Which massive gravity ? dRGT and bigravity

..why ? Ghost-free theory

Which type of cosmological sols are we after ? FRLW

..why ? Simplicity (need to? No..)

What do we ask of these solutions? Stable, compatible with observations

Linearly

Fierz and Pauli
Proc.Roy.Soc.Lond. A173 (1939) 211-232

$$\underbrace{-\frac{1}{4}h^{\mu\nu}\hat{\mathcal{E}}^{\alpha\beta}_{\mu\nu}h_{\alpha\beta}}_{\text{E-H}} \quad \underbrace{-\frac{1}{8}m^2\left(h^2_{\mu\nu}-h^2\right)}_{\text{F-P}} \quad ; \quad \text{2+3 = 5 healthy dof of a massive spin 2}$$

↳ breaks diff invariance: $h_{\mu\nu} \to h_{\mu\nu} + \partial_{(\mu}\xi_{\nu)}$

vDVZ discontinuity

Gravitational exchange amplitude btw 2 sources

$$\text{Theory}_{m\to 0} \neq \text{Theory}_{m=0}$$

Linear theory at odds with observations !

Linear mGR result off by 25% w.r.t. extremely well-tested GR results

Non-linearities better play a crucial role, they do

Non Linearly

Vainshtein: non-linearities screen helicity-0 mode in the presence of matter

$$r_V = \left(\frac{M}{M_P^2 m^2} \right)^{1/3}$$

Most of what is verified is static and spherical

screening in an area within r_V, where GR is recovered, the linearized theory good outside ✓

Not so fast: within most n-l lies a ghost

$$\mathcal{L}^{n-l}_{\mathrm{F-P}} = -m^2 M_{\mathrm{Pl}}^2 \sqrt{-g} \left([(\mathbb{I} - \mathbb{X})^2] - [\mathbb{I} - \mathbb{X}]^2 \right)$$

where $\mathbb{X}^\mu_\nu = g^{\mu\alpha} \tilde{f}_{\alpha\nu}$

Focusing on the helicity-0 mode π

$$\mathbb{X}^\mu_\nu = \delta^\mu_\nu - \frac{2}{M_{\mathrm{Pl}} m^2} \partial^\mu \partial_\nu \pi + \frac{1}{M_{\mathrm{Pl}}^2 m^4} \partial^\mu \partial_\alpha \pi \partial^\alpha \partial_\nu \pi$$

Generically non-linear interaction will carry an Ostrogradski ghost, or Boulware-Deser here

Non Linear and with Special Structure, *dRGT*

de Rham, Gabadadze (2010)
de Rham, Gabadadze, Tolley (2010)

$$S_{\rm mGR} = \frac{M_{\rm Pl}}{2}\int d^4x\sqrt{-g}\left(R[g] + \frac{m^2}{2}\sum_{n=0}^{4}\alpha_n\mathcal{L}_n[\mathcal{K}[g,f]]\right)$$

where $\mathcal{K}^\mu_\nu = \delta^\mu_\nu - \sqrt{g^{\mu\alpha}f_{\alpha\nu}}$ and

$$\begin{aligned}
\mathcal{L}_0[\mathcal{K}] &= 4! \\
\mathcal{L}_1[\mathcal{K}] &= 3!\,[\mathcal{K}] \\
\mathcal{L}_2[\mathcal{K}] &= 2!([\mathcal{K}]^2 - [\mathcal{K}^2]) \\
\mathcal{L}_3[\mathcal{K}] &= ([\mathcal{K}]^3 - 3[\mathcal{K}][\mathcal{K}^2] + 2[\mathcal{K}^3]) \\
\mathcal{L}_4[\mathcal{K}] &= ([\mathcal{K}]^4 - 6[\mathcal{K}]^2[\mathcal{K}^2] + 3[\mathcal{K}^2]^2 + 8[\mathcal{K}][\mathcal{K}^3] - 6[\mathcal{K}^4])
\end{aligned}$$

No ghosts verified at full non-linear level ✓

Hassan, Rosen (2011,2011)

High enough cutoff so that hierarchy of scales: linear, non-linear, quantum ✓

Screening in the non-linear regime that restores continuity with G.R. ✓

Hints structure of the potential preserved under quantum corrections ✓

Is is stable?

Is is observationally viable?

Stability bound

$$H = \alpha\, p^2 + \beta\, q^2 + \gamma (\nabla q)^2 + \ldots$$

coefficient of kinetic term > 0

tachyon inst.

gradient inst.

Quickest route to the Higuchi bound in dS:

"In the the linear (massive) theory there exist a unitary spin 2 representation of the dS group iff:"

$m^2 = 0$ G.R.

$m^2 = 2H^2$ Partially massless theory

Higuchi bound in massive g. → $m^2 > 2H^2$ Massive

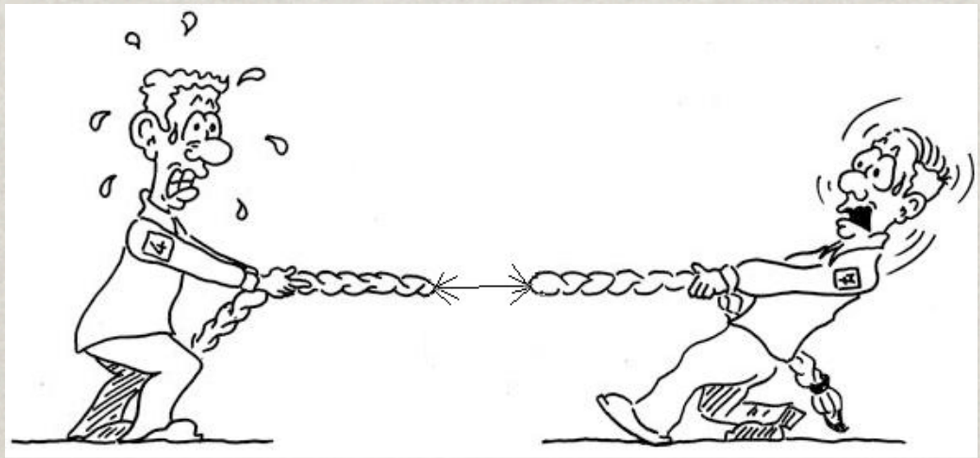
Bound from Observations

Before Dark Energy epoch sets in, G.R. good description:

$$3H^2 = \Lambda + 3m^2 \times \Theta(1) + ...$$

$$\boxed{m^2 \lesssim H^2}$$

<u>combining</u> Stability and Observations then:



want our theory to be stable

$$m^2 > 2H^2$$

GR over many cosmo epochs

$$m^2 \lesssim H^2$$

No FRW solutions in dRGT if "f" Minkowski

D'Amico, de Rham, Dubovsky,
Gabadadze, Pirtskhalava, Tolley

Inhomogeneities/Anisotropies

D'amico et al '11, Koyama '11, Gratia et al '12, Kobayashi et al '12,
Volkov '13, DeFelice '11 '13, Tasinato et al '12, Maeda et al '13

Modify Assumptions to allow for FRW

Open Universe solutions (Gumrukcuoglu et al 2013, Vakili et al 2012) unstable

Make reference metric dS (MF, Tolley) ? (1)

Generalized massive gravity, but keep the 5 d.o.f. (de Rham, MF, Tolley)... later (3)

Make the reference metric dynamical - Bigravity/Bimetric... later (2)

Significant modifications of the theory to allow for FRW

Quasi-dilaton, Lorentz-violating, Multi-vierbeins,
Extended massive gravity, Non-local massive gravity

① Usual starting point

$$S_{\rm mGR} = \frac{M_{\rm Pl}}{2}\int d^4x\sqrt{-g}\left(R[g] + \frac{m^2}{2}\sum_{n=0}^{4}\alpha_n\mathcal{L}_n[\mathcal{K}[g,f]]\right)$$

Allow different “f”, find that solutions exist for “f” dS, FRW

MF, Tolley (2012)

$$\mathcal{K}^\mu_\nu = \delta^\mu_\nu - \sqrt{g^{\mu\alpha}f_{\alpha\nu}}$$

Add matter content

$$\mathcal{L}_M \sim \int d^4x\sqrt{-g}\left[\frac{1}{2}(\partial\phi)^2 + V(\phi)\right]$$

Existence ✓

Stability ?

Generalized Higuchi:

$$\tilde{m}^2(H) = m^2 \frac{H}{H_f}\left((3+3\alpha_3+\alpha_4) - 2(1+2\alpha_3+\alpha_4)\frac{H}{H_f} + (\alpha_3+\alpha_4)\frac{H^2}{H_f^2} \right) \geq 2H^2$$

N.B. independent on precise form of matter

Friedman Side:

$$m^2\left(\frac{2}{3}(-6-4\alpha_3-\alpha_4) + 2\left(\frac{H}{H_f}\right)(3+3\alpha_3+\alpha_4) - 2\left(\frac{H}{H_f}\right)^2(1+2\alpha_3+\alpha_4) + \frac{2}{3}\left(\frac{H}{H_f}\right)^3(\alpha_3+\alpha_4)\right) \ll 2H^2$$

Combined:

$$\frac{\mathrm{poly}_1^{(k)}(z)}{\mathrm{poly}_2^{(k)}(z)} \gg 1$$

Hard to satisfy even using α_3, α_4

Impossible when we account $H = H(t)$

② What now? Go bigravity !

Hassan, Rosen

$$\mathcal{L} = \frac{1}{2}\sqrt{-g}\left[M_P^2\, R(g) - m^2 \sum_{n=0}^{4} \beta_n\, \mathcal{U}_n\left(g^{-1} f\right)\right] + \frac{1}{2}\sqrt{-f}\, M_f^2\, R(f) + \mathcal{L}_M$$

$$g_{\mu\nu} \leftrightarrow f_{\mu\nu},\ \ M_P \leftrightarrow M_f,\ \ \beta_n \leftrightarrow \beta_{4-n}$$

This fact must be reflected on the bound itself

Proven to be ghost-free

Stability Bound in Bigravity

g ⇦⇨ f symmetric form

Stability bound

$$\tilde{m}^2\left[1+\left(\frac{H_f/M_f}{H/M_P}\right)^2\right]\geq 2H^2$$

Minisuperspace action ✓

Decoupling limit ✓

PM check ✓

Symmetric ✓

Recover Massive gravity bound in the limit $M_f \to \infty,\ M_P, H_f$ finite.

Not possible before: $\boxed{\frac{H_f}{M_f} \gg \frac{H}{M_P}}$ not directly invoking m

Friedman side

$$H^2 = \frac{1}{3M_P^2}\left[\rho(a) + \sum_{n=0}^{3} \frac{3m^2\beta_n}{(3-n)!n!}\left(\frac{H}{H_f}\right)^n\right] \quad ; \quad H_f^2 = \frac{1}{3M_f^2}\left[\sum_{n=0}^{3} \frac{3\beta_{n+1}}{(3-n)!n!}\left(\frac{H}{H_f}\right)^{(n-3)}\right]$$

$m^2 \times \Theta(1) \ll H^2$ it's the only direct requirement on m, but now:

In the $\frac{H_f}{M_f} \gg \frac{H}{M_P}$ region with $\beta_1 \neq 0$ solve for $\tilde{m}^2$, Hf, bound reads:

$$3H^2 > 2H^2 \checkmark$$

The (most pressing) stability vs observations tension is resolved in bigravity !

Stable Self-accelerating Solution

Akrami, Koivisto, Sandstad (2012,2013)

Set: $\beta_2 = 0 = \beta_3;\ \beta_1 = 2M_P^2$

$$H^2 = \frac{1}{6M_P^2}\left(\rho(a) + \sqrt{\rho(a)^2 + \frac{12\,m^4 M_P^6}{M_f^2}}\right)$$

Model	B_0	B_1	B_2	B_3	B_4	Ω_m	χ^2_{min}	p-value	log-evidence
ΛCDM	free	0	0	0	0	free	546.54	0.8709	-278.50
(B_1, Ω_m^0)	0	free	0	0	0	free	551.60	0.8355	-281.73

Observationally viable (?) ! Small part of the whole table

Stability bound? It reduces to

$$\left(\frac{1}{M_P^2} + \frac{12M_f^2}{m^4\beta_1^2}H^4\right) > 0 \quad \checkmark$$

Stable as well.

Further work on bigravity Cosmo Solutions

Perturbations in bigravity solutions

Comelli, Crisostomi, Pilo (2012,2014)

Our focus on unitarity (Higuchi), but ought to check gradient instability

$$H = \alpha\, p^2 + \beta\, q^2 + \gamma(\nabla q)^2$$

Gradient instability analysis has been done, a mild problem awaiting a full resolution !

Vainshtein? (in progress...)

Konnig, Akrami, Amendola, Motta, and Solomon (2014)

Ways around it

A viable (in a reduced parameter space region) bigravity model put forward by

De Felice, Gumrukcuoglu, Mukohyama, Tanahashi, Tanaka (2014)
De Felice, Nakamura, Tanaka (2013)

the key is to push instability scale higher than the cutoff

Bounds on the effective Planck mass at late times, confine parameter space to a region very close to GR MF, Ribeiro (2015)

...in the low-energy regime of De Felice et al

Gumrukuoglu, Kuroyanagi, Lin, Mukohyama, Tanahashi (2012)

Cusin, Durrer, Guarato, Motta (2014)

Lagos, Ferreira (2014)

Amendola, Koennig, Martinelli, Pettorino, Zumalacarregui, (2015)

Johnson, Terrana (2015)

Cusin, Durrer, Guarato, Motta (2015)

reheating temperature constraints

Access to initial conditions ?

③ Generalized Massive Gravity

De Rham, MF, Tolley (2014)

Let the $\alpha_n(\beta_n)$ now depend on the Stueckelbergs fields as

$$S = \frac{M_{Pl}^2}{2}\int d^4x\sqrt{-g}\left[R[g] + \frac{m^2}{2}\sum_{n=0}^{4}\tilde{\alpha}_n(\tilde{\phi}^a\tilde{\phi}_a)\mathcal{U}_n[\mathcal{K}]\right] + \mathcal{L}_{\rm matter}[g,\psi^{(i)}]$$

In pure mGR, isometry group of the reference metric is Poincare'

Now:

keep { Lorentz ✓

give up { Translations ✗

Gen. mGR theory still has 5 d.o.f. proven ghost-free

De Rham, Keltner, Tolley (2014)

Cosmology

Simple Form + Preserve Lorentz → open universe solutions, $k<0$

↓

$SO(3,1)$ acts on open spatial slicing H^3

Stueckelbergs will describe open slices of Minkowski

Gumrukcuoglu, Lin, Mukohyama (2011)

$$\tilde{\phi}^0 = f(t)\sqrt{1+|k|\bar{x}^2}\,, \quad \tilde{\phi}^i = \sqrt{|k|}f(t)x^i$$

$$\tilde{\phi}^a\tilde{\phi}_a = -f(t)^2\,, \quad f_{\mu\nu}\mathrm{d}x^\mu\mathrm{d}x^\nu = -\dot{f}(t)^2\mathrm{d}t^2 + |k|f^2\, d\Omega^2_{H^3}$$

$$\mathrm{d}\Omega^2_{H^3} = (\mathrm{d}\vec{x})^2 - \frac{|k|}{1+|k|\vec{x}^{\,2}}(\vec{x}.\mathrm{d}\vec{x})^2$$

...

Stuckbs:

$$\alpha_n = \alpha_n(\chi(t))$$

$$f_{\mu\nu} = -\dot{\chi}(t)^2\mathrm{d}t^2 + \mathrm{d}\vec{x}^2$$

$$\alpha_n \leftrightarrow \beta_n$$

In unitary gauge $\alpha_n(\chi(t)) \to \alpha_n(t)$

New action terms look like those of a time-dependent mass mGR on f Minkowski

but, crucially, this is the zero-curvature scaling limit of a L-I theory, so L-I it is

Friedman Eq.

$$3M_{Pl}^2 H^2 = \rho + \frac{m^2 M_{Pl}^2}{4}\left[24\beta_0(\chi) + 18\frac{\beta_1(\chi)}{a} + 6\frac{\beta_2(\chi)}{a^2} + \frac{\beta_3(\chi)}{a^3}\right]$$

Constraint Eq. for

χ

$$m^2 H\left[\frac{3}{2}\beta_1 a^2 + \beta_2 a + \frac{1}{4}\beta_3\right] = \frac{m^2}{2a}\left[4\beta_0' a^3 + 3\beta_1' a^2 + \beta_2' a + \frac{1}{6}\beta_3'\right]$$

= 0 in pure massive gravity,
hence the well known lack of solution for f Minkowski

Before taking a useful scaling limit (DL)

(what do we know already at this stage?)

Constraint equation with $\beta_{n,2} = 0$ is

$$m^2 H \left[\Sigma_{1,0} + m\chi\Sigma_{1,1}\right] = \frac{m^3}{a}\Sigma_{0,1}$$

Plug back($\bar{\beta}_{0,1}$, $\bar{\beta}_{1,1}$, $\bar{\beta}_2 \neq 0 = \forall_{\text{else}}$) hints of self-acceleration (a)

$$3M_{\rm PL}^2 H^2 = \rho + \frac{m^3 M_{\rm PL}^2}{2H}\left[\frac{\left(4\bar{\beta}_{0,1} + 3\bar{\beta}_{1,1}a^{-1}\right)^2}{\bar{\beta}_{1,1}}\right] - 2m^2 M_{\rm PL}^2 \frac{\bar{\beta}_{0,1}\bar{\beta}_2}{a\bar{\beta}_{1,1}}$$

H scales ~ m so the Friedman equation is finite in DL (b)

$$3\frac{H^2}{m^2} = \frac{\rho}{m^2 M_{\rm Pl}^2} + \frac{m}{2H}\left[\frac{\left(4\bar{\beta}_{0,1} + 3\bar{\beta}_{1,1}a^{-1}\right)^2}{\bar{\beta}_{1,1}}\right] - 2\frac{\bar{\beta}_{0,1}\bar{\beta}_2}{a\bar{\beta}_{1,1}}\,,$$

DL is a good probe of cosmological behaviour

Decoupling Limit and Stability Analysis

Any space-time manifold can split into open chart around x^μ where metric looks like Minkowski

(for sufficiently small chart)

How small? $\sqrt{R_{abcd}R^{abcd}} \sim H^2$ ok within $L \lesssim 1/H$

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}/M_{\rm Pl}$$

DL : leading contribution to an EFT describing physics within a given chart of size $L \lesssim 1/H$

In our Setup

$$m \to 0\,, \qquad M_{\rm Pl} \to \infty\,, \quad \text{keeping} \quad (m^2 M_{\rm Pl})^{1/3} \equiv \Lambda \to \text{fixed}\,, \quad T^{\mu\nu}/M_{\rm Pl} \to \text{fixed}$$

Coefficients expansion $\beta_n(\chi) = \bar\beta_n + \bar\beta_{n,1} m\chi + \frac{m^2}{\Lambda^2}\beta_{n,2}(\chi)$

$$\phi^a = x^a - \frac{V^a}{mM_{\rm Pl}} - \frac{\eta^{ab}\partial_b\pi}{\Lambda^3} = x^a - m\frac{V^a}{\Lambda^3} - \frac{\eta^{ab}\partial_b\pi}{\Lambda^3} \quad \to \quad \alpha_n(\chi) \to \alpha_n(\phi^0) \to \alpha_n(t + \frac{\dot\pi}{\Lambda^3})$$

Familiar Pieces + New Terms

$$\mathcal{L}^{(0)}_{\rm DL} = -\frac{1}{4}h^{\mu\nu}\hat{\mathcal{E}}^{\alpha\beta}_{\mu\nu}h_{\alpha\beta} + \frac{\Lambda^3}{8}\sum_{n=0}^{4}\bar{\alpha}_n h^{\mu\nu}\left((4-n)X^{(n)}_{\mu\nu} + nX^{(n-1)}_{\mu\nu}\right) + m\mathcal{L}^{\rm Vectors,(0)}_{\rm DL} + \frac{h_{\mu\nu}}{2M_{\rm Pl}}T^{\mu\nu}$$

New scalar self-interactions and new scalar-vector interactions

$$\mathcal{L}^{(\text{New } \pi \text{ terms})}_{\rm DL} = \frac{\Lambda^4}{4}\sum_{n=0}^{4}\left[(4-n)\alpha_{n,2}(t+\dot{\pi}/\Lambda^3) + (n+1)\alpha_{n+1,2}(t+\dot{\pi}/\Lambda^3)\right]\mathcal{L}^{(3{\rm d})}_n$$

$$\mathcal{L}^{(\text{New s}-\text{v terms})}_{\rm DL} = \frac{\Lambda^3}{4}\sum_{n=0}^{4}\left[n\bar{\alpha}_{n,1}(t+\dot{\pi}/\Lambda^3)\partial^\mu V^\nu X^{(n-1)}_{\mu\nu}\right]$$

Appropriate decoupling limit requires $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}/M_{\rm Pl}$

$$\mathrm{d}s^2_g = -N^2(T)\mathrm{d}T^2 + a^2(T)\mathrm{d}\vec{X}^2 \qquad ; \qquad \mathrm{d}s^2_f = -\mathrm{d}T^2 + \mathrm{d}\vec{X}^2$$

FERMI NORMAL COORDINATES

$$X^i = \frac{1}{a(T)}x^i \equiv \phi^i_{\rm FN}(t,x)$$

$$T = \chi(t) - \frac{1}{2}\frac{H(\chi(t))}{N(\chi(t))}x_i x^i \equiv \phi^0_{\rm FN}(t,x^i)$$

$$\mathrm{d}s^2_g = -\left[1 - \left(\frac{\dot{H}}{N} + H^2\right)r^2\right]\mathrm{d}t^2 + \left(1 + H^2r^2\right)\mathrm{d}r^2 + r^2\mathrm{d}\Omega^2_2 \qquad \mathrm{d}s^2_f = \partial_\mu\phi^a_{\rm FN}\partial_\nu\phi^b_{\rm FN}\eta_{ab}\mathrm{d}x^\mu\mathrm{d}x^\nu$$

Friedman recovered in the DL

$$3M_{\rm Pl}H^2 = \frac{\rho}{M_{\rm Pl}} - \frac{3}{2}\Lambda^3 \sum_{n=0}^{4} \bar{\alpha}_n \left[(4-n)\left(1-a^{-1}\right)^n + n\left(1-a^{-1}\right)^{n-1} \right]$$ ✓

Perturbation theory in DL and read off sign from coefficient (Higuchi, grad. inst)

$\bar{\beta}_{1,2,3} \geq 0$ Vector sector (sufficient cond)

$\bar{\beta}_{1,2} \geq 0\,; \quad \bar{\beta}_3 = 0$ coupling with matter (sufficient cond)

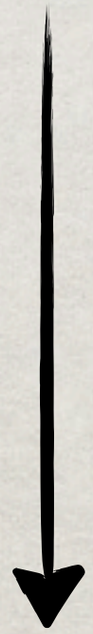
$$\frac{1}{4}\left[24\beta''_{0,2} + 18\frac{\beta''_{1,2}}{a} + 6\frac{\beta''_{2,2}}{a^2} + \frac{\beta''_{3,2}}{a^3}\right] + \frac{3}{4}\frac{H}{m}\frac{a+N}{a^2N^2}S_{2,1} + \frac{2}{a^4c_2}\Sigma_{1,1}S_{1,1} < 0$$ Higuchi bound (suff.)

$$\frac{1}{4}\left[6\beta''_{1,2} + 4\frac{\beta''_{2,2}}{a} + \frac{\beta''_{3,2}}{a^2}\right] + \frac{H(a+N)}{2maN^2}S_{3,1} < \frac{\dot{H}}{2m^2aN}\Sigma_{2,0}$$ Gradient instability (suff)

...Tensor as usual (after diagonalization)

Conclusions

Extra



Late time constraints

Solar system tests

point source $\Psi(r) = -h_{00} = -\dfrac{M}{4\pi M_{\rm Pl}}\dfrac{1}{r}$ but in massive gravity helicity-0 π joins in, DL :

$$h_{\mu\nu}^{\text{massive gravity}} = h_{\mu\nu}^{\text{Einstein}} + \pi_0\eta_{\mu\nu} + \frac{\alpha}{\Lambda_3^3}\partial_\mu\pi_0\partial_\nu\pi_0 \rightsquigarrow \delta\Psi = -\pi_0$$

fractional change in the potential

$$\epsilon(r) = \frac{\delta\Psi}{\Psi} = \frac{\pi_0(r)}{M_{\rm Pl}}\frac{r}{r_S}$$

elliptical orbits witness additional precession of the perihelion, advance per orbit:

$$\delta\phi \sim R_0\frac{\mathrm{d}}{\mathrm{d}r}\left(r^2\frac{\mathrm{d}}{\mathrm{d}r}(r^{-1}\epsilon)\right)\Big|_{R_0} \sim \left(\frac{r}{r_*}\right)^2 \sim 10^{-16}/\text{orbit}$$

current reach is ~10^{-11}/orbit

[E-M system, quartic Galileon, m ~ H_0]

a bit of cheating to give an idea on the order; the bound is not exactly on m, rather, as usual it's on $m_{\rm eff}$

A bit of cheating to give an idea on the order; the bound is not exactly on m, rather, as usual it's on meff

$$\mathcal{L}_\pi = -\frac{3}{4}(\partial\pi)^2 + \frac{3\alpha}{4\Lambda_3^3}\mathcal{L}^{(3)}_{(\mathrm{Gal})} - \frac{1}{4}\left(\frac{\alpha}{\Lambda_3^3}\right)^2 \mathcal{L}^{(4)}_{(\mathrm{Gal})} + \frac{1}{M_{\mathrm{Pl}}}\left(\pi T + \frac{\alpha}{\Lambda_3^3}\partial_\mu\pi\partial_\nu\pi T^{\mu\nu}\right)$$

[massive gravity where (diag..)]
$\alpha_3 + \alpha_4 = 0\,; \alpha \equiv -(1 + 3/2\alpha_3)$

what one is really probing here is actually m^2/α , we could call it $m^{(\mathrm{late})}_{\mathrm{eff}}$

Just as the decoupling limit of massive gravity illuminates what's happening on solar s. scales,

so does the decoupling limit of bigravity

MF, Tolley (2013)
Ondo, Tolley (2013)

For Vainshtein m. in bigravity:
Babichev, Crisostomi (2013)

For Vainshtein in general:

Babichev, Deffayet ('10-'11-'13)

Bloomfield, Burrage, Davis (2014)

Scalar coupled to matter

$$\phi = \phi_0 + \delta\phi$$

$$\mathcal{L} \sim -Z_{\mu\nu}(\phi_0, \partial\phi_0 \dots)\partial^\mu\delta\phi\partial^\nu\delta\phi - m^2_{eff}(\phi_0)\delta\phi^2 - \beta(\phi_0)\frac{\delta\phi}{M_P}T + \dots$$

Vainshtein

Chameleon

Symmetron/dilaton

Minisuperspace

(gravity w/ metric and fields function of time alone)

Start with

$$ds_g^2 = -N(t)^2 dt^2 + a(t)^2 d\vec{x}^2$$

$$\sqrt{g^{-1}f} = \begin{pmatrix} \frac{\dot{\phi}}{N} & 0_j \\ 0_i & \frac{b(\phi)}{a}\delta_{ij} \end{pmatrix}$$

Action

$$S = V_3 \int dt N a^3 \left[-3M_P^2 \left(\frac{\dot{a}^2}{N^2 a^2} \right) - \frac{\dot{\phi}}{N} \sum_{n=0}^{3} A_n \left(\frac{b(\phi)}{a} \right)^n - \sum_{n=0}^{3} B_n \left(\frac{b(\phi)}{a} \right)^n - \rho(a) \right]$$

Notation

$$B_0 = m^2 M_P^2 (-6 - 4\alpha_3 - \alpha_4) = \frac{1}{2} m^2 \beta_0$$

$$B_1 = 3m^2 M_P^2 (3 + 3\alpha_3 + \alpha_4) = \frac{3}{2} m^2 \beta_1$$

$$B_2 = 3m^2 M_P^2 (-1 - 2\alpha_3 - \alpha_4) = \frac{3}{2} m^2 \beta_2$$

$$B_3 = m^2 M_P^2 (\alpha_4 + \alpha_3) = \frac{1}{2} m^2 \beta_3 \quad ; \quad A_n(3-n) = B_{n+1}(n+1)$$