Cosmological black holes and accretion: Causal structure and models for the dark sector

Daniel C. Guariento

based on

A. Maciel, M. Fontanini, DCG, E. Abdalla, [1207.1086], [1212.0155]

E. Abdalla, N. Afshordi, M. Fontanini, DCG. E. Papantonopoulos, [1312.3682]

N. Afshordi, M. Fontanini, DCG, [1408.5538], A. Maciel, DCG, C. Molina, [1502.01003]

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- Field theory:
  - Consistency analysis
  - Evolution and interaction from equations of motion

- Cosmological black holes: McVittie solution [G. C. McVittie, *MNRAS* **93**,325 (1933)]

$$ds^2 = -\frac{\left[1 - \frac{m(t)}{2a(t)\hat{r}}\right]^2}{\left[1 + \frac{m(t)}{2a(t)\hat{r}}\right]^2}\,dt^2 + a^2(t)\left[1 + \frac{m(t)}{2a(t)\hat{r}}\right]^4 \left(d\hat{r}^2 + \hat{r}^2\, d\Omega^2\right)$$

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- Unique solution that satisfies (with m constant)

  - Spherical symmetry
  - Perfect fluid
  - Shear-free
  - Asymptotic FLRW behavior
  - Central singularity

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[N. Kaloper, M. Kleban, D. Martin, *PRD* **81** 104044 (2010), 1003.4777]

$$\mathrm{d}s^2 = -R^2\,\mathrm{d}t^2 + \left\{\frac{\mathrm{d}r}{R} - \left[\frac{\dot{a}}{a} + \frac{\dot{m}}{m}\left(\frac{1}{R} - 1\right)\right] r\,\mathrm{d}t\right\}^2 + r^2\,\mathrm{d}\Omega^2$$

where $\left(R = \sqrt{1 - \frac{2m}{r}}\right)$

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- Past spacelike singularity at $r = 2m$
- Event horizons only defined if $\dot{a}/a$ constant as $t \to \infty$

- McVittie is a solution of a scalar field with a modified kinetic term minimally coupled to GR

[E. Abdalla, N. Afshordi, M. Fontanini, DCG, E. Papantonopoulos, *PRD* **89** 104018 (2014), 1312.3682]

Scalar sources

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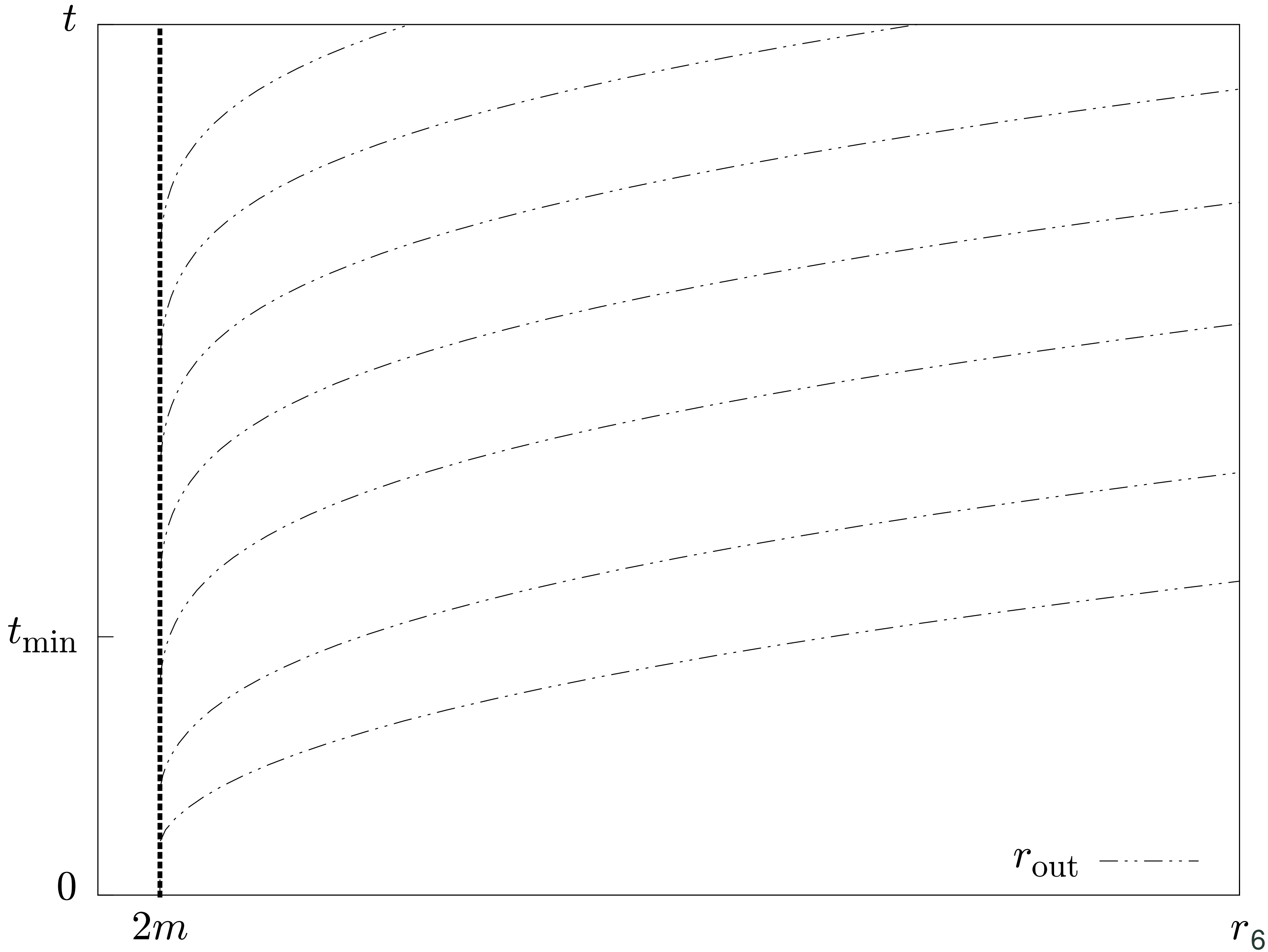
- Field imposes CMC foliation, which the McVittie class satisfies

$$K^{\alpha}{}_{\alpha} = \frac{1}{\mu^2} \frac{\mathrm{d}V}{\mathrm{d}\phi} = 3H(t)$$

Light cones and apparent horizons



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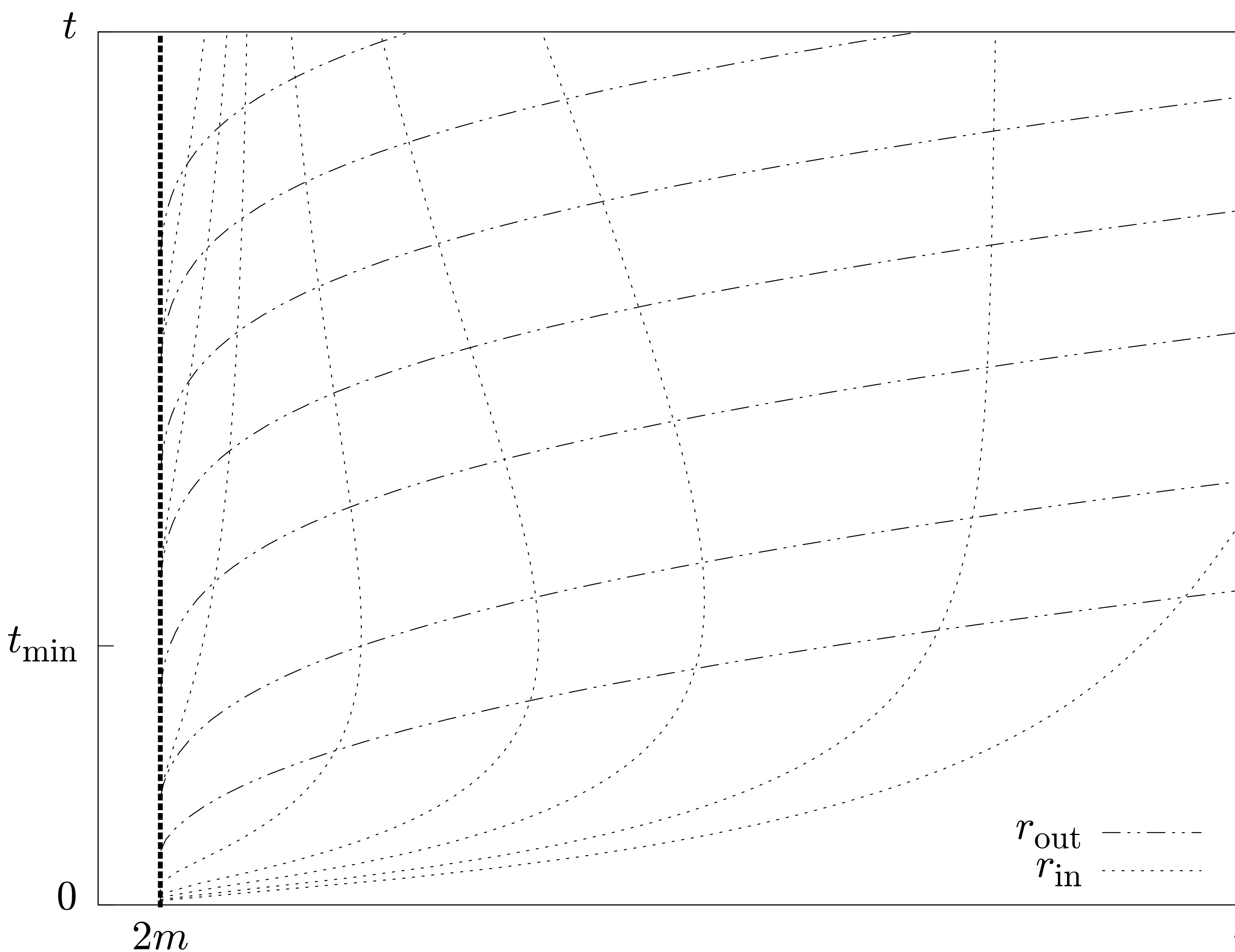

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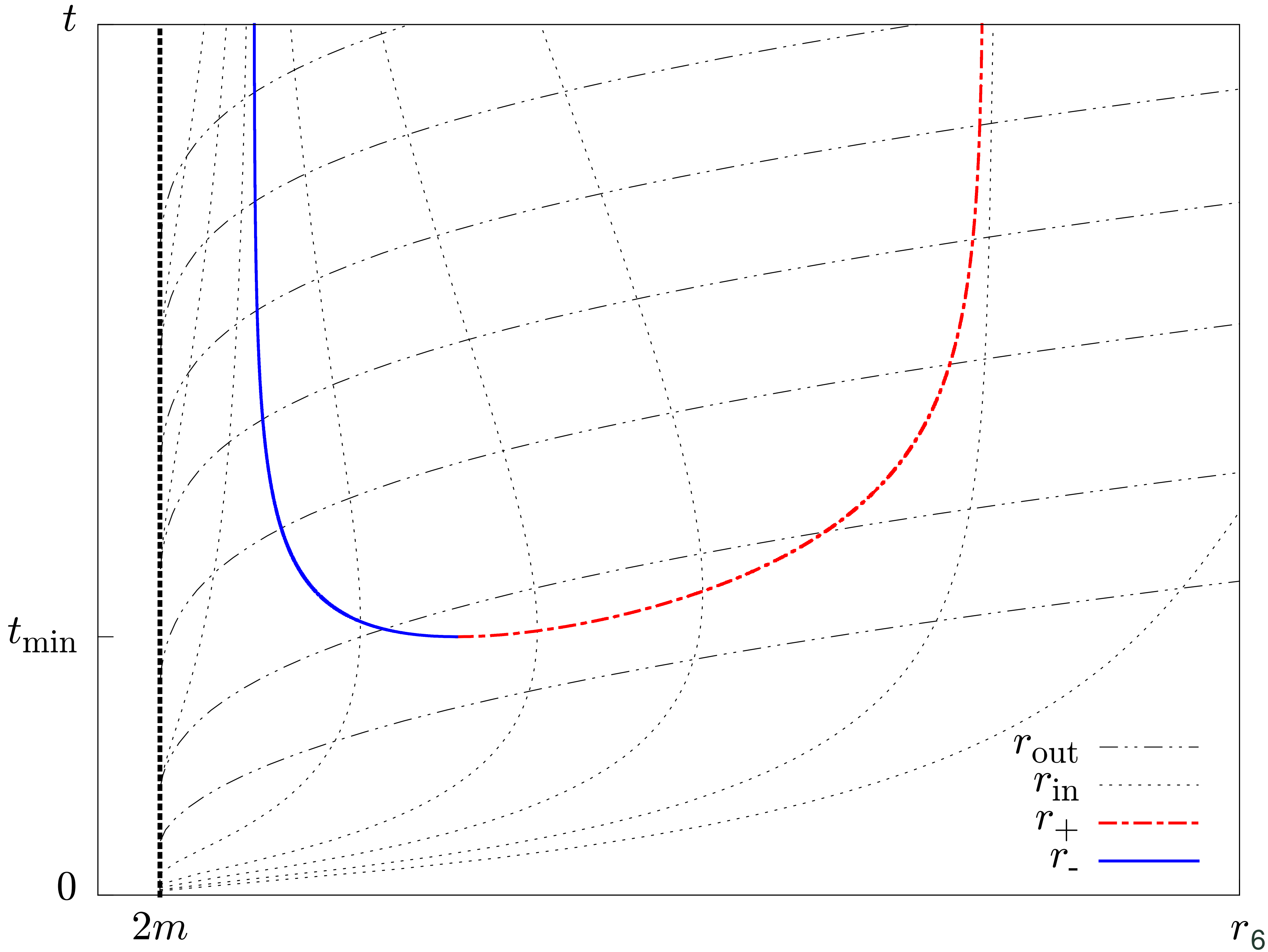

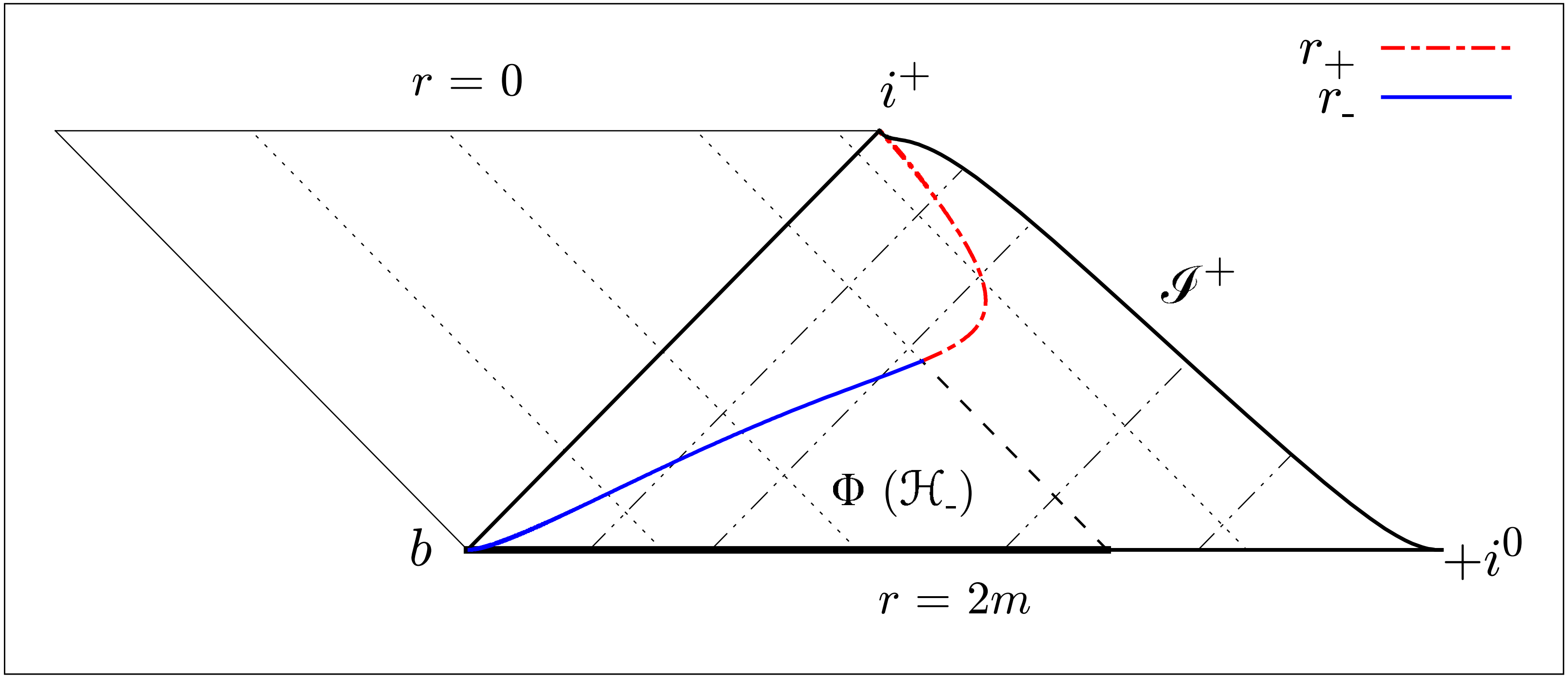

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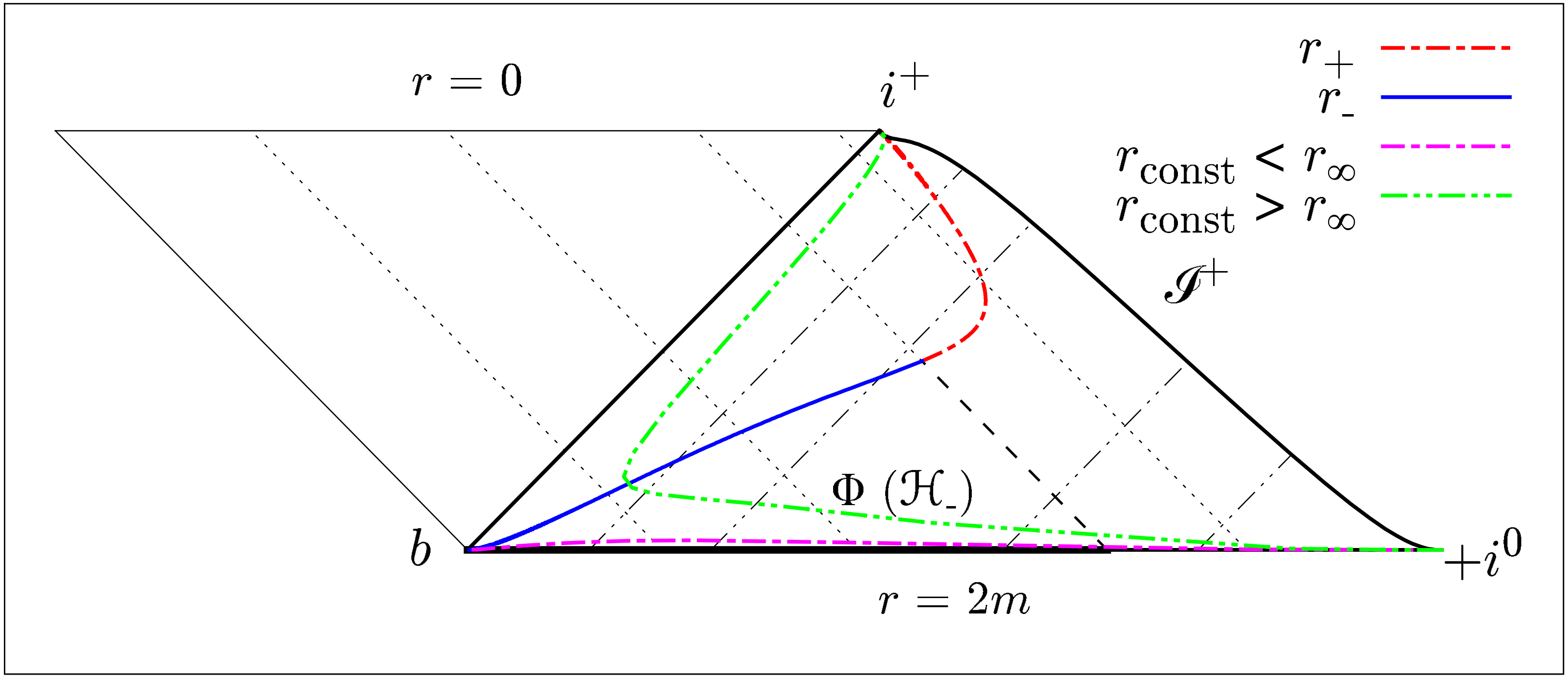

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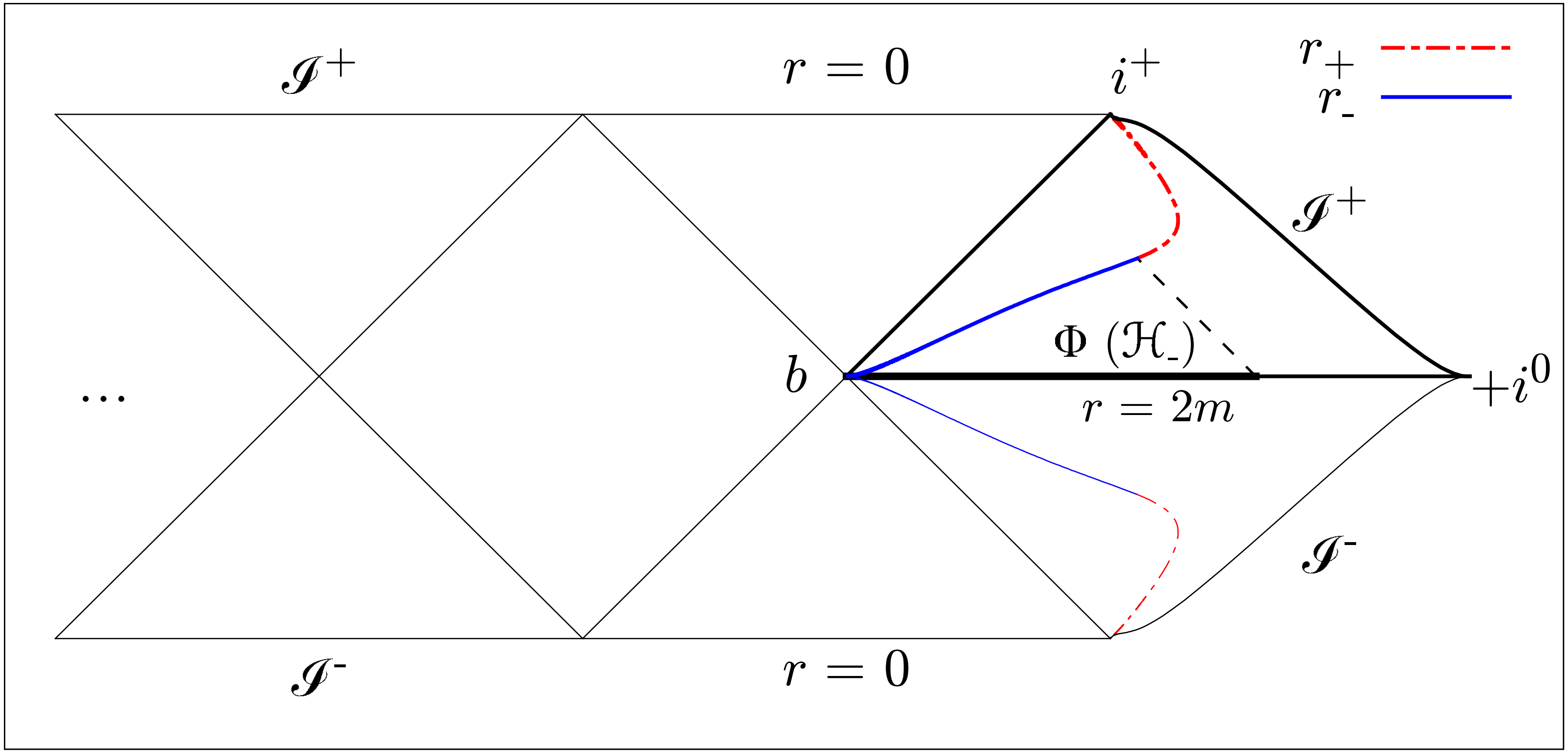
Penrose diagrams



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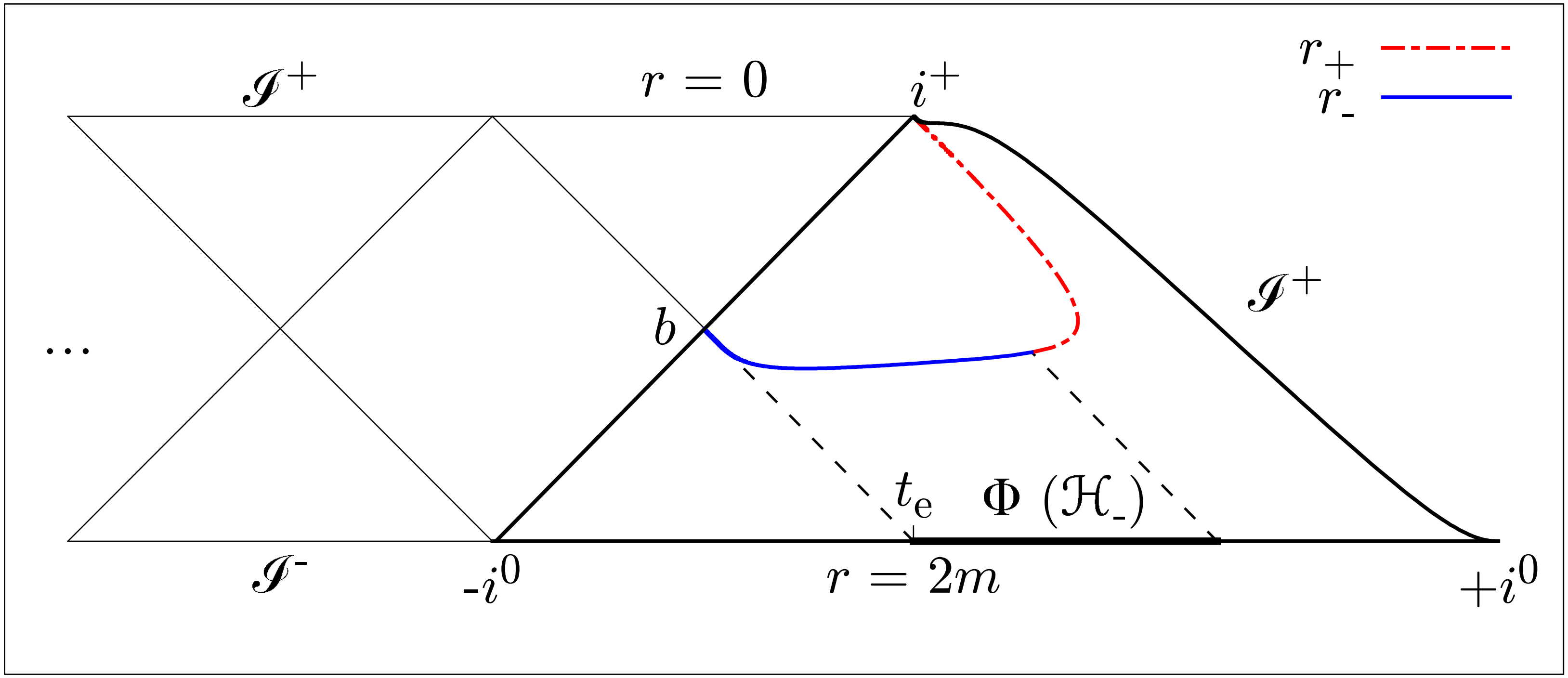

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Horndeski action

- We can also have a field as source for McVittie with $\dot{m} \neq 0$

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Reduces to McVittie/Cuscuton when $G = 0$ ($\dot{m} = 0$)

- Energy-momentum tensor of the KGB term

$$T_{\mu\nu} = \left(K - G_{;\alpha}\varphi^{;\alpha}\right) g_{\mu\nu} + \left(K_{,X} + \Box\varphi G_{,X}\right) \varphi_{;\mu}\varphi_{;\nu} + 2G_{(;\mu}\varphi_{;\nu)}$$

- Equivalent fluid four-velocity u^μ (time direction)

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- Solution of the field equations

$$\begin{aligned} G &= g_0(\varphi)\ln X + g_1(\varphi) \\ K &= f_1(\varphi) + f_2(\varphi)\sqrt{X} + 2X\left[(2 - \ln X)g_0' - g_1' - 24\pi g_0^2\right] \end{aligned}$$

Field equations

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$$\text{where} \begin{cases} g_0 = -\frac{1}{8\pi\dot\varphi}M \\ f_1 = -\frac{3}{8\pi}\left(H - M\right)^2 \\ f_2 = \frac{\sqrt{2}}{4\pi\dot\varphi}\left[H - M + 3M\left(\dot H - \dot M\right)\right] \end{cases}$$

Causal structure

- Radial null geodesics [A. Maciel, DCG, C. Molina, *PRD* **91** 084043 (2015), 1502.01003]

$$\frac{\mathrm{d}r}{\mathrm{d}t} = -R_\infty \left[\alpha \frac{r - r_\infty}{r_\infty} - \xi(t) \right] + o(\delta) \,,$$

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	$\dot{\xi}(t) \to 0^-$	$\dot{\xi}(t) \to 0^+$
$\left\lvert\int^\infty e^{(\alpha H_0 - \sigma)u}\xi(u)\,\mathrm{d}u\right\rvert < \infty$	black hole and white hole	black hole and white hole
$\left\lvert\int^\infty e^{(\alpha H_0 + \sigma)u}\xi(u)\,\mathrm{d}u\right\rvert \to \infty$	black hole only	white hole only

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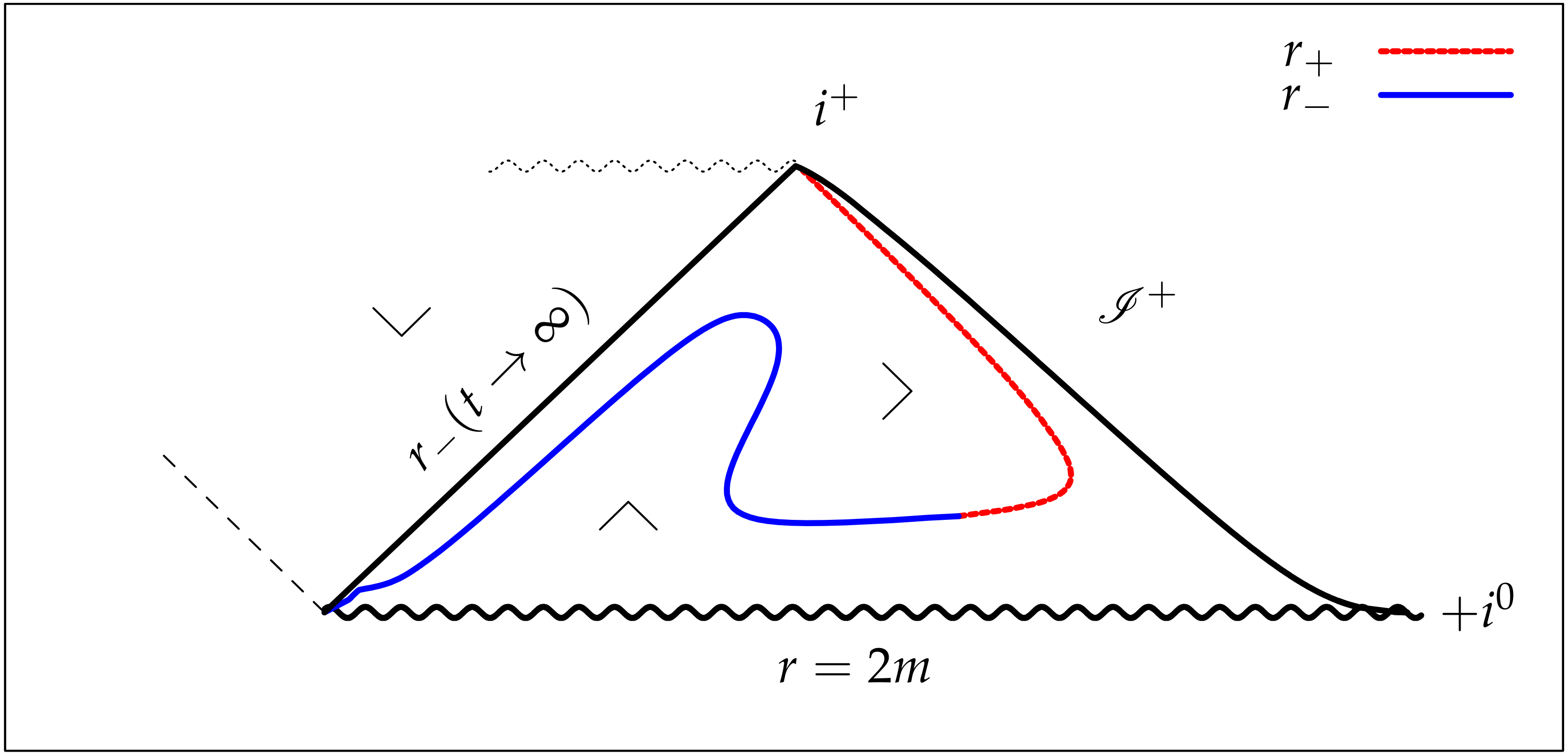

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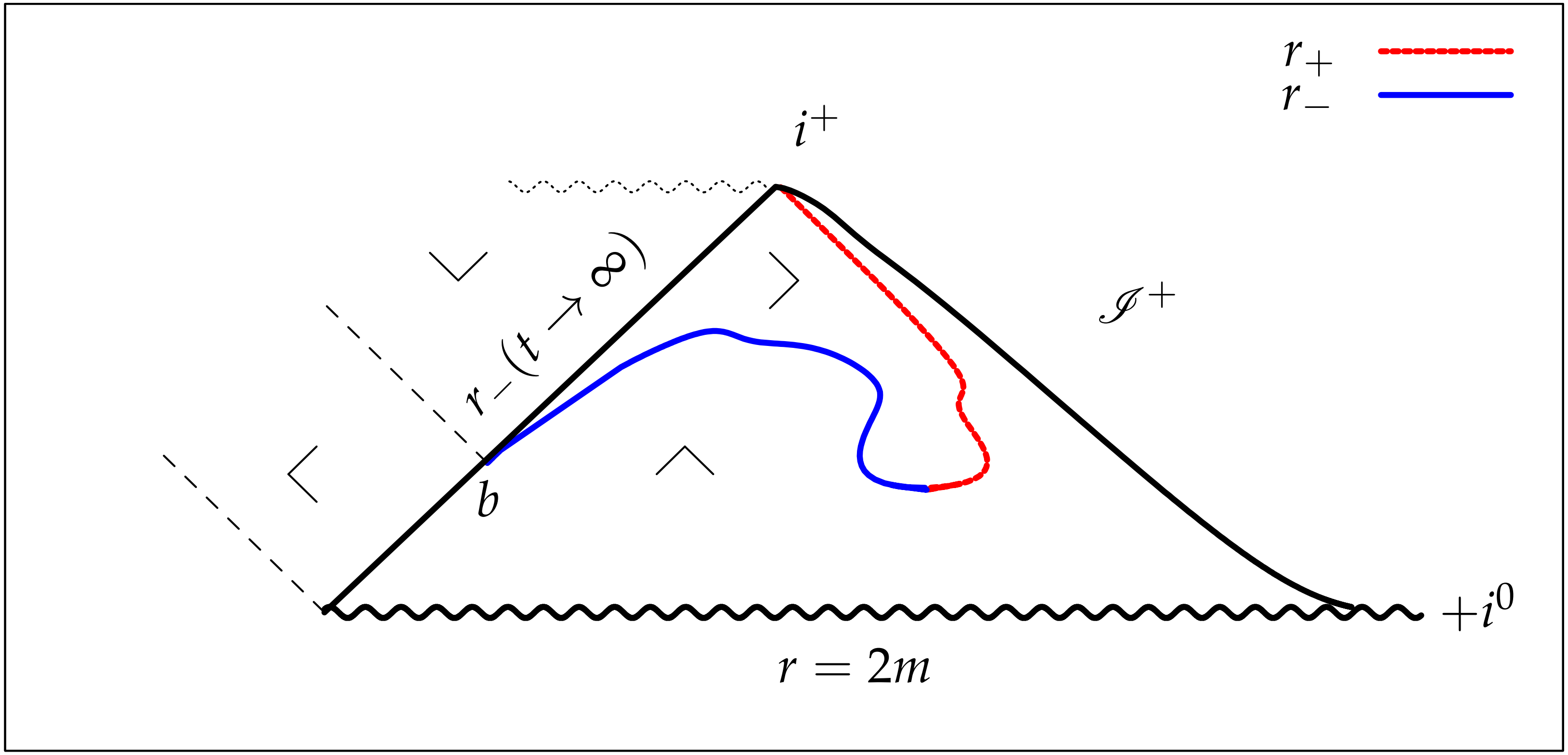

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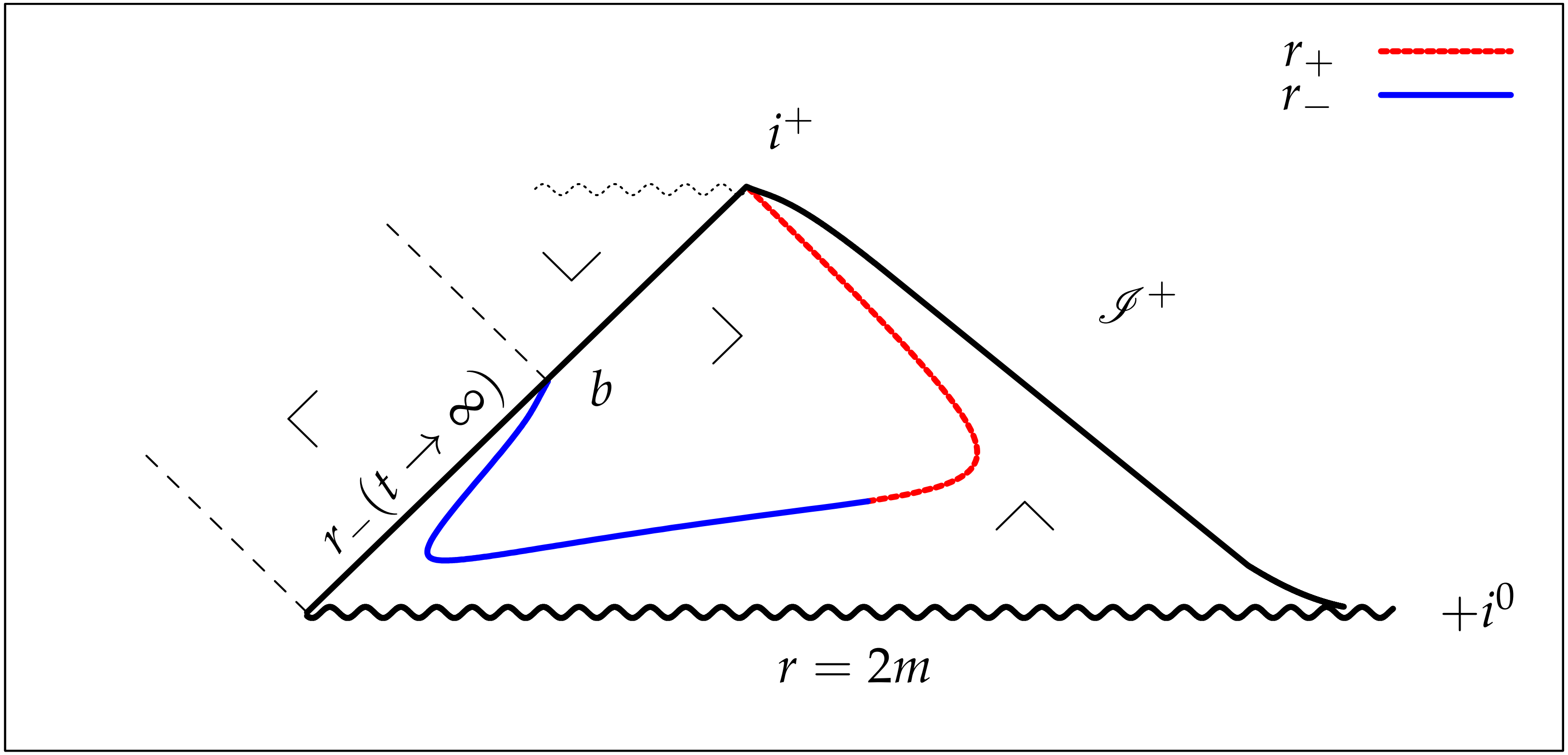

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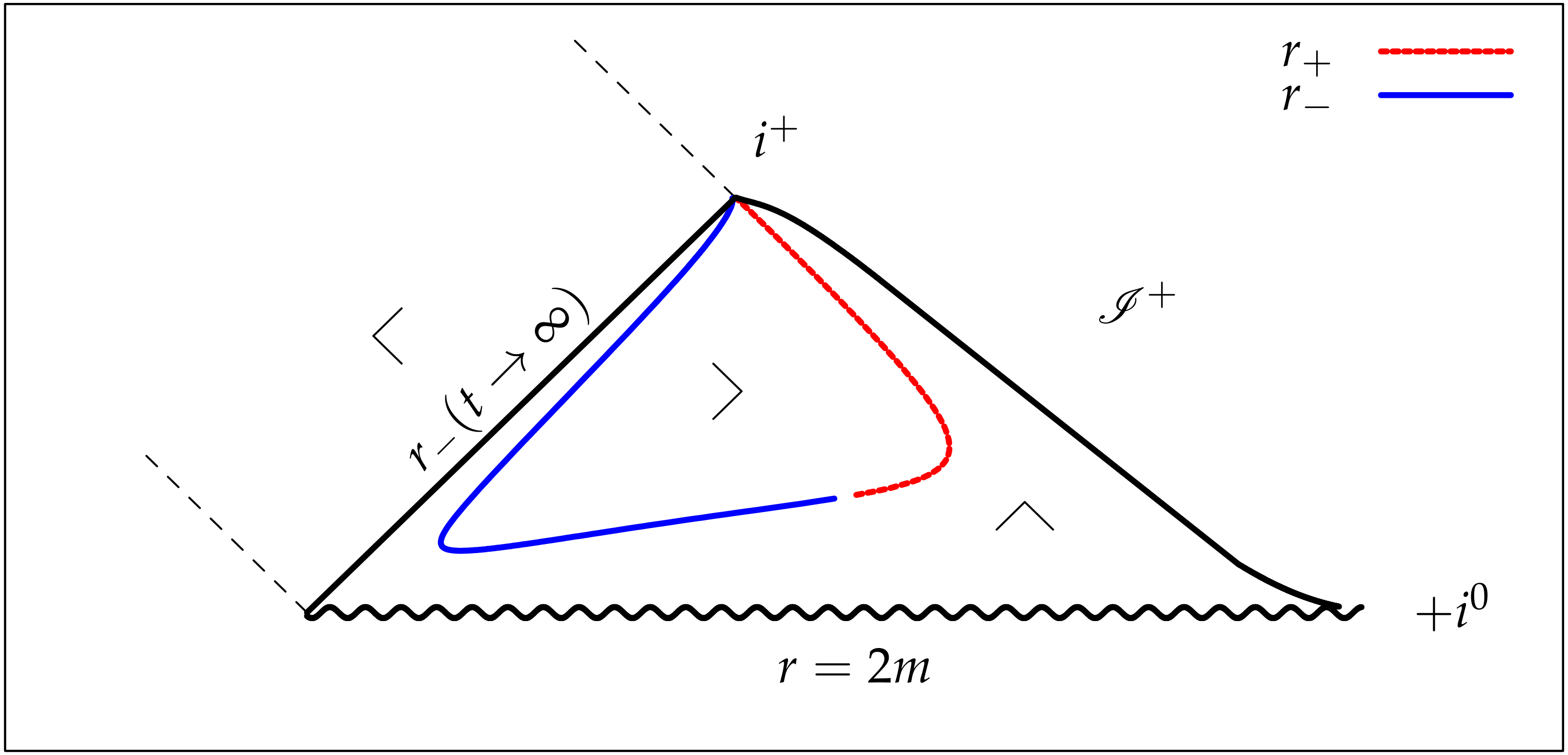

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- Causal structure of McVittie metrics with charge and non-flat spatial foliation
  - ☐ Apparent horizons
  - ☐ Cauchy horizons
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Thank you!

Accretion of general fluids

The McVittie metric
Accreting solutions
Conclusion
Extra slides
Heat flow
Full Horndeski action

- McVittie class does not admit accretion of perfect fluids
- Imperfect fluid (heat conductivity χ, ~~bulk viscosity ζ~~, ~~shear viscosity η~~)
  - ☐ McVittie class is shear-free
  - ☐ Bulk viscosity reabsorbed into pressure

$$T^r{}_r = T^\theta{}_\theta = T^\phi{}_\phi = p - 3\zeta\left(H + \frac{2\dot{m}}{2ar - m}\right)$$

- An imperfect fluid gives an exact solution to Einstein equations

[DCG, M. Fontanini, A. M. da Silva, E. Abdalla, *PRD* **86** 124020 (2012), 1207.1086]

- Expansion scalar and fluid density are related via (t, t) component of field equations

$$\rho(r,t) = \frac{3}{8\pi}\left[H + \frac{2\dot{m}}{2ar - m}\right]^2 = \frac{1}{8\pi}\frac{\Theta^2}{3}$$

$$S = \int \mathrm{d}^4x \sqrt{-g} \left(\frac{1}{2} R + \sum_{n=0}^{3} \mathcal{L}^{(n)} \right)$$

where

$$\begin{aligned}
\mathcal{L}^{(0)} &= K(X,\varphi) \\
\mathcal{L}^{(1)} &= G(X,\varphi)\,\Box\varphi \\
\mathcal{L}^{(2)} &= G^{(2)}(X,\varphi)_{,X} \left[(\Box\varphi)^2 - \varphi_{;\alpha\beta}\varphi^{;\alpha\beta} \right] + R G^{(2)}(X,\varphi) \\
\mathcal{L}^{(3)} &= G^{(3)}(X,\varphi)_{,X} \left[(\Box\varphi)^3 - 3\Box\varphi\,\varphi_{;\alpha\beta}\varphi^{;\alpha\beta} + 2\varphi_{;\alpha\beta}\varphi^{;\alpha\rho}\varphi_{;\rho}{}^{\beta} \right] \\
&\quad - 6 G_{\mu\nu}\varphi^{;\mu\nu} G^{(3)}(X,\varphi)
\end{aligned}$$

- $\mathcal{L}^{(2)}$ and $\mathcal{L}^{(3)}$ components of $T_{\mu\nu}$ have non-vanishing anisotropic stress