

*Corpuscular Considerations on
Cosmological Observables
and Eternal Inflation*



Florian Kühnel

COSMO-15

Warsaw, 10th of September 2015

work in particular with

Roberto Casadio,
Alessio Orlandi,
Marit Sandstad,
Bo Sundborg

*Corpuscular Considerations on
Cosmological Observables,
Eternal Inflation,
and Primordial Black Holes*

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★ **Recall:** The metric is a **classical** object

$$g_{\mu\nu}(x)$$

space-time coordinate

★ Now, in **quantum** theory there are **operators** acting on **states**
(with certain **occupation number** N)

$$\hat{a}^\dagger |0\rangle = |1\rangle$$

★ Key point (see Gia's talk):



Understand gravitational backgrounds, of characteristic wavelength R , as states on Minkowski space of a certain occupation number N of gravitons.

R being the R_S for black holes or R_H for cosmology.

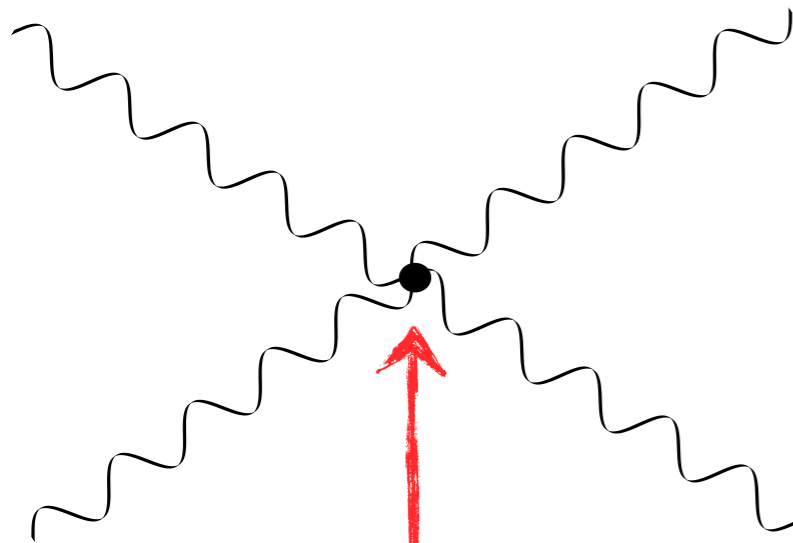
★ Occupation number:

$$N \sim \frac{R^2}{L_P^2}$$

(... smells holographic...)



- ★ Gravitational quanta (of General Relativity), gravitons, have mass $m = 0$ and spin = 2, with **momentum-dependent** self-interactions.



$$\alpha \sim \frac{\hbar G_N}{\lambda^2}$$

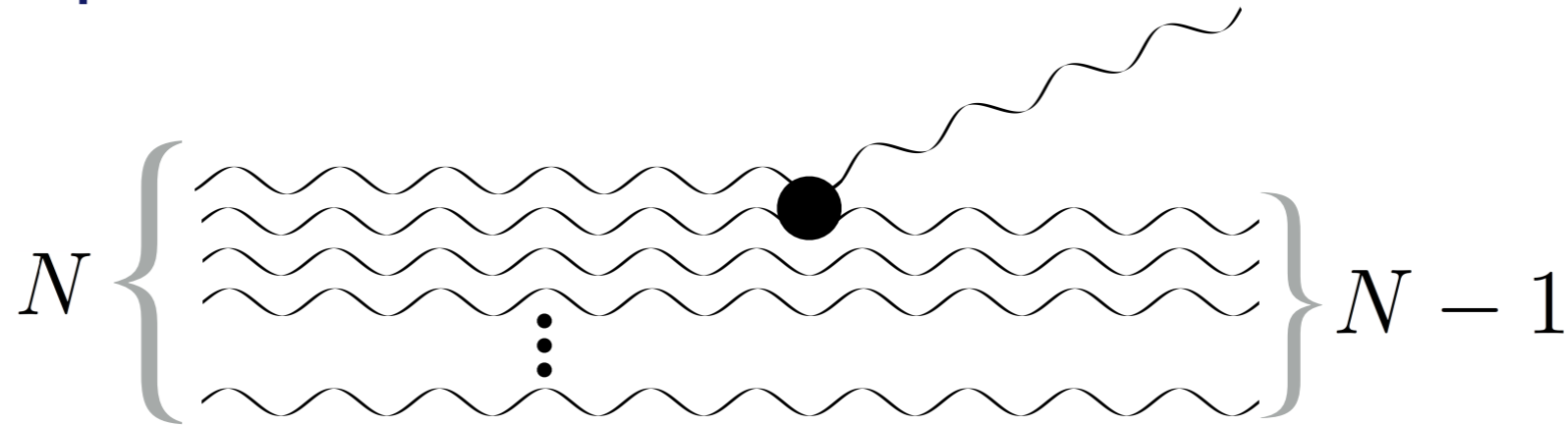
$$\Rightarrow \alpha = \frac{L_P^2}{\lambda^2}$$

- ★ Quantum entities:

$$L_P = \sqrt{\hbar G_N} \quad , \quad M_P = \frac{\hbar}{L_P}$$

- ★ Classical limit: $(L_P, M_P, \alpha) \xrightarrow{\hbar \rightarrow 0} 0$

★ Quantum depletion:



$$\rightarrow \frac{dN}{dt} \sim -\frac{1}{\sqrt{N}} + \mathcal{O}(N^{3/2})$$

(in Planck units) or, with $M \sim \sqrt{N}$

$$\rightarrow \frac{dM}{dt} \sim -M^{-2}$$

→ Hawking's result at leading order in $1/N$!

★ Note that particle creation is **not a vacuum process!**

★ States are characterised by a **single number**

N

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N

★ Mass: $M = \sqrt{N}$

★ Wavelength: $\lambda = \sqrt{N}$

★ Coupling: $\alpha = \frac{1}{N}$

★ Overview:

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3. **Quantum:** $\hbar \neq 0$, $\frac{1}{N} \neq 0$

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a) $\alpha N \neq 1$ (non-critical systems)

b) $\alpha N = 1$ (critical systems)

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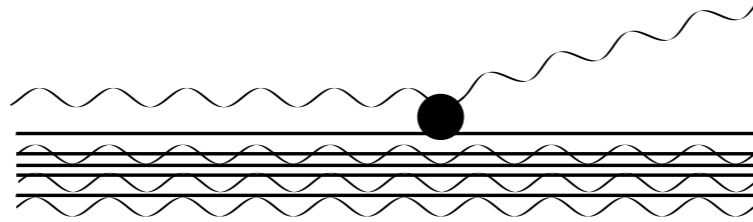
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a) $\alpha N \neq 1$ (non-critical systems)

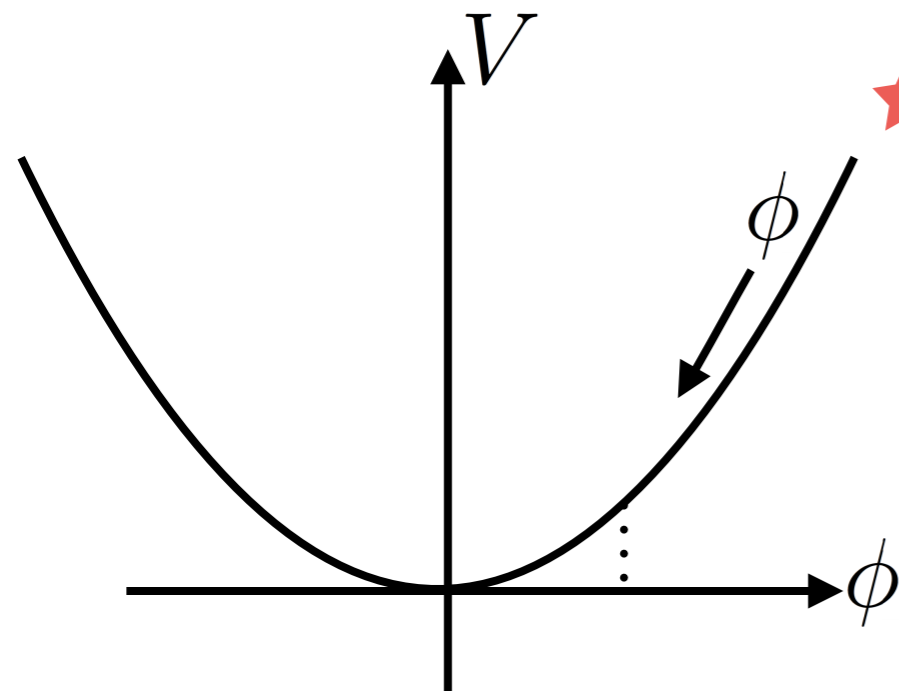
b) $\alpha N = 1$ (critical systems)

★ What does adding **extra species**, e.g. **inflaton**, do?



Enhancement

$$\frac{dN_{\text{grav}, \phi}}{dt} \sim \frac{dN_{\text{grav}}}{dt} \frac{N_{\phi}}{N_{\text{grav}}}$$



★ While for gravitons $N_{\text{grav}} \sim R_{\text{H}}^2$, we have **far more inflatons** during inflation:

$$\frac{N_{\phi}}{N_{\text{grav}}} \sim \frac{1}{\sqrt{\epsilon}}$$

$$\ddot{\phi} + 3H\dot{\phi} + V' = 0$$

$$\eta = \frac{V''}{V}$$

$$\epsilon = \left(\frac{V'}{V} \right)^2$$

★ Time evolution:

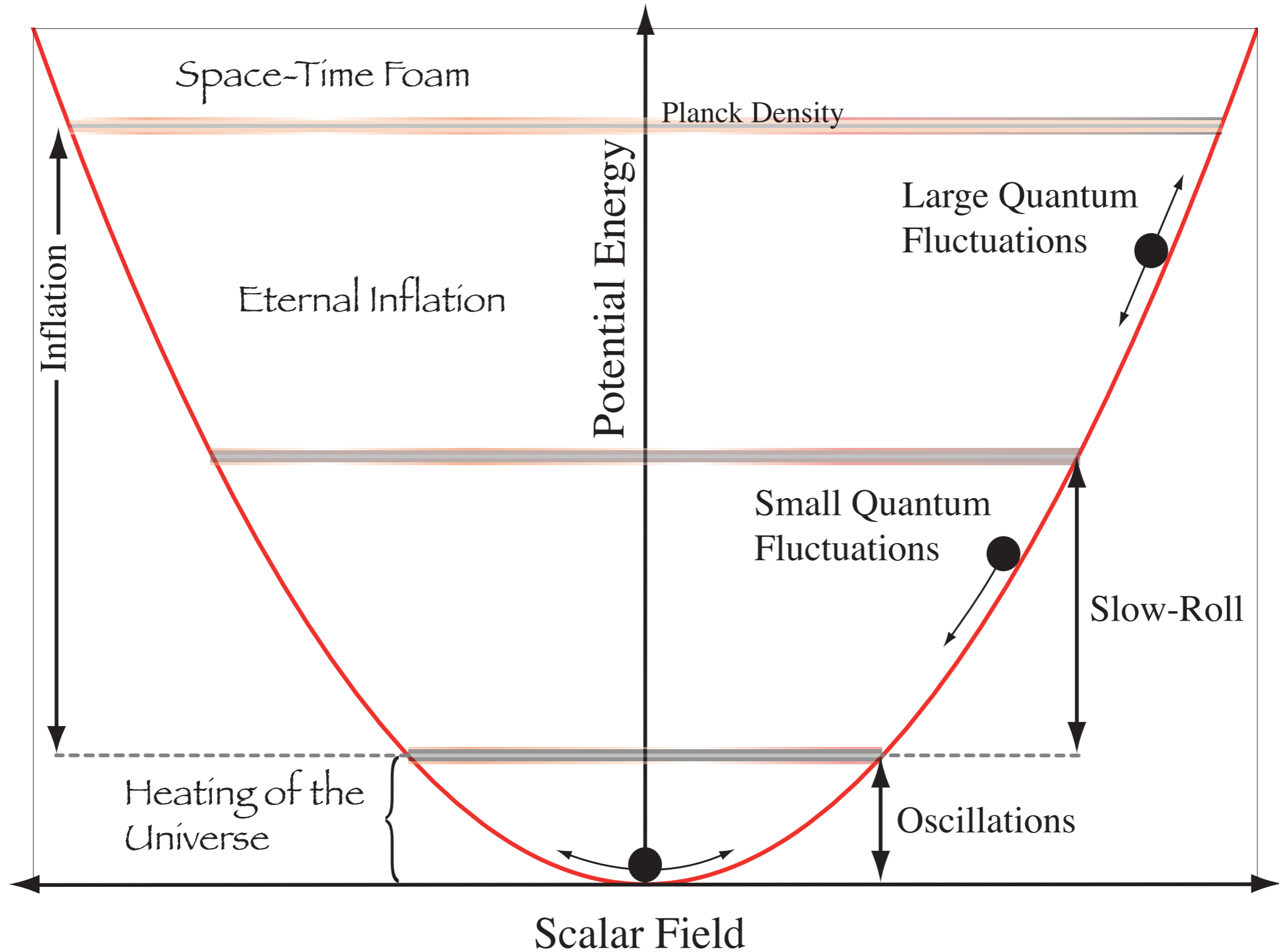
$$\frac{\dot{N}}{N} \sim \varepsilon H - \frac{1}{\sqrt{N}}$$

★ Two clocks: **classical** and **quantum**
(refilling the reservoir) (emptying the reservoir)



Limit on the **total** number of e-foldings!

Corpuscular Eternal Inflation?



Corpuscular Eternal Inflation?

★ Three contributions for $V = \lambda_n \phi^n$

★ typical quantum jumps:

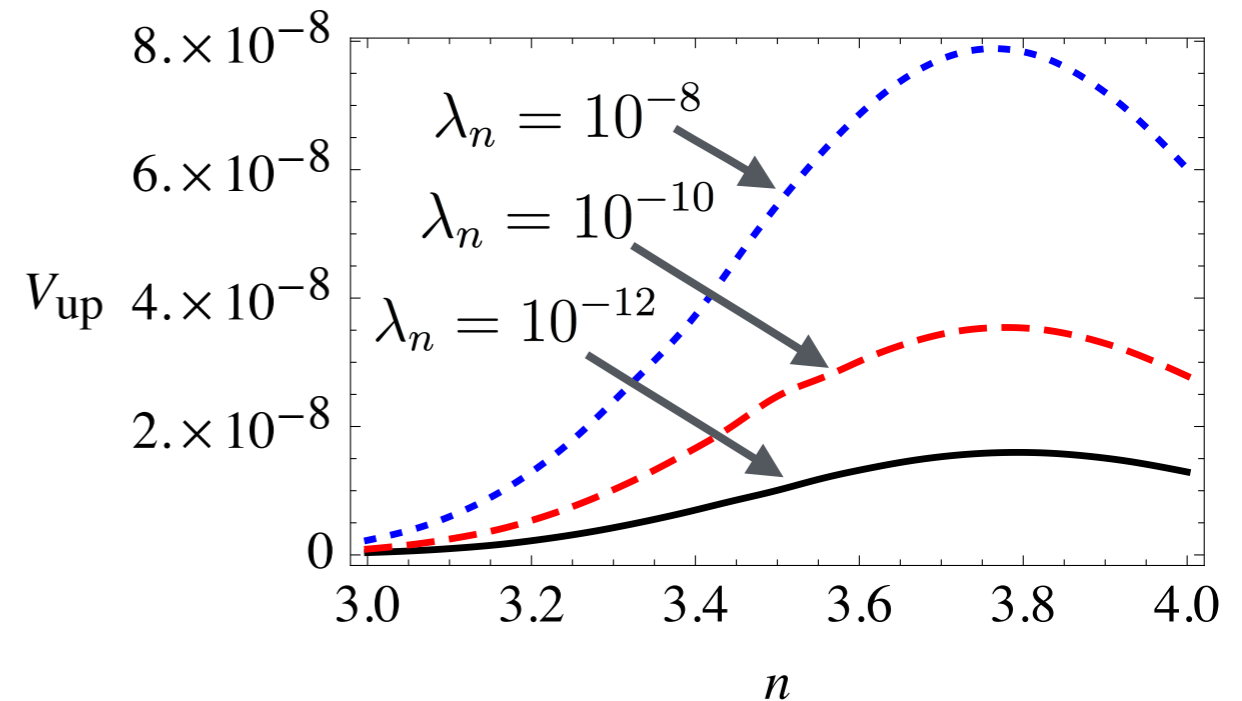
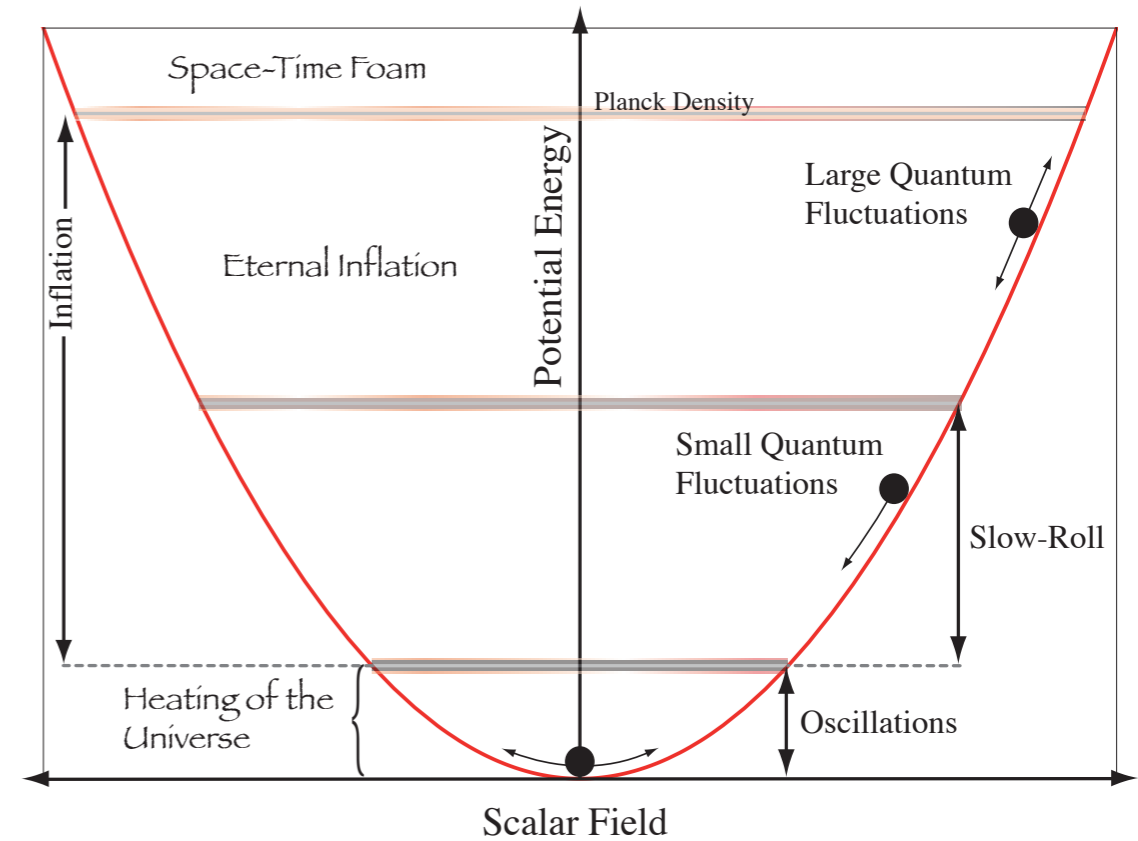
$$\dot{N}_{\text{qf}} \simeq \frac{n}{2\pi} \sqrt[n]{\frac{\lambda_n}{3n!}} N^{\frac{1}{n}}$$

★ quantum depletion:

$$\dot{N}_{\text{dep}} \simeq \sqrt{\frac{3}{n|n-1|}} \sqrt[n]{\frac{3n!}{\lambda_n}} N^{-\frac{2+n}{2n}}$$

★ classical drift:

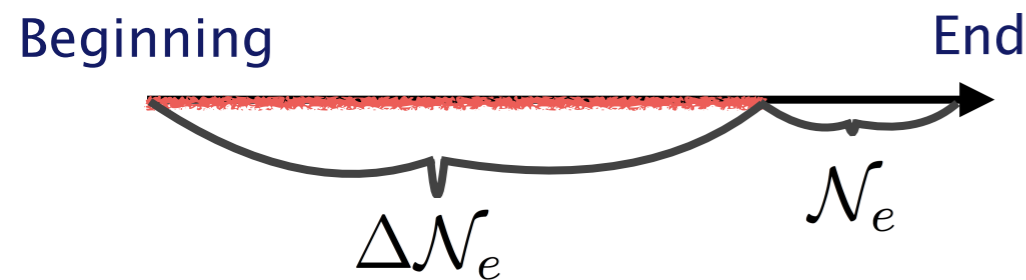
$$\dot{N}_{\text{cl}} \simeq n^2 \left(\frac{\lambda_n}{3n!}\right)^{\frac{2}{n}} N^{\frac{4+n}{2n}}$$



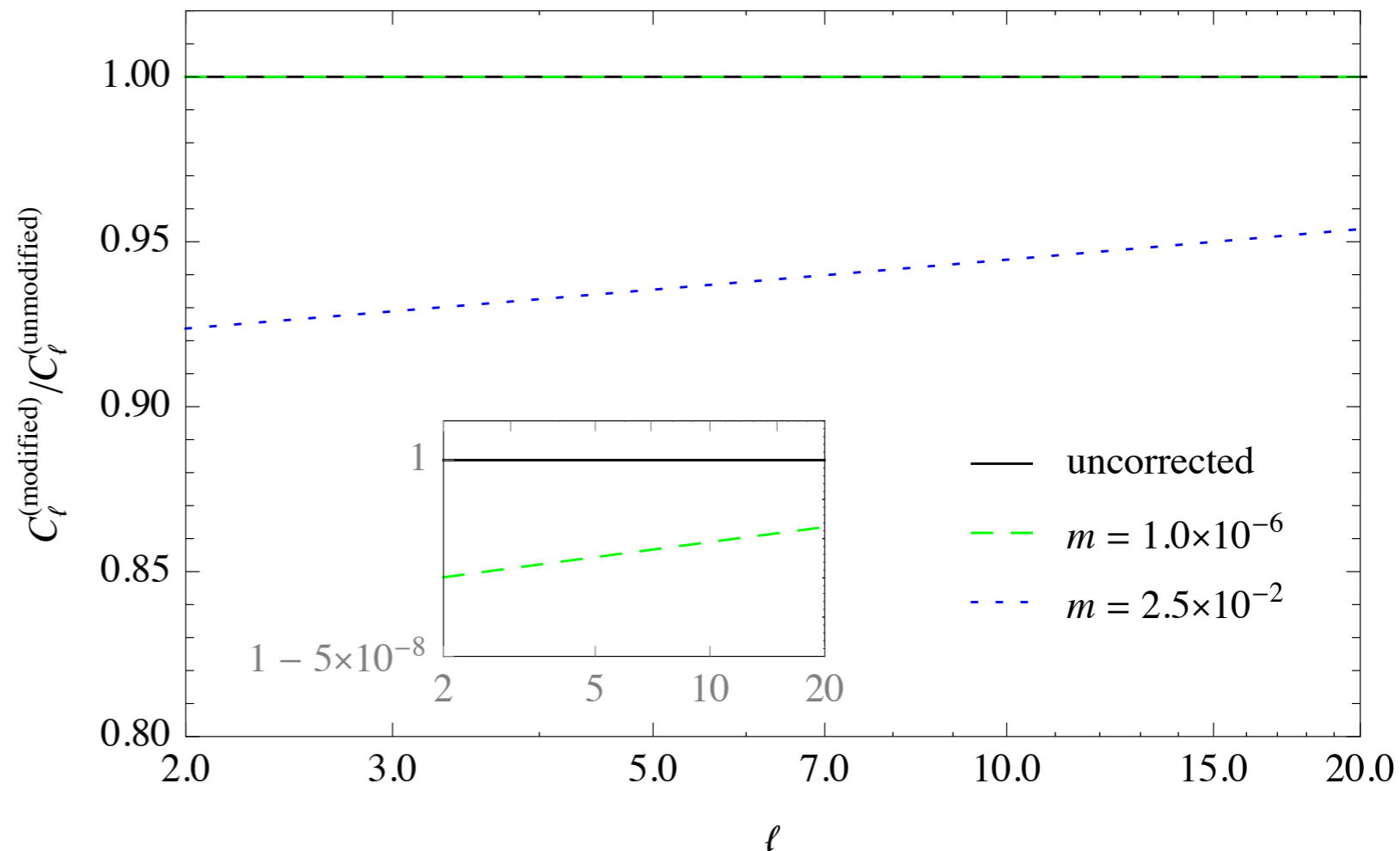
- ★ Quantum depletion can have strong impacts on observables:
[Dvali-Gomez '14]

$$r \simeq \frac{N}{N_\phi} \left[1 - \frac{\Delta \mathcal{N}_e}{N} \right]$$

$$(1 - n_s) \simeq \frac{3}{2} \varepsilon - \eta - \frac{1}{\sqrt{\varepsilon N}}$$



- ★ ... and also on **CMB** multipole moments:



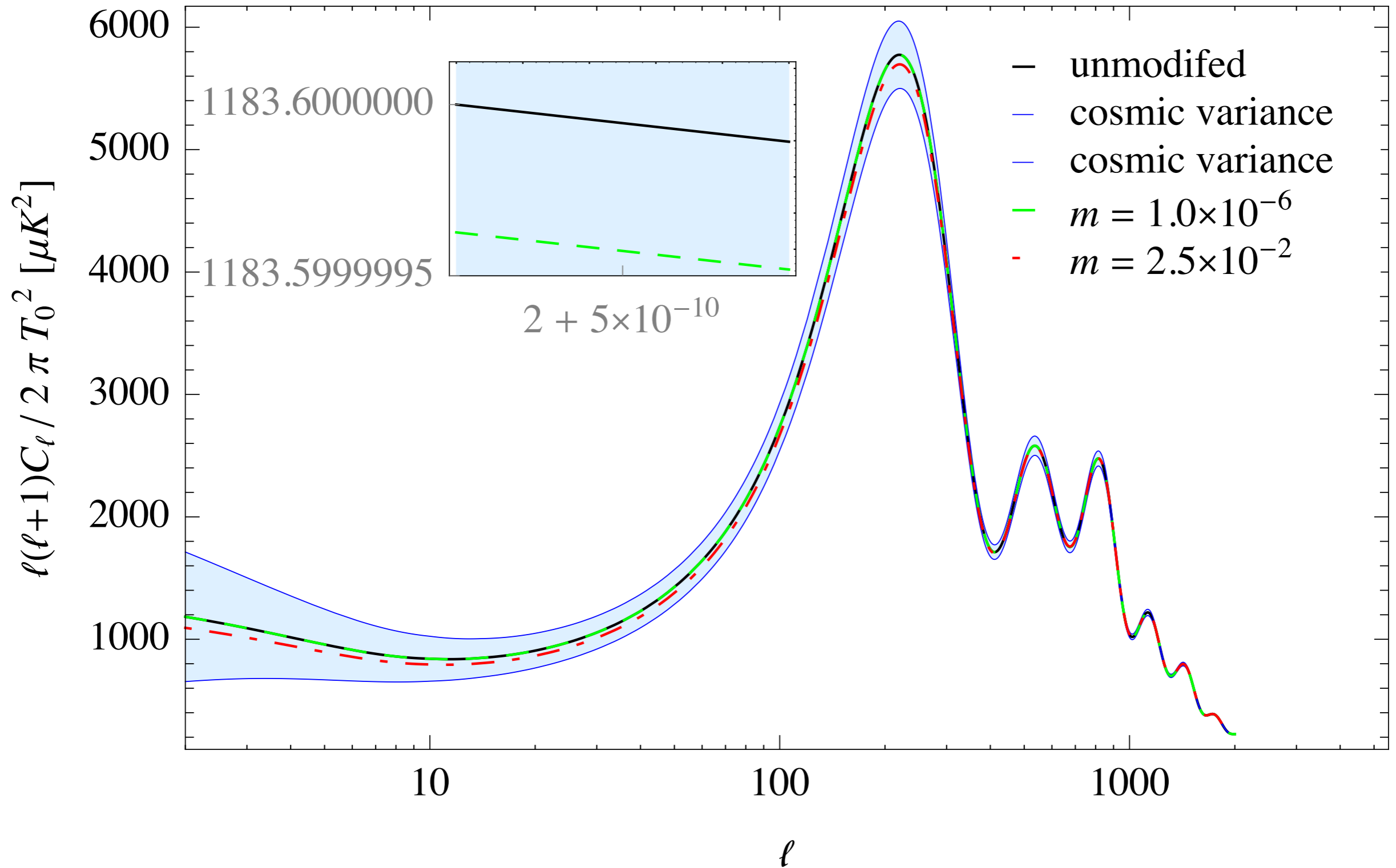
★ The inflatons increase the number of depletion channels

→ Leads to a **faster depletion rate** and naturally explains a small **tensor-to-scalar ratio...**

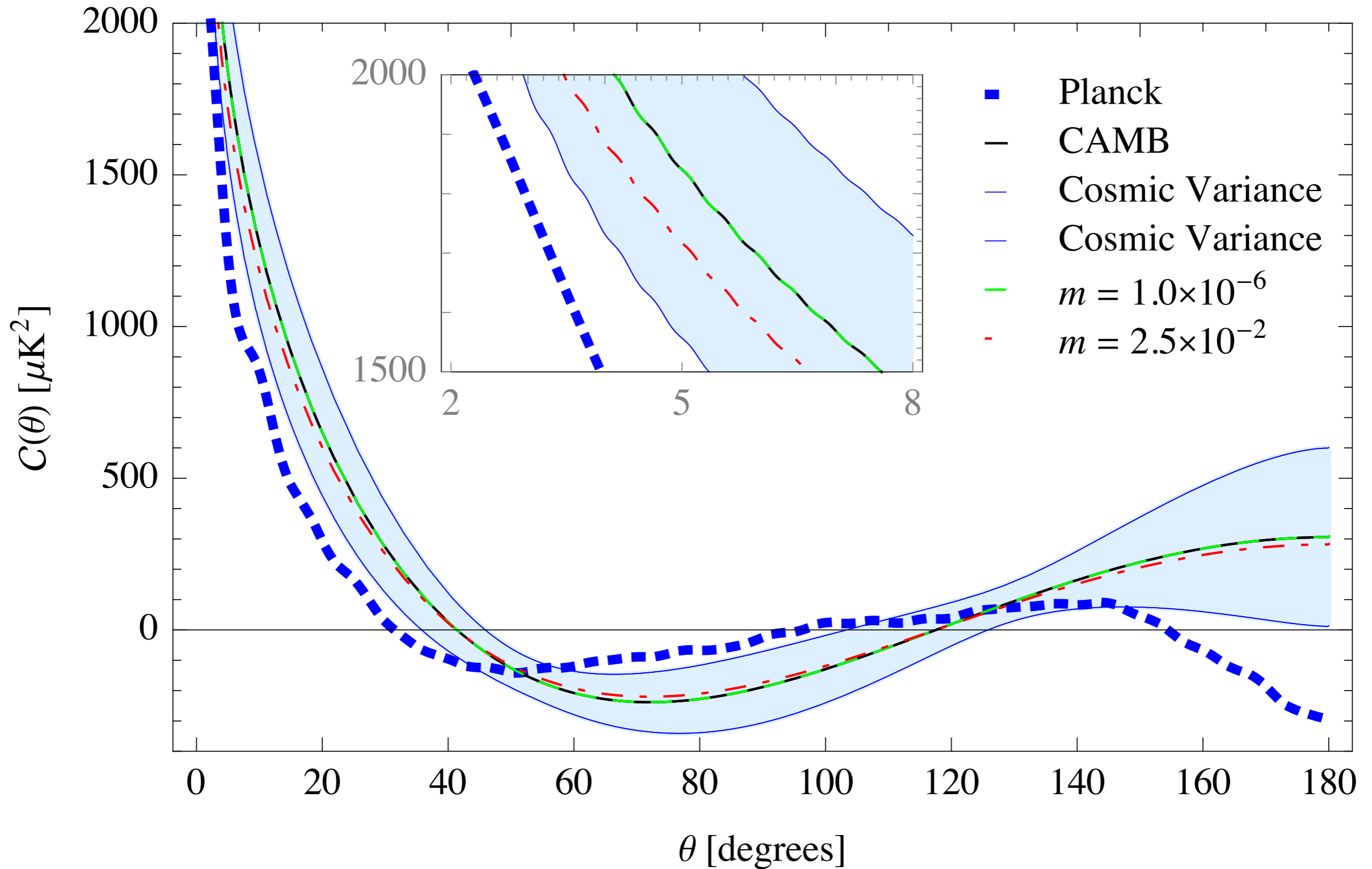
★ ... which is furthermore **suppressed** with the mass scale:

$m[m_p]$	10^{-6}	10^{-5}	10^{-4}	10^{-3}	$5 \cdot 10^{-3}$	10^{-2}	$1.5 \cdot 10^{-2}$	2×10^{-2}
ratio								
corrected / uncorrected	1	1	1	0.93	0.43	0.25	0.18	0.14

Power Spectrum



Angular Temperature Auto-Correlation



★ PBHs are black holes formed in the (very) **early Universe**.

★ **Possibly formed by:** inhomogeneity, bubble collisions, cosmic strings, domain walls, ...

★ **Probe a huge range of scales:**

$M \sim 10^{-5} \text{g}$: **Quantum gravity** (Planck relics, Brane cosmology, TeV QG, ...)

$M \lesssim 10^{15} \text{g}$: **Early Universe** (Baryogenesis, Nucleosynthesis, Reionisation, ...)

$M \sim 10^{15} \text{g}$: **High-energy physics** (Ultra-high cosmic rays, ...)

$M \gtrsim 10^{15} \text{g}$: **Gravity** (CDM, gravitational waves, LSS, BH in galactic nuclei, ...)

★ **Connection to critical phenomena** [Choptuik '93]:

$$\frac{M_{BH}}{M_H} = K (\delta - \delta_c)^\gamma$$

horizon mass \rightarrow M_H \leftarrow density contrast δ

★ Now add **baryons** and consider the master equations:

[Dvali-Gomez '13]

$$\dot{N} \simeq - \frac{1}{L_P \sqrt{N}} \quad \text{gravitons}$$

$$\dot{N}_B \simeq - \frac{1}{L_P \sqrt{N}} \frac{N_B}{N} \quad \text{baryons}$$

→ For $N_B \ll N$ the graviton depletion rate will be much larger than that of the baryons.

→ Baryon density increases till a critical value is reached.

→ Formation of a new state!

★ Predictions (for $n_c \equiv 1$ baryon/fm³):

★ **Mass:** $M \simeq 3 \cdot 10^{23}$ kg

★ **Formation time:** $t_{\text{form}} \sim 10^{-12}$ s

★ Applied to **astrophysical black-hole formation**,
using the same critical baryon density, and
estimating $N_{B,\text{initial}} \sim M_{\text{Star}}/M_{\text{Proton}}$

→ **Lowest possible mass of the bound state**

$$M \geq \frac{c^3}{4} \sqrt{\frac{3 \cdot 10^{57}}{2\pi M_{\odot} n_c}} G_{\text{N}}^{-3/2} \approx 3 M_{\odot}$$

which roughly **reproduces the Chandrasekhar limit!**