Vector and Tensor Contributions to ζ

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Contents

- What is ζ?
- Why do we care about ζ?
- The many faces of ζ
- Evolution of ζ
- Summary (of ζ)

Cosmological Perturbation Theory

Start with Friedmann-Lemaître-Robertson-Walker

$$ds^2 = g^{(0)}_{\mu\nu} dx^\mu dx^\nu = a(\tau)^2(-d\tau^2 + d\vec{x}^2)$$

And perturb it

$$g_{\mu\nu} = g^{(0)}_{\mu\nu} + \delta g_{\mu\nu},$$

$$\delta g_{00} = -2a^2\phi\,, \qquad \delta g_{i0} = a^2\left(B_{,i} + \boldsymbol{B_{Vi}}\right),$$

$$\delta g_{ij} = 2a^2\left(-\boldsymbol{\psi}\delta_{ij} + E_{,ij} + \boldsymbol{F}_{(\boldsymbol{i},j)} + \boldsymbol{h_{ij}}\right).$$

Curvature perturbation **Vectors** **Tensors**

Cosmological Perturbation Theory

Perturbations are dependent on the choice of coords., called a **gauge choice**.

To make that choice one must fix:

- 1 time coordinate – **Slicing**
- 3 spatial coordinates – **Threading**

} **Gauge Fixing** = 4 conditions

And hence define **gauge invariants!**

What is ζ?

Long name:

Curvature Perturbation on Uniform Density Hypersurfaces

$$\boldsymbol{\zeta}$$

Definition:

$$\zeta = -\psi, \quad \text{in the gauge with} \quad \delta\rho = 0$$

At first order, this means

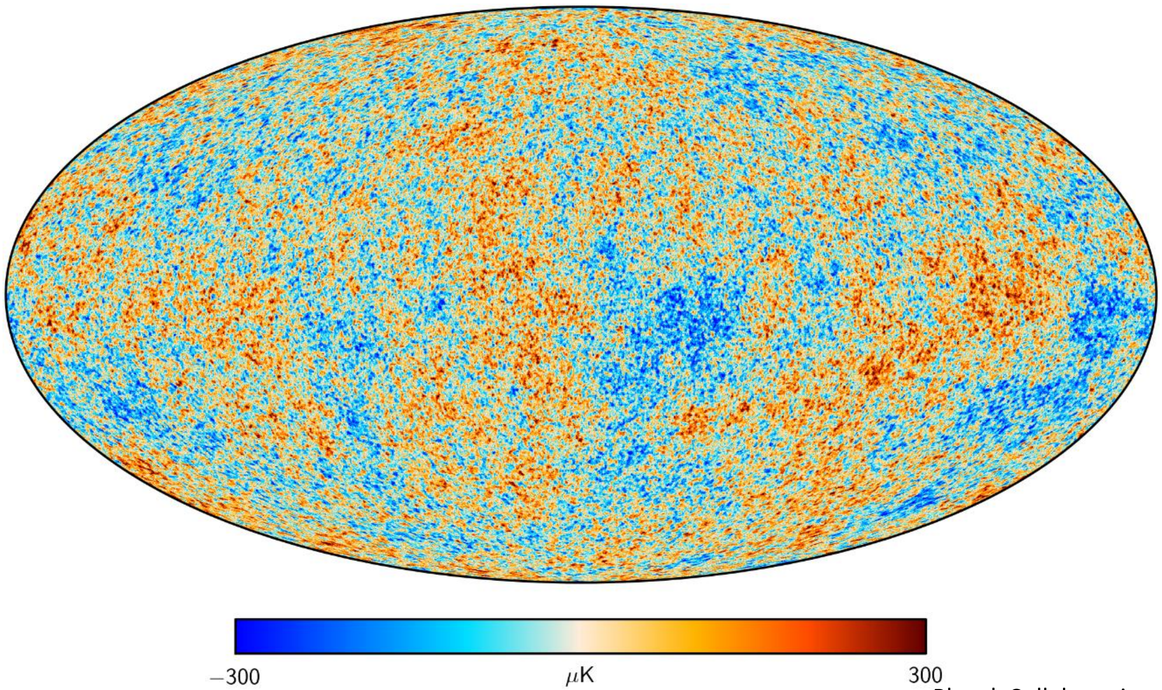
$$\zeta_1 = -\psi_1 - \frac{\mathcal{H}}{\rho'}\delta\rho_1$$

Gauge Invariant

Why do we care about ζ?

The **Curvature Perturbation** – ζ – links observations to fundamental theory:

$$\langle \delta T \delta T \rangle \sim \langle \zeta\zeta \rangle \sim \langle \delta\varphi\delta\varphi \rangle$$



Planck Collaboration

Why do we care about ζ?

With future surveys – Euclid, SKA, LSST – observables will be measured to high accuracy. E.g. f_{NL}:

$$f_{NL} \sim \frac{\langle \zeta\zeta\zeta \rangle}{\langle \zeta\zeta \rangle \langle \zeta\zeta \rangle} \sim O(1)$$

Theoretical predictions must keep up with this accuracy.

Non-linear (2nd order) contributions are required!

The many faces of ζ

The definition of ζ:

$$\zeta = -\psi, \quad \text{in the gauge with} \quad \delta\rho = 0$$

is not **unique**, for two reasons:

- There are other ways to split g_{ij} and define ψ;
- $\delta\rho = 0$ is not enough to fix the gauge at all orders, the threading must also be fixed.

The many faces of ζ

Four different versions of ψ:

- Original [Mukhanov, Feldman, Brandenberger, 1992]

$$\delta g_{ij} = 2a^2\left(-\psi\delta_{ij} + E_{,ij}\right)$$

- Trace of δg_{ij} [Bardeen, 1980]

$$\psi_T = \mathrm{tr}\left(\delta g_{ij}/(2a^2)\right)$$

- Determinant of g_{ij} [Salopek, Bond, 1990]

$$e^{6\psi_D} = \det(g_{ij}/a^2)$$

- Determinant of g^{ij}

$$e^{-6\psi_I} = \det(g^{ij}a^2)$$

The many faces of ζ

Two main choices for fixing the threading:

- Flat threading

  Fixes spatial metric components:

$$E = F_i = 0$$

- Comoving threading

  Fixes velocity perturbations:

$$v = v_{Vi} = 0$$

- And many more…

The many faces of ζ

In summary: There are many definitions for ζ!

We studied 6, but show only 2:

- Original convention and flat threading:

$$\zeta_2 = \underbrace{-\psi_2 - \frac{\mathcal{H}}{\rho'}\delta\rho_2}_{\text{Linear}} + \underbrace{\chi^i{}_i + \partial^{-2}\partial_i\partial_j\chi^{ij}}_{\text{Non-linear}}$$

- Determinant of g^{ij}, comoving threading:

$$\zeta_{I2}^{(v)} = \zeta_2 - 2\zeta_1^2 - \frac{2}{3}h_{1ij}h_1^{ij} + \frac{1}{3}W_{1i}W_1^i$$
$$+\frac{1}{3}\int \partial_i V_2^i\, d\tau + 2\partial_i\zeta_1 \int V_1^i\, \mathrm{d}\tau + \frac{2}{3}\int\left\{\partial_i\left[V_1^i \Upsilon\right] + \partial_i\partial_j V_1^j \int V_1^i d\tau'\right\}d\tau$$

Momentum perturbation Velocity perturbation Lapse perturbation

Evolution of ζ

We study the evolution of ζ using solely:

$$\nabla_\mu T^{\mu\nu} = 0,$$

with $T^{\mu\nu}$ evaluated in the energy frame ($q^\mu = 0$):

$$T^{\mu\nu} = (\rho + P)u^\mu u^\nu + P g^{\mu\nu} + \pi^{\mu\nu},$$

$$\pi^{\mu\nu} u_\nu = 0, \qquad \pi^\mu{}_\mu = 0.$$

The field equations are not necessary.

Evolution of ζ

There are only two possibilities at first order:

- Flat threading

$$\zeta_1' = -\frac{1}{3}\partial_i V_1^i - \mathcal{H}\,\frac{\delta P_{\mathrm{nad}\,1}}{\rho + P}$$

- Comoving threading

$$\zeta_1^{(v)\,\prime} = -\mathcal{H}\,\frac{\delta P_{\mathrm{nad}\,1}}{\rho + P}$$

Evolution of ζ

Up to second order, we find:

- Original convention, flat threading

$$\left(-\zeta_2 + 2\zeta_1^2 + \frac{2}{3}h_{1ij}h_1^{ij} - \frac{1}{3}W_{1i}W_1^i\right)' = \frac{1}{3}\partial_i V_2^i + \frac{2}{3}\partial_i\left[V_1^i \Upsilon\right]$$

$$+2\partial_i\zeta_1 V_1^i + \mathcal{H}\frac{\delta P_{\mathrm{nad}\,2}}{\rho + P} - 2\mathcal{H}\frac{\delta P_{\mathrm{nad}\,1}^2}{(\rho + P)^2} - \frac{2\mathcal{H}}{a^2\rho'}\pi_{1ij}\left(\partial^j V_1^i + h_1^{ij\prime}\right)$$

- Determinant of g^{ij}, comoving threading

$$-\zeta_{I2}^{(v)\prime} = \mathcal{H}\frac{\delta P_{\mathrm{nad}\,2}^{(v)}}{\rho + P} - 2\mathcal{H}\frac{\delta P_{\mathrm{nad}\,1}^2}{(\rho + P)^2} - \frac{2\mathcal{H}}{a^2\rho'}\pi_{1ij}\left(\partial^j V_1^i + h_1^{ij\prime}\right)$$

In agreement with [Enqvist, Högdahl, Nurmi, Vernizzi, 2007]

Evolution of ζ

On large scales ($\partial \to 0$), the results reduce to:

$$-\zeta_{I2}^{(v)\prime} = \left(-\zeta_2 + 2\zeta_1^2 + \frac{2}{3}h_{1ij}h_1^{ij} - \frac{1}{3}W_{1\mathrm{V}i}W_{1\mathrm{V}}^{i}\right)' =$$

$$\mathcal{H}\frac{\delta P_{\mathrm{nad}\,2}}{\rho + P} - 2\mathcal{H}\frac{\delta P_{\mathrm{nad}\,1}^2}{(\rho+P)^2} - \frac{2\mathcal{H}}{\rho'}\Pi_{1ij}h_1^{ij\prime}$$

With negligible δP_{nad} and Π_{ij}, there exists a conserved quantity:

$$\zeta_{I2}^{(v)} = \zeta_2 - 2\zeta_1^2 - \frac{2}{3}h_{1ij}h_1^{ij} + \frac{1}{3}W_{1\mathrm{V}i}W_{1\mathrm{V}}^{i}$$

Some ζ's are more conserved than other ζ's.

Summary

- There are many different versions of ζ.

- In general, the evolution is different for each version and is dependent on the vector and tensor modes.

- $\zeta_I^{(v)}$, based on the determinant of g^{ij}, is **exactly** conserved if both $\delta \mathrm{P}_{\mathrm{nad}} = 0$ and $\pi_{ij} = 0$.

Vector and Tensor Contributions to ζ

Questions?

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