

Dark energy and non-linear power spectrum

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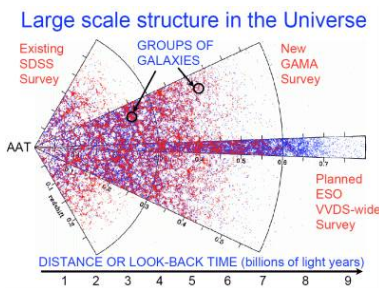
Based on S. G. Biern and [JG](#), 1505.02972 [astro-ph.CO]

Outline

- 1 Introduction
- 2 Formulation of perturbation theory
 - Newtonian theory
 - Relativistic theory
- 3 Relativistic theory with homogeneous dark energy
 - Effects of dark energy
 - Non-linear power spectrum with dark energy
- 4 Conclusions

Why GR in LSS?

Planned galaxy surveys: DESI, HETDEX, LSST, Euclid, WFIRST...



Larger and larger volumes, eventually accessing the scales comparable to the horizon: beyond Newtonian gravity, fully general relativistic approach (or any modification) is necessary

Why dark energy in non-linear regime?

- DE was negligible at very early times
- DE becomes significant at later stage when non-linearities in cosmic structure are developed

Naturally DE affects the evolution of gravitational instability, so that its effects emerge more prominently at non-linear level

What are the effects of DE in non-linear regime of LSS?

Newtonian theory

3 basic equations for density perturbation $\delta \equiv \delta\rho/\bar{\rho}$, peculiar velocity \mathbf{u} and gravitational potential Φ with a *pressureless* fluid

$$\dot{\delta} + \frac{1}{a} \nabla \cdot \mathbf{u} = -\frac{1}{a} \nabla \cdot (\delta \mathbf{u}) \quad \text{continuity eq}$$

$$\dot{\mathbf{u}} + H\mathbf{u} + \frac{1}{a} \nabla \Phi = -\frac{1}{a} (\mathbf{u} \cdot \nabla) \mathbf{u} \quad \text{Euler eq}$$

$$\frac{\Delta}{a^2} \Phi = 4\pi G \bar{\rho} \delta \quad \text{Poisson eq}$$

Newtonian system is closed at 2nd order

$$\ddot{\delta} + 2H\dot{\delta} - 4\pi G \bar{\rho} \delta = -\frac{1}{a^2} \frac{d}{dt} [a \nabla \cdot (\delta \mathbf{u})] + \frac{1}{a^2} \nabla \cdot (\mathbf{u} \cdot \nabla \mathbf{u})$$

→ at linear order, $\delta_+ \propto a$ (growing) and $\delta_- \propto a^{-3/2}$ (decaying)

Basic non-linear equations

Based on the ADM metric

$$ds^2 = -N^2(dx^0)^2 + \gamma_{ij}(N^i dx^0 + dx^i)(N^j dx^0 + dx^j)$$

the fully non-linear equations are (Bardeen 1980)

$$R - \bar{K}^i_j \bar{K}^j_i + \frac{2}{3} K^2 - 16\pi G E = 0$$

$$\bar{K}^j_{i;j} - \frac{2}{3} K_{,i} = 8\pi G J_i$$

$$\frac{K_{,0}}{N} - \frac{K_{,i} N^i}{N} + \frac{N^i_{;i}}{N} - \bar{K}^i_j \bar{K}^j_i - \frac{1}{3} K^2 - 4\pi G(E + S) = 0$$

$$\frac{\bar{K}^i_{j,0}}{N} - \frac{\bar{K}^i_{j;k} N^k}{N} + \frac{\bar{K}^j_{k;N^i;k}}{N} - \frac{\bar{K}^i_k N^k_{;j}}{N} = K \bar{K}^i_j - \frac{1}{N} \left(N^i_{;j} - \frac{\delta^i_j}{3} N^i{}^k{}_{;k} \right) + \bar{R}^i_j - 8\pi G \bar{S}^i_j$$

$$\frac{E_{,0}}{N} - \frac{E_{,i} N^i}{N} - K \left(E + \frac{S}{3} \right) - \bar{K}^i_j \bar{S}^j_i + \frac{(N^2 J^i)_{;i}}{N^2} = 0$$

$$\frac{J_{i,0}}{N} - \frac{J_{i;j} N^j}{N} - \frac{J_j N^j_{;i}}{N} - K J_i + \frac{E N_{,i}}{N} + S^j_{i;j} + \frac{S^j_i N_{,j}}{N} = 0$$

Fluid quantities: $E \equiv n_\mu n_\nu T^{\mu\nu}$, $J_i \equiv -n_\mu T^{\mu}_i$, $S_{ij} \equiv T_{ij}$

Einstein-de Sitter universe

Usually, structure formation is described in EdS

$$T_{\mu\nu} = \rho_m u_\mu u_\nu \longrightarrow J_i = S_{ij} = 0$$

- Linear growth factor is all: $D_1 = a$, $D_2 = 3D_1^2/7$ and so on
- Comoving gauge ($\gamma = 0$ and $T^0_i = 0$) gives identical equations to the Newtonian counterparts up to 2nd order
- Pure GR contribution appears from 3rd order and is totally sub-dominant (Jeong, [JG](#), Noh & Hwang 2011, Biern, [JG](#) & Jeong 2014)
- In e.g. synchronous gauge ($g_{00} = -1$ and $g_{0i} = 0$) we can have another Newtonian correspondence (Hwang, Noh, Jeong, [JG](#) & Biern 2015)

Linear power spectrum is obtained by solving the Boltzmann eq (e.g. CAMB) and is used iteratively to obtain non-linear contributions

Putting dark energy on the table

Previous strategy is not complete

- Λ CDM power spectrum in EdS background
- Matter domination all the way

But we know the universe has been dominated by DE for a long time

$$\rho = \rho_m \longrightarrow \rho = \rho_m + \rho_{de} \quad \text{with} \quad p_{de} = w\rho_{de}$$

For simplicity

- 1 No DE perturbation: $\rho_{dm} = \bar{\rho}_{de}$ (cf. Park, Hwang, Lee & Noh 2009)
- 2 Comoving gauge: $T^0_i = 0$

Dark energy changes the game

DE provides different BG from both EdS and Λ CDM:

$$\mathcal{H}^2 = \frac{8\pi G}{3} a^2 (\bar{\rho}_m + \bar{\rho}_{de}) \quad \text{and} \quad \mathcal{H}' = -\frac{1}{2} \mathcal{H}^2 (1 + 3w)$$

DE permeates *all* order in perturbation: e.g. energy conservation

$$\delta' - \kappa(1 - \lambda) = (\text{non-linear terms}) \quad \text{where} \quad \lambda \equiv (1 + w) \left(1 - \frac{1}{\Omega_m} \right)$$

Thus away from EdS ($\Omega_m = 1$) and Λ CDM ($w = -1$) the effects of general, dynamical DE are *manifest*: we use the parametrization

(Chevallier & Polarski 2001, Linder 2003)

$$w(a) = w_0 + (1 - a)w_a$$

Non-linear solutions with DE

Curvature perturbation is **not** conserved: from energy constraint

$$\varphi = -\frac{\mathcal{H}^2 f}{1-\lambda} \left[1 + \frac{3}{2}(1-\lambda) \frac{\Omega_m}{f} \right] \Delta^{-1} \delta \neq \text{constant}$$

Thus δ receives a) curvature evolution effects from 3rd order and b) general, dynamical DE effects from BG and linear order:

$$\delta'' + \left(\mathcal{H} + \frac{\lambda'}{1-\lambda} \right) \delta' - \frac{3}{2}(1-\lambda) \mathcal{H}^2 \Omega_m \delta = \underbrace{\mathcal{N}_N + \mathcal{N}_\varphi + \mathcal{N}_{\varphi'} + \mathcal{N}_\lambda}_{=\text{non-linear source terms}}$$

	Newtonian	EdS	Λ CDM	DE
\mathcal{N}_N	O	O	O	O
\mathcal{N}_φ	X	O	O	O
$\mathcal{N}_{\varphi'}$	X	X	X	O
\mathcal{N}_λ	X	X	X	O

Relativistic kernels

2nd and 3rd order solutions are (Biern & [IG](#) 2015)

$$\delta_2(\mathbf{k}, a) = D_1^2 \sum_{i=a}^b c_{2i}(a) \int \frac{d^3 q_1 d^3 q_2}{(2\pi)^3} \delta^{(3)}(\mathbf{k} - \mathbf{q}_{12}) F_{2i}(\mathbf{q}_1, \mathbf{q}_2) \delta_1(\mathbf{q}_1) \delta_1(\mathbf{q}_2)$$

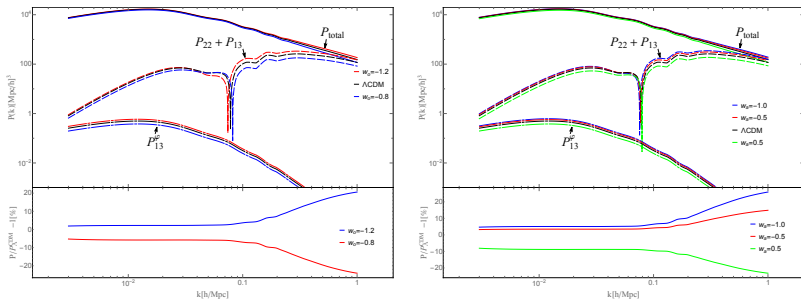
$$\delta_3(\mathbf{k}, a) = D_1^3 \sum_{i=a}^f c_{3i}(a) \int \left[\dots F_{3i} \dots 3 \delta_1' s \right] \quad c_{ni} \equiv \frac{D_{ni}}{D_1^n}$$

$$+ D_1^3 \mathcal{H}^2 \sum_{i=a}^b c_{3i}^\varphi(a) \int \left[\dots F_{3i}^\varphi \dots 3 \delta_1' s \right] \quad c_{3i}^\varphi \equiv \frac{D_{3i}^\varphi}{D_1^3 \mathcal{H}^2}$$

In the EdS universe c 's are fixed as certain numbers ($c_{2a} = 3/7\dots$) and (also in Λ CDM) c_{ni} terms become purely Newtonian [Kamionkowski & Buchalter 1999 (2nd) and Takahashi 2008 (3rd)] and only c_{3i}^φ terms remain relativistic

N. B. λ is completely entangled and cannot be separated like φ

One-loop corrected power spectrum: versus Λ CDM

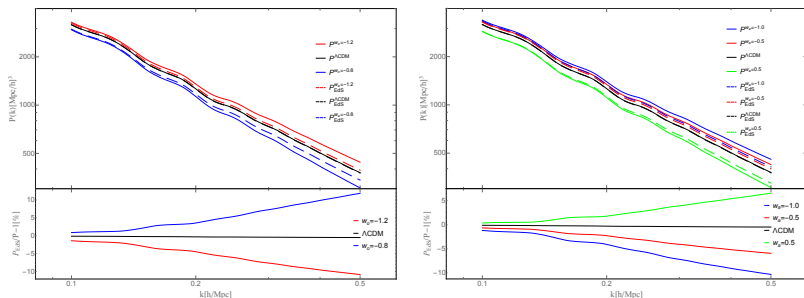


- Overall almost constant deviation on large scales ($k \lesssim 0.1 h/\text{Mpc}$)
- Deviation becomes significant on $k \gtrsim 0.1 h/\text{Mpc}$, close to baryon acoustic oscillations
- $w_0 > -1$ / $w_a > 0$ ($w_0 < -1$ / $w_a < 0$) give smaller (larger) $P(k)$

One-loop corrected power spectrum: versus EdS

In Newtonian studies, usually EdS power spectrum is transferred to an arbitrary DE model by replacing $a \rightarrow D_1(a)$:

$$P(k, a) = D_1^2(a) P_{11}(k) + D_1^4(a) [P_{22}(k) + P_{13}(k)]_{\text{EdS}}$$



- For Λ CDM, only φ drives difference so almost identical to EdS
- For general DE, the difference notably increases from $k \approx 0.1 h/\text{Mpc}$

To redshift space

Observations are made i.t.o. redshift (Kaiser 1987, Heavens, Matarrese & Verde 1998)

$$\delta_s = \delta_r - \partial_{\parallel} U + \text{higher order terms}$$

$$\text{where } \delta_r = b\delta, U \equiv \frac{\hat{\mathbf{n}} \cdot \mathbf{v}}{\mathcal{H}} \quad \text{and} \quad \partial_{\parallel} \equiv \hat{\mathbf{n}} \cdot \nabla$$

Then the observable galaxy power spectrum in the redshift space

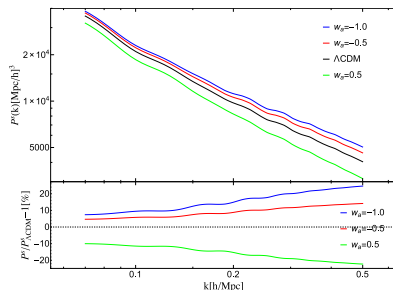
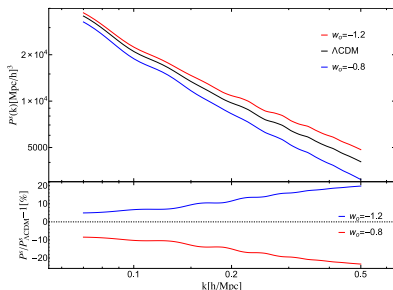
$$P_s(k, \mu, a) = P_{s11}(k, \mu, a) + P_{s22}(k, \mu, a) + P_{s13}(k, \mu, a)$$

with $\mu \equiv \hat{\mathbf{n}} \cdot \mathbf{k}/k$, thus no longer isotropic

- $\mu = 1$: line-of-sight direction, most dominant
- $\mu = 0$: perp to LoS

Thus the deviation from Λ CDM becomes larger for LoS spectrum

One-loop corrected LoS power spectrum



Deviation is enhanced as large as 10% at around BAO scales

$w_a = 0$ and varying w_0			$w_0 = -1$ and varying w_a			
$k [h/\text{Mpc}]$	$w_0 = -1.2$	$w_0 = -0.8$	$k [h/\text{Mpc}]$	$w_a = -1.0$	$w_a = -0.5$	$w_a = 0.5$
0.1	6.8%	-10.2%	0.1	9.5%	5.8%	-11.5%
0.2	11.6%	-15.0%	0.2	14.9%	8.8%	-15.3%
0.3	16.0%	-19.4%	0.3	20.1%	11.6%	-19.0%

Conclusions

- As galaxy surveys become deeper and deeper, fully GR description is relevant
- With general dark energy:
 - Dark energy background greatly affects GR contributions
 - Notable difference of a few percent near BAO scales
 - Detectable signatures of judging Λ or not