Consistency relations for features in the primordial spectra

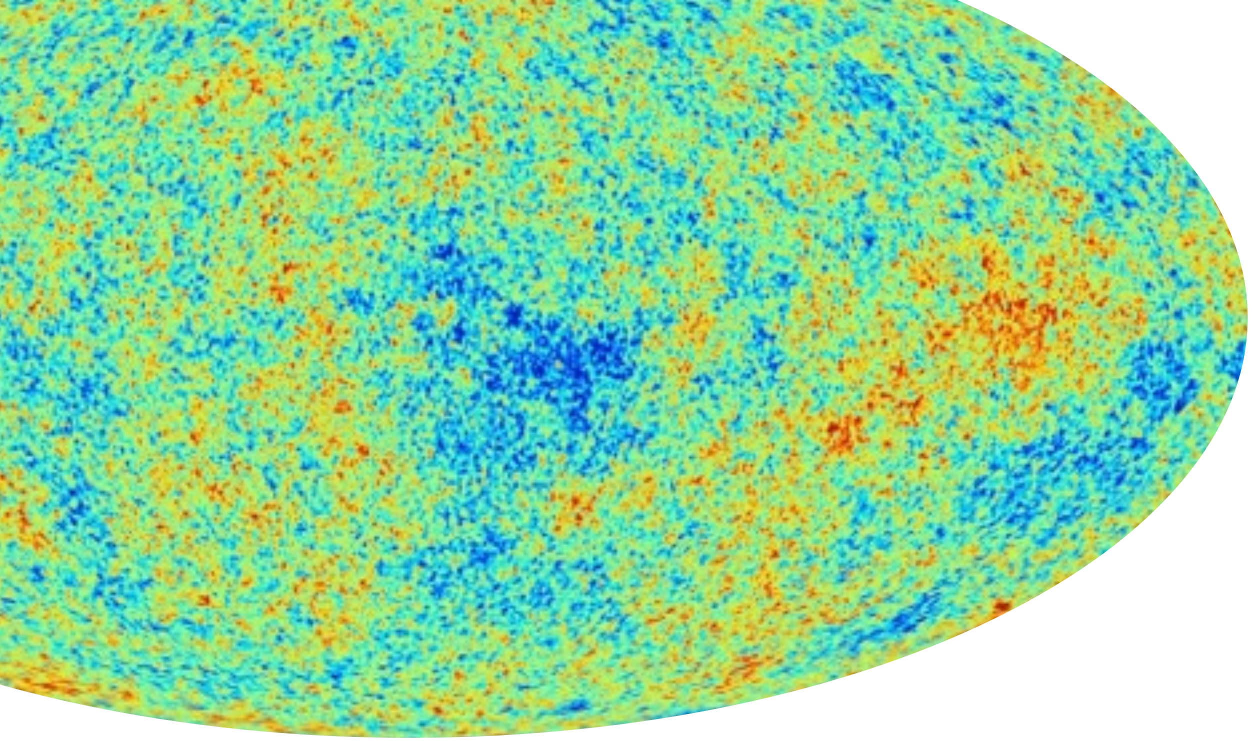
Gonzalo A. Palma

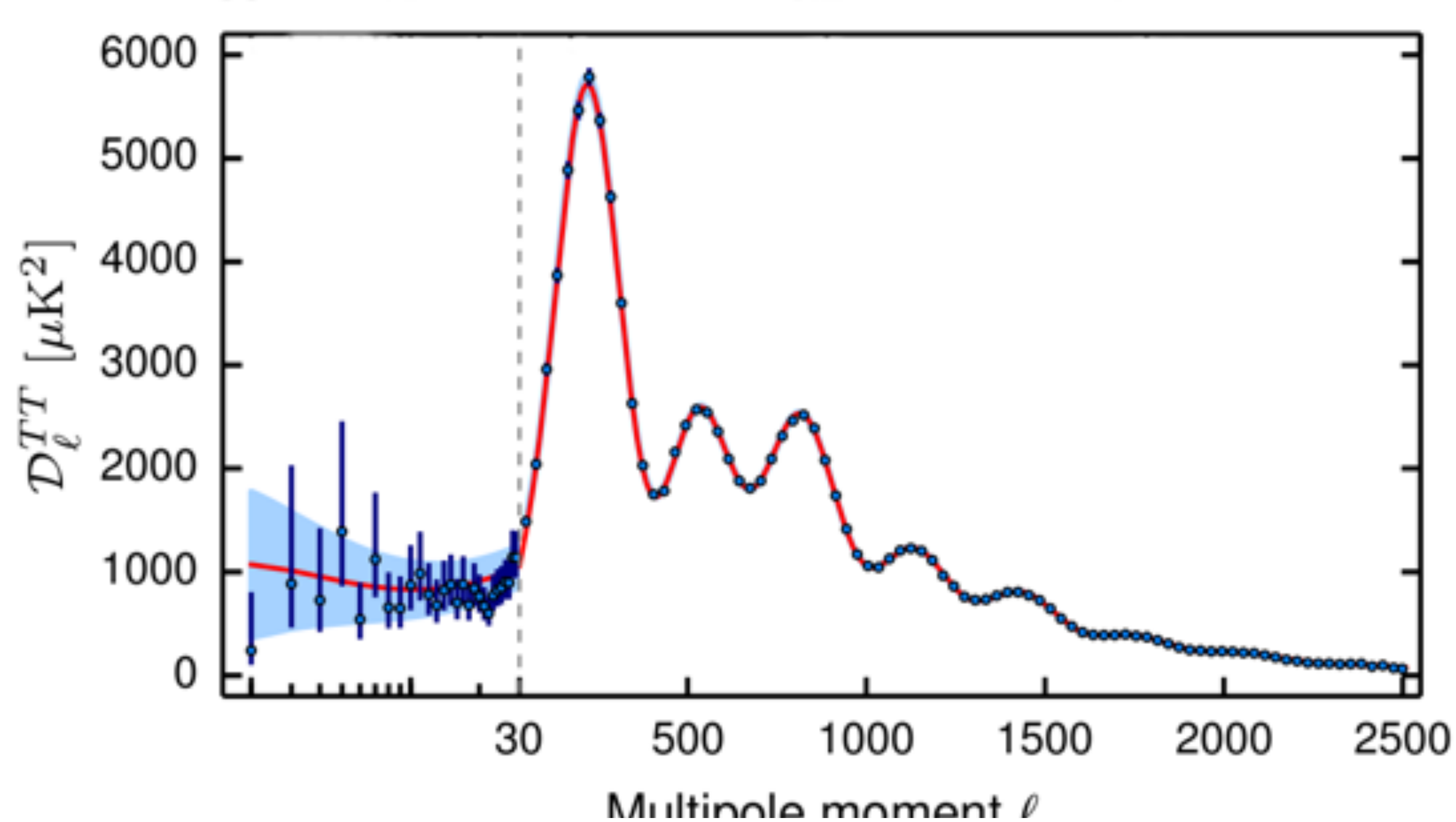
Group of Cosmology & Theoretical Astrophysics
FCFM, Universidad de Chile

work in collaboration with:

Sander Mooij
Grigoris Panotopoulos
& Alex Soto

arXiv: 1507.08481



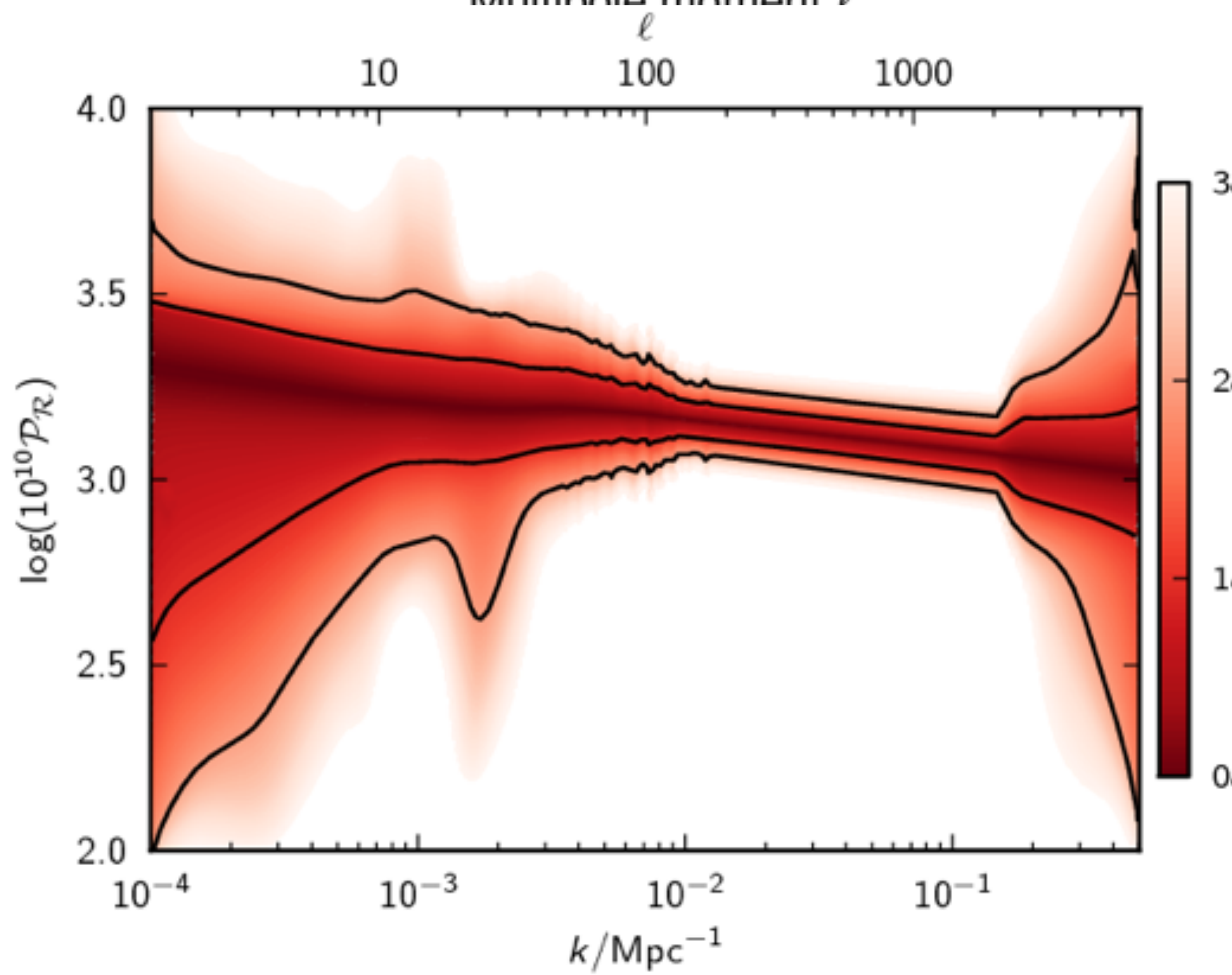


$$D_\ell = \int dk \Delta_\ell(k) \mathcal{P}(k)$$

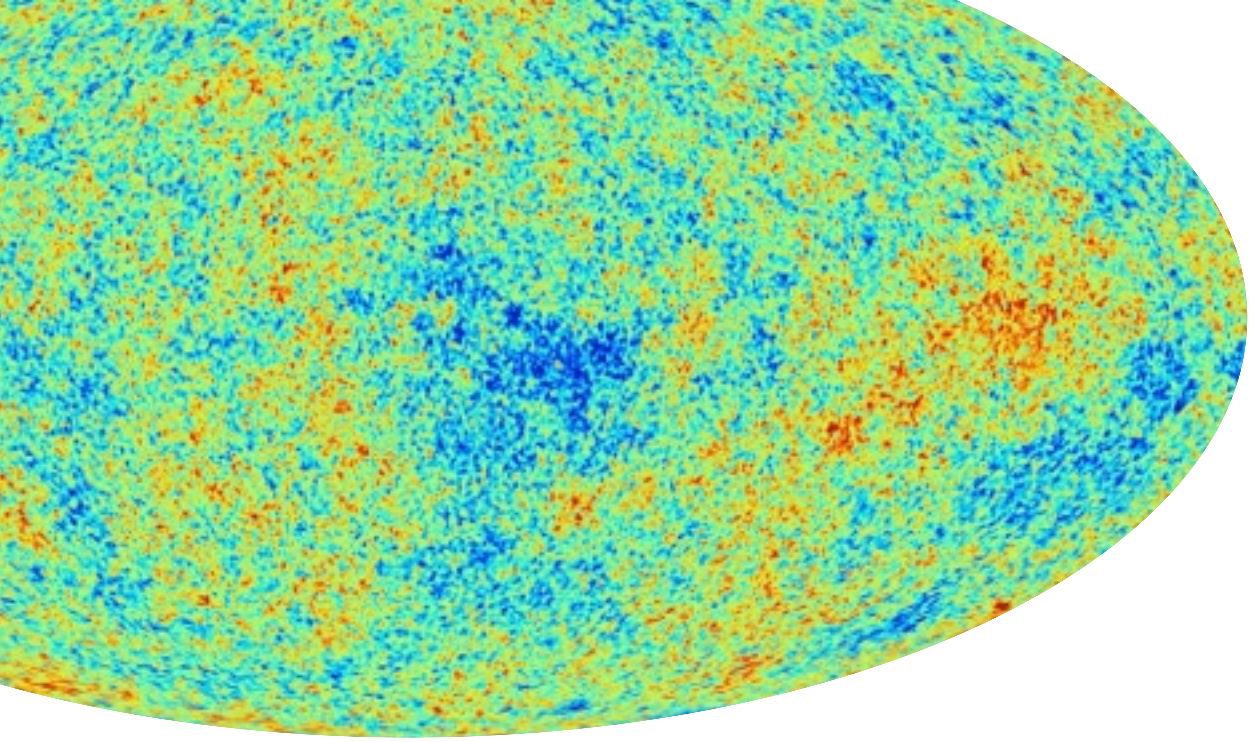
$$\langle \mathcal{R}(\mathbf{k}) \mathcal{R}(\mathbf{k}') \rangle = \delta(\mathbf{k} + \mathbf{k}') \mathcal{P}(k)$$

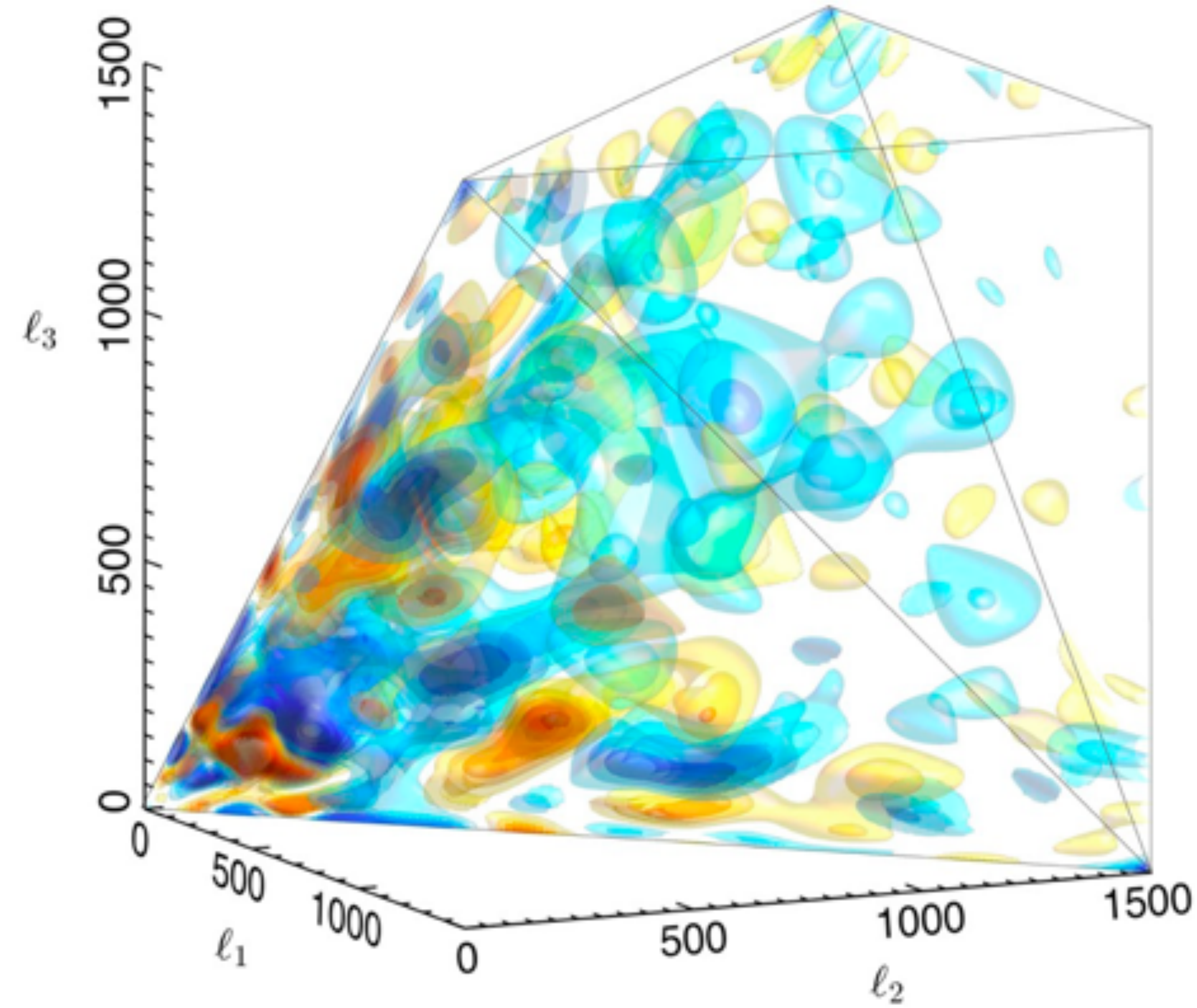
$$ds^2 = -dt^2 + a^2(t) e^{2\mathcal{R}} d\mathbf{x}^2$$

Observations compatible with a featureless power spectrum



Planck (2015)





Observations compatible
with non Gaussian statistics

$$\langle \mathcal{R}(\mathbf{k}_1)\mathcal{R}(\mathbf{k}_2)R(\mathbf{k}_3)\rangle = \delta(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)B(k_1,k_2,k_3)$$

This talk is about stuff we don't observe,
but might be there

- **Observations are so far consistent with featureless primordial spectra**
- These observations are well addressed by the slow roll inflationary paradigm
- However: What if there are features hidden in the primordial spectra?
- Of course, it would rule out the slow roll paradigm
- On the other hand, it would open the path to new scales, and new tests to inflation

- Observations are so far consistent with featureless primordial spectra

- **These observations are well addressed by the slow roll inflationary paradigm**

- However: What if there are features hidden in the primordial spectra?

- Of course, it would rule out the slow roll paradigm

- On the other hand, it would open the path to new scales, and new tests to inflation

- Observations are so far consistent with featureless primordial spectra
- These observations are well addressed by the slow roll inflationary paradigm
- **However: What if there are features hidden in the primordial spectra?**
- Of course, it would rule out the slow roll paradigm
- On the other hand, it would open the path to new scales, and new tests to inflation

- Observations are so far consistent with featureless primordial spectra
- These observations are well addressed by the slow roll inflationary paradigm
- However: What if there are features hidden in the primordial spectra?
- **Of course, it would rule out the slow roll paradigm**
- On the other hand, it would open the path to new scales, and new tests to inflation

- Observations are so far consistent with featureless primordial spectra
- These observations are well addressed by the slow roll inflationary paradigm
- However: What if there are features hidden in the primordial spectra?
- Of course, it would rule out the slow roll paradigm
- **On the other hand, it would open the path to new scales, and new tests to inflation**

In this talk I will advertise two results:

1. There exists a general relation linking sharp features in the power spectrum with those in the bispectrum

$$\Delta B = b_0 \Delta \mathcal{P} + b_1 \frac{d}{d \ln k} \Delta \mathcal{P} + b_2 \frac{d^2}{d \ln k^2} \Delta \mathcal{P}$$

2. There is a general relation linking rapid variations of η and the speed of sound c_s

$$\eta = \eta_0 + \frac{\alpha}{2H} \frac{d}{dt} (1 - c_s^2)$$

We needed 2 to show 1

In this talk I will advertise two results:

1. There exists a general relation linking sharp features in the power spectrum with those in the bispectrum

$$\Delta B = b_0 \Delta \mathcal{P} + b_1 \frac{d}{d \ln k} \Delta \mathcal{P} + b_2 \frac{d^2}{d \ln k^2} \Delta \mathcal{P}$$

2. There is a general relation linking rapid variations of η and the speed of sound c_s

$$\eta = \eta_0 + \frac{\alpha}{2H} \frac{d}{dt} (1 - c_s^2)$$

We needed 2 to show 1

I will not discuss techniques to search for features

(See Moritz Münchmeyer's talk)

- Vast literature on how to search for features in the primordial power spectrum $\mathcal{P} = \mathcal{P}_0 + \Delta\mathcal{P}$

Covi et al. astro-ph/0606452
Hunt & Sarkar, arXiv:1308.2317
Meerburg et a. arXiv:1406.0548
Hazra et al. arXiv:1406.4827
Hu & Torrado, arXiv:1410.4804
Fergusson et al. arXiv:1410.5114
Fergusson et al. arXiv:1412.6152
Gariazzo et al. arXiv:1506.05251
Cai et al. arXiv:1507.05619
Chluba et al. arXiv:1505.01834
Ade et al. arXiv:1502.02114

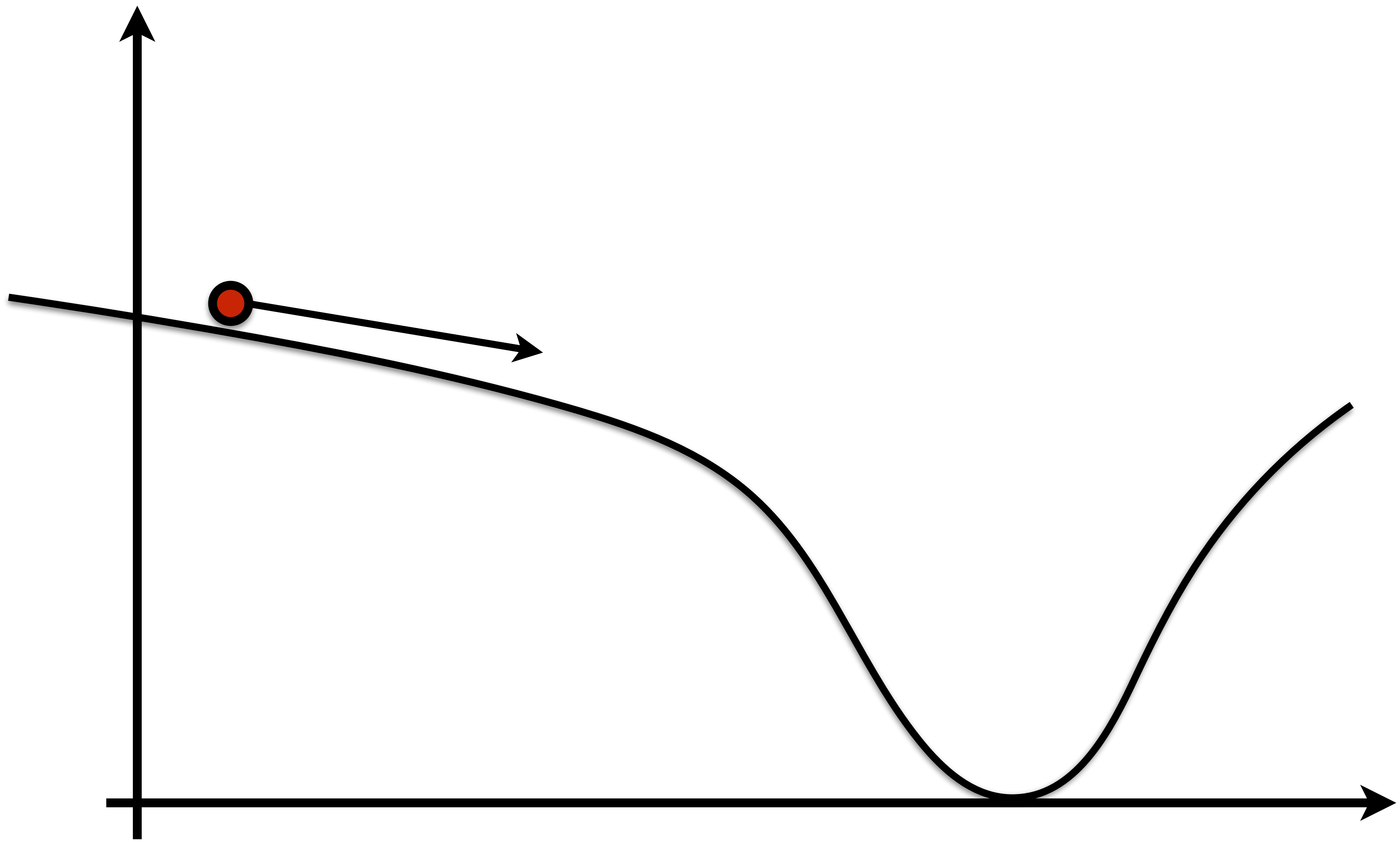
I will not discuss techniques to search for features

(See Moritz Münchmeyer's talk)

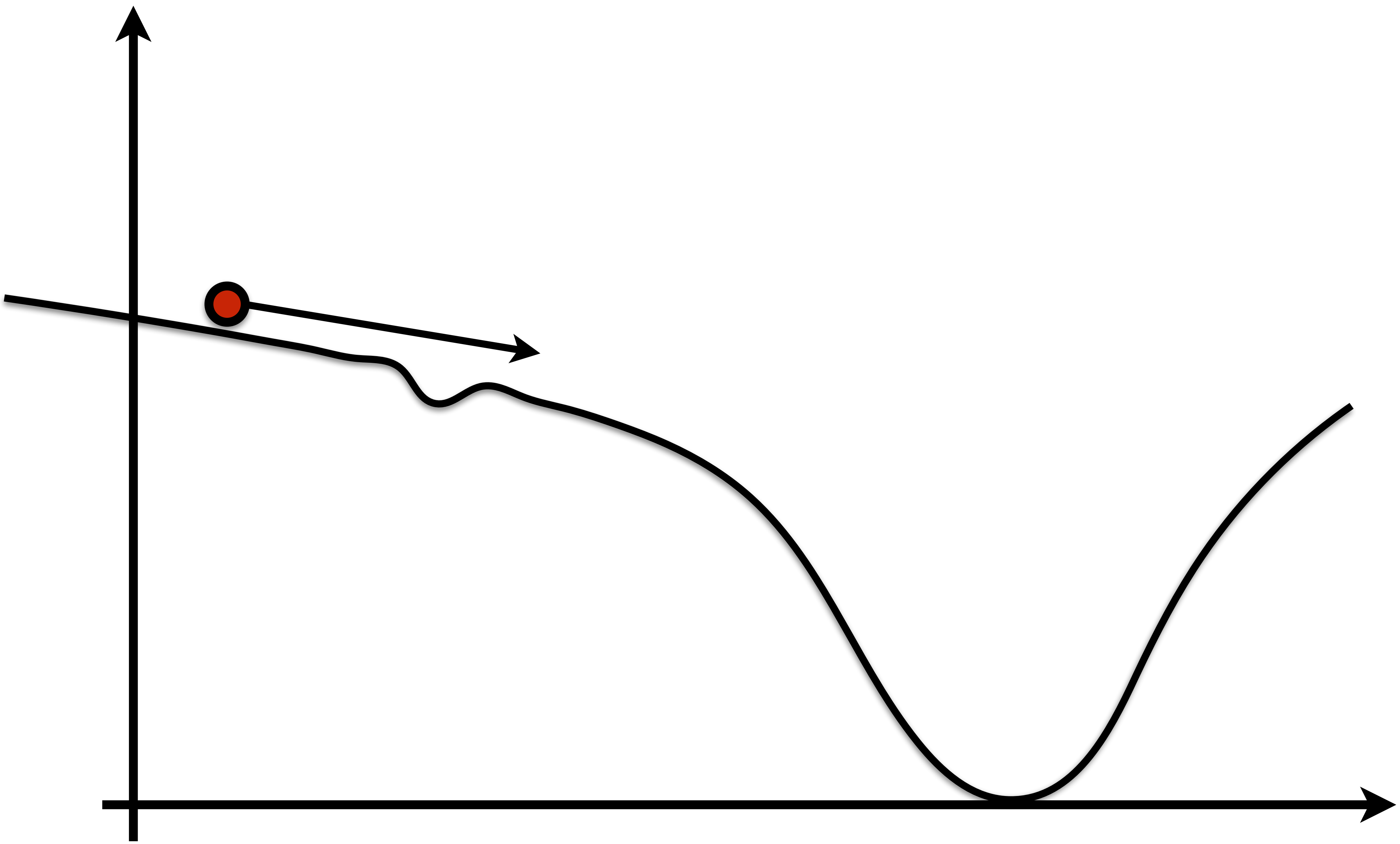
- Important progress in how to parametrize features in the bispectrum $B = B_0 + \Delta B$

Adshead et al. arXiv:1110.3050
Adshead et al. arXiv1102.3435
Meerburg et al. arXiv:1308.3704
Achucarro et al. arXiv:1311.2552
Hu & Torrado, arXiv:1410.4804
Fergusson et al. arXiv:1410.5114
Fergusson et al. arXiv:1412.6152
Münchmeyer et al. arXiv:1412.3461
Fergusson et al. arXiv:1412.6152
Gariazzo et al. arXiv:1506.05251
Chluba et al. arXiv:1505.01834
Ade et al. arXiv:1502.02114
Ade et al. arXiv:1502.01592

If features are present, where would they come from?

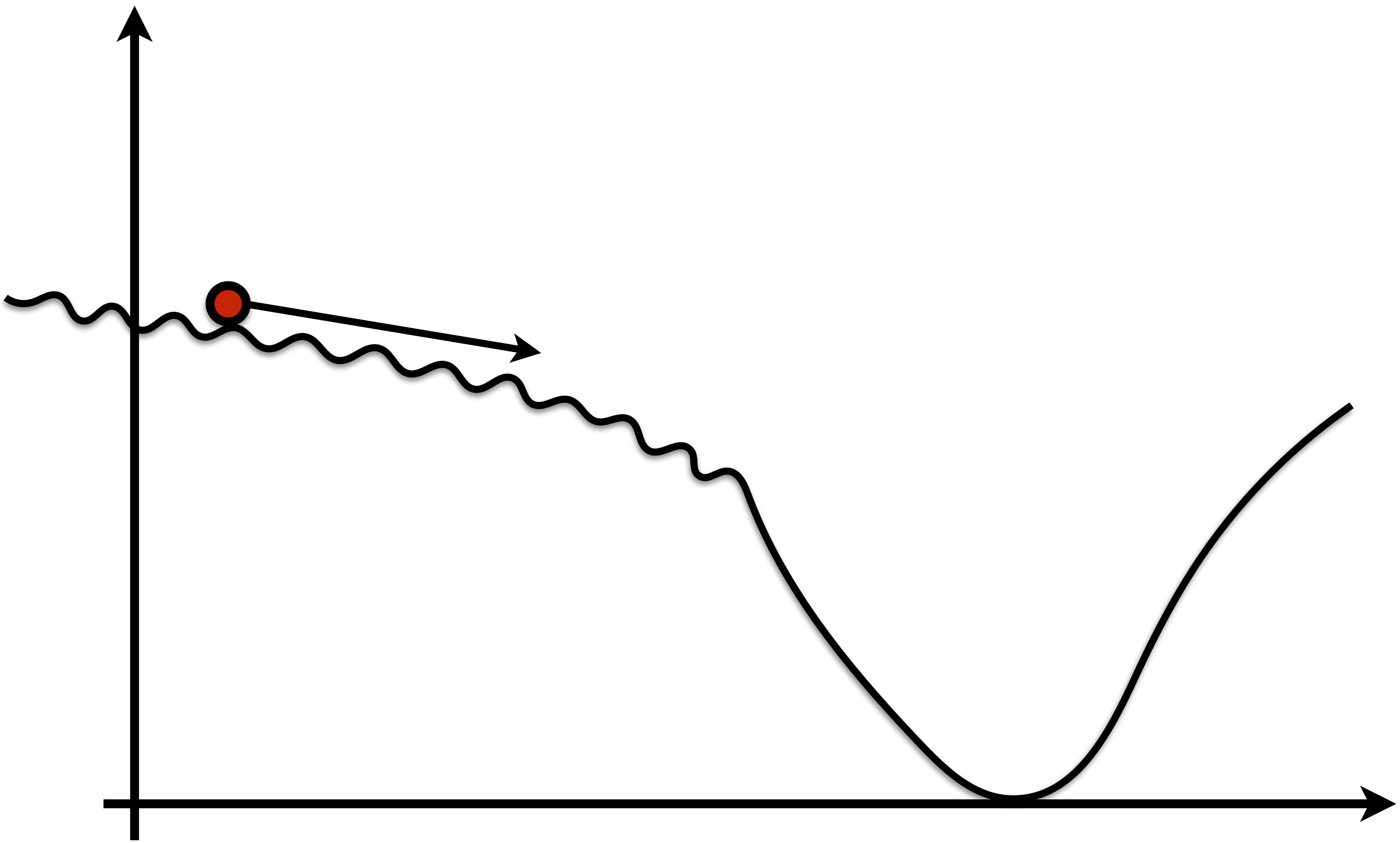


If features are present, where would they come from?



Starobinsky (1992) and many others thereafter

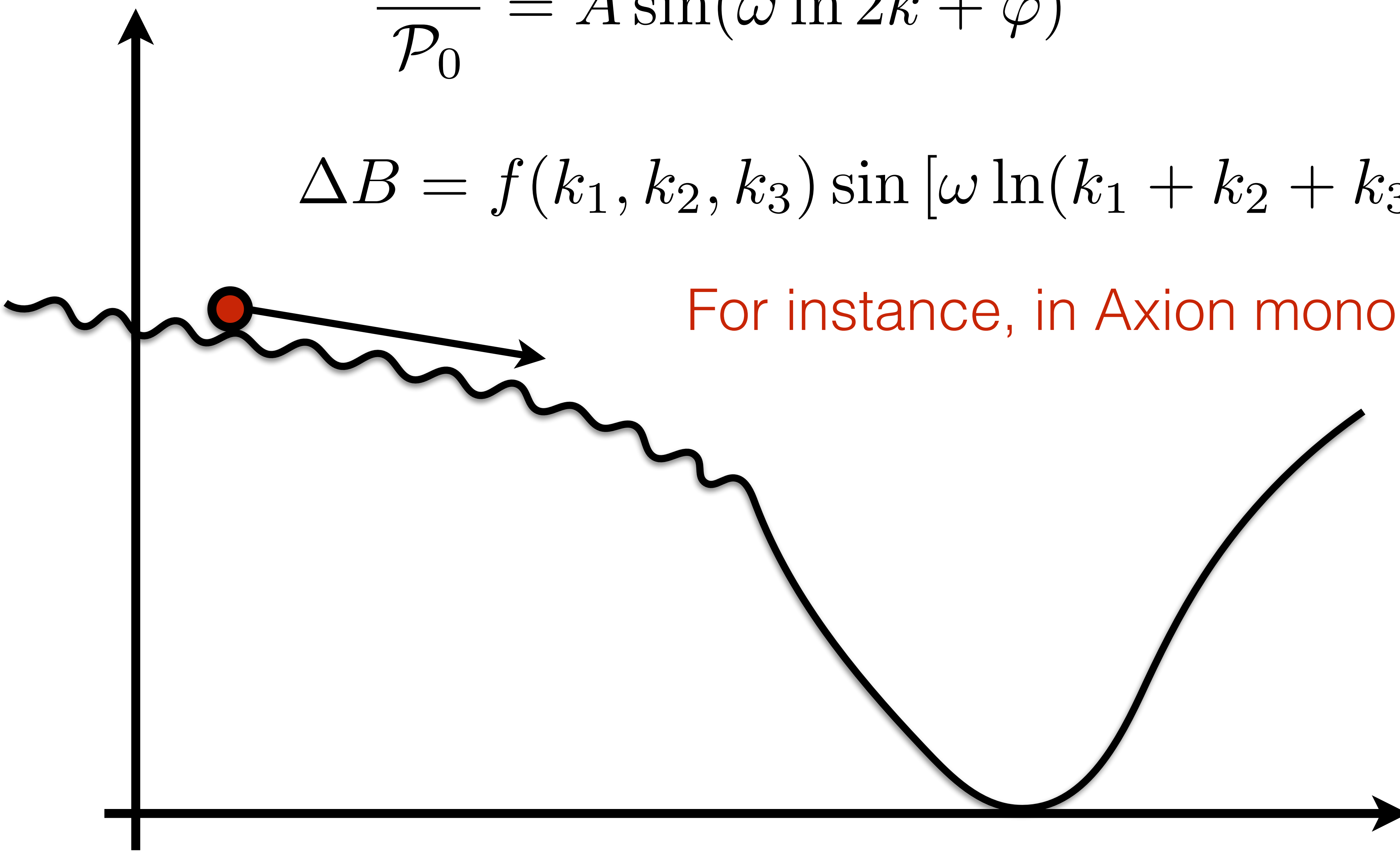
If features are present, where would they come from?



If features are present, where would they come from?

$$\frac{\Delta \mathcal{P}}{\mathcal{P}_0} = A \sin(\omega \ln 2k + \varphi)$$

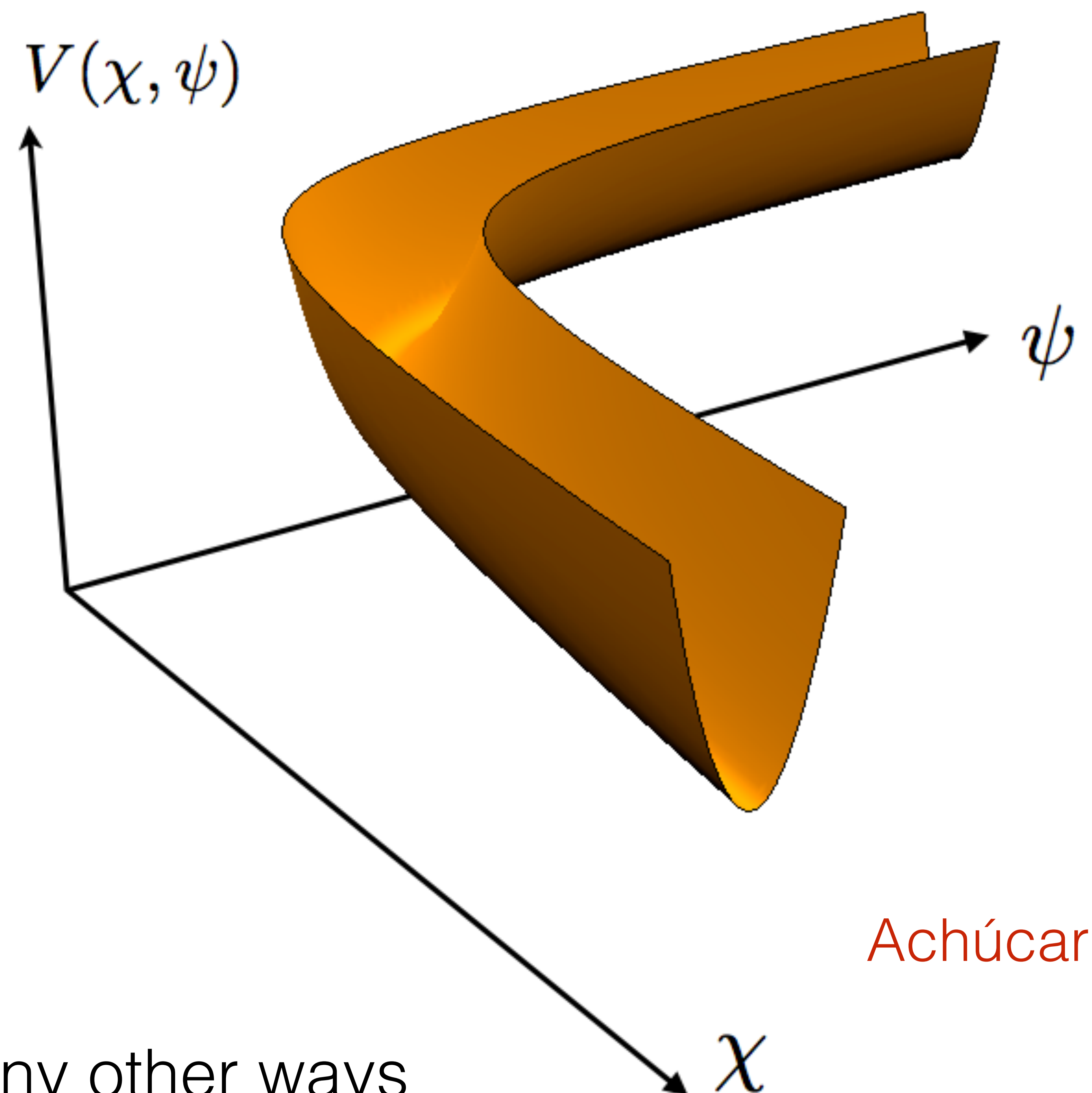
$$\Delta B = f(k_1, k_2, k_3) \sin\left[\omega \ln(k_1 + k_2 + k_3) + \varphi\right]$$



For instance, in Axion monodromy

Flauger & Pajer (2011)

Turns in multi-fiel inflation



Achúcarro et al. (2010)

And many other ways

- We are not interested in the specific model responsible for features.
- Then, we may use the EFT of inflation approach to study the appearance of features in a model independent way

$$S^{(2)} = \int d^4x \, a^3 \epsilon \left[\frac{1}{c_s^2} \dot{\mathcal{R}}^2 - \frac{1}{a^2} (\nabla \mathcal{R})^2 \right]$$

$$S^{(3)} = \int d^4x \, a^3 \epsilon \Bigg[\frac{1}{c_s^4} \left[3(c_s^2 - 1) + \epsilon - \eta \right] \mathcal{R} \dot{\mathcal{R}}^2 + \frac{1}{c_s^2 a^2} \left((1 - c_s^2) + \eta + \epsilon - \frac{2\dot{c}_s}{H c_s} \right) \mathcal{R} (\nabla \mathcal{R})^2 + \frac{1}{H} \left(\frac{1 - c_s^2}{c_s^4} - \frac{2\lambda}{\epsilon H^2} \right) \dot{\mathcal{R}}^3 + \cdots \Bigg]$$

- We are not interested in the specific model responsible for features.
- Then, we may use the EFT of inflation approach to study the appearance of features in a model independent way

$$S^{(2)} = \int d^4x\, a^3 \epsilon \left[\frac{1}{c_s^2} \dot{\mathcal{R}}^2 - \frac{1}{a^2} (\nabla \mathcal{R})^2 \right]$$

$$\begin{aligned} S^{(3)} = \int d^4x\, a^3 \epsilon \Bigg[& \frac{1}{c_s^4} \left[3(c_s^2 - 1) + \epsilon - \eta \right] \mathcal{R} \dot{\mathcal{R}}^2 \\ & + \frac{1}{c_s^2 a^2} \left((1 - c_s^2) + \eta + \epsilon - \frac{2 \dot{c}_s}{H c_s} \right) \mathcal{R} (\nabla \mathcal{R})^2 \\ & + \frac{1}{H} \left(\frac{1 - c_s^2}{c_s^4} - \frac{2\lambda}{\epsilon H^2} \right) \dot{\mathcal{R}}^3 + \cdots \Bigg] \end{aligned}$$

- We are not interested in the specific model responsible for features.
- Then, we may use the EFT of inflation approach to study the appearance of features in a model independent way

$$S^{(2)} = \int d^4x\, a^3 \epsilon \left[\frac{1}{c_s^2} \dot{\mathcal{R}}^2 - \frac{1}{a^2} (\nabla \mathcal{R})^2 \right]$$

$$S^{(3)} = \int d^4x\, a^3 \epsilon \Bigg[\frac{1}{c_s^4} \left[3(c_s^2 - 1) + \epsilon - \eta \right] \mathcal{R} \dot{\mathcal{R}}^2 + \frac{1}{c_s^2 a^2} \left((1 - c_s^2) + \eta + \epsilon - \frac{2\dot{c}_s}{H c_s} \right) \mathcal{R} (\nabla \mathcal{R})^2 + \frac{1}{H} \left(\frac{1 - c_s^2}{c_s^4} - \frac{2\lambda}{\epsilon H^2} \right) \dot{\mathcal{R}}^3 + \cdots \Bigg]$$

We may simplify things by adopting a few assumptions. The three main assumptions are:

- We assume that the sound speed has small departures from unity

$$1 - c_s^2 \ll 1$$

- We assume that epsilon is always much smaller than one

$$\epsilon \ll 1$$

- Variations happen rapidly when compared to slow roll

$$\left| \frac{dA}{dN} \right| \gg |A|$$

We may simplify things by adopting a few assumptions. The three main assumptions are:

- We assume that the sound speed has small departures from unity

$$1 - c_s^2 \ll 1$$

- We assume that epsilon is always much smaller than one

$$\epsilon \ll 1$$

- Variations happen rapidly when compared to slow roll

$$\left| \frac{dA}{dN} \right| \gg |A|$$

We may simplify things by adopting a few assumptions. The three main assumptions are:

- We assume that the sound speed has small departures from unity

$$1 - c_s^2 \ll 1$$

- We assume that epsilon is always much smaller than one

$$\epsilon \ll 1$$

- Variations happen rapidly when compared to slow roll

$$\left| \frac{dA}{dN} \right| \gg |A|$$

We finally arrive to a very simple cubic interaction that depend on c_s and η

$$S_{\rm int}^{(3)} = -\int_x a^3 \epsilon_0 \left\{ \left[3(1-c_s^2)+\eta\right]\mathcal{R}\dot{\mathcal{R}}^2 + \frac{1}{a^2}\left(\frac{2c_s\dot{c}_s}{H}+\eta\right)\mathcal{R}(\nabla\mathcal{R})^2 \right\}.$$

Now there are only two operators in the cubic interaction action

Let's see an idea introduced in 2012 by Achúcarro, Gong, Palma & Patil (very schematic):

$$S^{(2)} = \int_x \left[\frac{1}{c_s^2}\dot{\mathcal{R}}^2 - (\nabla\mathcal{R})^2\right] = \underbrace{\int_x \left[\dot{\mathcal{R}}^2 - (\nabla\mathcal{R})^2\right]}_{S_0} + \underbrace{\int_x \left(\frac{1}{c_s^2} - 1\right)\dot{\mathcal{R}}^2}_{S_{int}^{(2)}}$$

Then, just use the in-in perturbation theory formalism to deduce the power spectrum with features

$$\delta(\mathbf{k}+\mathbf{k}')\Delta\mathcal{P}(k) = \int_t [H_{int}^{(2)}, R(\mathbf{k})R(\mathbf{k}')]$$

$$\Delta\mathcal{P}(k) = \int_t e^{itk}(1 - c_s^{-2})$$

Achúcarro et al. (2012)

You can do the same with the cubic part of the theory, and find a similar expression for the bispectrum

$$\Delta B(k_1, k_2, k_3) = \int_t e^{-it(k_1+k_2+k_3)}(1 - c_s^{-2}) \times (\text{other terms})$$

You can do the same with the cubic part of the theory, and find a similar expression for the bispectrum

$$\Delta B(k_1, k_2, k_3) = \int_t e^{-it(k_1+k_2+k_3)}(1 - c_s^{-2}) \times (\text{other terms})$$

But now you may Fourier invert the power spectrum

$$\Delta \mathcal{P}(k) = \int_t e^{itk}(1 - c_s^{-2}) \qquad (1 - c_s^{-2}) = \int_k e^{-itk} \Delta \mathcal{P}(k)$$

And then replace $(1 - c_s^{-2})$ in the bispectrum:

$$\Delta B = b_0 \Delta \mathcal{P} + b_1 \frac{d}{d \ln k} \Delta \mathcal{P} + b_2 \frac{d^2}{d \ln k^2} \Delta \mathcal{P}$$

This can be repeated for cases where the variation happens only in the expansion rate (no speed of sound)

$$\Delta\mathcal{P}(k) = \int_t e^{itk}\,\eta \qquad\qquad \eta(t) = \int_k e^{-itk}\,\Delta\mathcal{P}(k)$$

$$\Delta B(k_1, k_2, k_3) = \int_t e^{-it(k_1+k_2+k_3)}\eta \times (\text{other terms})$$

- We obtain the same type of result:

$$\Delta B = b_0 \Delta\mathcal{P} + b_1 \frac{d}{d\ln k}\Delta\mathcal{P} + b_2 \frac{d^2}{d\ln k^2}\Delta\mathcal{P}$$

- But with different coefficients

G. A. Palma (2014)

But, more generally, we have a problem. Too many parameters to be able to correlate features in the spectra

$$\Delta \mathcal{P}(k) = \int_t e^{itk} \, \left[a_1 \eta + a_2 (1 - c_s^{-2})\right]$$

$$\Delta B(k_1, k_2, k_3) = \int_t e^{-it(k_1+k_2+k_3)} \left[b_1 \eta + b_2 (1 - c_s^{-2})\right] \times (\cdots)$$

As a consequence, you cannot correlate the features in the bispectrum with those of the power spectrum

See also Gong, Schalm and Shiu (2014)

But, more generally, we have a problem. Too many parameters to be able to correlate features in the spectra

$$\Delta\mathcal{P}(k) = \int_t e^{itk} \left[a_1\eta + a_2(1 - c_s^{-2})\right]$$

$$\Delta B(k_1, k_2, k_3) = \int_t e^{-it(k_1+k_2+k_3)} \left[b_1\eta + b_2(1 - c_s^{-2})\right] \times (\cdots)$$

As a consequence, you cannot correlate the features in the bispectrum with those of the power spectrum

See also Gong, Schalm and Shiu (2014)

A crucial observation allowing us to avoid this obstacle: The slow roll parameters vary in synchrony.

$$\epsilon(t) - \epsilon_0 \propto 1 - c_s^2(t)$$

$$\eta(t) = \eta_0 + \frac{\alpha}{2}\frac{d}{dN}\left(1 - c_s^2(t)\right)$$

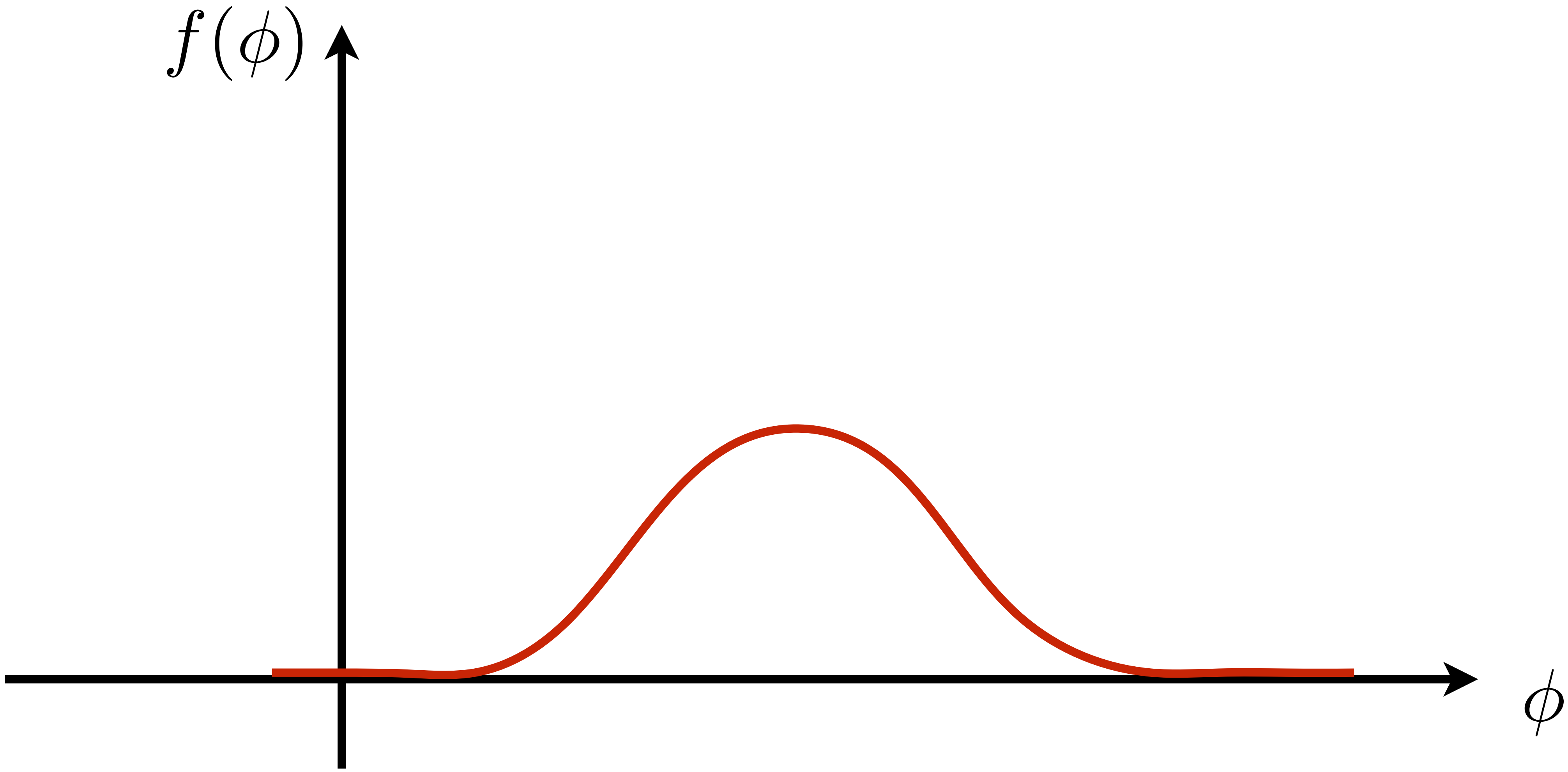
We have not been able to prove this relation. But we have reasonable arguments.

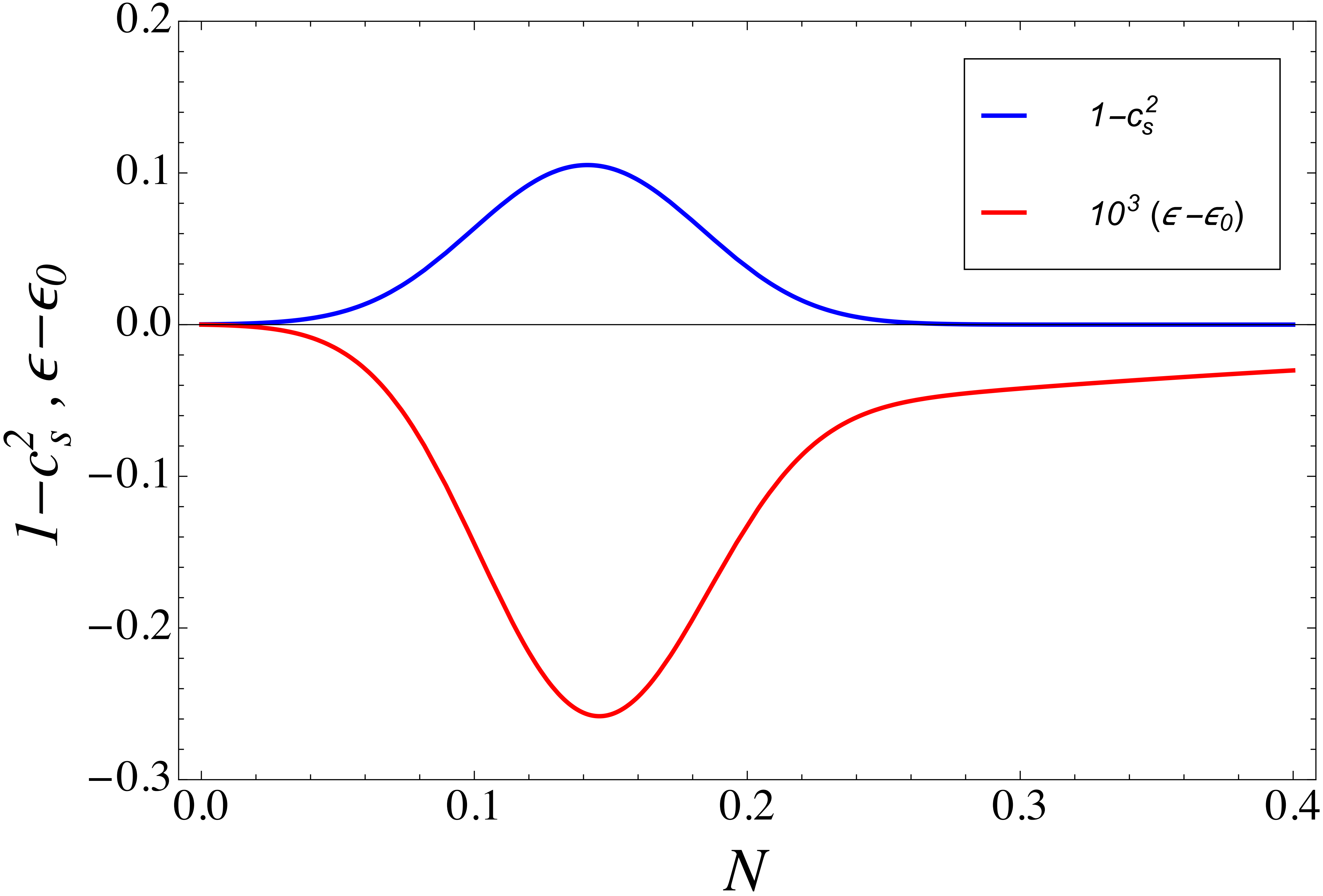
Example: DBI with features

$$P(X,\phi) = \frac{1}{f(\phi)}\left[1 - \sqrt{1 - 2f(\phi)X}\right] - V(\phi)$$

Silverstein & Tong (2004)

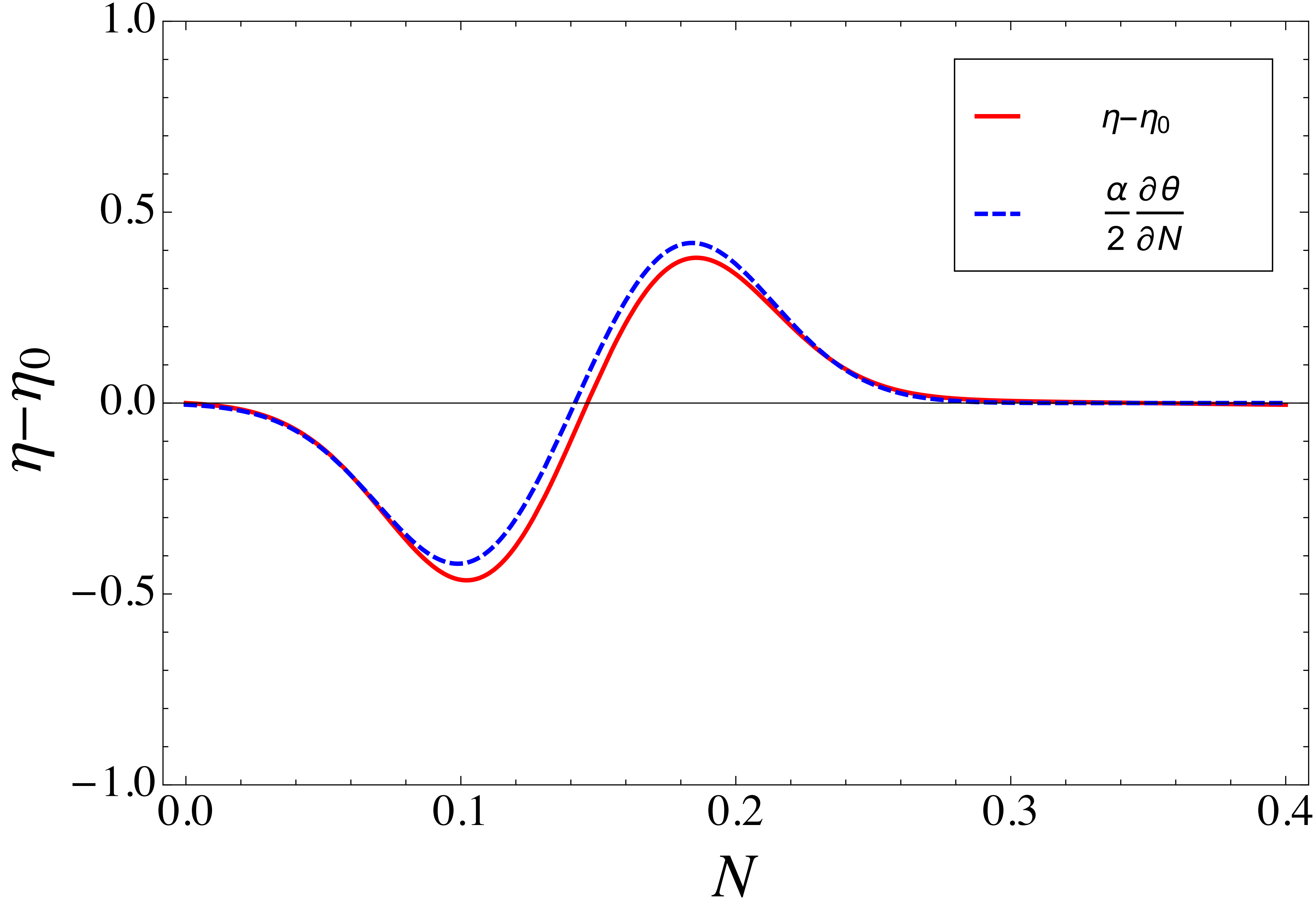
- We may introduce features by playing with the warping factor

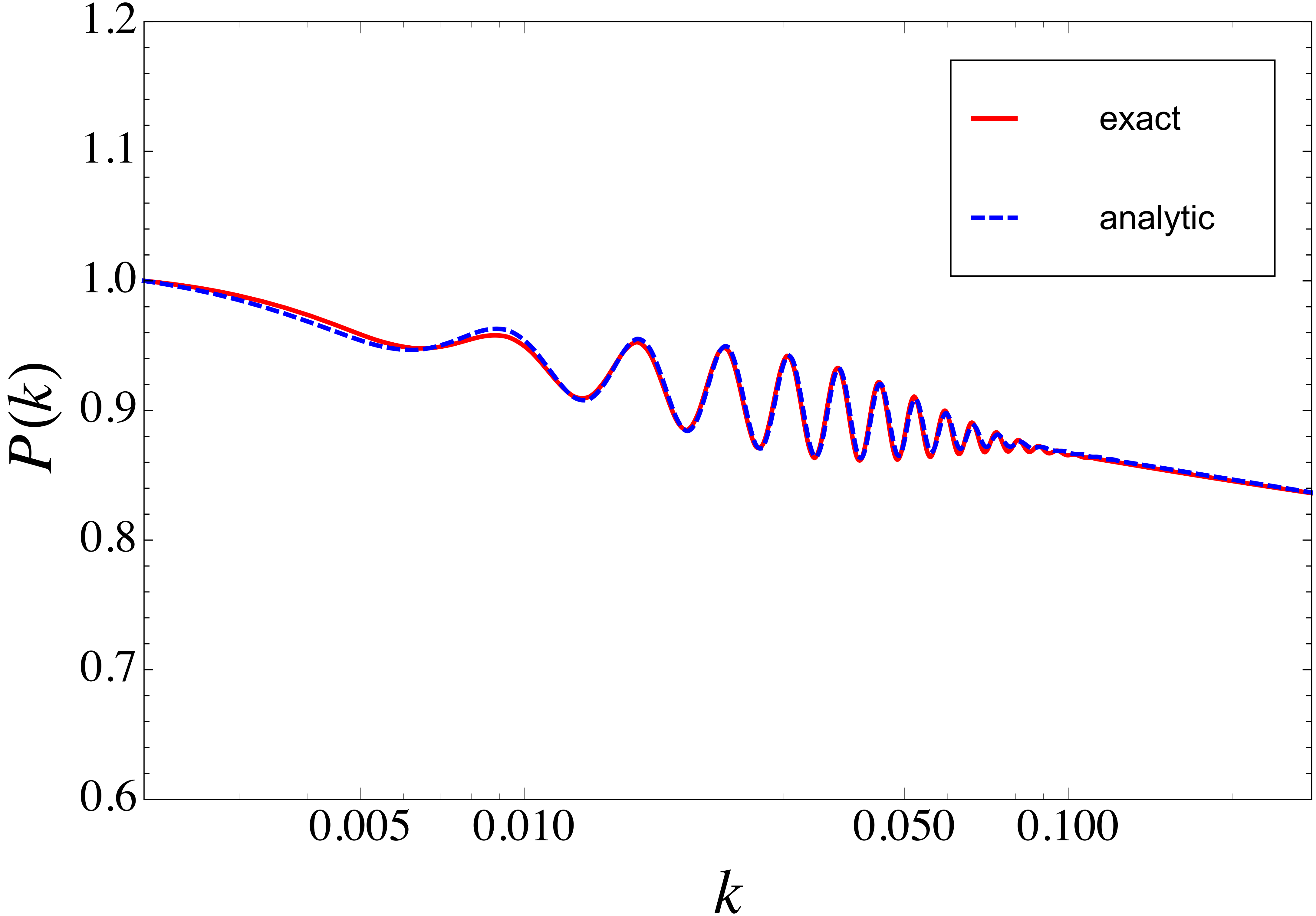




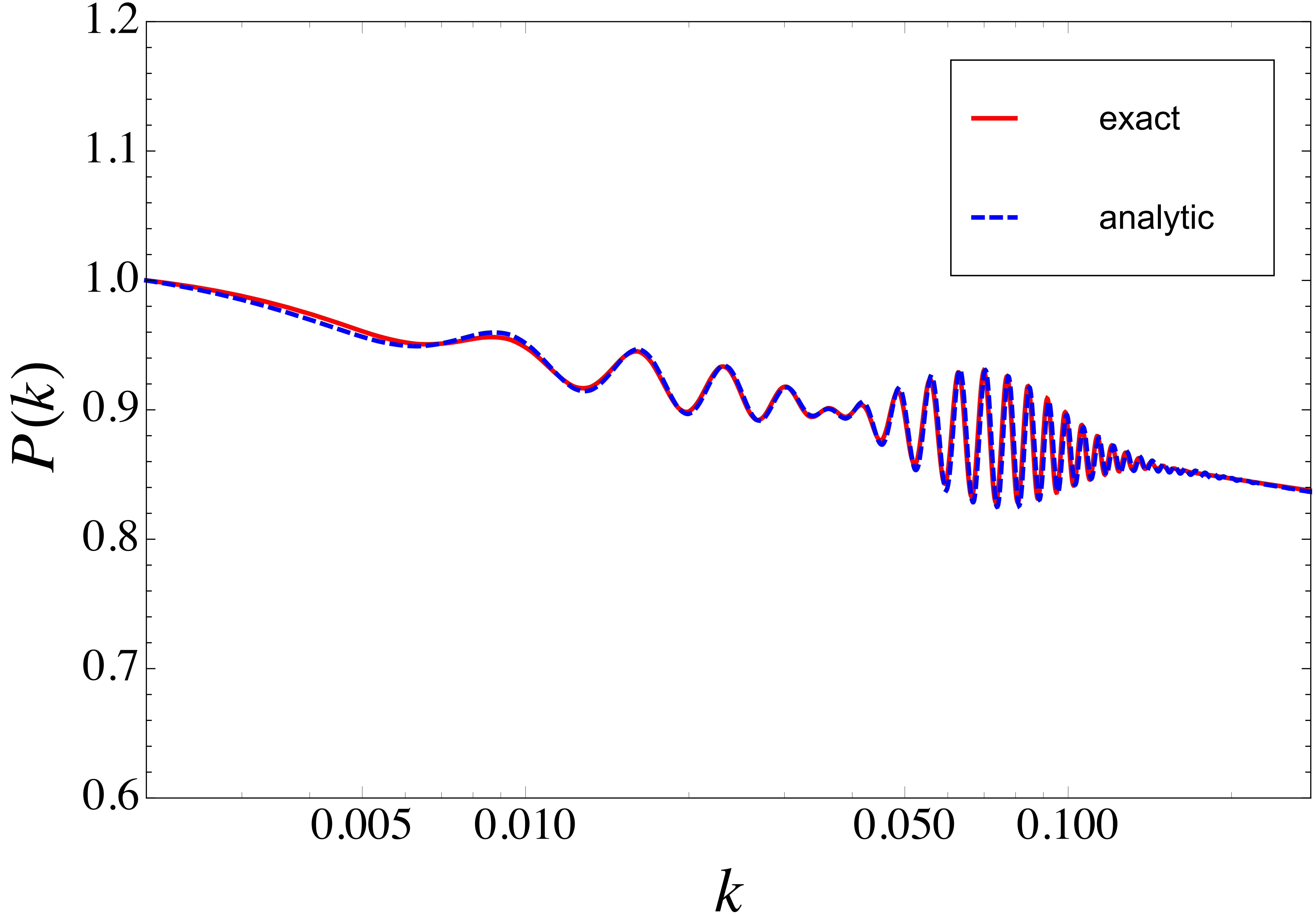
0.2
0.1
0.0
−0.1
−0.2
−0.3
0.0
0.1
0.2
0.3
0.4
N
$1-c_s^2$, $\epsilon-\epsilon_0$
$1-c_s^2$
$10^3\ (\epsilon-\epsilon_0)$

$$\alpha = -0.53$$




exact
analytic
1.2
1.1
1.0
0.9
0.8
0.7
0.6
0.005
0.010
0.050
0.100
$P(k)$
k

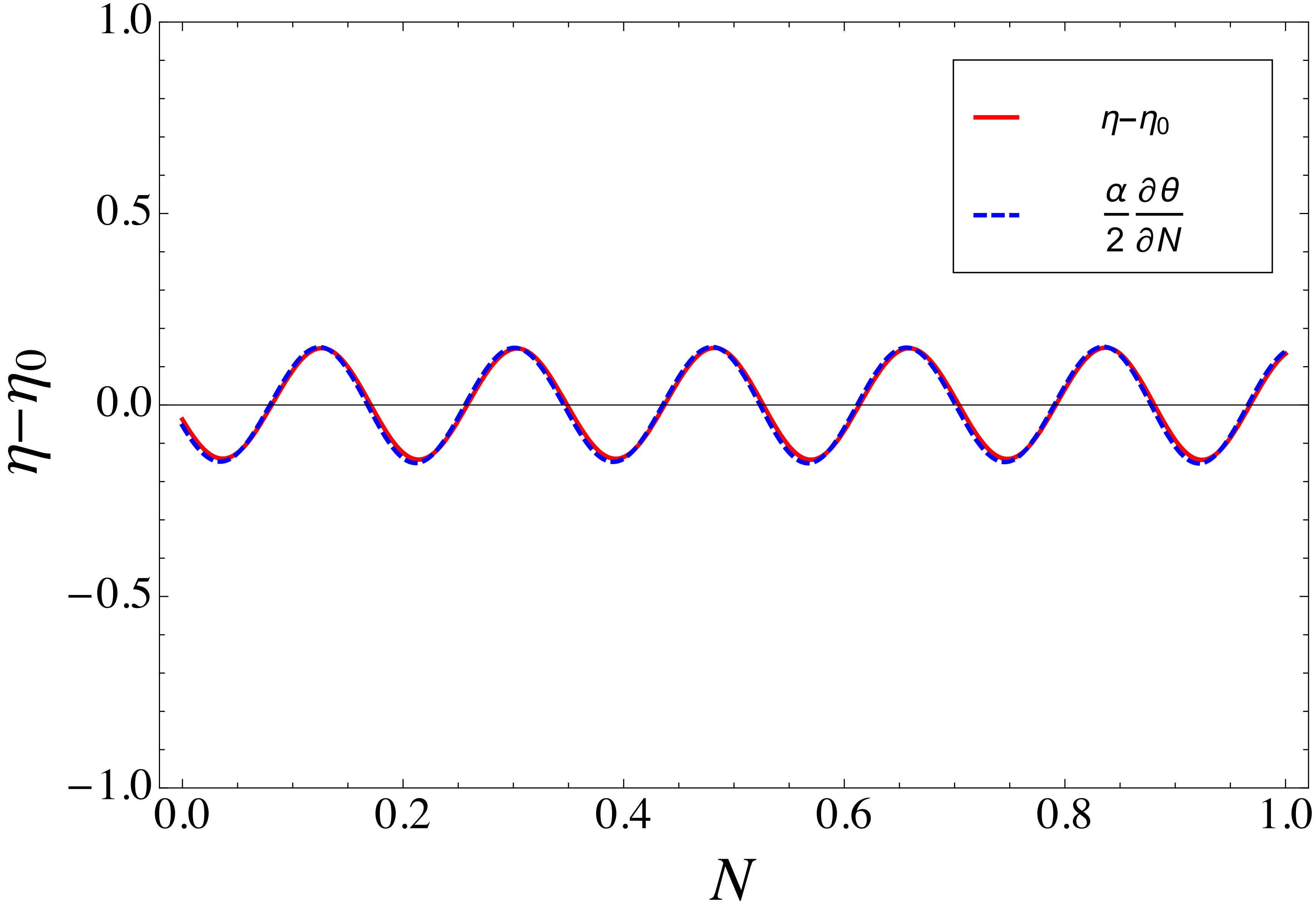
Another example of DBI with two bumps:



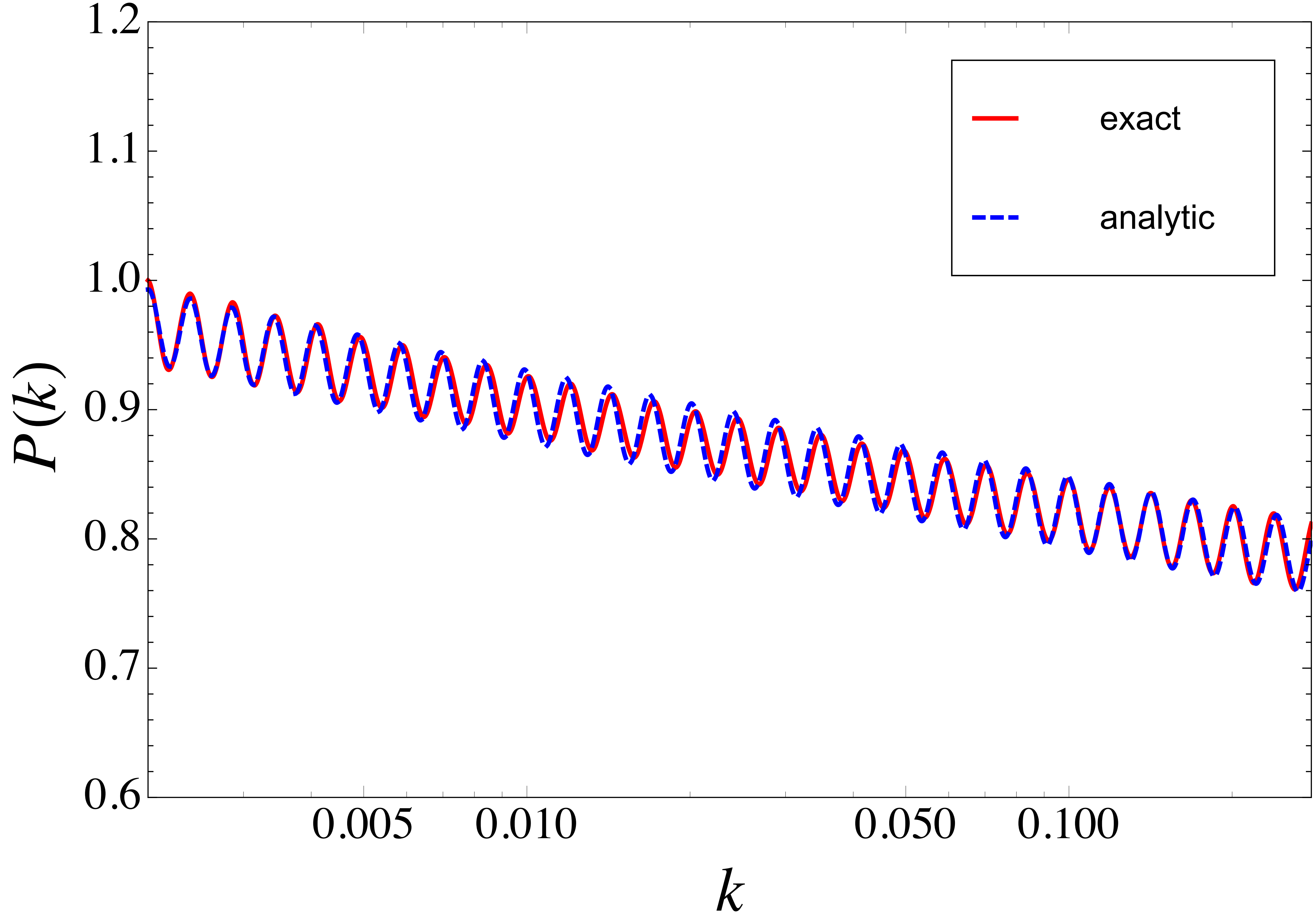
Example 2: DBI inflation with resonances

$$P(X,\phi)=\frac{1}{f(\phi)}\left[1-\sqrt{1-2f(\phi)X}\right]-V(\phi)$$

$$f(\phi)=A\left[1+\cos\left(2\pi\frac{(\chi-\chi_0)}{\Delta\chi}\right)\right]$$



Example 2: DBI inflation with resonances



We tried many other models and our formula works impressively well

$$\eta = \eta_0 + \frac{\alpha}{2} \frac{d}{dN}(1 - c_s^2)$$

Now that you have this relation, you may obtain a general expression correlating features in the bispectrum with dose in the power spectrum

We find:

$$\Delta B = \frac{(2\pi)^4 \mathcal{P}_0^2}{8(k_1 k_2 k_3)^2} \frac{1}{1+\alpha} \left[\alpha + 2 \frac{k_1^2 + k_2^2 + k_3^2}{(k_1 + k_2 + k_3)^2} \right] \frac{d^2 \Delta \mathcal{P}(k)}{d \ln k^2}$$

$$+ \cdots$$

Conclusions

Two main results:

- We uncovered a relation between the expansion rate and the speed of sound

$$\eta = \eta_0 + \frac{\alpha}{2H}\frac{d}{dt}(1 - c_s^2)$$

- We have a general map between features in the power spectrum and bi-spectrum

$$\Delta B = b_0 \Delta \mathcal{P} + b_1 \frac{d}{d\ln k}\Delta \mathcal{P} + b_2 \frac{d^2}{d\ln k^2}\Delta \mathcal{P}$$

Conclusions

Two main results:

- We uncovered a relation between the expansion rate and the speed of sound

$$\eta = \eta_0 + \frac{\alpha}{2H}\frac{d}{dt}(1 - c_s^2)$$

- We have a general map between features in the power spectrum and bi-spectrum

$$\Delta B = b_0 \Delta\mathcal{P} + b_1 \frac{d}{d\ln k}\Delta\mathcal{P} + b_2 \frac{d^2}{d\ln k^2}\Delta\mathcal{P}$$