

# Spacetime curvature and Higgs stability before and after inflation

arXiv:1407.3141 (PRL 113, 211102);

arXiv:1506.04065

Tommi Markkanen<sup>1,2</sup> Matti Herranen<sup>3</sup> Sami Nurmi<sup>2</sup>  
Arttu Rajantie<sup>1</sup>

<sup>1</sup>Imperial College

<sup>2</sup>University of Helsinki

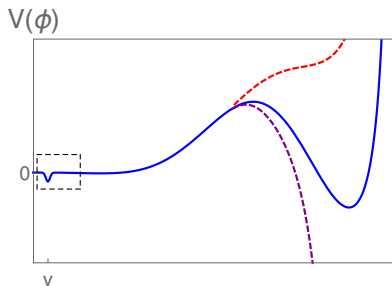
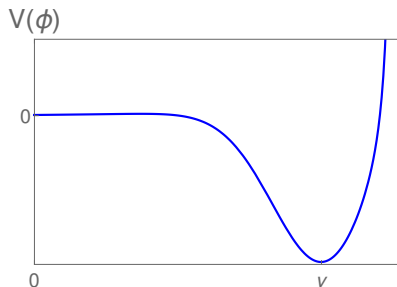
<sup>3</sup>Niels Bohr International Academy, Copenhagen



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- 2 Higgs stability during inflation
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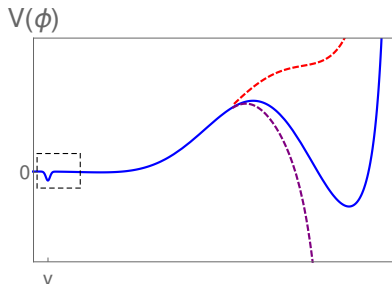
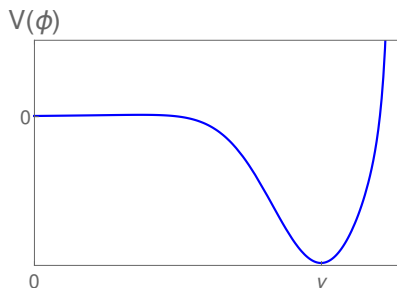
# Standard Model Higgs potential



● Sensitive to  $M_h$  and  $M_t$

- A vacuum at  $\phi \neq v$  incompatible with observations
- *Meta*stable at 99% CL

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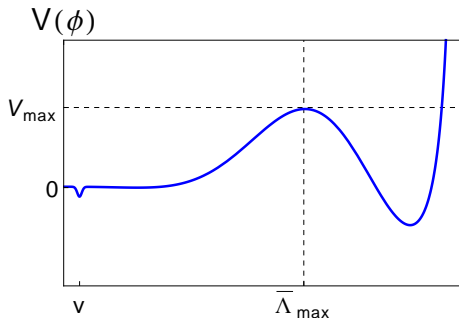


- Sensitive to  $M_h$  and  $M_t$

- A vacuum at  $\phi \neq v$  incompatible with observations
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- What about the early Universe (**inflation, reheating**)?
- New physics needed to stabilize the vacuum?

# The early Universe and the Standard Model

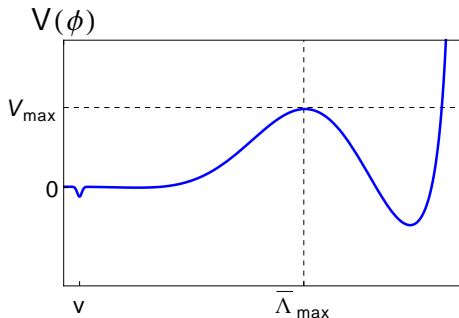
- Assume the SM to be valid and decoupled from inflation
  - Higgs subdominant,  $\phi \ll M_{\text{pl}}$
- Curvature can induce fluctuations of the Higgs  $\Delta\phi \sim H$ 
  - Important if  $\bar{\Lambda}_{\text{max}} \lesssim H$
  - State of the art calculations [1]:  $\bar{\Lambda}_{\text{max}} \sim 10^{11} \text{ GeV}$



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BICEP2/Keck/Planck

$$H \lesssim 10^{14} \text{ GeV}$$

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# Potential during inflation

- Fluctuations may be treated as stochastic variables [2]
- Probability density  $P(t, \phi)$  from the Fokker-Planck equation

$$\dot{P}(t, \phi) = \frac{1}{3H} \frac{\partial}{\partial \phi} [P(t, \phi) V'_{\text{eff}}(\phi)] + \frac{H^3}{8\pi^2} \frac{\partial^2}{\partial \phi^2} P(t, \phi)$$

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- Leading potential contributions:

Flat space,  $\phi \gg m$

$$V_{\text{eff}}(\phi) \approx \frac{\lambda(\phi)}{4} \phi^4$$

Curved space,  $H \gg \phi \gg m$

$$V_{\text{eff}}(\phi) \approx \frac{\lambda(H)}{4} \phi^4 + \frac{\xi(H)}{2} R \phi^2$$

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# Full 1-loop result

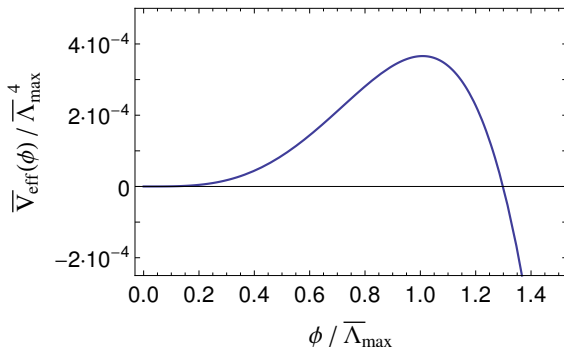
$$V_{\text{eff}}(\phi, R) = -\frac{1}{2}m^2(\mu)\phi^2 + \frac{1}{2}\xi(\mu)R\phi^2 + \frac{1}{4}\lambda(\mu)\phi^4$$

$$+ \sum_{i=1}^9 \frac{n_i}{64\pi^2} M_i^4(\phi) \left[ \log \frac{|M_i^2(\phi)|}{\mu^2} - c_i \right] \quad ; \quad \begin{aligned} M_i^2(\phi) &= \kappa_i \phi^2 - \kappa'_i + \theta_i R \\ \mu^2 &= \phi^2 + R \end{aligned}$$

$\Phi$	$i$	$n_i$	$\kappa_i$	$\kappa'_i$	$\theta_i$	$c_i$
$W^\pm$	1	2	$g^2/4$	0	1/12	3/2
	2	6	$g^2/4$	0	-1/6	5/6
	3	-2	$g^2/4$	0	-1/6	3/2
$Z^0$	4	1	$(g^2 + g'^2)/4$	0	1/12	3/2
	5	3	$(g^2 + g'^2)/4$	0	-1/6	5/6
	6	-1	$(g^2 + g'^2)/4$	0	-1/6	3/2
t	7	-12	$y_t^2/2$	0	1/12	3/2
$\phi$	8	1	$3\lambda$	$m^2$	$\xi - 1/6$	3/2
$\chi_i$	9	3	$\lambda$	$m^2$	$\xi - 1/6$	3/2



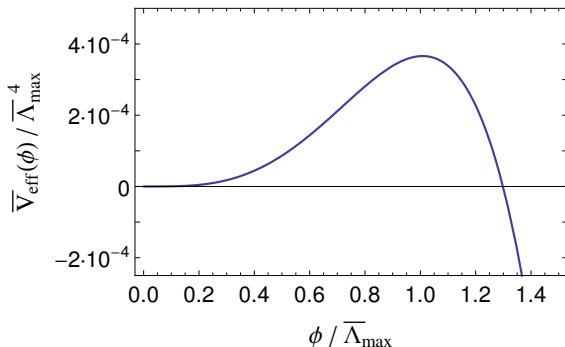
# Stability (Minkowski)



- For large  $H$  ( $\sim 10^3 \bar{\Lambda}_{\text{max}}$ ), the SM is not stable [5]

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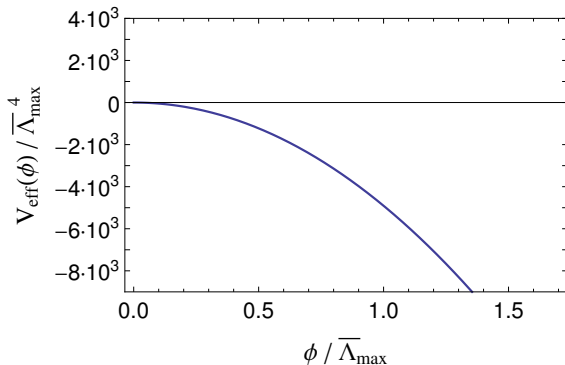
- First attempt, set  $\xi_{EW} = 0$  and  $H \sim 10^3 \bar{\Lambda}_{\max}$

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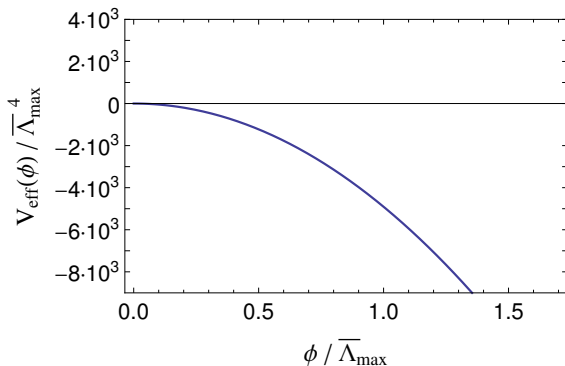
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- For large  $H$  one has  $\lambda(\mu) < 0$ , since  $\mu^2 = \phi^2 + R$
- $\xi$  Can become positive or negative depending on  $\xi_{EW}$

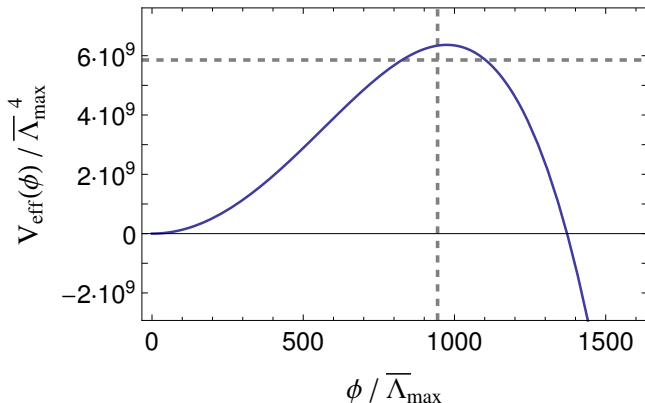
# Stability results (curved space) II

- Now choosing  $\xi_{EW} = 0.1$  [6]

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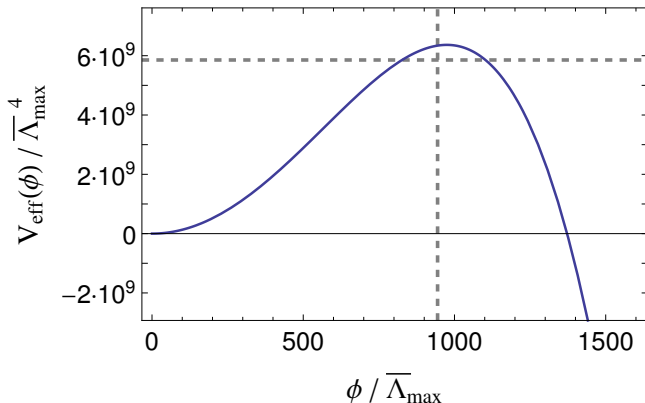
$$V_{\text{max}}^{1/4} \simeq V_{\text{tree}}^{1/4}(\Lambda_{\text{max}})$$

- $V_{\text{max}}(\text{curved}) \gg V_{\text{max}}(\text{flat})$  (and at a higher scale)

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$$P \sim \exp\left[-8\pi^2 (V_{\text{max}}/3H^4)\right] \Rightarrow \text{Stable!}$$

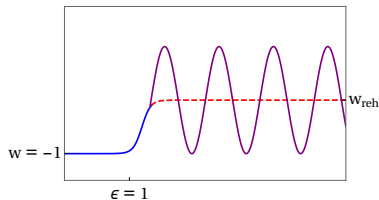
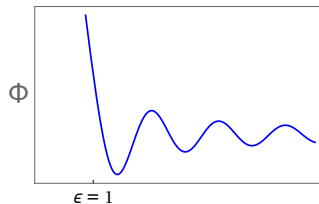
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# Reheating

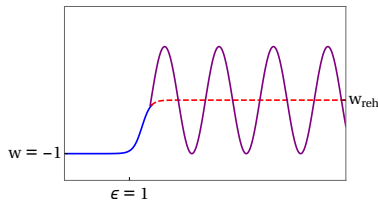
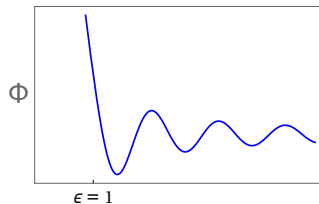
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- The Higgs always feels reheating via gravity

$\Rightarrow$  New stability constraints !

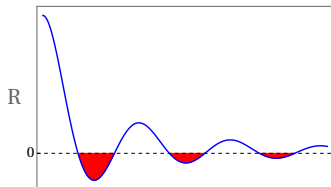
- Two effects:
  - A rapid drop in  $w$ , *on average*
  - Oscillations in the complete solution

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# Oscillating $R$

- The curvature oscillates during reheating (Planck units)

$$G_{\mu\nu} = T_{\mu\nu} \quad \Rightarrow \quad R = \left[ 4V_{\text{inf}}(\Phi) - \dot{\Phi}^2 \right]$$

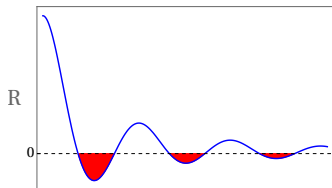


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- *Parametric (tachyonic) resonance* via the Mathieu equation

$$\frac{d^2 f(z)}{dz^2} + \left[ A_{\mathbf{k}} - 2q \cos(2z) \right] f(z) = 0, \quad z = t M_{\text{inf}}$$

⇒ Exponential amplification,  $n_{\mathbf{k}} \propto \exp \{ \mu_{\mathbf{k}} \}$  [8]

[8] Kofman, Dufaux, Felder, Peloso & Podolsky (2006); Bassett & Liberati (1997)

# Fluctuations from parametric resonance

- Resonance may give large fluctuations,  
⇒ Instabilities ?!
- After *one* oscillation, with  $\mathbf{k} < aH$

$$n_{\mathbf{k}} \sim \exp \left\{ \sqrt{\xi} \frac{2\Phi_0}{M_{\text{pl}}} \right\}$$

- For example for quadratic inflation

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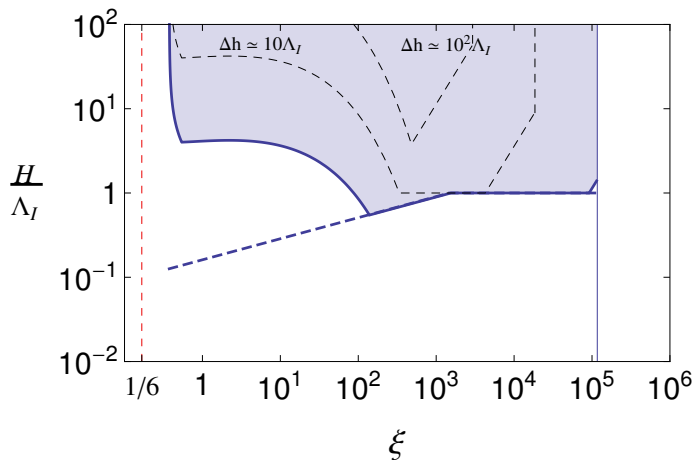
Superhorizon modes,  $\mathbf{k} < aH$

$$\Rightarrow \Delta\phi^2 \sim \left( \frac{H}{2\pi} \right)^2 \frac{\exp \left\{ \sqrt{\xi} \frac{2\Phi_0}{M_{\text{pl}}} \right\}}{\sqrt{\xi}}$$

- Potentially a **huge** effect,  $\Delta\phi \gg \Lambda_I$

- However, the resonance may be shut off by **backreaction**

# Stability results, reheating



## Interactions

$$\lambda \langle \hat{\phi}^2 \rangle \ll \xi R,$$

if  $\lambda > 0$

## Gravity

$$\rho_{\text{Hig}} \ll 3M_{\text{pl}}^2 H^2$$

$\Rightarrow$  For  $H \gtrsim \Lambda_I \sim 10^{11} \text{ GeV}$ ,  $\xi$  is constrained to be  $\sim 1/6$



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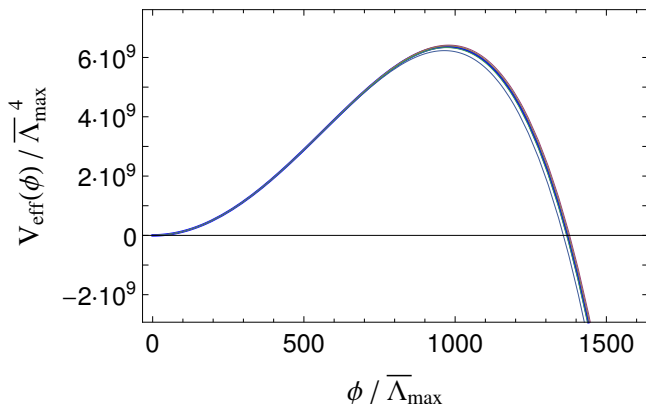
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$$\mu^2 = \alpha\phi^2 + \beta R \quad \alpha, \beta \in \{0.1 \dots 10\}$$



## Quantum field theory on a curved background

- **Example:** self-interacting scalar field in curved space

$$S = - \int d^4x \sqrt{-g} \left[ \frac{1}{2} (\partial_\mu \phi)^2 + \frac{1}{2} m^2 \phi^2 + \frac{\xi}{2} R \phi^2 + \frac{\lambda}{4!} \phi^4 \right] ; R = 12H^2$$

- For a constant field

$$\frac{\delta S}{\delta \phi} = 0 \quad \Leftrightarrow \quad V'(\phi) = 0 \quad \Rightarrow \quad V(\phi) = \int^\phi V'(\phi)$$



# Potential during inflation II

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- Quantum correction to LO in fluctuations,

$$\hat{\phi} \rightarrow \langle \hat{\phi} \rangle + \hat{\phi} \equiv \phi + \hat{\phi}$$

$$V'(\phi) = \underbrace{m^2 \phi + \xi R \phi + \frac{\lambda}{6} \phi^3}_{\text{classical}} + \underbrace{\frac{\lambda}{2} \phi \langle \hat{\phi}^2 \rangle}_{\text{quantum}}$$

- Same calculation in flat and curved space
  - The only difference is the form of the mode

## Flat space

$$ds^2 = -dt^2 + d\mathbf{x}^2$$

$$\left[ -\square + M(\phi)^2 \right] \hat{\phi} = 0$$

$$f_{\mathbf{k}} = \frac{1}{\sqrt{\omega}} e^{-i\omega t}$$

$$\omega^2 = \mathbf{k}^2 + M(\phi)^2$$

## Curved space (FLRW)

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# Renormalization group improvement for $\lambda\phi^4$ theory

- Large  $\log$ 's perturbatively problematic
- However, the physical result must not depend on  $\mu$ :

$$\frac{d}{d\mu} V_{\text{eff}}(\phi) = 0 \quad \Leftrightarrow \quad \left\{ \mu \frac{\partial}{\partial \mu} + \beta_\lambda \frac{\partial}{\partial \lambda} + \gamma_\phi \phi \frac{\partial}{\partial \phi} \right\} V_{\text{eff}}(\phi) = 0$$

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- Leads to *running parameters*

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$$\frac{d}{d\mu} V_{\text{eff}}(\phi) = 0 \quad \Leftrightarrow \quad \left\{ \mu \frac{\partial}{\partial \mu} + \beta_\lambda \frac{\partial}{\partial \lambda} + \gamma_\phi \phi \frac{\partial}{\partial \phi} \right\} V_{\text{eff}}(\phi) = 0$$

- Leads to *running parameters*
- We must choose  $\mu$  to optimize the expansion [9]

optimal choice, flat space

$$\mu^2 = m^2 + \frac{\lambda}{2} \phi^2$$

optimal choice, curved space

$$\mu^2 = m^2 + \frac{\lambda}{2} \phi^2 + \left( \xi - \frac{1}{6} \right) R$$

[9] Ford, Jones, Stephenson & Einhorn (1993)

# Renormalization group improvement for $\lambda\phi^4$ theory

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⇒ Curvature induces running of the constants ! [10]

[9] Ford, Jones, Stephenson & Einhorn (1993)

[10] Zurek, Kearney & Yoo (2015); TM (2014)

# Stability results (curved space) II

$$V_{\text{eff}}(\phi) \approx \frac{\lambda(\mu)}{4}\phi^4 + \frac{\xi(\mu)}{2}R\phi^2$$

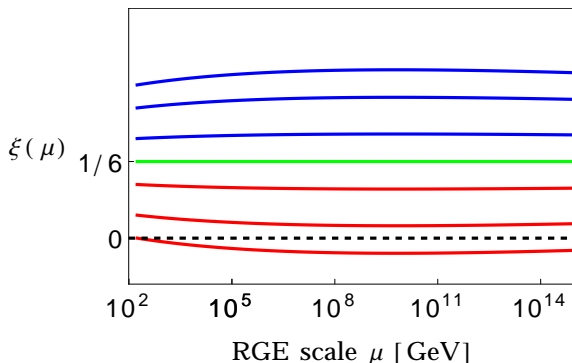
- For large  $H$  one has  $\lambda(\mu) < 0$ , since  $\mu^2 = \phi^2 + R$



# Stability results (curved space) II

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- For large  $H$  one has  $\lambda(\mu) < 0$ , since  $\mu^2 = \phi^2 + R$
- $\xi$  Can become positive or negative depending on  $\xi_{EW}$



$\xi_{EW}$   
0, 0.05, 0.12, 1/6,  
0.22, 0.28, 0.33