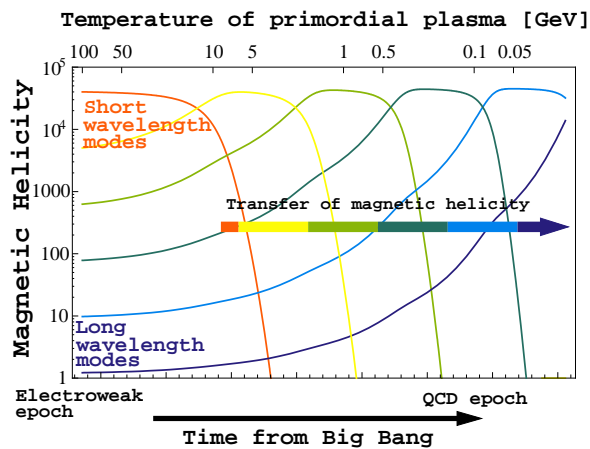


# Generation and evolution of cosmological magnetic fields at the electroweak epoch

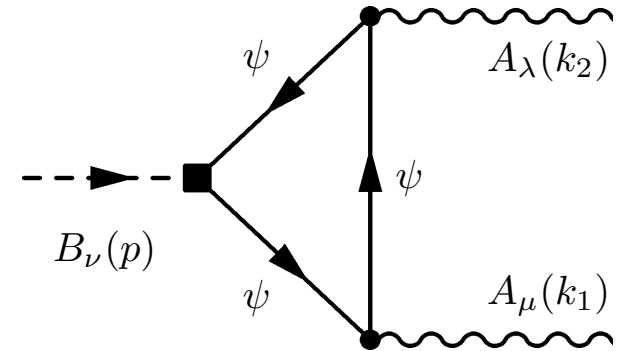


Oleg  
RUCHAYSKIY



COSMO 2015

September 10, 2015



Phys. Rev. Lett. **108** (2012) 031301 [arXiv:1109.3350] (with A. Boyarsky and J. Fröhlich)

Phys. Rev. Lett. **109** (2012) 111602 [arXiv:1204.3604] (with A. Boyarsky and M. Shaposhnikov)

Phys. Rev. D (2015) [arXiv:1504.04854] (with A. Boyarsky and J. Fröhlich)

# Magnetic fields in the Universe

---

- Our Universe today is magnetized: (earth, stars, galaxies, clusters of galaxies)
- Magnetic fields in the spiral (i.e. rotating) galaxies can be generated by turbulent effect from tiny primordial seeds (“dynamo mechanism”)
- Magnetic fields in **elliptical galaxies**?
- Magnetic fields in **clusters of galaxies**  $\neq$  sum of magnetic fields of galactic magnetic fields
- Even **intergalactic medium** seems to be filled with magnetic fields
- Are we observing the evidence of process **in the very early Universe**?

Neronov &  
Vovk'10;  
Dolag et al.'10  
Tavecchio et  
al.'11

# Magnetic fields in the early Universe

---

- Magnetic fields affect every important process in the early Universe:
  - Change the nature of electroweak phase transition
  - Affect baryogenesis
  - Leave its imprints in production of gravity waves
  - Affect BBN
  - Leave its imprints in CMB
  - Affect structure formation
  - Could have played a role of seeds of galactic magnetic fields
- Many mechanisms of generation of primordial magnetic fields exist, usually associated with violent events (at inflation, electroweak transition, QCD transition, etc.)
- It is commonly believed that **noticeable magnetic fields** are **not generated in the Universe filled with Standard Model particles**

Many, many works...  
Recent reviews:  
Widrow et al.'11;  
Kandus et al.'11;  
Yamazaki et al.'12;  
Durrer & Neronov '13

## Main message

---

- Primordial plasma (MeV-GeV temperatures) behaves very different from a laboratory plasmas
- In fact this is a rare situation when **quantum effects** affect dynamics at **macroscopic** (arbitrarily large) scales
- What I discuss in this talk **does not rely on any new physics** and is true fully within the Standard Model of particle physics
- Who should care about this?
  - People who study cosmological magnetic fields
  - People who work on leptogenesis and related themes

## Axial symmetry

---

- Massless fermions can be left and right-chiral (left and right moving):

$$(i\gamma^\mu \partial_\mu - \cancel{m})\psi = \begin{pmatrix} \cancel{m} & i(\partial_t + \vec{\sigma} \cdot \vec{\nabla}) \\ i(\partial_t - \vec{\sigma} \cdot \vec{\nabla}) & \cancel{m} \end{pmatrix} \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix} = 0$$

where  $\gamma_5 \psi_{R,L} = \pm \psi_{R,L}$  and  $\gamma_5 = i\gamma_0\gamma_1\gamma_2\gamma_3$ .

- Number of left  $N_L = \int d^3x \psi_L^\dagger \psi_L$  and right  $N_R = \int d^3x \psi_R^\dagger \psi_R$  particles is conserved **independently**
- Electric charge:  $Q = N_L + N_R$  – gauge symmetry
- **Axial charge**:  $Q_5 = N_L - N_R$  – global symmetry

# Axial anomaly

---

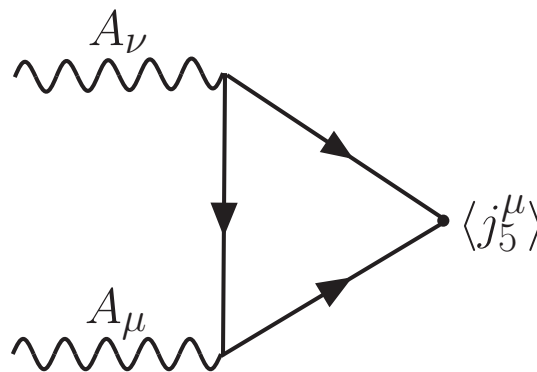
- Gauge interactions respects chirality ( $D_\mu = \partial_\mu + eA_\mu$ )...

$$\begin{pmatrix} \cancel{-m}^0 & i(D_t + \vec{\sigma} \cdot \vec{D}) \\ i(D_t - \vec{\sigma} \cdot \vec{D}) & \cancel{-m}^0 \end{pmatrix} \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix} = 0$$

- ... but the difference of left and right-movers is **not conserved** once the quantum corrections are taken into account — **axial anomaly**

$$\frac{dQ_5}{dt} = \frac{d(N_L - N_R)}{dt} = \int d^3\vec{x} (\partial_\mu j_\mu^5) = \frac{\alpha}{\pi} \int d^3\vec{x} \vec{E} \cdot \vec{B}$$

$\alpha \equiv \frac{e^2}{4\pi}$  — fine-structure constant



# QM interpretation: spectral flow

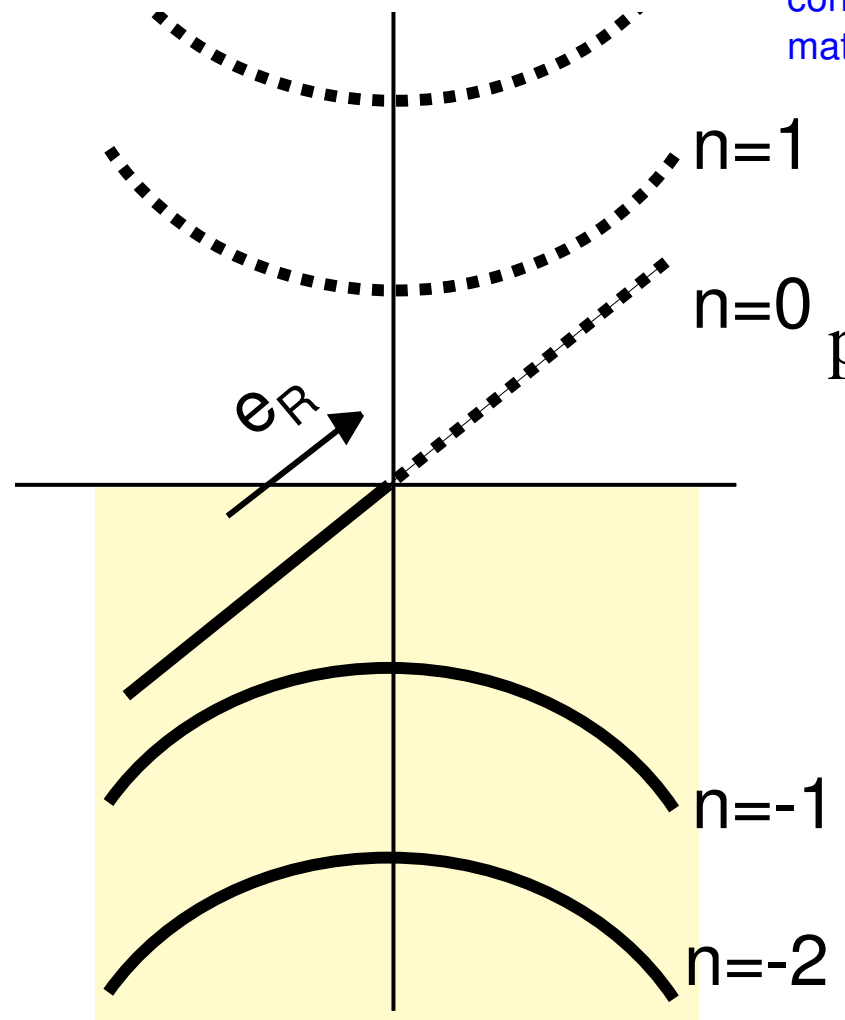
$$\text{Landau levels: } E^2 = p_z^2 + e|B|(2n + 1) + 2e\vec{B} \cdot \vec{s}$$

G.E. Volovik,  
cond-  
mat/9802091

- Only particles with  $(\vec{B} \cdot \vec{s} < 0)$  have **massless** branches:

$$E = \begin{cases} -p_z & \text{left branch} & (\vec{p} \cdot \vec{s}) > 0 \\ p_z & \text{right branch} & (\vec{p} \cdot \vec{s}) < 0 \end{cases}$$

- Electric field  $\vec{E} = E\hat{z}$  **creates right particle** (because  $p_z(t) = p_z(0) + eEt$ )
- Electric field destroys **left particles**
- Total number** does not change
- Difference** of **left** minus **right** appears – **chiral anomaly!**



# Axial anomaly $\leftrightarrow$ change of helicity

---

- If we introduce **magnetic helicity**

$$\mathcal{H} = \int d^3x \vec{A} \cdot \vec{B}$$

- ... then

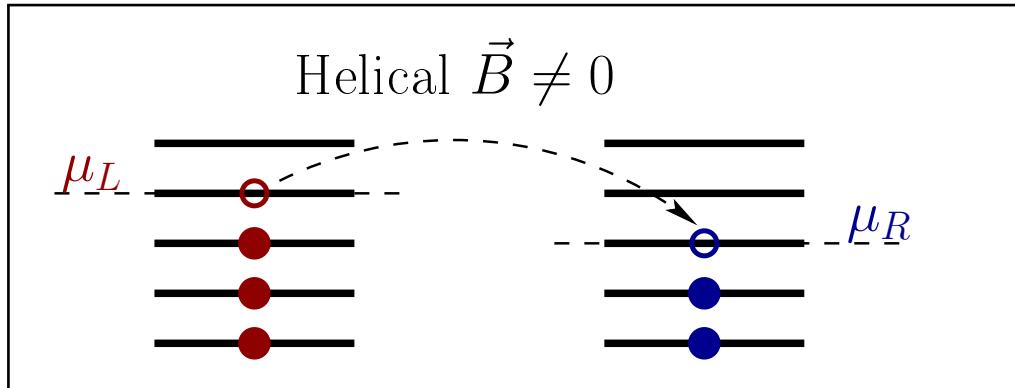
$$\int d^3x \vec{E} \cdot \vec{B} = -\frac{1}{2} \frac{d\mathcal{H}(t)}{dt}$$

- Axial anomaly couples change of **left-right asymmetry** to the change of magnetic helicity:

$$\frac{d}{dt} \left( (N_L - N_R) + \frac{\alpha}{\pi} \mathcal{H} \right) = 0$$



# Axial anomaly at finite densities



- $\mu_L$  – Fermi level for **left fermions**
- $\mu_R$  – Fermi level for **right fermions**

- Change helicity of  $\vec{B}$ -field  $\Rightarrow$  due to axial anomaly of  $N_{L,R}$  changes:

$$\delta N_{L,R} = \int_{t_i}^{t_f} dt \dot{N}_{L,R}(t) = \mp \int dt \frac{\alpha}{\pi} \int dV E \cdot B$$

- The energy:  $\delta \mathcal{E} = \delta N_L \mu_L + \delta N_R \mu_R = \frac{\alpha(\mu_L - \mu_R)}{2\pi} \int dV A \cdot B$

Nielsen &  
Ninomiya  
(1983);

Rubakov  
(1986)

- Free energy of magnetic fields in plasma:  $\delta \mathcal{F} = \int dV \frac{\alpha(\mu_L - \mu_R)}{2\pi} A \cdot B$

## Chern-Simons term

---

- In coordinate space  $\Pi_2 \neq 0$  leads to a **Chern-Simons term**:

$$\mathcal{F}[\vec{A}] = \frac{1}{2} \int d^3x \left( \vec{B}^2 + \frac{\alpha}{\pi} \mu_5 \vec{A} \cdot \vec{B} \right)$$

- Consider configuration  $\vec{A} = A_0 (\cos(kz), \sin(kz), 0)$
- The magnetic field  $\vec{B} = -k\vec{A}$  (*maximally helical* configuration)
- The effective action:  $\mathcal{F}[A] = \frac{1}{2} \int d^3x \left( k^2 - k \frac{\alpha}{\pi} \mu_5 \right) A_0^2 < 0$
- Increasing amplitude for  $k < \frac{\alpha}{\pi} \mu_5$  we lower energy  $\Rightarrow$  **Instability**

# Chiral Magnetic effect

---

- In coordinate space  $\Pi_2 \neq 0$  leads to a **Chern-Simons term**:

$$\mathcal{F}[\vec{A}] = \frac{1}{2} \int d^3x \left( \vec{B}^2 + \frac{\alpha}{\pi} \mu_5 \vec{A} \cdot \vec{B} \right)$$

- **Chiral magnetic effect**:

$$\vec{j} = \frac{\delta \mathcal{F}[\vec{A}]}{\delta \vec{A}} = \frac{2\alpha}{\pi} \mu_5 \vec{B}$$

Vilenkin'78;

Redlich &  
Wijewardhana  
(1985);

Rubakov'86;

Cheianov,  
Fröhlich,  
Alexeev'98;

Fröhlich &  
Pedrini'00,'02;

Fukushima,  
Kharzeev,  
Warringa'08

Son &  
Surowka'09

# Maxwell equations

---

- The presence of difference of chemical potential of left and right fermions leads to additional terms in the effective Lagrangian for electromagnetic fields – **Chern-Simons term**

- As a result Maxwell equations contain current, **proportional to**  $\mu_5$  Kharzeev'11  
— MHD turns into **chiral MHD**:

$$\text{curl } \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\text{curl } \vec{B} = \sigma \vec{E} + \frac{2\alpha}{\pi} \mu_5 \vec{B} \quad \leftarrow \text{Chiral magnetic effect}$$

Vilenkin  
(1978)

Fröhlich &  
Pedrini  
(2000–2001)

Joyce &  
Shaposhnikov  
(1997)

- **In addition**,  $\mu_5$  should be allowed to **become dynamical**:

$$\frac{d(N_L - N_R)}{dt} \propto \frac{d\mu_5}{dt} \propto \frac{\alpha}{\pi} \int d^3x \vec{E} \cdot \vec{B}$$

$$\text{curl } \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\text{curl } \vec{B} = \sigma \vec{E} + \frac{2\alpha}{\pi} \mu_5 \vec{B} \quad \leftarrow \text{Chiral magnetic effect}$$

$$\frac{\partial \mu_5}{\partial t} \propto \frac{2\alpha}{\pi} \int d^3x \vec{E} \cdot \vec{B} - \Gamma_{\text{flip}} \mu_5 \quad \leftarrow \text{chirality-flipping due to finite mass of fermions}$$

- Without  $B$  chirality flipping reactions drive  $\mu_5 \rightarrow 0$  ( $\mu_5 = \mu_0 e^{-\Gamma_{\text{flip}} t}$ )
- Without  $\mu_5$  finite conductivity drives  $B \rightarrow 0$  ( $B_k = B_0 e^{-\frac{k^2 t}{\sigma}}$ ). This is called **magnetic diffusion**

# Instability

---

- Maxwell equations with  $\mu_5$  are **unstable**:

$$\frac{\partial B}{\partial t} = \underbrace{\frac{1}{\sigma} \nabla^2 B}_{\text{magnetic diffusion}} + \underbrace{\frac{\alpha \mu_5}{\pi} \text{curl } B}_{\text{instability}}$$

magnetic diffusion

- Exponential growth for  $k < \frac{\alpha}{\pi} \mu_5$  (for one of the circular polarizations depending on the sign of  $\mu_5$ ) — **generation of helical magnetic fields**

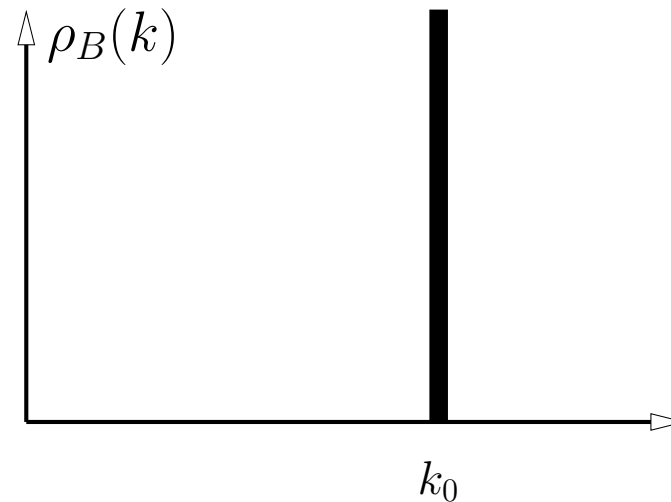
$$B_{\pm} = B_0 \exp\left(-\frac{k^2}{\sigma} t \pm \frac{\alpha k \mu_5}{\pi \sigma} t\right)$$

# Attractor solution

- Consider **sharply peaked at  $k_0$  maximally helical field**

$$\frac{d\mu_5}{dt} = -\rho_B (\mu_5 - \bar{\mu}_5) - \cancel{\Gamma_{\text{flip}} \mu_5}$$

$$\frac{d\rho_B}{dt} = \frac{\rho_B}{t_\sigma} \left( \frac{\mu_5}{(\bar{\mu}_5)} - 1 \right)$$



where  $t_\sigma = \frac{2\sigma}{k_0^2}$  and  $\rho_B \gg \Gamma_{\text{flip}}$

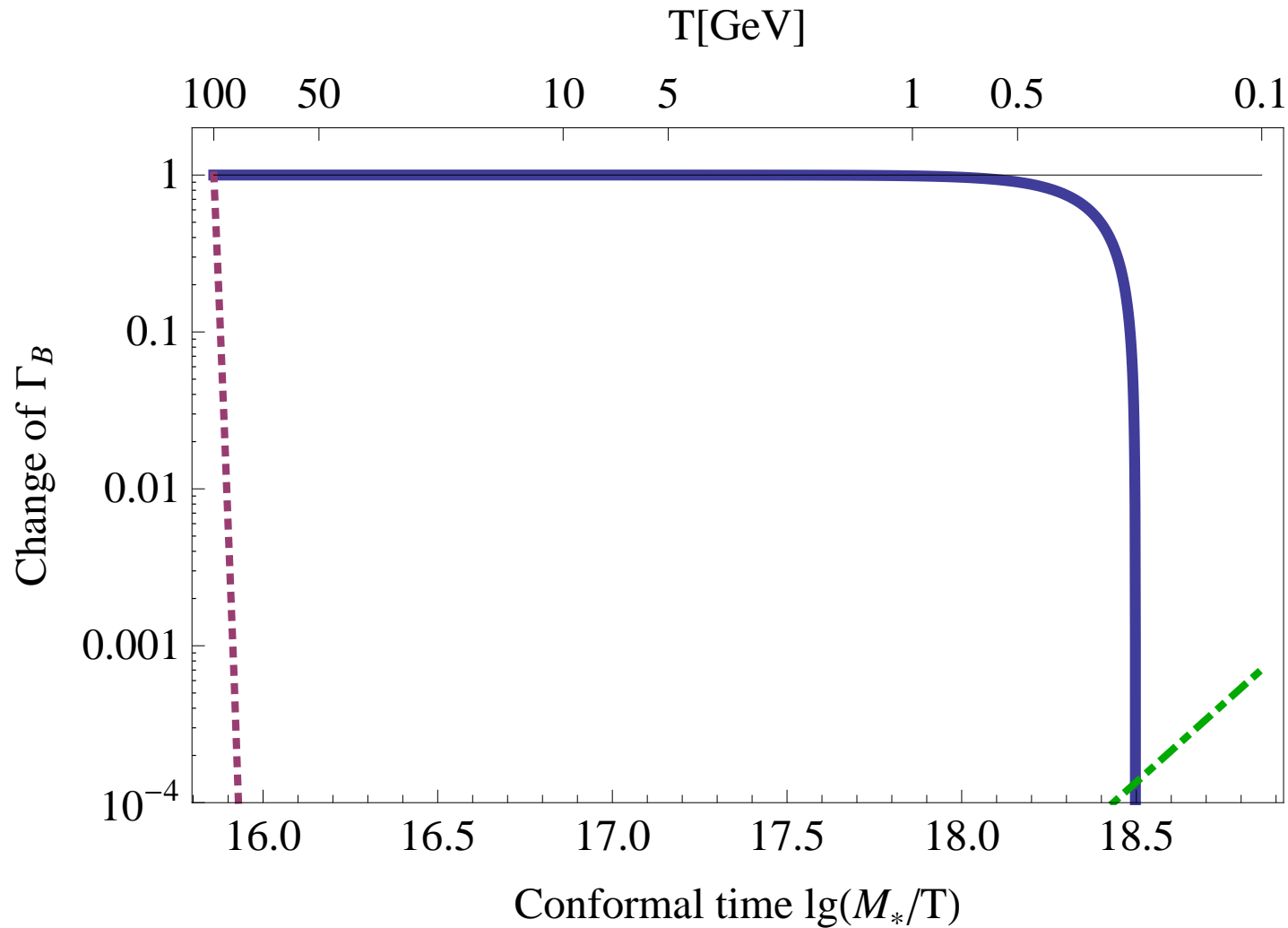
- Large  $\rho_B$  drives  $\mu_5$  to an **attractor solution**

$$\bar{\mu}_5 = \frac{2\pi k_0}{\alpha}$$

- Electric conductivity of the plasma is **finite** but **magnetic diffusion is compensated by the presence of  $\mu_5$**

O.R. with  
A. Boyarsky,  
J. Fröhlich  
PRL 2012  
[1109.3350]

# Evolution of magnetic energy density



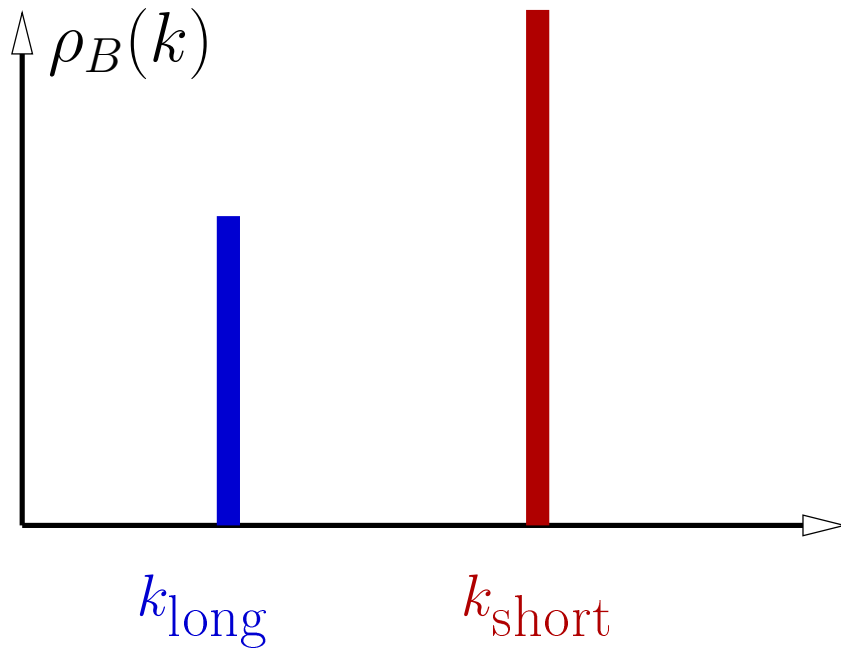
PRL (2012)  
[1109.3350]

X-axis – log of time

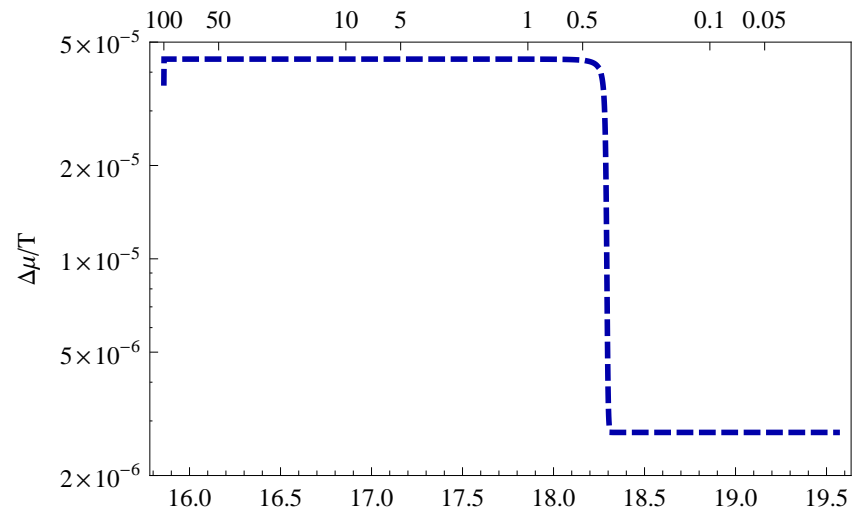
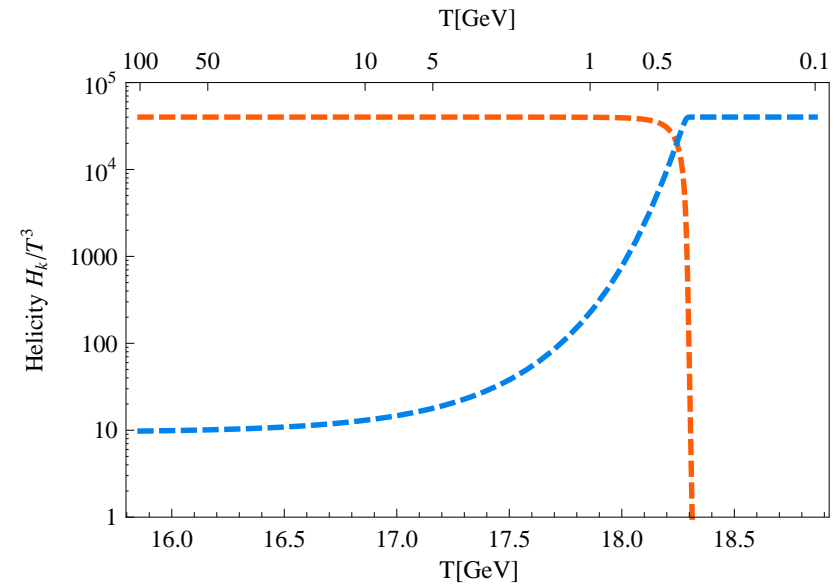
See also  
Tashiro et al.  
[1206.5549]



## Two modes

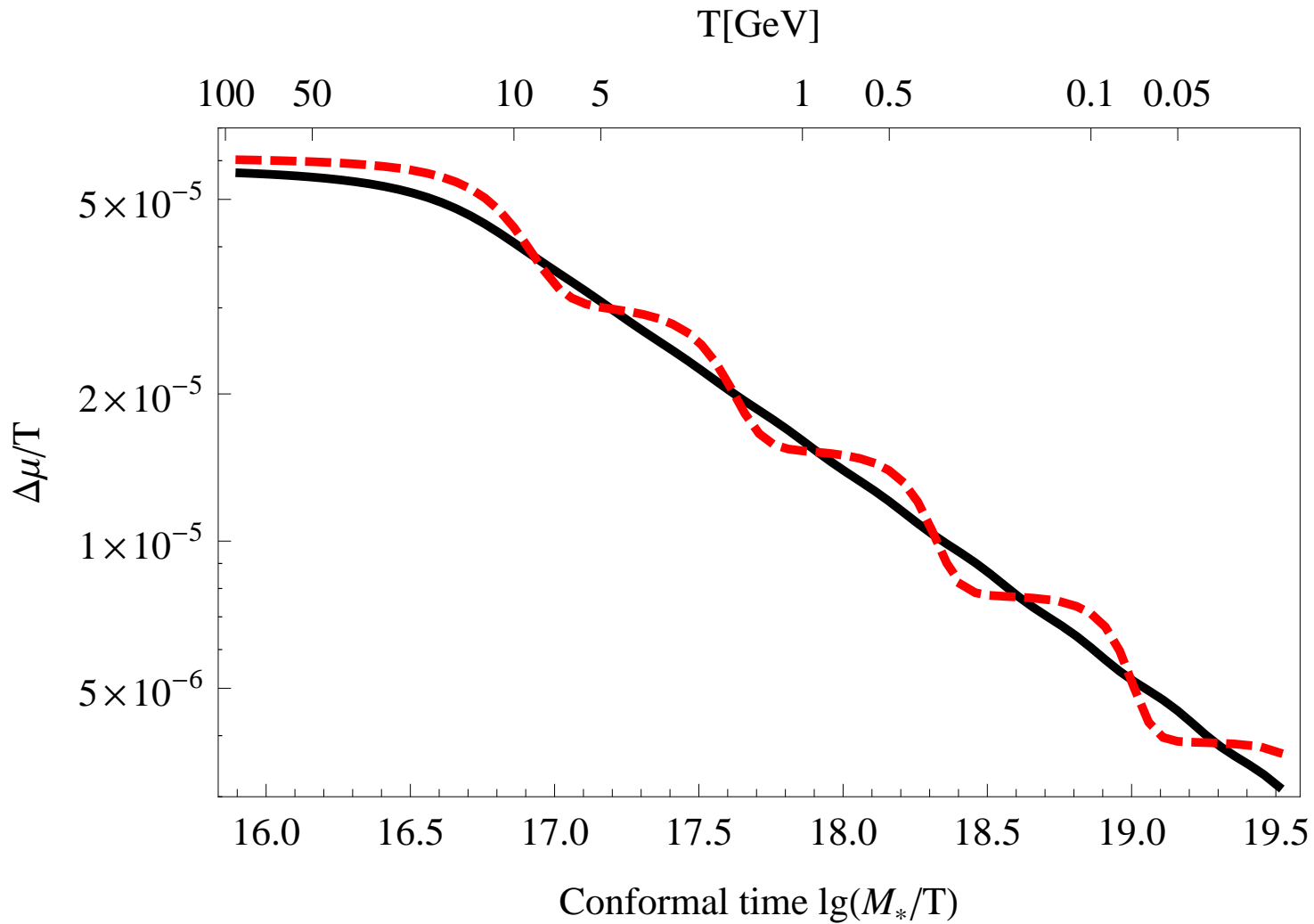


- In case of two modes the helicity gets transferred from the **shorter one** to the **longer one**
- Chemical potential follows the wave-number of the mode with higher helicity  $\mu = \frac{2\pi k}{\alpha}$



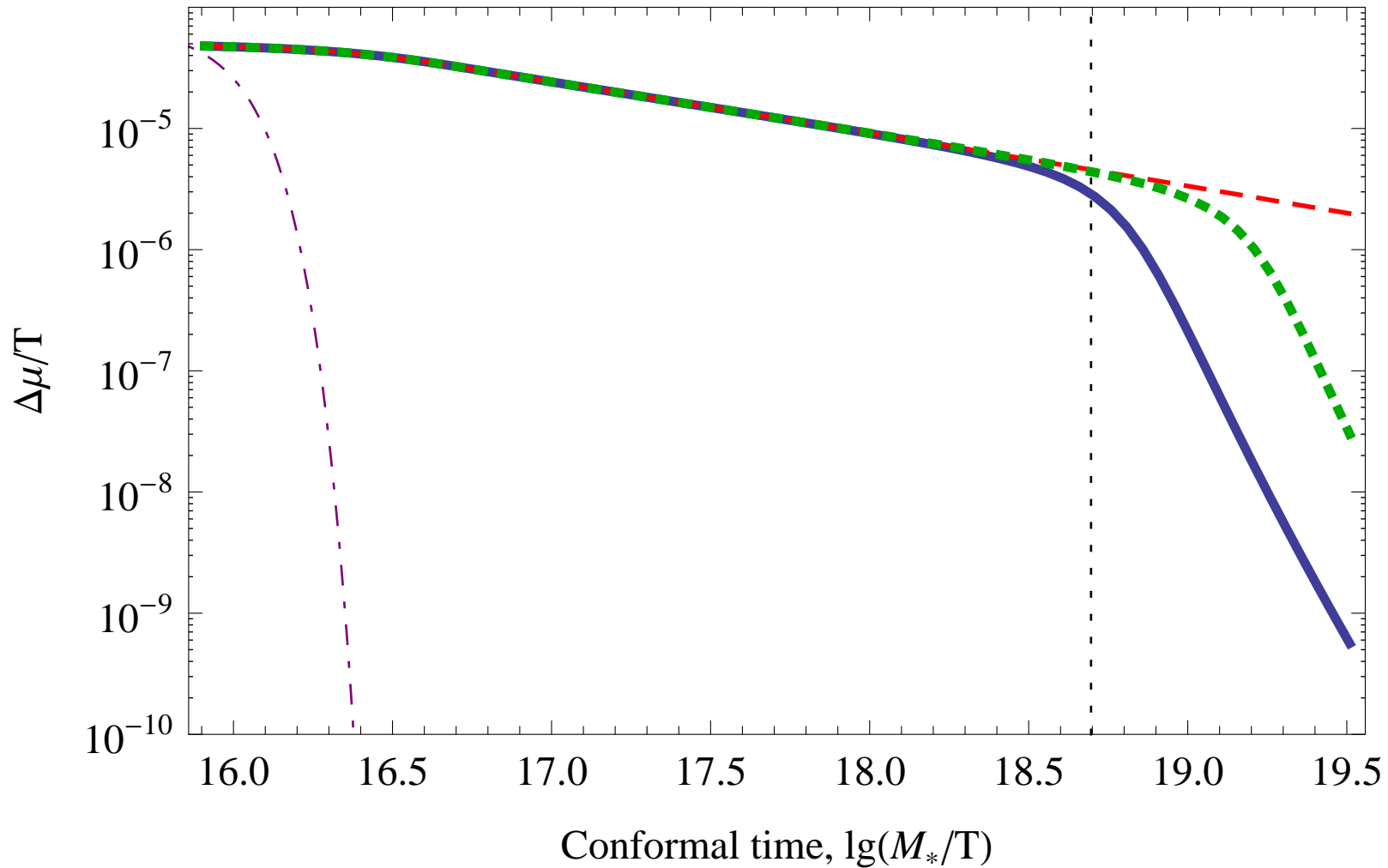
# Evolution of chemical potential

---



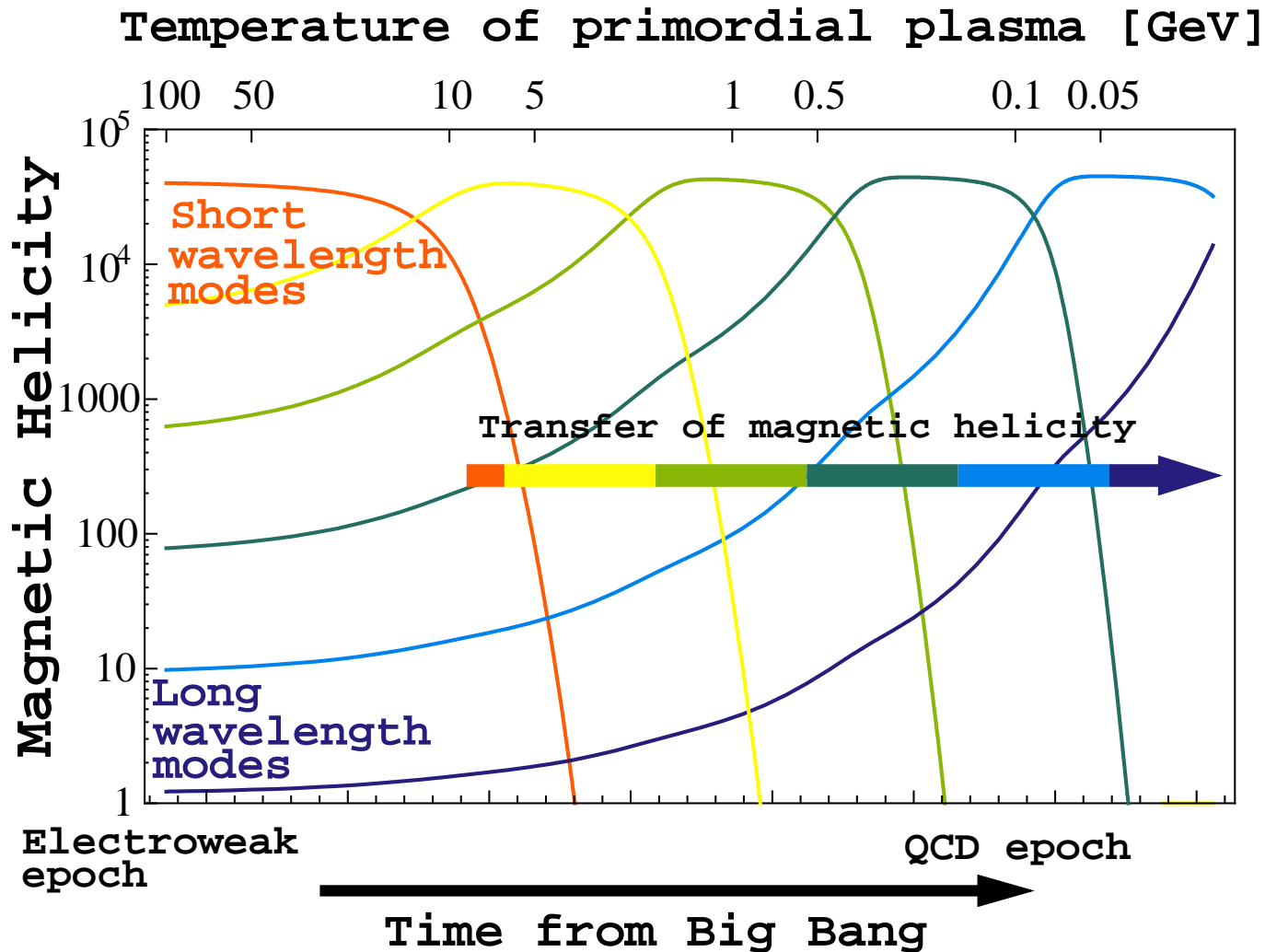
Process continues while  $\rho_B \gg \Gamma_{\text{flip}}$

# Evolution of chemical potential



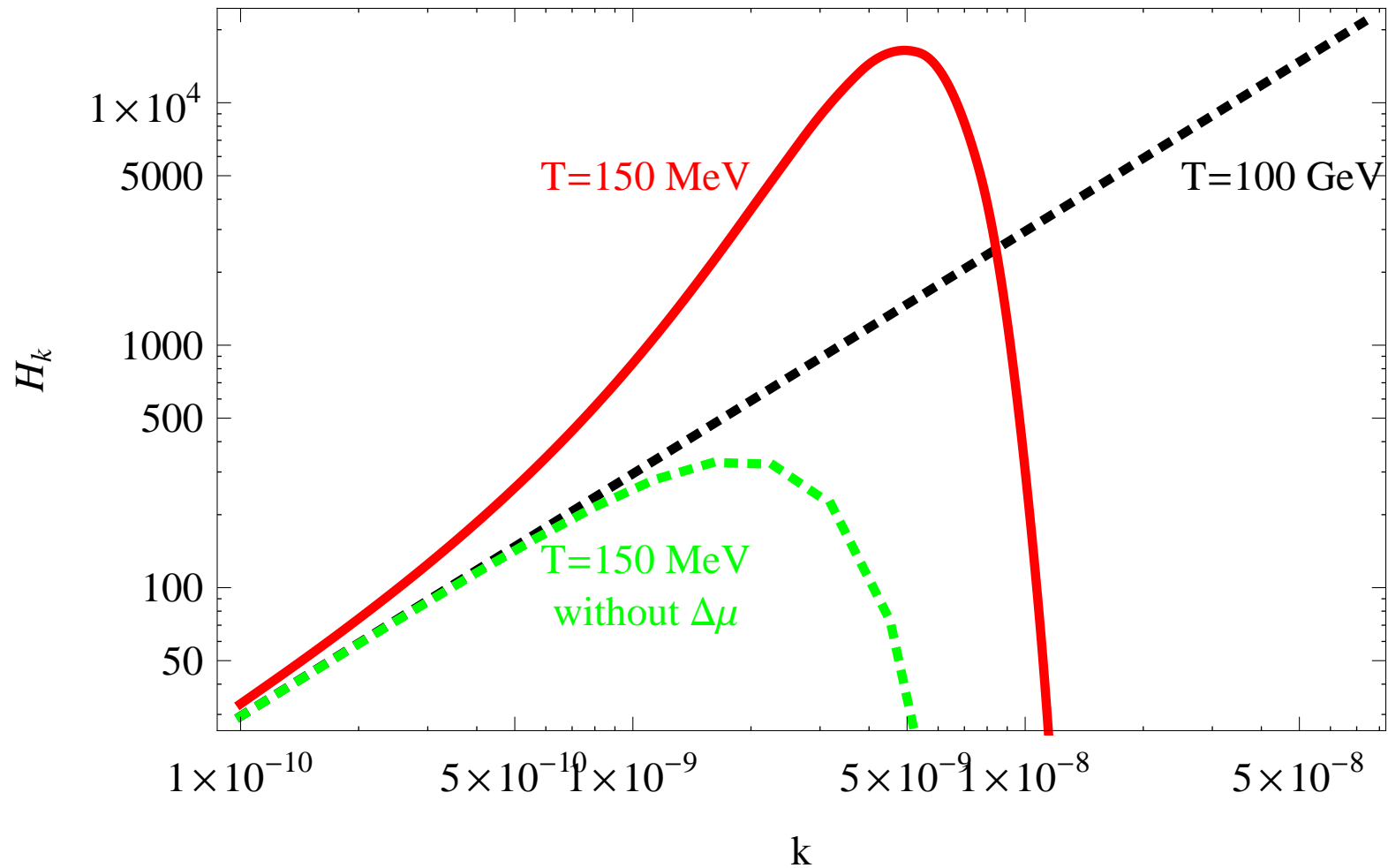
Continuous initial spectrum with  $\mathcal{H}_k \propto k$  and fraction of magnetic energy density  $5 \times 10^{-5}$  (blue) or  $5 \times 10^{-4}$  (green). Red – evolution without flip

# Evolution of helicity spectrum



Process continues while  $\Gamma_B \gg \Gamma_{\text{flip}}$  (recall that  $\Gamma_B \propto \rho_B$ )

# Evolution of helicity spectrum



Inverse cascade without turbulence!

# Inhomogeneous dynamics of $\mu_5$

---

- Back-reaction of magnetic fields on chemical potential (via axial anomaly) will make  $\mu_5$  inhomogeneous (i.e.  $\mu_5(x)$ )

Boyarsky et al.,  
[1504.04854]

$$\text{curl } \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\text{curl } \vec{B} = \sigma \vec{E} + \frac{2\alpha}{\pi} \mu_5(\vec{x}, t) \vec{B} + \text{gradient terms?}$$

$$\frac{\partial \mu_5(\vec{x}, t)}{\partial t} + \mathcal{O}(\nabla \mu_5) \propto \frac{2\alpha}{\pi} \vec{E}(\vec{x}, t) \cdot \vec{B}(\vec{x}, t) - \Gamma_{\text{flip}} \mu_5$$

- We are missing some gradient terms in a free energy that would make it gauge invariant (make current conserved)
- Can there be terms, proportional  $\vec{E}$ ?
- Recall:  $\vec{j}_{\text{CME}}$  is a  $\mathcal{T}$ -odd current, and  $\vec{E}$  is  $\mathcal{T}$ -even

- Let us introduce a pseudo-scalar field such that

$$\dot{\theta} = \mu_5$$

See also Nicolis, Son, Dubovsky, et al. (2011–2013)

- $\theta$  is  $\mathcal{T}$ -odd and therefore one can now write currents like  $\nabla\theta \times \vec{E}$ :

$$j_{\text{CME}}^\mu = \frac{\alpha}{\pi} \epsilon^{\mu\nu\lambda\rho} \partial_\nu \theta F_{\lambda\rho}$$

- We started from a thermodynamical system with **only** Standard Model particles (electrons, photons).
- It turns out that if the system has chiral anomaly, it is not described by the usual magneto-hydrodynamics, rather it is described by “chiral” (or axion) hydrodynamics with an **extra degree of freedom**
- Axion obeys a second order in time differential equation – **diffusion-type** equation for  $\mu_5$ , as it should be

# Full system of chiral MHD equations

---

$$\begin{aligned}
 \text{Chiral anomaly} & : \frac{\partial \mu_5}{\partial t} = D_5 \Delta \mu_5 + \Lambda^{-2} \vec{E} \cdot \vec{B}, \\
 \text{Axion} \leftrightarrow \text{chemical potential} & : \frac{\partial \Theta}{\partial t} + \vec{U} \cdot \nabla \Theta = \frac{\alpha}{\pi} \mu_5, \\
 \text{Maxwell's eq. with CME} & : \nabla \times \vec{B} = \frac{4\pi}{c} \vec{J} + \frac{\vec{B}}{c} \frac{\partial \Theta}{\partial t} + (\nabla \Theta) \times \vec{E}, \\
 \text{Bianchi identity} & : \nabla \times \vec{E} = -\frac{1}{c} \frac{\partial \vec{B}}{\partial t}, \\
 \text{Ohm's law} & : \vec{J} = \sigma \left( \vec{E} + \frac{1}{c} \vec{U} \times \vec{B} \right), \\
 \text{Continuity eqn.} & : \frac{D\rho}{dt} = 0, \\
 \text{Navier-Stokes eqn.} & : \rho \frac{D\vec{U}}{dt} = -c_s^2 \nabla p + \vec{F} + \frac{1}{c} \vec{J} \times \vec{B} \\
 & \qquad \qquad \qquad \frac{Df}{dt} \equiv \dot{f} + \vec{U} \cdot \nabla f
 \end{aligned}$$



## In short

---

$$\frac{d}{dt} \left( (N_L - N_R) + \frac{\alpha}{\pi} \mathcal{H} \right) = 0$$

**Change in magnetic helicity** is coupled to the chiral chemical potential  $\mu_5(t)$

Change of magnetic helicity

$\implies$

Change of  $\mu_L - \mu_R$

- **Change in magnetic helicity** excites  $\mu_5$  and then evolution of **already existing** magnetic fields occurs differently
- Presence of  $\mu_5$  **excites helical magnetic fields**

Change of  $\mu_L - \mu_R$

$\implies$

Change of magnetic helicity

# Generation of magnetic fields

---

- The presence of **chiral imbalance** excites magnetic fields (dynamo effect)
- What is the origin of **chiral imbalance**?
- All charged fermions are massive
- Mass breaks left/right symmetry

$$(i\gamma^\mu\partial_\mu - m)\psi = \begin{pmatrix} -m & i(\partial_t + \vec{\sigma} \cdot \vec{\nabla}) \\ i(\partial_t - \vec{\sigma} \cdot \vec{\nabla}) & -m \end{pmatrix} \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix} = 0$$

- In the early Universe when particles have  $\langle E \rangle \sim T \gg m$  – can we think about  $\mu_5$  as approximately conserved? – **No!**
- Chirality-flipping reaction:  $e_L + \gamma \rightarrow e_R + \gamma$

## Generation of magnetic fields

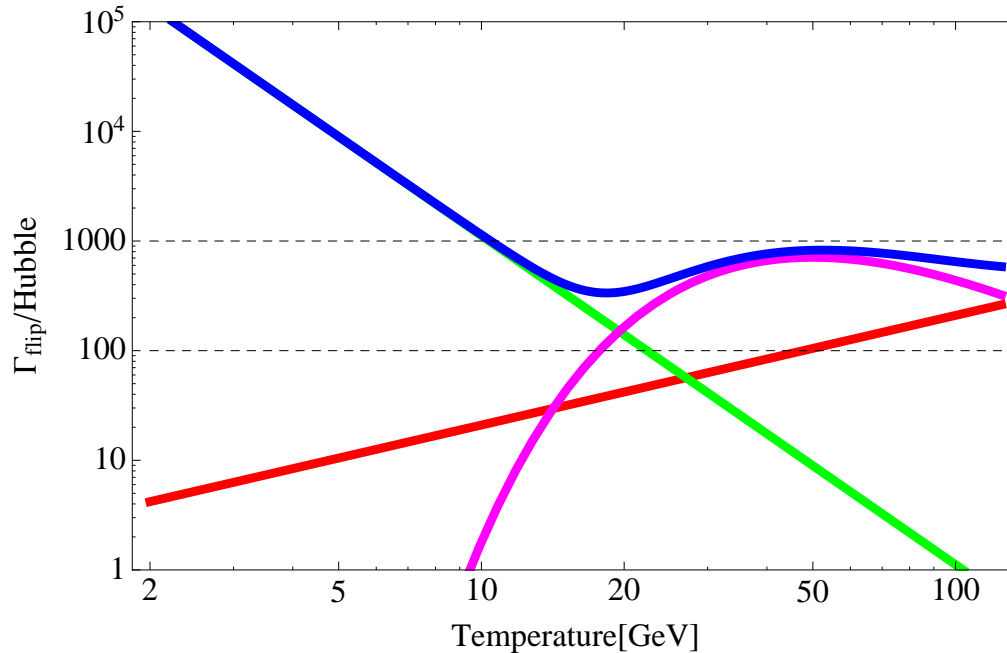
---

- Although  $T \gg m$  and these reactions are suppressed as  $(m/T)^2$  as compared to chirality-preserving reactions after long time they will wash out  $\mu_5$ :

$$\frac{d\mu_5}{dt} = -\Gamma_f \mu_5$$

- Starting from  $T \sim 80$  TeV chirality flipping processes are in equilibrium ( $\Gamma_{\text{flip}}(T) \gg H(T) = T^2/M_*$ ) [Cambell et al. \(1992\)](#)
- Although  $(\frac{m_e}{80 \text{ TeV}})^2 \sim 10^{-17}$  chirality flipping reactions are in thermal equilibrium for  $T < 80$  TeV and drive  $\mu_L - \mu_R$  to zero **exponentially fast** (suppression of at least  $e^{-1000}$  over one Hubble time)

# Generation of magnetic fields



Symmetric phase:

- $\Gamma_{\text{high-temp}} \sim \frac{80 \text{ TeV}}{M_*} T$

Broken phase:

- $\Gamma_{\text{EM}} \propto \alpha^2 T \left(\frac{m_e}{3T}\right)^2$

- $\Gamma_{\text{W}} \propto G_F^2 T^5 \left(\frac{m_e}{3T}\right)^2$

- If a **source** creates an asymmetry in a left sector  $\Rightarrow$  its transition to a right sector is not immediate ( $e^{-\Gamma_{\text{flip}} t}$ ) but instead can be long and be accompanied by the generation of magnetic fields (after which the whole system slowly relaxes to the expected minimum as  $1/t^a$ )

## Summary

---

- System with chiral anomaly when put at non-zero temperature/finite density **necessarily** contains an effective degree of freedom that couples to the change of magnetic helicity
- This additional **IR degree of freedom** is the difference of chemical potentials of left and right particles
- If such a degree of freedom gets excited – this leads to an instability and exponential growth of the helical magnetic field. This instability is **always present**, however time of development may be long
- Evolution of relativistic systems with magnetic fields **are not described** by the standard MHD equations (as was previously believed) but rather by chiral MHD

---

Thank you for your  
attention!

---

# MORE REALISTIC ANALYSIS?

- LOCAL  $\mu_5(t, x)$
- VELOCITIES

# Analysis of non-linear equations

---

- The non-linear system of equations:

$$\left\{ \begin{array}{l} \text{curl } \vec{B} = \sigma \vec{E} + \frac{\alpha}{\pi} (\dot{\theta}_5 \vec{B} + \nabla \theta_5 \times \vec{E}) \\ \Lambda^2 \ddot{\theta}_5 + \text{gradient terms} = \frac{2\alpha}{\pi} \vec{E} \cdot \vec{B} + \text{chirality flip}, \\ \vec{B} \cdot \nabla \dot{\theta}_5 = 0, \\ \mu_5 = \dot{\theta}_5 \end{array} \right.$$

- Tracking solution?
- Take  $\vec{B}(x) = B(t) (\sin(kz), \cos(kz), 0)$ . In this case  $\vec{E} \cdot \vec{B}$  is **constant in space**
- $\Rightarrow \mu_5 = \dot{\theta}_5$  depends **on time only**. The gradient terms are not important. But the equation is still non-linear!



# Analysis of non-linear equations

---

$$\vec{B}(x) = B(t)(\sin(kz), \cos(kz), 0)$$

Boyarsky,  
Fröhlich,  
Ruchayskiy  
[1504.04854]

- The solution at any time is given by

$$\left\{ \begin{array}{l} \text{amplitude } B(t) = \frac{C_1}{\sqrt{1 + C_2 \exp\left(\frac{2(k^2 - \gamma^2)}{\sigma} t\right)}}, \\ \mu_5(t) = \mu_5^0 - \frac{\alpha}{2\pi} \frac{B^2(t) - B_0^2}{k\Lambda^2} \end{array} \right.$$

$$\text{where } \gamma^2 = \frac{\alpha}{\pi} k \mu_5^0 + \frac{\alpha^2 B_0^2}{2\pi^2 \Lambda^2}.$$

- If  $\gamma > k$  we get a non-trivial static solution at  $t \rightarrow \infty$

$$B_\infty^2 = B_0^2 \left[ 1 + \left( \frac{2k^2}{\beta_0^2} \right) \left( \frac{\mu_5^0}{\mu_5^\infty} - 1 \right) \right]$$

$$\text{where } \mu_5^\infty \equiv \mu_5(t \rightarrow \infty) = \frac{\pi k}{\alpha}$$

# Analysis of non-linear equations

---

- Do we see energy transfer from short scale to longer scales? —  
**Yes**

- Linearized analysis: perturb force-free configuration with the wavenumber  $k$  and the amplitude  $B_\infty$  ( $\vec{B} = B_\infty(\sin(kz), \cos(kz), 0)$ ) by longer mode  $q$

- Homogeneous case: every  $q < k$  is unstable (energy transfer to longer wavelength)

- Inhomogeneous case: unstable  $q$ :

$$\frac{k - \sqrt{k^2 - 4\beta^2}}{2} < q < \frac{k + \sqrt{k^2 - 4\beta^2}}{2}, \quad \text{where } \beta^2 = \frac{\alpha^2 B^2}{\pi^2 T^2}$$

- $\Rightarrow$  Longer wavelength modes start to grow exponentially!

Boyarsky,  
Fröhlich,  
Ruchayskiy  
[1504.04854]